

Q/ Are the following binary operations comm. or not?

a) $a * b = a + b - ab \quad \forall a, b \in (\mathbb{R}, *)$

Sol.: Let $a, b \in \mathbb{R}$

$$a * b = a + b - ab \quad \dots \textcircled{1}$$

$$b * a = b + a - ba \quad \dots \textcircled{2}$$

$$\therefore \textcircled{1} = \textcircled{2}$$

$\therefore *$ is comm.

b) $(a, b) * (c, d) = (a + c, b + d) \quad \forall (a, b), (c, d) \in (\mathbb{R} \times \mathbb{R}, *)$

Sol.: Let $(a, b), (c, d) \in \mathbb{R} \times \mathbb{R}$

$$(a, b) * (c, d) = (a + c, b + d) \quad \dots \textcircled{1}$$

$$(c, d) * (a, b) = (c + a, d + b) \quad \dots \textcircled{2}$$

$$\therefore \textcircled{1} = \textcircled{2}$$

$*$ is comm.

c) $a * b = 0 \quad \forall a, b \in (\mathbb{Q}, *)$

Sol.: Let $a, b \in \mathbb{Q}$

$$a * b = 0 \quad \dots \textcircled{1}$$

$$b * a = 0 \quad \dots \textcircled{2}$$

$$\therefore \textcircled{1} = \textcircled{2}$$

$*$ is comm.