

② \cdot Associative is satisfy

③ Identity element

$$\text{Let } c+d\sqrt{3} = e \quad \forall a+b\sqrt{3} \in \mathbb{Z}\sqrt{3}$$

$$a * e = e * a = a$$

$$(a+b\sqrt{3}) + (c+d\sqrt{3}) = (a+b\sqrt{3})$$

$$(a+c) + (b+d)\sqrt{3} = (a+b\sqrt{3})$$

$$a+c = a \Rightarrow \boxed{c=0}$$

$$b+d = b \Rightarrow \boxed{d=0}$$

$$\therefore e = c+d\sqrt{3} = 0+0\sqrt{3}$$

④ Inverse

$$\forall a+b\sqrt{3} \in \mathbb{Z}\sqrt{3}$$

$$\exists (a+b\sqrt{3})^{-1} \in \mathbb{Z}\sqrt{3} \Rightarrow \text{let } a^{-1} = c+d\sqrt{3}$$

$$\text{i.e. } a * a^{-1} = a^{-1} * a = e$$

$$(a+b\sqrt{3}) + (c+d\sqrt{3}) = (0+0\sqrt{3})$$

$$(a+c) + (b+d)\sqrt{3} = 0+0\sqrt{3}$$

$$a+c=0 \Rightarrow c=-a$$

$$b+d=0 \Rightarrow d=-b$$

$$\therefore a^{-1} = c+d\sqrt{3} = -a-b\sqrt{3}$$

$\therefore (\mathbb{Z}\sqrt{3}, +)$ is group.