3.3. Path

A digital path (or curve) from pixel p with coordinate (x,y)to pixel q with coordinate (s,t) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1),..., \quad (x_n, y_n),$$

where

$$(x_0, y_0)=(x,y),$$

and $(x_n, y_n)=(s,t)$

And $(x_i, y_i)$is adjacent pixel $(x_{i-1}, y_{i-1})$for $1\leq i \leq n$,

In this case, n is the length of the path.

•If $(x_0, y_0)=(x_n, y_n)$:the path is closed path.

•We can define 4- ,8- , or m-paths depending on the type of adjacency specified.

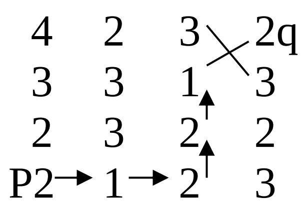
Example

Consider the image segment shown in figure. Compute length of the shortest-4, shortest-8 & shortest-m paths between pixels p& q where, v={1,2}

4	2	3	2q
3	3	1	3
2	3	2	2
P2	1	2	3

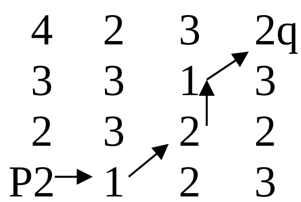
SOL:

1- shortest-4path



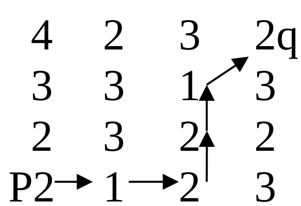
So ,path does not exist

2- shortest-8path



-8path =4

3- shortest-m paths



So, shortest-m path =5

3.4. Connectivity

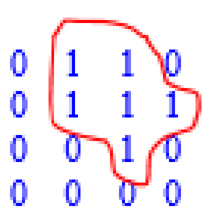
Pixels are said to be connected if there exists a path between them .

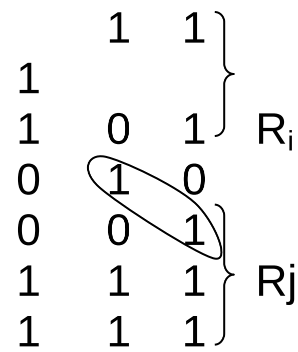
•Let S represent a subset of pixels in an image,
Two pixels p and q are said to be connected in S if there exists a path between them.
Consisting entirely of pixels 'S'.
For any pixel p in S , the set of pixels that are connected to it in S is called a **connected component** of S .

3.5. Region

• Let **R** to be a subset of pixels in an image, R is called a region if every pixel in R is connected to any other pixel in R.

.



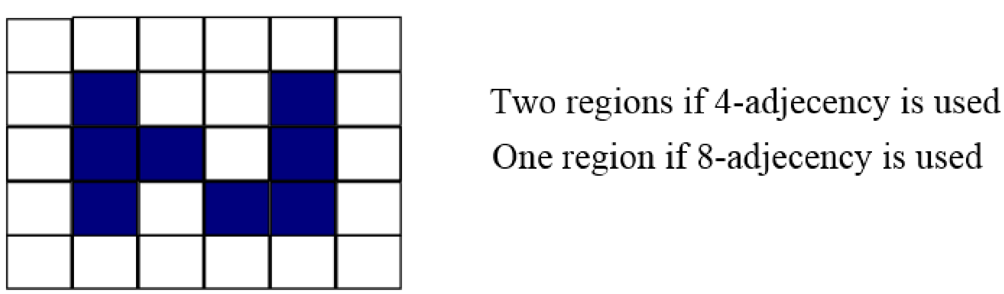


Two regions, R_i and R_j are said to be adjacent if their union forms a connected set. Region that are not adjacent are said to be **disjoint.**

We consider 4-and 8- adjacency when referring to regions,
EX1 : Below regions are adjacent only if 8-adjacency(connectivity) is used.

4-path between the two regions does not exist, (so their union is not a connected set).

EX2:



3.6. Boundary (border)

Boundary (border or contour) of a region R is the set of points that are adjacent to points in the complement of R (another way: the border of a region is the set of pixels in the region that have at least are background neighbor).

Ex : The circled point is part of the boundary of the 1-valued pixels only if 8-adjacency between the region and background is used

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	(1)	1	0
0	1	1	1	0
0	0	0	0	0

3.7. Distance Measures

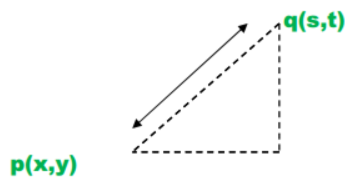
For pixels p, q, and z, with coordinates (x, y), (s, t), and (v, w),respectively, D is a distance function or metric if

- (a) $D(p,q) \geq 0 \quad (D(p,q) = 0 \quad iff \quad p = q)$
- (b) $D(p,q) = D(q,p)$
- (c) $D(p,z) \leq D(p,q) + D(q,z)$

1) The Euclidean distance between p and q is define as

$$D_e(p,q) = \left[(x-s)^2 + (y-t)^2\right]^{1/2}$$

For this distance measure, the pixels having a distance less than or equal to some value r from (x,y) are the points contained in the disk of radius r centered at (x,y)



2) The D_4 distance (called the city-block distance) between p and q is defined as:

$$D_4(p,q) = |x-s| + |y-t|$$

In this case, Pixels having a D_4 distance from (x,y) less than or equal to some value r from a Diamond centered at(x, y),.

Example:

2	1	2
1	0	1
2	1	2

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

the pixels with D_4=1 are the 4-nighbors of (x, y).

3) The D_8 distance (called the chessboard distance) between p and q is defined

$$D_8(p,q) = max(|x-s|,|y-t|)$$

In this case, the pixels having a D_8 distance from (x,y) less than or equal to some value r from a square centered at (x, y).

Example:

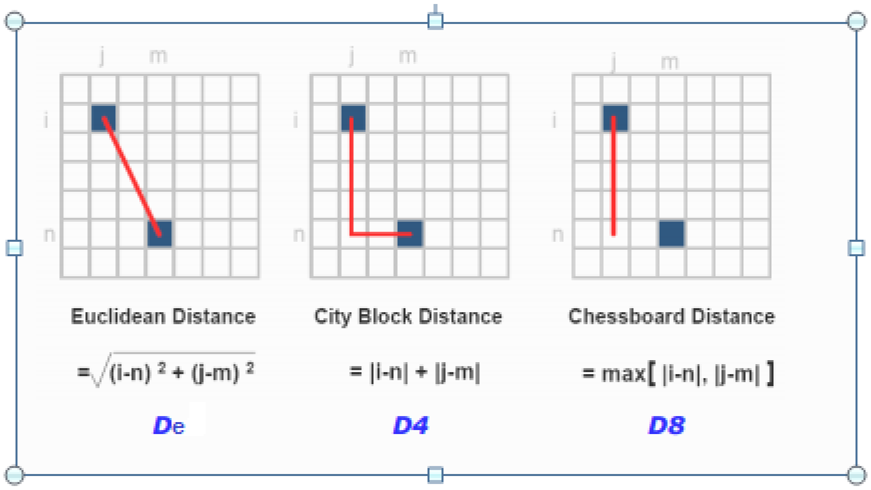
1	1	1
1	0	1
1	1	1

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

The pixels with D_8=1 are the 8-nieghbors of (x,y).

Note: that the D_4 distance & D_8 distance are independent of any paths that might exist between the points because these distances involve only the coordinates of the points.

أن المسافة D_4 و المسافة D_8 مستقلة عن أي المسارات التي قد توجد بين النقاط لأن هذه المسافات تنطوي فقط إحداثيات نقطة.



4. D_m – distance between two points is defined as the shortest m-path between the point.

In this case the distance two pixels will depend on the values of the pixels along the path as well as the values of their neighbors.

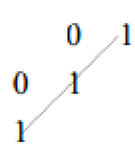
Example : consider the following arrangement of pixels and assume that p,p_2 and p_4 have value 1 and that p_1 and p_3 can have a value of 0 or 1:

	p_3	p_4
p_1	p_2	
p		

Suppose that we consider adjacency of pixels valued 1(i.e. v={1})

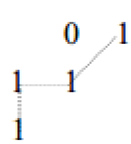
1) If p_1 and p_3 are 0

   The m- path (D_m distance between p and $p_4$is <u>**2**</u>



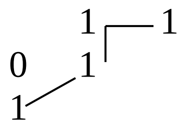
2) If p_1 is 1

   The D_m distance between p p_1 p_2 p_4 is <u>**3**</u>

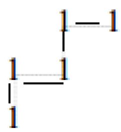


3) If $p_3=1$ and $p_1=0$

The D_m distance between p p_2 p_3 p_4 is **<u>3</u>**



4) If p_1 and p_3 are 1



The D_m distance between p p_1 p_2 p_3 p_4 is **<u>4</u>**

Example

Compute the distance between the two pixels using the three distances methods:

	1	2	3
1	q		
2		P	
3			

q : (1,1) = (i =1, j =1)

P_{center} : (2,2) = (n=2 , m=2)

Euclidian distance ***D_e*** : $((1-2)^2+(1-2)^2)^{1/2}$ = sqrt(2)=1.41

D4(City Block distance): |1-2| +|1-2| =2

D8(Chessboard distance) : max(|1-2|,|1-2|)= 1

(because it is one of the 8-neighbors)

Example :

Use the city block distance to prove 4-neighbors ?

Pixel **a** : | 2-2| + |1-2| = 1

Pixel **b**: | 3-2|+|2-2|= 1

Pixel **c**: |2-2|+|2-3| =1

Pixel **d**: |1-2| + |2-2| = 1

	1	2	3
1		d	
2	a	P	c
3		b	

<u>Homework</u>: Try the chessboard distance to proof the 8-neighbors