Method [6] Integration By Parts

Consider $w = u \cdot v$

$$dw = u\,dv + v\,du \Rightarrow u\,dv = dw - v\,du$$

$$\int u\,dv = \int dw - \int v\,du = w - \int v\,du$$

$$\boxed{\int u\,dv = u \cdot v - \int v \cdot du}$$

EX.(1) Find $I = \int \ln x\,dx$

$u = \ln x \quad , dv = dx$

$du \frac{1}{x} dx \quad , v = x$

$$I = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - \int dx = x \ln x - x + C.$$

EX.(2) Find $I = \int \tan^{-1} x\,dx$

$u = \tan^{-1} x \quad , \quad dv = dx$

$du = \frac{dx}{1+x^2} \quad , \quad v = x$

$$I = x \tan^{-1} x - \int \frac{x\,dx}{1+x^2} = x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C.$$

EX.(3) Find $I = \int x \ln x\,dx = \int \ln x \cdot x\,dx$

$u = \ln x \quad , \quad dv = x\,dx$

$du = \frac{dx}{x} \quad , \quad v = \frac{1}{2} x^2.$

$$I = \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x\,dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C.$$

EX.(4) Evaluate $I = \int x e^x\,dx$

$u = x \quad , \quad dv = e^x\,dx$

$du = dx \quad , \quad v = e^x$

$$I = x e^x - \int e^x\,dx = x e^x - e^x + C.$$

H.W

EX.(5) Evaluate $I = \int \sin^3 4x\,dx.$

EX. (6) Find $I = \int x^2 e^x \, dx$

$u = x^2 \quad , \quad dv = e^x \, dx$

$du = 2x \, dx \quad , \quad v = e^x$

$I = x^2 e^x - \int 2x e^x \, dx$

use EX. (4), we have $\Rightarrow I = x^2 e^x - 2(x e^x - e^x + C)$

$= x^2 e^x - 2x e^x + 2e^x + K$

where $K = -2C$.

Tabular Integration

Consider the integral of the form : $\int f(x) g(x) \, dx$ in which $f(x)$ can be differentiable repeatedly to become zero and $g(x)$ can be integrated repeatedly without difficulty.

Tabular integration saves a great deal of work as natural method for integration by parts.

The method can be illustrated as follows:-

$f(x)$ and its derivatives	sign	$g(x)$ and its integrals
		$g(x)$
$f(x)$	$+$	$\int g(x) \, dx = g_1(x)$
$f'(x)$	$-$	$\int g_1(x) \, dx = g_2(x)$
$f''(x)$	$+$	$\int g_2(x) \, dx = g_3(x)$
$f'''(x)$	$\vdots$	$\vdots$
$\vdots$		
$f^{(n-1)}(x)$	$\pm$	$\int g_{n-1}(x) \, dx = g_n(x)$
$f^{(n)}(x) = 0$		

$$\int f(x)g(x)\,dx = f(x)g_1(x) - f'(x)g_2(x) + f''(x)g_3(x) + \cdots \pm f^{(n-1)}(x)g_n(x) + C.$$

EX.(7) Back to EX.(6), we have $\int x^2 e^x\,dx$.

Let $f(x) = x^2$ and $g(x) = e^x$

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2		e^x
$2x$	$+$	e^x
2	$-$	e^x
0	$+$	e^x

$$\int x^2 e^x\,dx = x^2 e^x - 2xe^x + 2e^x + C.$$

EX.(8) Evaluate $I = \int (x^3-2x^2+3x+)\sin 2x\,dx$

Let $f(x) = x^3 - 2x^2 + 3x + 1$, $g(x) = \sin 2x$

$f(x)$ and its derivatives		$g(x)$ and its integrals
		$\sin 2x$
x^3-2x^2+3x+1	$+$	$-\frac{1}{2}\cos 2x$
$3x^2-4x+3$	$-$	$-\frac{1}{4}\sin 2x$
$6x-4$	$+$	$\frac{1}{8}\cos 2x$
6	$-$	$\frac{1}{16}\sin 2x$
0		

$$I = \int (x^3-2x^2+3x+1)\sin 2x\,dx = (x^3-2x^2+3x+1)\left(-\tfrac{1}{2}\cos 2x\right) + \tfrac{1}{4}(3x^2-4x+3)\sin 2x + \tfrac{1}{8}(6x-4)\cos 2x - \tfrac{6}{16}\sin 2x.$$

EX.(9) Evaluate $I=\int e^x \sin x\, dx$.

$u=e^x$, $dv=\sin x \Rightarrow du=e^x dx$, $v=-\cos x$

$I=-e^x\cos x+\int e^x \cos x\, dx = -e^x\cos x + I_1$

where $I_1=\int e^x \cos x\, dx$

$u=e^x \quad dv=\cos x\, dx$

$du=e^x dx \quad v=\sin x$

$I_1= e^x \sin x-\int e^x \sin x\, dx = e^x \sin x - I$

$\therefore I = -e^x\cos x + e^x \sin x - I$

$2I = -e^x \cos x + e^x \sin x$

$I = \frac{e^x}{2}(\sin x - \cos x) + C$.

Exercises To Solve [NO.6]

① $\int x^2 \ln(x+1)\, dx$ ② $\int x^{-2} \tan^{-1} x\, dx$ ③ $\int_0^1 x\sqrt{1-x}\, dx$

④ $\int_0^\infty x^3 e^{-x}\, dx$ ⑤ $\int \sin(\ln x)\, dx$ ⑥ $\int \sinh 2x \cosh 5x\, dx$

⑦ $\int e^{-x} \sin x\, dx$ ⑧ $\int x[\ln(x)]^2 dx$ ⑨ $\int \sin\sqrt{2x}\, dx$

⑩ $\int x^2 \tan^{-1} x\, dx$ ⑪ $\int \ln(x+\sqrt{1+x^2})\, dx$ ⑫ $\int \sqrt{1-x^2}\, \sin^{-1}x\, dx$.

Method [7] Integration of Rational Functions

Defn. A rational function is a quotient of two polynomials, written as

$$R(x) = \frac{P_n(x)}{Q_m(x)}, \quad Q_m(x) \neq 0 \quad \text{where } P_n(x) \text{ and } Q_m(x) \text{ are polynomials of degree } n \text{ and } m \text{ respectively.}$$

If $n > m$, we perform a long division until we obtain a rational function whose numerator degree less than or equal to the denominator degree

For Example
$$I = \int \frac{x^5 - 6x^4 - 2x^2 - 3x + 4}{x^3 + 2x + 3}\, dx$$

$$\begin{array}{r|l}
 & x^2 - 6x - 2 \\ \hline
x^3 + 2x + 3 & x^5 - 6x^4 - 2x^2 - 3x + 4 \\
 & \mp x^5 \mp 2x^3 \mp 3x^2 \\ \hline
 & -6x^4 - 2x^3 - 5x^2 - 3x + 4 \\
 & \pm 6x^4 \pm 12x^2 \pm 18x \\ \hline
 & -2x^3 + 7x^2 + 15x + 4 \\
 & \pm 2x^3 \pm 4x \pm 6 \\ \hline
 & 7x^2 + 19x + 10
\end{array}$$

$$\therefore \quad I = \int \left(x^2 - 6x - 2 + \frac{7x^2 + 19x + 10}{x^3 + 2x + 3} \right) dx$$

$$= \frac{1}{3}x^3 - 3x^2 - 2x + \int \frac{7x^2 + 19x + 10}{x^3 + 2x + 3}\, dx .$$

If $n \leq m$, we shall discuss the three cases of separating $\frac{P_n(X)}{Q_m(X)}$ in a sum of partial fractions.

Case I If the m factors of $Q_m(X)$ are all different and simple, that is,

$Q_m(X) = (X-a_1)(X-a_2) \cdot \cdots \cdot (X-a_m)$. Then we assign the sum of m partial fractions to these factors as follows

$$\frac{A_1}{(X-a_1)} + \frac{A_2}{(X-a_2)} + \cdots + \frac{A_m}{(X-a_m)}$$

, where $A_1, A_2, \dots, A_m$ are constants must be evaluated.

EX. (1) Find $I = \int \frac{x^2+3X+3}{x^3-X} dx = \int \frac{(x^2+3X+3)\, dx}{X(X-1)(X+1)}$

$$\frac{x^2+3X+3}{X(X-1)(X+1)} = \frac{A}{X} + \frac{B}{X-1} + \frac{C}{X+1}$$

$$= \frac{A(X-1)(X+1) + BX(X+1) + C(X-1)X}{x(X-1)(X+1)}$$

$$X^2+3X+3 = A(X-1)(X+1) + BX(X+1) + CX(X-1)$$

at $X=0 \Rightarrow 3 = A(0-1)(0+1)+0+0 \Rightarrow A=-3$

at $X=1 \Rightarrow 7 = 0 + B(1)(1+1) + 0 \Rightarrow B = \frac{7}{2}$

at $X=-1 \Rightarrow 1 = 0+0+C(-1)(-1-1) \Rightarrow C = \frac{1}{2}$

OR

$$X^2+3X+3 = AX^2 - A + BX^2 + BX + CX^2 - CX$$
$$= (A+B+C)X^2 + (B-C)X - A$$

$$\left.\begin{aligned} A+B+C &= 1 \\ B-C &= 3 \\ -A &= 3 \end{aligned}\right\} \Rightarrow A=-3, \quad B = \frac{7}{2}, \quad C = \frac{1}{2}$$

$$I = \int \left[\frac{-3}{X} + \frac{7/2}{x-1} + \frac{1/2}{X+1}\right] dx = -3\ln X + \frac{7}{2}\ln(X-1) + \frac{1}{2}\ln(X+1) + C.$$

Case II Repeated factors of $Q_m(X)$

Suppose $(x-a)^r$ is the highest power of $(x-a)$ which divides $Q_m(X)$. Then to this factor we assign the sum of r partial fraction as follows:

$$\frac{A_1}{(x-a)} + \frac{A_2}{(x-a)^2} + \cdots + \frac{A_r}{(x-a)^r}$$

Where $A_1, A_2, \ldots, A_r$ are constants must be evaluated.

for example

$$\frac{x^3-3x^2+4x-2}{x(x-1)^2(x+1)(x+2)^3} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} + \frac{D}{(x+1)} + \frac{E}{(x+2)} + \frac{F}{(x+2)^2} + \frac{G}{(x+2)^3}.$$

EX.(2) Evaluate $I = \int \frac{(x+5)dx}{(x+2)(x-1)^2}$

$$\frac{x+5}{(x+2)(x-1)^2} = \frac{A}{(x+2)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2 + B(x-1)(x+2) + C(x+2)}{(x+2)(x-1)^2}$$

$$x+5 = A(x-1)^2 + B(x-1)(x+2) + C(x+2)$$

at $x=1 \Rightarrow 6 = 0+0+3C \Rightarrow C=2$

at $x=-2 \Rightarrow 3 = 9A+0+0 \Rightarrow A=\frac{1}{3}$

at $x=0 \Rightarrow 5 = \frac{1}{3} - 2B + 4 \Rightarrow B = -\frac{1}{3}$

$$\therefore I = \int \left[\frac{\frac{1}{3}}{x+2} - \frac{\frac{1}{3}}{x-1} + \frac{2}{(x-1)^2} \right] dx = \frac{1}{3}\ln(x+2) - \frac{1}{3}\ln(x-1) - \frac{2}{(x-1)} + C.$$

Case III Quadratic factors of $Q_m(X)$

Suppose $(x^2+ax+b)^r$ is the highest power of (x^2+ax+b) which divides $Q_m(X)$, Then to this factor we assign the sum of r partial fractions as follows:—

$$\frac{A_1x+B_1}{x^2+ax+b}+\frac{A_2x+B_2}{(x^2+ax+b)^2}+\cdots+\frac{A_rx+B_r}{(x^2+ax+b)^r}\text{, where}$$

$A_1, A_2, \ldots, A_r$, $B_1, B_2, \ldots, B_r$ are constants must be evaluated

for example

$$\frac{x^2+2x-5}{x^2(x-1)(x^2+1)(x^2+2x+2)^2}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x-1)}+\frac{Dx+E}{x^2+1}+\frac{Fx+G}{x^2+2x+2}$$

$$+\frac{Hx+I}{(x^2+2x+2)^2}.$$

Ex. (B) Evaluate $I=\int\frac{x^5-x^4-3x+5}{x^4-2x^3+2x^2-2x+1}\,dx$

$$\begin{array}{r|l} & x+1 \\ \hline x^4-2x^3+2x^2-2x+1 & x^5-x^4-3x+5 \\ & \mp x^5 \pm 2x^4 \mp 2x^3 \pm 2x^2 \mp x \\ \hline & x^4-2x^3+2x^2-4x+5 \\ & \mp x^4 \pm 2x^3 \mp 2x^2 \pm 2x \mp 1 \\ \hline & -2x+4 \end{array}$$

$$I=\int\left(x+1+\frac{-2x+4}{x^4-2x^3+2x^2-2x+1}\right)dx=\frac{x^2}{2}+x+\int\frac{(-2x+4)\,dx}{x^4-2x^3+2x^2-2x+1}$$

$$\frac{-2x+4}{(x-1)^2(x^2+1)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{Cx+D}{x^2+1}$$

$$-2x+4=A(x-1)(x^2+1)+B(x^2+1)+(Cx+D)(x-1)^2$$

at $x=1 \Rightarrow 2=0+2B+0 \Rightarrow B=1$

at $x=0 \Rightarrow 4=A(-1)(1)+B(1)+D(-1)^2 \Rightarrow 4=-A+B+D$

$3=-A+D$

at $x=-1 \Rightarrow 6=A(-2)(2)+2B+(-C+D)(4) \Rightarrow 1=-A-C+D$

at $x=2 \Rightarrow 0=A(1)(5)+B(5)+(2C+D)(1) \Rightarrow -5=5A+2C+D$

$A=-2$, $B=1$, $C=2$, $D=1$.

Let $I_1 = \int \frac{(-2x+4)\,dx}{x^4-2x^3+2x^2-2x+1} = \int \left(\frac{-2}{x-1} + \frac{1}{(x-1)^2} + \frac{2x+1}{x^2+1} \right) dx$

$$= -2\ln(x-1) - \frac{1}{x-1} + \int \frac{2x\,dx}{x^2+1} + \int \frac{dx}{x^2+1}$$

$$= -2\ln(x-1) - \frac{1}{x-1} + \ln(x^2+1) + \tan^{-1}x + C$$

$$\therefore I = \frac{x^2}{2} + x - 2\ln(x-1) - \frac{1}{x-1} + \ln(x^2+1) + \tan^{-1}x + C.$$

Exercises To Solve [No. 7]

(1) $\int \frac{x^2+3x+4}{x-2}\,dx$ (2) $\int \frac{x^3-x^2+2x+2}{x^2+3x+2}\,dx$

(3) $\int \frac{x^4+1}{x^3-x}\,dx$ (4) $\int_0^{\sqrt{3}} \frac{5x^2}{x^2+1}\,dx$

(5) $\int \frac{dx}{x^2(x^2+1)^2}$ (6) $\int_{\pi/3}^{\pi/2} \frac{\sin\theta\,d\theta}{\cos^2\theta+\cos\theta-2}$

(7) $\int \frac{x^2+3x+3}{(x+1)(x^2+1)}\,dx$. (8) $\int_0^{\ln 2} \frac{e^x\,dx}{e^{2x}+3e^x+2}$

Method [8] Integration of Irrational Funtions

Case I : If the integral contain a single irrational expression of the form $\sqrt[q]{(ax+b)} = (ax+b)^{\frac{1}{q}}$.

Let $z = (ax+b)^{\frac{1}{q}} \Rightarrow z^q = ax+b \Rightarrow q\, z^{q-1} dz = a\, dx$

$\Rightarrow dx = \frac{q}{a} z^{q-1} dz$

EX.(1) $I = \int \frac{2x+3}{\sqrt{x+2}} dx = \int \frac{2x+3}{(x+2)^{1/2}} dx$

Let $z = (x+2)^{1/2} \Rightarrow z^2 = x+2 \Rightarrow 2z\, dz = dx$

$$I = \int \frac{2(z^2-2)+3}{z} \cdot 2z\, dz = 2\int (2z^2-1) dz$$

$$= 2\left[\frac{2}{3} z^3 - z\right] + C$$

$$= 2\left[\frac{2}{3}(x+2)^{3/2} - (x+2)^{1/2}\right] + C.$$

EX.(2) find $I = \int \frac{\sqrt[3]{x+1}}{x} dx = \int \frac{(x+1)^{1/3}}{x} dx$

Let $z = (x+1)^{1/3} \Rightarrow z^3 = (x+1) \Rightarrow 3z^2 dz = dx$

$$I = \int \frac{z \cdot 3z^2\, dz}{z^3-1} = 3\int \frac{z^3}{z^3-1} dz = 3\int \left(1 + \frac{1}{z^3-1}\right) dz$$

$$= 3z + \int \frac{3\, dz}{z^3-1}$$

let $I_1 = \int \frac{3}{z^3-1} dz = \int \frac{3\, dz}{(z-1)(z^2+z+1)}$

$$\frac{3}{(z-1)(z^2+z+1)} = \frac{A}{z-1} + \frac{Bz+C}{z^2+z+1}$$

$$3 = A(z^2+z+1) + (Bz+C)(z-1)$$

$$A=1 \quad , \quad B=-1 \quad , \quad C=-2$$

$$I_1 = \int \left(\frac{1}{z-1} - \frac{z+2}{z^2+z+1} \right) dz = \ln(z-1) - \frac{1}{2}\int \frac{2z+1-1}{z^2+z+1}\,dz - \int \frac{2}{z^2+z+1}\,dz$$

$$= \ln(z-1) - \frac{1}{2}\int \frac{(2z+1)\,dz}{z^2+z+1} + \frac{1}{2}\int \frac{dz}{z^2+z+1} - \int \frac{2\,dz}{z^2+z+1}$$

$$= \ln(z-1) - \frac{1}{2}\ln(z^2+z+1) + \frac{3}{2}\int \frac{dz}{z^2+z+1}$$

$$= \ln(z-1) - \frac{1}{2}\ln(z^2+z+1) - \frac{3}{2}\int \frac{dz}{(z+\frac{1}{2})^2+\frac{3}{4}}$$

$$I_1 = \ln(z-1) - \frac{1}{2}\ln(z^2+z+1) - \frac{3}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{z+1/2}{\sqrt{3}/2}\right) + C$$

$$I = 3(x+1)^{1/3} + \ln[(x+1)^{1/3}-1] - \frac{1}{2}\ln[(x+1)^{2/3} + (x+1)^{1/3}+1]$$
$$- \sqrt{3}\tan^{-1}\left[\frac{2(x+1)^{1/3}}{\sqrt{3}}\right] + C$$

H.W

Ex.(3) Find $\int \frac{dx}{\sqrt[3]{x^2} + \sqrt{x}}$

H.W

Ex.(4) Find $\int \frac{\sqrt{x}}{1+\sqrt[4]{x}}\,dx$

Case II If a single irrational expression of the form $\sqrt{a^2-x^2}$ or $\sqrt{a^2+x^2}$ or $\sqrt{x^2-a^2}$, the z substitution to the radical reduces the given integral to that of a rational function.

EX.(5)

Find $I = \int \frac{\sqrt{4-x^2}}{x^3}\, dx = \int \frac{(4-x^2)^{1/2}}{x^3}\, dx$

$$z = (4-x^2)^{1/2} \Rightarrow z^2 = 4-x^2 \Rightarrow x^2 = 4-z^2 \Rightarrow 2x\,dx = -2z\,dz$$

$$\Rightarrow x\,dx = -z\,dz$$

$$I = \int \frac{(4-x^2)^{1/2}}{x^3}\, dx = \int \frac{(4-x^2)^{1/2}}{x^4} \cdot x\,dx = \int \frac{z\cdot(-z)\,dz}{(4-z^2)^2}$$

$$= -\int \frac{z^2}{(z+2)^2(z-2)^2}\, dz$$

$$\frac{-z^2}{(z+2)^2(z-2)^2} = \frac{A}{(z+2)} + \frac{B}{(z+2)^2} + \frac{C}{(z-2)} + \frac{D}{(z-2)^2}$$

$$A = \frac{1}{8}, \quad B = -\frac{1}{4}, \quad C = -\frac{1}{8}, \quad D = -\frac{1}{4}$$

$$I = \int \left[\frac{\frac{1}{8}}{z+2} - \frac{\frac{1}{4}}{(z+2)^2} - \frac{\frac{1}{8}}{z-2} - \frac{\frac{1}{4}}{(z-2)^2} \right] dz$$

$$= \frac{1}{8} \ln(z+2) + \frac{1}{4}\,\frac{1}{z+2} - \frac{1}{8} \ln(z-2) + \frac{1}{4}\,\frac{1}{z-2} + C$$

$$= \frac{1}{8} \ln\left(\frac{z+2}{z-2}\right) + \frac{1}{4}\left[\frac{1}{z+2} + \frac{1}{z-2}\right] + C$$

$$= \frac{1}{8} \ln\left(\frac{z+2}{z-2}\right) + \frac{1}{4}\left[\frac{2z}{z-4}\right] + C$$

$$= \frac{1}{8} \ln\left(\frac{z+2}{z-2}\right) - \frac{1}{2}\left[\frac{z}{4-z^2}\right] + C$$

$$= \frac{1}{8} \ln\left(\frac{\sqrt{4-x^2}+2}{\sqrt{4-x^2}-2}\right) - \frac{1}{2}\left[\frac{\sqrt{4-x^2}}{x^2}\right] + C$$

Exercises To Solve [No. 8]

① $\int \frac{\sqrt{x}+2}{\sqrt{x}-1}\,dx$ ② $\int \frac{2x+1}{(x+2)^{2/3}}$ ③ $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

④ $\int \frac{dx}{\sqrt[3]{x}+\sqrt[4]{x}}$ ⑤ $\int \frac{\sqrt{x^2+9}}{x^3}\,dx$

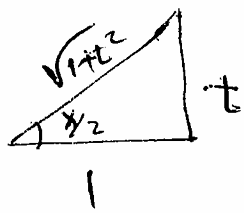
Method [9] Integration of Rational Functions of Trigonometric

If the integrand is a rational function of trigonometric, the substitution of $t = \tan\frac{x}{2}$ will reduce the integral to a rational function of t which can be handle by method [7].

Mathematically Speaking :-

$$t = \tan\frac{x}{2} \Rightarrow \frac{x}{2} = \tan^{-1} t \Rightarrow \frac{1}{2}dx = \frac{dt}{1+t^2}$$

$$\Rightarrow \boxed{dx = \frac{2\,dt}{1+t^2}}$$



Since $\sin\frac{x}{2} = \frac{t}{\sqrt{1+t^2}}$, $\cos\frac{x}{2} = \frac{1}{\sqrt{1+t^2}}$

$$\boxed{\begin{aligned} \sin x &= 2\sin\frac{x}{2}\cos\frac{x}{2} = \frac{2t}{1+t^2} \\ \cos x &= \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \frac{1-t^2}{1+t^2} \end{aligned}}$$

EX.(1)

$$I=\int \frac{dx}{5-4\cos x} = \int \frac{\frac{2\,dt}{1+t^2}}{5-4\,\frac{1-t^2}{1+t^2}} = \int \frac{2\,dt}{5(1+t^2)-4(1-t^2)}$$

$$= 2\int \frac{dt}{1+9t^2} = \frac{2}{3}\int \frac{3\,dt}{1+(3t)^2} = \frac{2}{3}\tan^{-1}3t + C$$

$$= \frac{2}{3}\tan^{-1}\left[3\left(\tan\frac{x}{2}\right)\right] + C.$$

H.W

EX.(2)

$$I=\int \frac{dx}{3\cos x + 4\sin x}$$

Exercises To solve [No. 9]

① $\int \frac{dx}{2-\sin x}$ ② $\int \frac{\cos x\,dx}{5+4\cos x}$ ③ $\int_0^{\pi} \frac{dx}{1+\sin x}$

④ $\int_{\pi/2}^{\pi} \frac{dx}{1-\cos x}$ ⑤ $\int \frac{dx}{2-\cos x}$ ⑥ $\int \frac{dx}{5+4\cos x}$