أسس رياضيات ٢

مرحلة الاولى

قسم الرياضيات

كلية التربية الأساسية

الجامعة المستنصرية

د. عبير جبار سعيد

القسم العلمي: الرياضيات　　　المرحلة : الاولى　　　الفصل الدراسي : الثاني

المادة : اسس الرياضيات ٢　　　عدد الوحدات : ٢　　　عدد الساعات : النظري: ١ العملي:٢

مفردات المادة

الاسبوع	عنوان المفردة
الاول	مفهوم التطبيق
الثاني	تطبيقات خاصة (الذاتي , الثابت)
الثالث	أنواع التطبيقات (التباين ،الشامل ،التقابل) مع امثلة
الرابع	أنواع التطبيقات (التباين ،الشامل ،التقابل) مع امثلة
الخامس	تركيب التطبيقات مع امثلة
السادس	مبرهنات حول تركيب التطبيقات
السابع	امتحان الشهر الاول
الثامن	معكوس التطبيق مع امثلة
التاسع	مبرهنات حول معكوس التطبيق
العاشر	مفهوم العملية الثنائية مع خواصها وامثلة ,
الحادي عشر	مفهوم النظام الرياضي
الثاني عشر	شبة الزمرة
الثالث عشر	امتحان الشهر الثاني
الرابع عشر	مفهوم الزمرة مع امثلة
الخامس عشر	بعض المبرهنات حول الزمرة

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Chapter one: Functions (mappings)

الدوال (التطبيقات)

1. mappings — مفهوم التطبيق
2. Constant mapping and Identity mapping — تطبيقات خاصة
3. Types of mappings — أنواع التطبيقات
4. Composition of mappings — تركيب التطبيقات
5. Direct image and inverse image. — الصورة المباشرة والصورة العكسية
6. Some functions applications. — بعض تطبيقات الدوال

Mapping (Function) — التطبيق أو الدالة

Let A and B be two nonempty sets. A relation f from A to B ($f \subseteq A \times B$) is called a mapping or function if each element in A is related to a unique element in B. This relation is denoted by $f: A \longrightarrow B$.

لتكن كل من A و B مجموعة غير خالية. يقال العلاقة f التي تربط كل عنصر $x \in A$ بعنصر وحيد $y \in B$ بأنها تطبيق من A الى B (الدالة) ويرمز لها بالرمز $f: A \longrightarrow B$

Mathematically

$$f: A \longrightarrow B \text{ is a mapping} \iff \forall x \in A \ \exists! y \in B \text{ s.t. } f(x) = y$$

1. y is the image of x and x is called the preimage of y.
2. The set A is called the Domain of f (D_f) مجال الدالة
3. The set B is called the Codomain of f Cod_f
4. The set of all images of the elements of A is called the Range of f $R_f = f(A) = \{y \in B : f(x) = y \ \forall x \in A\} \subseteq Cod_f$

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Remark: ① A mapping is (generally) denoted by $f, F, g, G, \dots$
2. Every function is a relation but not every relation is a function.

Example 1: let $A = \{1,2,3,4\}$ and $B = \mathbb{Z}$. Which of the following relations is mapping?

1. $R_1 = \{(x,y) \in A \times B : y = 2x\}$

Solution:

2. $R_2 = \{(1,1), (1,2), (2,0), (3,-1), (4,1)\}$

Solution:

3. $R_3 = \{(1,1), (2,0), (3,3)\}$

Solution:

Mapping can be defined in another way: شروط تطبيق

Definition: let A and B be two non empty sets.
A relation f from A to B ($f \subseteq A \times B$) is called **mapping** if it satisfies two conditions:
1. Closure الانغلاق : $\forall x \in A \longrightarrow f(x) \in B$
2. well-defined حسنة التعريف :
if $x_1 = x_2$ then $f(x_1) = f(x_2)$ $\forall x_1, x_2 \in A$

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Example 2: Determine whether $f: Z \to R$ is a function or not:

ⓐ $f(x) = \sqrt{x^2+1}$

Solution:

① $\forall x \in Z \overset{?}{\Rightarrow} f(x) \in R$

let $x \in Z \Rightarrow x^2+1 \in Z^+ \Rightarrow \sqrt{x^2+1} \in R$

$\therefore$ f is closure

② let $x_1, x_2 \in Z$ s.t $x_1 = x_2 \Rightarrow$

$x_1^2 = x_2^2 \Rightarrow x_1^2+1 = x_2^2+1$

$\Rightarrow \sqrt{x_1^2+1} = \sqrt{x_2^2+1}$

$\Rightarrow f(x_1) = f(x_2)$

$\therefore$ f is well defined

$\therefore$ f is mapping (function)

ⓑ $g(x) = \begin{cases} -x & , x \le 1 \\ x & , x \ge 1 \end{cases}$

Solution:

g is not well defined because $x = 1 \in Z$

but $g(1) = 1$ and $g(1) = -1$ يوجد عنصر له صورتان

$\therefore$ g is not a mapping

ⓒ $h(x) = \frac{1}{x}$

Solution:

Closure Condition is not hold because

$x = 0 \in Z$ but $h(0) = \frac{1}{0} \notin R$

$\therefore$ h is not a mapping

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تطبيقات خاصة :

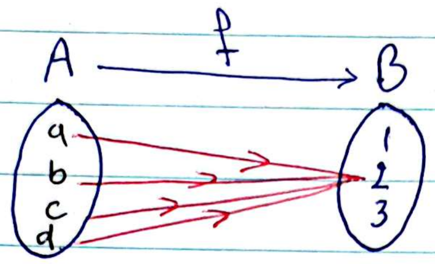
1. Constant mapping التطبيق الثابت

let f be a map from A to B. f is called Constant map on A if and only if there exist uniqe element c in B such that it is image for all element in A.

i.e

$f: A \to B$ is Constant map $\Leftrightarrow \exists!\, c \in B$ s.t $f(x) = c \quad \forall x \in A$

$\Leftrightarrow R_f = \{c\}$



Constant mapping

2. Identity mapping الدالة (التطبيق الذاتي)

let f be a map from A to A. f is called Identity mapping on A if and only if image of all element is equal it-self element. denoted by i_A, I_A

i.e

$f: A \to A$ is identity map $\Leftrightarrow \forall x \in A \Rightarrow f(x) = x$

Example 3: ① let $f = \{(1,1), (3,3), (0,0), (-6,-6)\}$

f is identity function defined on $A = \{0, 1, 3, -6\}$

$\therefore f = i_A$

② $I_N = f: N \to N$ s.t $f(x) = |x| \quad \forall x \in N$

$f(x) = x \quad \forall x \in N$

Types of mappings

أنواع التطبيقات

1. Injective mapping:

let f be amap from A to B, f is (1-1) injcetive on A if and only if for deference elements is deference image or equal image is implies for equal elements.

i.e $f: A \longrightarrow B$ is 1.1 $\Leftrightarrow \forall x_1, x_2 \in A$ s.t $f(x_1) = f(x_2)$ then $x_1 = x_2$

or $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$

2. Surjective mapping

A function $f: A \rightarrow B$ is called (onto) or Surjective if every element in B has at least one relating element in A

i.e $f: A \longrightarrow B$ is onto $\Leftrightarrow R_f = Cod_f$

$\Leftrightarrow \forall y \in Cod_f \; \exists x \in D_f$ s.t $f(x) = y$

3. Bijective mapping

Afunction $f: A \longrightarrow B$ is called bijective if and only if f is injective and surjective mapping.

i.e $f: A \longrightarrow B$ is bijective $\Leftrightarrow$ f is 1-1 and onto

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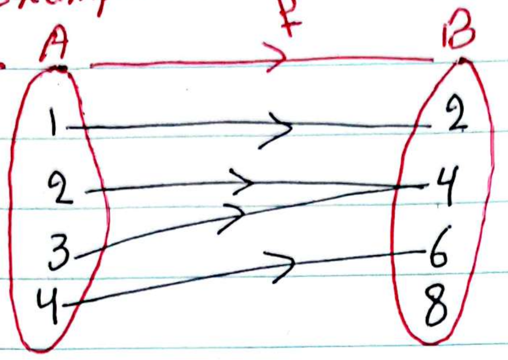
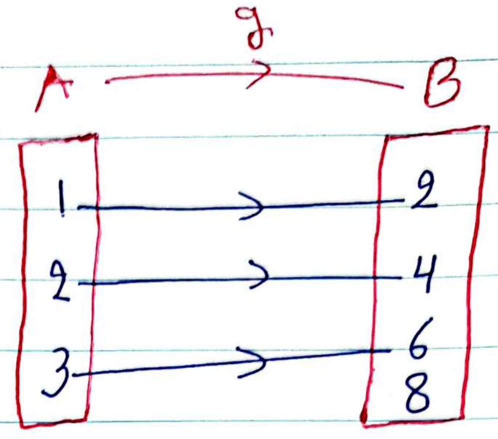
Example:

1.



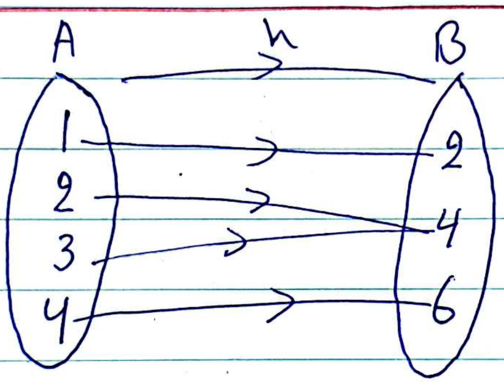
f is

2.

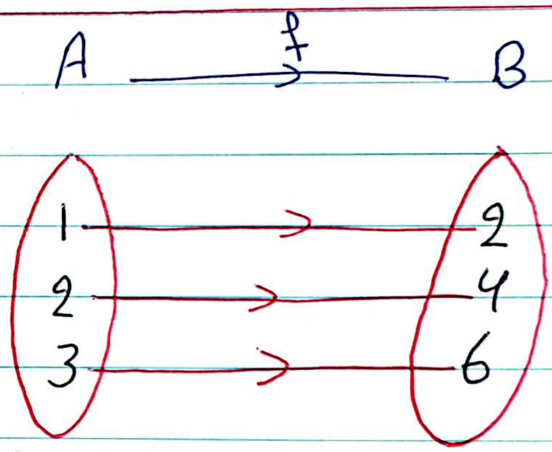


g is

3.



h is



f is

مثال: لتكن $f: R \to R$ دالة معرفة على R
بحيث أن $f(x) = 5x+1$ ما نوع الدالة؟

الحل: ① let $x_1, x_2 \in R$ s.t

$$f(x_1) = f(x_2)$$

$$5x_1 \cancel{+1} = 5x_2 \cancel{+1}$$

$$[5x_1 = 5x_2] \div 5$$

$$x_1 = x_2$$

f is 1-1

② $R_f \subseteq Cod_f$?

نريد ان نبرهن أن $Cod_f \subseteq R_f$

let $y \in Cod_f = R$ s.t $f(x) = y$

مطلوب ايجاد قيمة x ؟؟

$$f(x) = y$$

$$5x+1 = y$$

$$5x = y-1$$

$$x = \frac{y-1}{5} \in R$$

f is onto

$\therefore$ f is Bij.

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Example: Which of the following functions are injective? surjective? Bijective?

1. $f: R \longrightarrow R$ s.t $f(x) = \sqrt{x^2+9}$

2. $f: R \longrightarrow [1,\infty)$ s.t $f(x) = |x-4| + 1$ (H.w)

Solution 1:

$f: R \longrightarrow R$ s.t $f(x) = \sqrt{x^2+9}$

$D_f = \mathbb{R}$ and $Cod_f = R$

① we need to find R_f

when $x \in R \Longrightarrow f(x) = y \geqslant 3$

$R_f = \{y : y \geqslant 3\} = [3,\infty) \neq R = Cod_f$

$\therefore f$ is not surjective (not onto)

② let $f(x_1) = f(x_2) \xrightarrow{?} x_1 = x_2$

$$f(x_1) = f(x_2) \Longrightarrow \sqrt{x_1^2+9} = \sqrt{x_2^2+9}$$
$$\Longrightarrow x_1^2+9 = x_2^2+9$$
$$\Longrightarrow x_1^2 = x_2^2$$
$$\Longrightarrow x_1 = \mp x_2$$
$$\Longrightarrow x_1 = x_2 \text{ or } \boxed{x_1 = -x_2} *$$

from (*), f is not injective

$\therefore f$ is not bijective

$\boxed{\text{or}}$ let $x_1 = 4 \neq x_2 = -4$ but

$$f(x_1) = f(x_2)$$
$$\sqrt{4^2+9} = \sqrt{(-4)^2+9} = \sqrt{25} = 5$$

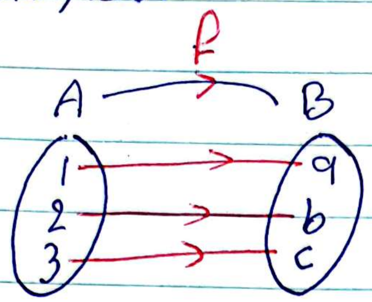
$\therefore f$ is not (1-1)

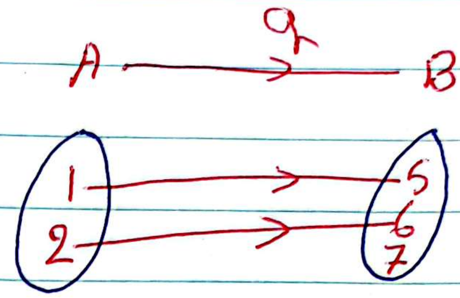
Inverse mapping — الدالة العكسية

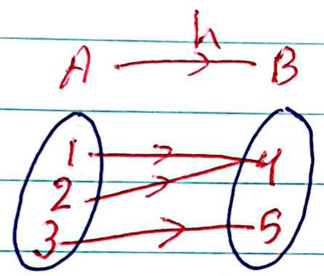
let f be a bijective mapping from A to B then f^{-1} is a mapping from B to A such that $f^{-1}(y)=x$

$f:A\to B$ s.t $f(x)=y$ $\forall x\in A$ is bijective $\Leftrightarrow$ $\exists f^{-1}:B\to A$ is a map s.t $f^{-1}(y)=x$

Example:-







Example:- let $f:R^+\to R^+$ s.t $f(x)=\frac{x}{2}$ Find f^{-1} (if exist)?

Solution:- f^{-1} exist $\Leftrightarrow$ f is bijective

① let $f(x_1)=f(x_2)$ $\forall x_1,x_2\in R^+$ we want to show that $x_1=x_2$

$$\frac{x_1}{2}=\frac{x_2}{2}\Rightarrow x_1=x_2\Rightarrow f \text{ is } 1\text{-}1$$

② let $y\in R^+$ $\overset{?}{\exists}\, x\in R^+$ s.t $f(x)=y=\frac{x}{2}$

$x=2y\in R^+$

$R_f=\text{Cod}f$ so f is onto

f is bijective $\Rightarrow$ f^{-1} is exist and a mapping

$$y = \frac{x}{2} \Rightarrow x = 2y$$

$$f^{-1}: R^{+} \longrightarrow R^{+} \quad \text{s.t} \quad f^{-1}(y) = 2y = x$$

Composition of Mappings تركيب التطبيقات

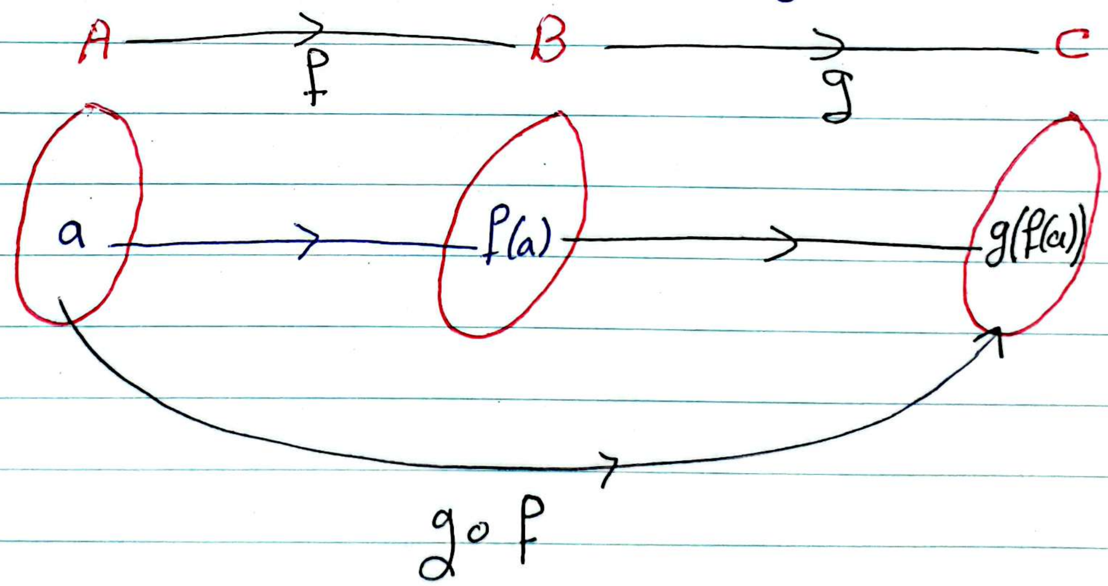
let $f: A \longrightarrow B$ be a mapping and $g: B \longrightarrow C$ be a mapping. The composition of g and f is a mapping denoted by $g \circ f$ and is defined as

$$(g \circ f)(x) = g(f(x)) \qquad \forall x \in A$$

i.e

If $f: A \longrightarrow B$ and $g: B \longrightarrow C$ then $g \circ f: A \longrightarrow C$ is a map $\Longleftrightarrow$ $\forall x \in A \ \exists! \ g(f(x)) \in C$ s.t

$$(g \circ f)(x) = g(f(x))$$



Remark: $g \circ f \neq f \circ g$ (in general)

مثال: لتكن $A=\{1,2,3\}$ ، $B=\{2,4,6\}$ ، $C=\{7,9,11\}$

حيث أن $f: A \longrightarrow B$; $f(a)=2a$ and $g: B \longrightarrow C$

$g(b)=b+5$.

جد $g\circ f(x)$ ، $g\circ f(1)$ ، $f\circ g(x)$ ؟

الحل:

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[rr, bend right, "g\circ f"'] & B \arrow[r, "g"] & C \end{tikzcd}$$

(1) $g\circ f(x)=g(f(x))=g(2x)=2x+5$

(2) $g\circ f(1)=2(1)+5=7$

(3) $f\circ g$ isn't define $\quad A \xrightarrow{f} B \xrightarrow{g} C \not\longrightarrow$

مثال: لتكن $f: N \to N$ حيث أن $f(x)=4x^2$

$g: N \to N$ حيث أن $g(x)=3x+1$

$\forall x \in N$ جد $g\circ f(x)$ ، $f\circ g(x)$ ؟

الحل:

$$\begin{tikzcd} N \arrow[r, "f"] \arrow[rr, bend right, "g\circ f"'] & N \arrow[r, "g"] & N \end{tikzcd}$$

(1) $g\circ f(x)=g(4x^2)$

$=3(4x^2)+1=12x^2+1$

$$\begin{tikzcd} N \arrow[r, "g"] \arrow[rr, bend right, "f\circ g"'] & N \arrow[r, "f"] & N \end{tikzcd}$$

(2) $f\circ g(x)=f(3x+1)$

$=4(3x+1)^2$

Example:- let $f: \mathbb{R} \longrightarrow [2,\infty)$ s.t $f(x) = x^4 + x^2 + 2$

$g: \mathbb{R}^+ \longrightarrow [-4,\infty)$ s.t $g(x) = \sqrt{x} - 4$

Find $f \circ g$ and $g \circ f$ (if exist).

Solution: $f \circ g$ exist when $R_g \subseteq D_f$

$R_g \subseteq [-4,\infty)$ and $D_f = \mathbb{R} \implies R_g \subseteq D_f$

$\therefore$ $f \circ g$ is defined

$$\begin{tikzcd}
\mathbb{R}^+ \arrow[r, "g"] \arrow[rr, bend right, "f \circ g"'] & \underset{\subseteq \mathbb{R}}{[-4,\infty)} \arrow[r, "f"] & {[2,\infty)}
\end{tikzcd}$$

$f \circ g: \mathbb{R}^+ \longrightarrow [2,\infty)$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x} - 4) = (\sqrt{x} - 4)^4 + (\sqrt{x} - 4)^2 + 2$

$g \circ f$ is defined when $R_f \subseteq D_g$

$R_f \subseteq [2,\infty) \subseteq \mathbb{R}^+ = D_g$

$\implies g \circ f$ is defined

$(g \circ f)(x) = g(f(x)) = g(x^4 + x^2 + 2) = \sqrt{x^4 + x^2 + 2} - 4$