Analysis of variance

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In statistics, analysis of variance (ANOVA) is a collection of statistical models, and their associated procedures, in which the observed variance in a particular variable is partitioned into components attributable to different sources of variation.

In its simplest form, ANOVA provides a statistical test of whether or not the means of several groups are all equal, and therefore generalizes *t*-test to more than two groups.

Doing multiple two-sample t-tests would result in an increased chance of committing a type I error.

For this reason, ANOVAs are useful in comparing two, three, or more means.

ANOVA is a particular form of statistical hypothesis testing heavily used in the analysis of experimental data.

In the typical application of ANOVA, the null hypothesis is that all groups are simply random samples of the same population.

This implies that all treatments have the same effect (perhaps none). Rejecting the null hypothesis implies that different treatments result in altered effects.

The terminology of ANOVA is largely from the statistical design of experiments.

The experimenter adjusts factors and measures responses in an attempt to determine an effect.

Factors are assigned to experimental units by a combination of randomization and blocking to ensure the validity of the results.

ANOVA can ascertain the magnitude of the contribution of each of these sources to the total variation.

ANOVA is used for two different purposes:

1- To estimate and test hypotheses about population variances

2- To estimate and test hypotheses about population means.

The conclusions regarding the means will depend on the magnitudes of the observed variances.

Variable of interest is the dependent variable (out come),

grouping variable is the independent variable (factor).

The different conditions of this variable are known as "levels"

Design-of-experiments terms

Balanced design

An experimental design where all cells (i.e. treatment combinations) have the same number of observations.

Blocking

A schedule for conducting treatment combinations in an experimental study such that any effects on the experimental results due to a known change raw materials, operators ., become concentrated in the levels of the blocking variable.

The reason for blocking is to isolate a systematic effect and prevent it from obscuring the main effects.

Blocking is achieved by restricting randomization.

Design

A set of experimental runs which allows the fit of a particular model and the estimate of effects.

Effect

How changing the settings of a factor changes the response.

The effect of <u>a single factor</u> is also called a <u>main effect</u>.

Experimental unit

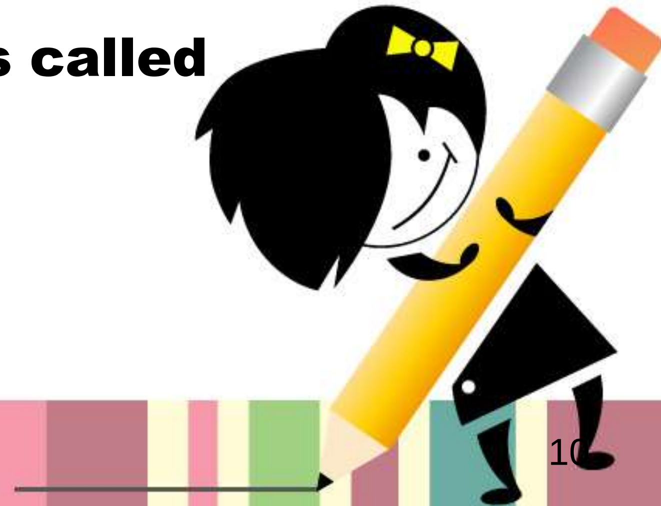
The entity to which a specific treatment combination is applied.

Factors

Process inputs an investigator manipulates to cause a change in the output.

Responses

The output(s) of a process. Sometimes called <u>dependent variable(s)</u> .



Treatment

A treatment is a specific combination of factor levels whose effect is to be compared with other treatments.

Logic of ANOVA

The calculations of ANOVA can be characterized as computing a number of means and variances, dividing two variances and comparing the ratio to a tab value to determine statistical significance.

Assumption

ANOVA uses information about the mean and SD in each group

Important assumptions are made :

1- the values of the dependent or out come variable are assumed to be normal distributed with each group that is , at each level of the factor or independent variable.

2- the pop variance is the same in each group.

3- the observations are random samples , and they are independent , the value of one observation is not related in any way to the value of other observations.

Hypotheses: **<u>Ho</u> that all group (population) or means are equal against the <u>HA</u> members of at least one pair are not equal.**

Ho = $\mu 1 = \mu 2 = \ldots\ldots.$

HA not all μ are equal.

Level of significance:

K-1 [NUMERATOR DEGREE OF FREEDOM]

N-K [DENOMINATOR DEGREE OF FREEDOM].

A) COMPLETELY RANDOMIZED DESIGN: ONE WAY- ANOVA

Because the observations are classified according to only one criterion.

This type of ANOVA is an extension to three or more population means of the two independent samples t- test for testing the equality of two population means.

Partitioning of the sum of squares

VARIANCE RATIO (VR):

$$VR = \frac{\text{Variance Among Groups}}{\text{Variance With in groups}} = \frac{MSA}{MSW} = \frac{\dfrac{SSA}{d.\ f\ [k-1]}}{\dfrac{SSW}{d.\ F\ [\ N-K]}}$$

$$SSA = [\sum (t2/n)] - (T2/N)$$

$$SSW = (\sum\sum X2) - [\sum (t2/n)]$$

Where:

MSA = mean square among (i.e. among study groups)

SSA = sum square among

t = total of the sub-group

T = grand total of the whole sample

K = no. of sub-groups

MSW = mean square within (i.e. we assume: one sample “no groups”)

SSW = sum square within

N = grand no. of the sample

SST = sum square total, either = (∑ X2) – (T2/N) or = SSA + SSW

$$F = \frac{\text{variance between treatments}}{\text{variance within treatments}}$$

The duration of stay (in days) in hospital following certain operation for patients operated by three different surgeons [A,B,C] .

Surgeon A: 4, 5, 5, 4, 6, 6, 4, 5 t A = 39

Surgeon B: 4, 5, 4, 3, 4, 5, 3, 3 t B= 31

Surgeon C: 5, 3, 3, 3, 3, 3, 4, 5 t C =29

T = 99

Mean A = 4.87 days
Mean B = 3.87 days
Mean C = 3.62 days

(N.B) if m A = m B = m C then we stop here, and if not; then we go on to test if the observed difference is significant or not.

Data: represent duration of stay in hospital among 24 patients , they under went same operation by 3 different surgeons with

MA =4.87 day , MB=3.87 day , MC= 3.62 day

Assumption: we assume that the data are normally distributed and the value of population variance between individuals is the same for each group.

Hypotheses:

Level of significance:

$\alpha = 0.05$

d. f = k – 1 = 3 – 1 = 2

d. f = N – k = 24– 3 =21

Tabulated F [at $\alpha = 0.05$ & d. f = 2,21] =3.47

Table of Probabilities for the F Distribution

Alpha = 0.05

D/N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	20	24	30	40	60	120
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	242.98	243.91	244.69	245.36	245.95	248.01	249.05	250.10	251.14	252.20	253.25
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41	19.42	19.42	19.43	19.45	19.45	19.46	19.47	19.48	19.49
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70	8.66	8.64	8.62	8.59	8.57	8.55
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86	5.80	5.77	5.75	5.72	5.69	5.66
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62	4.56	4.53	4.50	4.46	4.43	4.40
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94	3.87	3.84	3.81	3.77	3.74	3.70
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51	3.44	3.41	3.38	3.34	3.30	3.27
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22	3.15	3.12	3.08	3.04	3.01	2.97
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01	2.94	2.90	2.86	2.83	2.79	2.75
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85	2.77	2.74	2.70	2.66	2.62	2.58
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72	2.65	2.61	2.57	2.53	2.49	2.45
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62	2.54	2.51	2.47	2.43	2.38	2.34
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53	2.46	2.42	2.38	2.34	2.30	2.25
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46	2.39	2.35	2.31	2.27	2.22	2.18
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40	2.33	2.29	2.25	2.20	2.16	2.11
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35	2.28	2.24	2.19	2.15	2.11	2.06
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31	2.23	2.19	2.15	2.10	2.06	2.01
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27	2.19	2.15	2.11	2.06	2.02	1.97
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23	2.16	2.11	2.07	2.03	1.98	1.93
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.22	2.20	2.12	2.08	2.04	1.99	1.95	1.90
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.22	2.20	2.18	2.10	2.05	2.01	1.96	1.92	1.87
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15	2.07	2.03	1.98	1.94	1.89	1.84
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20	2.18	2.15	2.13	2.05	2.01	1.96	1.91	1.86	1.81
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.15	2.13	2.11	2.03	1.98	1.94	1.89	1.84	1.79
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16	2.14	2.11	2.09	2.01	1.96	1.92	1.87	1.82	1.77
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.12	2.09	2.07	1.99	1.95	1.90	1.85	1.80	1.75
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.17	2.13	2.10	2.08	2.06	1.97	1.93	1.88	1.84	1.79	1.73
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	2.09	2.06	2.04	1.96	1.91	1.87	1.82	1.77	1.71
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.08	2.05	2.03	1.94	1.90	1.85	1.81	1.75	1.70
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01	1.93	1.89	1.84	1.79	1.74	1.68
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92	1.84	1.79	1.74	1.69	1.64	1.58
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.89	1.86	1.84	1.75	1.70	1.65	1.59	1.53	1.47
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83	1.80	1.78	1.75	1.66	1.61	1.55	1.50	1.43	1.35

Right Tailed, D/N = df in denominator = down the rows, df in numerator = across the columns

Note: Table is for an alpha of 0.05

F-TABLE: VALUES OF F OF THE F-DISTRIBUTION (ALPHA = 0.01)

```
 |     ---
 |   --   ---
 |  -         ----       0.01
 | -              |:---- /
 |-               |::::---______
 +----------------+----------------
 0                F
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DEGREE OF NUMERATOR (V1)

V2	1	2	3	4	5	6	7	8	9	10
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98

SSA [∑ (t2/n)] – (T2/N)

t^2 A =1521

t^2 B =961 **T^2 =9801**

t^2 C =841

SSA=[1521/8 +961/8 +841/8] - 9801/24

SSA= 415.375 408.375=7

SSW = (∑ X2) – [∑ (t2/n)]

SSW =431- 415.375=15.625

$$MSA = \frac{SSA}{k - 1} = \frac{7}{3-1} = 3.5$$

$$MSW = \frac{SSW}{N - k} = \frac{15.625}{24-3} = 0.744$$

$$VR = \frac{MSA}{MSW} = \frac{3.5}{0.744} = 4.704$$

Calculated F > tabulated F → P < 0.05,

so we reject Ho and accept HA

Variance ratio (F) more than <u>one</u> indicates the variation is higher among study groups than within the grand sample readings, and thus the presence of grouping affects the value of each reading.

A larger value of V.R resulted from the fact that among groups mean squares was considerably larger than the within groups mean squares .

Since the among groups mean square is based on the dispersion of the sample means about mean, this quantity will be large when

there is a large discrepancy among the sizes of the sample means .

Because of this , then a significant value of V.R tell us to reject Ho that all population means are equal.

B) RANDOMIZED COMPLETE BLOCK DESIGN: TWO WAY- ANOVA:

The randomized complete block design is a design in which the treatments units (called experimental units) to which the are applied are subdivided into homogeneous groups called blocks .

The treatment are then assigned at random to the experiment units within each block.

It should be emphasized that each treatment appears in every block, and each block receive every treatment.

When blocking is used homogeneous blocks of experimental units.

Objective: is to isolate and remove from the error term the variation attributable to the blocks, while assuring that treatment means will be free of block effects.

The effectiveness of the design depends on the ability to form homogeneous blocks of experimental units.

When blocking is used effectively , the error mean square in the ANOVA table will be reduced, the V.R will be increased , and the

chance of rejecting the null hypothesis will improved.

Advantages: one of the advantages of the RCBD is that it is easily understood .

Certain complications that may arise in the course of an experiment are easily handled when this design is employed .

The paired comparisons analysis is a special case of the RCBD .

VARIANCE RATIO (VR):

$$VR = \frac{\text{Variance among Treatment groups}}{\text{Variance Residual}} = \frac{MSA}{MSE} = \frac{\dfrac{SSTr}{d.f\,Tr}}{\dfrac{SSE}{d.f\,E}} =$$

$$\frac{\dfrac{[\sum (t2/n)] - (T^2/N)}{\quad}}{\dfrac{SST - SS\,Bl - S\,ST\,r}{(n-1)(K-1)}}$$

d. f T r = K – 1 [treatment -1] [numerator df]

Denominator df [(n-1)x (k-1)]

Where n= [no. in each block]

Where:

SST=total sum of squares for the RCBD=

$\sum$ **[x– Mean of sum of means of groups]**^{2}

S ST r = treatment = no. of blocks [mean of each group –

Mean of sum of means of groups]^2

SSB=blocks= no. of groups $\sum$ **[mean of block**

– Mean of sum of means of groups]^2

SSE=error (residual)=SST- SSBI - S ST r

a physical therapist wished to compare three methods for teaching patients how to use a certain prosthetic device. He felt that the rate of learningwould be different for patients of different ages and wished to design an experiment in which the influence of age could be taken into account.

SOLUTION: **The randomized complete block design is the appropriate design**

Data: **three patients in each of five age groups were selected to participate in the experiment, and one patient in each age group was randomly assigned to each of the teaching methods.**

The methods of instruction (A, B & C) constitute our three treatments and the five age groups are the blocks.

Time in days required to learn the use of a certain prosthetic device

Teaching Method

Age group	A	B	C	TOTAL	Mean
Under 20	7	9	10	26	8.67
20-29	8	9	10	27	9.00
30-39	9	9	12	30	10.00
40-49	10	9	12	31	10.33
50 & over	11	12	14	37	12.33
Total	45	48	58	151	
mean	**9.0**	**9.6**	**11.6**		**10.07**

$SST = (7-10.07)^2 + (8-10.07)^2 + \ldots\ldots\ldots\ldots\ldots + (14-10.07)^2 = 46.9335$

$SSB = 3\,[(8.67-10.07)^2 + (9-10.07)^2 + \ldots\ldots\ldots\ldots + (12.33-10.07)^2 = 24.855$

$S\,ST\,r = 5\,[\,(9-10.07)^2 + (9.6-10.07)^2 + (11.6-10.07)^2 = 18.5335$

$SSE = SST - SSB - SS\,T\,r = 46.9333 + 24.855 + 18.5333 = 3.545$

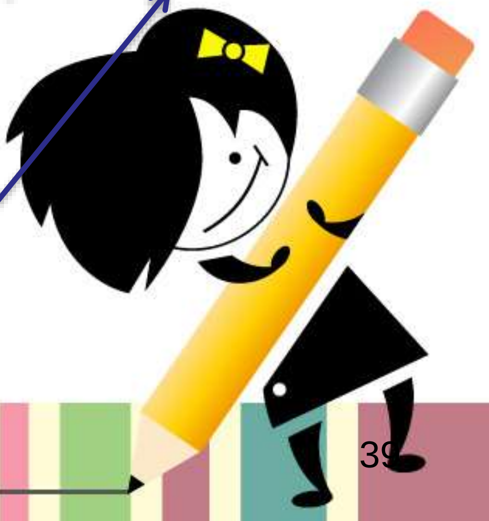
$$MSA = \frac{S\,S\,T\,r}{d.\,f\,T\,r} = \frac{18.5333}{2} = 9.2666$$

tab F df (5-1) (3-1)= 8

Denominator df[(n-1)x (k-1)]

tab F 4.46

[treatment -1] [numerator df]



$$MSE = \frac{SSE}{d.f_{E}} = \frac{3.545}{8} = 0.443$$

$$VR = \frac{MSA}{MSE} = 20.91$$

Measuring Effect Size in anova

- **Most common technique is "r^2"**
  - **Tells you what percent of the variance is due to the treatment**
  - **r^2 = SS between groups/SS total**