

t TEST AND t DISTRIBUTION

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Prof Dr NAJLAA FAWZI

When we do not know the population variance or standard deviation. we rely on the sample variance and standard deviation.

In this case the distribution will follow t-distribution instead of Z distribution.

$Z = (m - \mu) / (\delta / \sqrt{n})$, and $\delta = ?$ Then we use $S \rightarrow t = (m - \mu) / (S / \sqrt{n})$

t-distribution curve is characterized by:

1-It has a mean of zero.

2-It is symmetrical around its mean.

3-Its range lie between $-\alpha$ & $+\alpha$.

4-The quantity $(n-1)$ which is called the degree of freedom (d. f) ,is used in computing the sample variance.

5-Compared with normal distribution; it is of lower peak & higher tails.

This is because the variability is dependent upon S instead of δ and since the variability within the sample is larger than that within the population, then S is usually $>$ than δ .

6-It approaches normal distribution as $(n-1)$ approaches infinity.

As the sample size increases and approaches the normal distribution.

This is at $n > 200$), then we will shift from t-distribution to Z-distribution.

Notes about t-table:

1)The increase in the value of degree of freedom (d. f):

- From *1* to *30* is by *1* (i.e. 1, 2, 3...).**
- From *30* to *50* is by *5* (i.e. 30, 35, 40...).**
- From *50* to *100* is by *10* (i.e. 50, 60, 70...).**
- From *100* to *200* is by *20* (i.e. 120,140,160...).**

2) If we do not find the value of d. f that we want, we choose the nearest one, e.g. 41 → 40, 148 → 140, 96 → 100, and for 150 either 140 or 160.

3) The table stops at 200 and shifts to ∞ . Here we will shift to Z table & Z- distribution.

4) To find t-value we use t-table.

The tabulated value for t depends upon 2 factors; d .f (i.e. sample size

& the probability of error $\{\alpha\} \rightarrow (1 - \alpha/2)$.

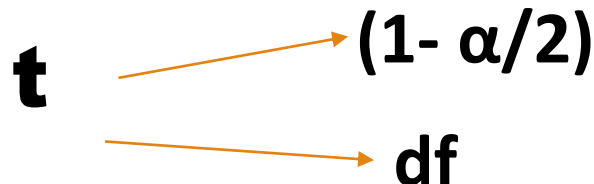
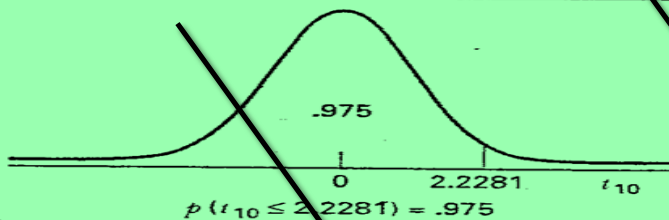
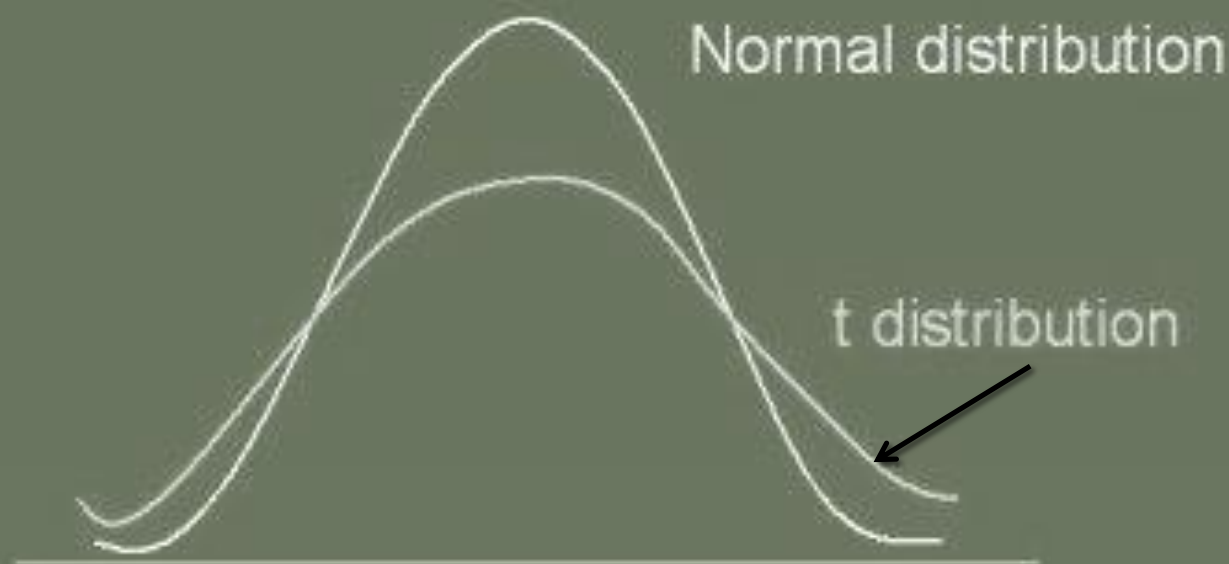


TABLE : Percentiles of the t Distribution



d.f.	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$
1	3.078	6.3138	12.706	31.821	63.657
2	1.886	2.9200	4.3027	6.965	9.9248
3	1.638	2.3534	3.1825	4.541	5.8409
4	1.533	2.1318	2.7764	3.747	4.6041
5	1.476	2.0150	2.5706	3.365	4.0321
6	1.440	1.9432	2.4469	3.143	3.7074
7	1.415	1.8946	2.3646	2.998	3.4995
8	1.397	1.8595	2.3060	2.896	3.3554
9	1.383	1.8331	2.2622	2.821	3.2498
10	1.372	1.8125	2.2281	2.764	3.1693
11	1.363	1.7959	2.2010	2.718	3.1058
12	1.356	1.7823	2.1788	2.681	3.0545
13	1.350	1.7709	2.1604	2.650	3.0123
14	1.345	1.7613	2.1448	2.624	2.9768
15	1.341	1.7530	2.1315	2.602	2.9467
16	1.337	1.7459	2.1199	2.583	2.9208
17	1.333	1.7396	2.1098	2.567	2.8982
18	1.330	1.7341	2.1009	2.552	2.8784
19	1.328	1.7291	2.0930	2.539	2.8609
20	1.325	1.7247	2.0860	2.528	2.8453
21	1.323	1.7207	2.0796	2.518	2.8314
22	1.321	1.7171	2.0739	2.508	2.8188
23	1.319	1.7139	2.0687	2.500	2.8073
24	1.318	1.7109	2.0639	2.492	2.7969
25	1.316	1.7081	2.0595	2.485	2.7874
26	1.315	1.7056	2.0555	2.479	2.7787
27	1.314	1.7033	2.0518	2.473	2.7707
28	1.313	1.7011	2.0484	2.467	2.7633
29	1.311	1.6991	2.0452	2.462	2.7564
30	1.310	1.6973	2.0423	2.457	2.7500
35	1.3062	1.6896	2.0301	2.438	2.7239
40	1.3031	1.6839	2.0211	2.423	2.7045
45	1.3007	1.6794	2.0141	2.412	2.6896
50	1.2987	1.6759	2.0086	2.403	2.6778
60	1.2959	1.6707	2.0003	2.390	2.6603
70	1.2938	1.6669	1.9945	2.381	2.6480
80	1.2922	1.6641	1.9901	2.374	2.6388
90	1.2910	1.6620	1.9867	2.368	2.6316
100	1.2901	1.6602	1.9840	2.364	2.6260
120	1.2887	1.6577	1.9799	2.358	2.6175
140	1.2876	1.6558	1.9771	2.353	2.6114
160	1.2869	1.6545	1.9749	2.350	2.6070
180	1.2863	1.6534	1.9733	2.347	2.6035
200	1.2858	1.6525	1.9719	2.345	2.6006
∞	1.282	1.645	1.96	2.326	2.576

Comparison of normal vs. t distributions



t -distribution is less peaked in the center and has higher tails compared to the normal distribution

CHOOSING Z or t the variability of the sample (S_2 & S) approaches that of the population as the sample size increases so the question will be : when we can use Z instead of t-distribution (i.e. when to state that $S \approx \delta$) :

There is no simple answer to the question. "how large is a large sample?"

We should be reasonably safe with inferences about means if the sample is greater than 100 for a single sample , or if both samples are greater than 50 for two samples .

The application of statistical methods is a matter of judgment as well as knowledge .

d. f \leq 30 \rightarrow we have to use t test

d. f 31-60 \rightarrow t test is preferred on Z

Calculated t & Confidence Intervals using t-distribution

For population mean: $t = (m - \mu) / (S/\sqrt{n})$

%C.I for $\mu = m \pm t \times S / \sqrt{n}$, where d. f = n-1

(df & $(1 - \alpha/2)$)

Example : using specimens obtained from 10 individuals for teeth contents calcium with $m = 35.7$ & $S = 0.7$.

What is the expected population mean?

**95%C.I for $\mu = m \pm (9 \text{ t } 0.975 \times S / \sqrt{n}) = 35.7 \pm (2.262 * 0.7/\sqrt{10}) =$
35.15 to 36.17**

From the rules of confidence intervals using t-distribution we notice that the interval is affected by sample size more than with using Z distribution as with t-distribution even the tabulated value is affected by the sample size and not only the standard error.

Criteria for applying t-test

- 1. Quantitative data**
- 2. Random samples (two samples only)**
- 3. Variables normally distributed**
- 4. Sample size less than 30.**

Applications of the t- test

1- One sample t-test, tests the mean of a single group against a known mean. { Sample mean, and population mean}.

2-Independent Samples t-test (pooled t – test)

Compare the means for two small groups.

3- Paired t-test compares means from the same group at different times.

What is really going on with t Tests?

- **Essentially the difference between the means of the two groups is being compared to the estimated standard error.**
- **$t = \text{difference between group means} / \text{estimated standard error}$**
- **$t = \text{variability due to chance} + \text{independent variable} / \text{variability due to chance alone}$**
- **The t distribution is the sampling distribution of differences between sample means. (comparing obtained difference to standard error of differences)**

Assumptions made in the use of t-test are:

1- Samples are randomly selected

2- Samples are independents

3-Variables follow normal distribution

4-Sample variances are same in both the groups under study. The variances of the two populations are equal.

Logic of Hypothesis Testing

- ❏ **Making inferences from sample means to population means**
- ❏ **Hypotheses about means**
- ❏ **$H_0 : \mu_1 = \mu_2 = \mu$**
- ❏ **$H_A : \mu_1 \neq \mu_2$**
- ❏ **Reject H_0 if its probability of being true is small (alpha level)**
- ❏ **Calculate sample statistic (example t value)**
- ❏ **Compare to critical value on the t distribution**
- ❏ **One tail vs. two-tailed tests**

Critical Value and Region for One Tailed Test

Note: SPSS gives two-tailed probability; divide by 2 to obtain One-tailed probability

$$\mathbf{df = 19}$$

$$\mathbf{\alpha = 0.05}$$

$$\mathbf{t = 1.729}$$

Critical Value and Region for Two Tailed Test

$$\mathbf{df = 19}$$

$$\mathbf{\alpha /2 .025}$$

$$\mathbf{t = 2.093}$$

Single Sample t-Test

Rationale: To test whether a sample mean differs significantly from a known population mean.

Mean Hb level of 25 preschool children was observed to be 10.6 g/dl with SD 1.15 g/dl ; by using α 0.05. Is it significantly different from mean Hb level of the population of 11 g/dl?

Data: represent the mean Hb level (10.6g/dl) of 25 preschool children, with SD of 1.15g/dl .And population mean Hb level was 11 g/ dl.

Assumption: we assume that the sample is randomly selected from normally distributed population.

Hypothesis:

Ho: there is no significant difference in mean Hb level of the sample from those in the population. $m = \mu$

HA: there is a significant difference between mean Hb level of the sample from that in the population. $m \neq \mu$

Level of significance:

$$\alpha = 0.05$$

5% chance factor effect

95% influencing factor effect

$$\text{d. f} = n - 1$$

$$\text{d. f} = 25 - 1 = 24$$

Critical point = tabulated t = d. f & (1- α /2) {from t distribution table}.

$$\text{d. f} (24) \text{ \& } 1 - \alpha / 2 (0.975) = 2.064$$

Testing significance:

calculated t = $m - \mu / (SD/\sqrt{n})$

calculated t = $m - \mu / SE$ of difference

SE = $SD/\sqrt{n} = 1.15 / \sqrt{25} = 0.23$

Cal t = $(10.6 - 11) = -1.74$

Decision and conclusion:

Cal t < tab t ($1.74 < 2.064$)

Cal t falls in acceptance area of Ho.

We not reject H0

P > 0.05

The difference between the mean Hb levels of 25 preschool children and that of population was not significant.

1: below are the heights of randomly selected 24 male in the age of 2, having sickle cell anemia. The mean Ht of that age and gender in that population is 86.5 cm. Dose the sample provide sufficient evidence about the effect of this disease on the heights of children?

Heights:84.4,89.9,89,81.4,87,78.5,84.1,86.3,80.6,80,81.3,86.8,83.4,89.8,85.9,80.6,85,82.5,80.7,84.3,85.4,85,58 & 81.9.

Data represent the heights of 24 male children suffer from sickle cell anemia with mean height 48.1 cm and standard deviation of 3.11 cm, and the standard height of children at that age and gender is 86.5cm.

Assumption: we assume that the sample is randomly selected from normally distributed population.

Hypothesis:

Ho: there is no significant difference in mean heights of the sample from those in the population.

HA: there is a significant difference between...

Level of significance:

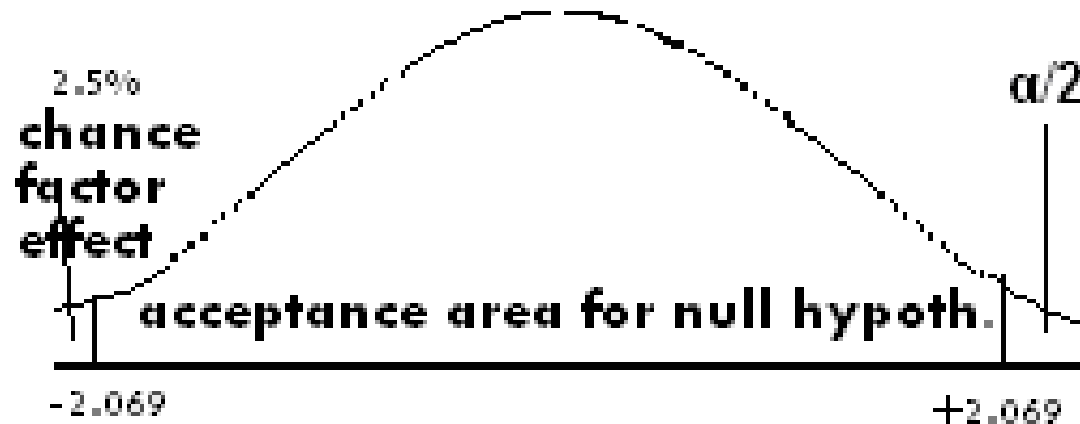
$\alpha = 0.05 \rightarrow 5\%$ chance factor effect

95% influencing factor effect

d. f = 24 - 1 = 23

Critical point = tabulated t = d.f t(1- α / 2)

= 23t0.975 = 2.069



**Testing significance: calculated $t = (m - \mu) / (S/\sqrt{n})$
 $= (84.1 - 86.5) / (3.11/\sqrt{24}) =$
- 3.81**

Conclusion: calculated t falls in the rejection area so we reject H_0 & accept H_A , i.e. there is a significant difference in heights between the sickled sample and the normal population. Or there is a significant effect of S.C.A on the height for that age and gender.

Independent Groups t-Test (pooled t -test)

Rationale : To test whether two independent sample means come from different populations

The two groups represent the independent variable

The two group means are statistics of the dependent variable

Advantages of Independent Sample Designs

- + Independent Designs have no carryover effects**
- + Independent designs do not suffer from fatigue or practice effects**
- + You do not have to worry about getting people to show up more than once**
- + Demand characteristics may be stronger in repeated measure studies than in independent designs**
- + Since more individuals participate in independent design studies, the results may be more generalizable**

 **This is a one-time test, not over time with multiple observations.**

Disadvantages of Independent Sample Designs

- **Usually requires more subjects (larger n)**
- **The effect of a variable cannot be assessed for each individual, but only for groups as a whole**
- **There will be more individual differences between groups, resulting in more variability**

Example : The mean values of B. wt are given below by socioeconomic status .

**Does the socioeconomic status have effect on the birth weight?
 α 0.01**

Data: n= 15; mean B. wt 2.91kg , SD 0.27[high social class];
n=10 ; mean B. wt 2.26 kg , SD 0.22 [low social class].

Assumption

Hypothesis

H0 m1 = m2 ; HA m1 and m2 not the same

Level of significant

α 0.01 df n1+ n2 -2 15+10 -2 = 23 tab t = 2.807

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \left[\frac{n_1 + n_2}{n_1 n_2} \right]}}$$

Pooled variance of the two groups

$$Sp = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

The nominator of the equation captures difference in means, while the denominator captures the variation within and between each group.

Important point: of interest is the difference between the sample means, not sample and population means.

However, rejecting the null means that the two groups under analysis have different population means.

Test criteria (t) : $t = m_1 - m_2 / SE (m_1 - m_2)$ [SE of difference]

Calculated $t = (m_1 - m_2) - (\mu_1 - \mu_2) / S_p \sqrt{(1/n_1 + 1/n_2)}$

$S_p = \sqrt{S_p^2} =$

(pooled variance) = $\sqrt{[(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2] / (n_1 + n_2 - 2)}$

$S_p = \sqrt{(0.27)^2(15-1) + (0.22)^2(10-1) / 15 + 10 - 2}$

$S_p = 0.25$

$S_p \sqrt{(1/n_1 + 1/n_2)}$

$SE = 0.25 \sqrt{1/15 + 1/10}$

$SE = 0.25 \times 0.407 = 0.101$

Calculated $t = 2.91 - 2.26 / 0.101 = 6.435$

Conclusion : Since $cal\ t > tab\ t$ $p < 0.01$ Reject H_0 Accepte H_A .

The socioeconomic statuts had effect on birth weight

A paired t test (also called a correlated pairs t-test, a paired samples t test or dependent samples t test) is where you run a t test on dependent samples.

Dependent samples are essentially connected — they are tests on the same person or thing.

For example:

- **Knee MRI costs at two different hospitals,**
- **Two tests on the same person before and after training,**
- **Two blood pressure measurements on the same person using different equipment.**

Paired-Groups t- Test

n refers to the number of pairs

df = n-1

Rationale:

- The two samples are not independent.**
- Subjects in one sample are paired with subjects in the second sample**

Examples of pairing

Matching on a characteristic (e.g., gender, age, IQ)

Pairing based on a specific relationship (e.g., twins, husband-wife)

Subjects appear in both samples of observations (e.g., repeated measures)

t –test is applied to paired data of observations from one sample only when each individual gives a pair of observations.

Examples is the comparison of the Hb values of persons estimated by two methods .

Another example is comparison of measurements of skin fold thickness or height of individuals before and after some experiment.

The use of the test is restricted for samples with matched pair of less than 30.

This technique is applied in order to eliminate as much as possible a maximum number of sources of variation by making the pairs similar [identical] to each other with respect to as many variables as possible [for the same persons two measurements one before and one after as the difference is mostly due to the effect of the factor itself].

Pairs of twins or siblings also can be use.

Reasons for Pairing : it frequently true that true

differences do not exist between two pop with respect to the variable of interest , but the presence of other [extraneous] sources of variations may cause rejection of the hypothesis of no difference.

On the other hand true differences may also be masked by the presence of extraneous factors.

Therefore the objective in paired comparison tests:

- 1- To eliminate a maximum number of sources of extraneous variation by making the pairs similar with respect to as many variable as possible.**
- 2- Fewer subjects are used with repeated measurements on each one , this is of value specially if subjects are scare or expensive to recruit.**

Disadvantages of Paired Comparisons:

- 1- Considerable time & expenses are needed in our trying to match individuals on one or more relevant variables.**
- 2- Loss of degrees of freedom , if we do not use paired observation, we have $(n_1-1) + (n_2-1)$, as compared to $n-1$ when we use the paired comparison procedure.**
- 3- Carry –over effect especially when one or more treatment are being evaluated with possibility of residual effect from previous treatment .**

This problem is solved by allowing a sufficient length of time between treatment .

4- Position effect; a subject response to a treatment experienced last in a sequence may be different from the response would have occurred if been first in sequence .

This problem can be solved by randomize the sequence of treatments independently for each subject.

Example :

A sample of 10 people with systolic B.P in mmHg before and after administering antihypertensive drug as shown below.

Is the drug significant in lowering blood pressure?

After	150	130	150	120	130	160	150	170	140	160
Before	180	140	170	130	130	180	170	160	170	190

Before	After	Difference (d)	d²
180	150	30	900
140	130	10	100
170	150	20	400
130	120	10	100
130	130	0	0
180	160	20	400
170	150	20	400
160	170	-10	100
170	140	30	900
190	160	30	900
1610(Ax)	1420(Ax_e)	160(Σd)	4200(Σd²)

Data: represent SBP measurements of 10 hypertensive patients before and after taking the hypotensive drug with mean SBP 161mmHg before and 142mmHg after & SD of difference is 13.49mmHg.

Assumption:

We assume that the sample of the 10 patients was randomly selected from normally distributed population.

Hypotheses:

H₀: no significant difference in means of SBP in relation to drug administration.

Or there is no significant effect of the drug on SBP.

HA: there is significant...

Level of significance: $\alpha = 0.05 \rightarrow 5\%$ chance factor effect

d. f = 10 - 1 = 9

95% influencing factor effect

Critical point = tabulated t = df t(1 - α / 2) = 9t0.975 = 2.2622

Cal t = mean of difference / SE of difference

Mean of difference = 160/10 = 16

SD of difference = 13.498

SE of difference = 4.268

Cal t = 16 / 4.268 = 3.748

(fall in the rejection area), so as calculated t > tabulated t ...

$\rightarrow P < 0.05$

We reject H0 and accepted HA.

Example : A certain breed of rats shows a mean weight gain of 65g during the 1st 3 months of life. A sample of 16 of these rats were fed a new diet from birth until the age of 3 months, the mean weight was 60.75g. If the *population variance* is 10, is there a reason to believe at 5% level of significance that the new diet induced a change in the average amount of gained weight?

Tabulated value = critical value = 1.96 (Z-distribution)

Calculated value = $Z = (m - \mu) / (\delta / \sqrt{n}) = (60.75 - 65) / (\sqrt{10} / \sqrt{16})$

= - 5.33 fall in the rejection area

Z (Normal Curve Test)

Z-TEST / Z-STATISTIC: used to test hypotheses about μ when the population standard deviation is known and population distribution is normal, or sample size is large

The only difference between the z- and t-tests is that the t-statistic estimates standard error by using the sample standard deviation, while the z-statistic utilizes the population standard deviation

Z (Normal Curve Test)

Normal test is utilized for differences between mean values based on large samples.

Examples for comparing

1- two sample means

2- sample mean with population mean

3- two sample proportions [or percentages]

4- sample proportion with population proportion

Comparison of Means of Two Samples

Example: Details of study on arm circumference of American and Indian preschool children are given. Can we inference that arm circumference is different between them?

	American	Indian
No. of children	625	625
mean	20.5	15.5
SD	5	5.4

Critical ratio $Z = \frac{M1-M2}{\sqrt{(SE1)^2 + (SE2)^2}}$

$$Z = \frac{20.5 - 15.5}{0.2947} = \frac{5}{0.2947} = 16.985$$

Z is greater than tab Z 2.58 , we reject Ho

Comparison of sample mean with population mean

When population variance is known and if it is desired to test the difference between sample mean and population mean , normal curve test is utilized .

N. B even for small sample size.

$Z = \frac{\text{mean} - \mu}{\sqrt{SD^2/n}}$; mean of sample , μ population mean,
SD² is the variance of the population and n is the sample size chosen at random.

Example : Mean serum vit A level of 10 students was 22mg /dl ;
**Dose this mean value differ from a mean of population 25gm/ dl
with population variance of 40?**

$$Z = \frac{\text{mean} - \mu}{\sqrt{SD^2/n}} = \frac{22-25}{\sqrt{40/10}} = \frac{3}{\sqrt{4}} = 1.5$$

Theoretical value at 5% = 1.96 1.5 < 1.96

P > 0.05 , NOT REJECT H₀

Differences between two sample proportions

For assessing the significance of differences between 2 large sample proportions.

normal curve test is utilized

Z = Differences in proportions/ SE of differences in proportions

$$Z = p_1 - p_2 / \sqrt{PQ (1/n_1 + 1/n_2)}$$

$$P = n_1 p_1 + n_2 p_2 / n_1 + n_2 \quad \text{and} \quad Q = 1 - P$$

Example : Details of population surveyed with nutritional deficiencies are given. Is there any sig of difference in the prevalence of nutrition deficiency between the villages?

& 0.01

	Village A	Village B
Sample size	n1 = 150	n2 =160
Subjects with clinical def	129	110
Proportion with clinical def	p1 129/150= 0.86	p2 110/ 160=0.6875

$$Z = \frac{p_1 - p_2}{\sqrt{PQ}} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)$$

$$P = \frac{p_1 n_1 + p_2 n_2}{n_1 + n_2}$$

$$P = \frac{239}{310} = 0.771$$

$$Q = 1 - 0.771 = 0.229$$

$$Z = \frac{0.86 - 0.6875}{\sqrt{(0.771)(0.229)}} \left[\frac{1}{150} + \frac{1}{160} \right]$$

$$Z = \frac{0.1725}{0.047755} = 3.61$$

Cal Z greater than tab Z

$P < 0.01$ reject H_0

Comparison of sample proportion with population proportion

Testing the validity of sample characteristics with these of population characteristics . p sample proportion , P population proportion .

$$Z = \frac{p - P}{\sqrt{PQ(1/n)}}$$

$$Q = 1 - P$$

Example: In a study among 2428 boys aged 7-12 years, 461 were overweight or obese . Based on this study can we conclude that more than (15%) of the boys aged 7-12 years in the sampled population are obese or overweight ? & 0.05

prop in sample = 461/ 2428= 0.189

Prop in population= 0.15

Q= 1-0.15= 0.85

Z = p- P/√PQ(1/n)

Z= 0.189- 0.15/ √(0.15)(0.85)(1/2428)

Z= 5.571