Analysis of variance

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In **statistics**, analysis of variance (ANOVA) is a collection of **statistical models**, and their associated procedures, in which the observed **variance** in a particular variable is partitioned into components attributable to different sources of variation.

In its simplest form, ANOVA provides a [statistical test](#) of whether or not the [means](#) of several groups are all equal, and therefore generalizes [*t*-test](#) to more than two groups.

Doing multiple two-sample t-tests would result in an

increased   chance of committing a [type I error](#).

For this reason, ANOVAs are useful in comparing two, three, or more means.

ANOVA is a form of <ins>statistical hypothesis testing</ins> heavily used in the analysis of experimental data.

In the typical application of ANOVA, the null hypothesis is that all groups are simply random samples of the same population.

This implies that all treatments have the same effect (perhaps none). Rejecting the null hypothesis implies that different treatments result in altered effects.

The terminology of ANOVA is largely from the statistical
 design of experiments.

 The experimenter adjusts factors and measures responses
  in an attempt to determine an effect.

 Factors are assigned to experimental units by a combination
 of randomization and blocking to ensure the validity of
  the results.

ANOVA can ascertain the magnitude of the contribution of each of these sources to the total variation.

ANOVA is used for two different purposes:

1- To estimate and test hypotheses about population variances

2- To estimate and test hypotheses about population means.

The conclusions regarding the means will depend on the magnitudes of the observed variances.

What are "Groups" or "Levels"?

Groups or levels are different groups in the same independent variable.

levels for "brand of cereal" might be Lucky Charms, Raisin Bran, Cornflakes — a total of three levels.

Variable of interest is the dependent variable (out come), grouping variable is the independent variable (factor).

The different conditions of this variable are known as "levels"

you are studying if [Alcoholics Anonymous](#) and individual counseling combined is the most effective treatment for lowering alcohol consumption.

 You might split the study participants into three groups or levels: medication only, medication and counseling, and counseling only.

 Your [dependent variable](#) would be the number of alcoholic beverages consumed per day.

Design-of-experiments terms

Balanced design

An experimental design where all cells (i.e. treatment combinations) have the same number of observations.

Blocking

A schedule for conducting treatment combinations in an experimental study such that any effects on the experimental results due to a known change in raw materials, operators ., become concentrated in the levels of the blocking variable.

The reason for blocking is to isolate a systematic effect and

prevent it from obscuring the main effects.

Blocking is achieved by restricting randomization.

Design

A set of experimental runs which allows the fit of a particular

model and the estimate of effects.

Effect

How changing the settings of a factor changes

the response.

The effect of <u>a single factor</u> is also called a

<u>main effect</u>.

Experimental unit

The entity to which a specific treatment combination is applied.

Factors

Process inputs an investigator manipulates to cause a change in the output.

Responses

The output(s) of a process. Sometimes called dependent variable(s) .

Treatment

A treatment is a specific combination of factor levels whose effect is to be compared with other treatments.

Logic of ANOVA

The calculations of ANOVA can be characterized as computing several means and   variances, dividing two variances and comparing the ratio to a tab value to determine statistical significance.

What Does "One-Way" or "Two-Way Mean?

One-way has one independent variable(iv) (with 2 [levels](#))
and two-way has two independent variables (can have
multiple levels).

For example, a one-way Analysis of Variance could have
one IV (brand of cereal) and a two-way Analysis of
Variance has two IVs (brand of cereal, calories).

Assumption

ANOVA uses information about the mean and SD in each group

Important assumptions are made :

1- the values of the dependent or out come variable are
   assumed   to be normal distributed with each group that is ,
   at each level of the factor or independent variable.

2- the pop variance is the same in each group.

3- the observations are random samples , and they are independent , the value of one observation is not related in any way to the value of other observations.

Hypotheses: <u>Ho</u> that all group (population) or means are equal against the <u>HA</u> members of at least one pair are not equal.

Ho = μ1 = μ2=.......

HA not all μ are equal.

Level of significance:

K-1 [NUMERATOR DEGREE OF FREEDOM]

N-K [DENOMINATOR DEGREE OF FREEDOM].

A) COMPLETELY RANDOMIZED DESIGN: ONE WAY- ANOVA

Because the observations are classified according to only one criterion.

This type of ANOVA is an extension to three or more population means of the two independent samples t- test for testing the equality of two population means.

The one-way analysis of variance (ANOVA) is used to determine whether there are any statistically significant differences between the means of three or more independent (unrelated) groups

When to use a one-way ANOVA

A group of psychiatric patients are trying three different therapies: counseling, medication and biofeedback. You want to see if one therapy is better than the others.

You have a group of individuals randomly split into smaller groups and completing different tasks.

For example, you might be studying the effects of tea on weight loss and form three groups: green tea, black tea, and no tea.

Partitioning of the sum of squares

VARIANCE RATIO (VR):

$$VR = \frac{\text{Variance Among Groups}}{\text{Variance Within groups}} = \frac{MSA}{MSW} = \frac{\dfrac{SSA}{d.\,f\,[k-1]}}{\dfrac{SSW}{d.\,F\,[\,N-K]}} =$$

$$SSA = [\textstyle\sum (t2/n)] - (T2/N)$$

$$SSW = (\textstyle\sum\sum X2) - [\textstyle\sum (t2/n)]$$

Where:

MSA = mean square among (i.e. among study groups)

SSA = sum square among

t = total of the sub-group

T = grand total of the whole sample

K = no. of sub-groups

MSW = mean square within (i.e. we assume: one sample "no groups")

SSW = sum square within

N = grand no. of the sample

SST = sum square total, either = ($\sum$ X2) − (T2/N) or = SSA + SSW

$$F = \frac{\text{variance between treatments}}{\text{variance within treatments}}$$

The duration of stay (in days) in hospital following certain operation for patients operated by three different surgeons [A,B,C] .

Surgeon A: 4, 5, 5, 4, 6, 6, 4, 5   t A = 39

Surgeon B: 4, 5, 4, 3, 4, 5, 3, 3   t B= 31   T = 99

Surgeon C: 5, 3, 3, 3, 3, 3, 4, 5   t C =29

Mean A = 4.87 days
Mean B = 3.87 days
Mean C = 3.62 days

(N.B) if m A = m B = m C then we stop here, and if not; then we go on to test if the observed difference is significant or not.

Data: represent duration of stay in hospital among 24 patients, they under went same operation by 3 different surgeons with

MA =4.87 day , MB=3.87 day , MC= 3.62 day

Assumption: we assume that the data are normally distributed and the value of population variance between individuals is the same for each group.

Hypotheses:

Level of significance:

α = 0.05

d. f = k – 1 = 3 – 1 = 2

d. f = N – k = 24– 3 =21

Tabulated F [at α = 0.05 & d. f = 2,21] =3.47

Table of Probabilities for the F Distribution

Alpha = 0.05

D/N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	20	24	30	40	60	120
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	242.98	243.91	244.69	245.36	245.95	248.01	249.05	250.10	251.14	252.20	253.25
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41	19.42	19.42	19.43	19.45	19.45	19.46	19.47	19.48	19.49
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70	8.66	8.64	8.62	8.59	8.57	8.55
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86	5.80	5.77	5.75	5.72	5.69	5.66
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62	4.56	4.53	4.50	4.46	4.43	4.40
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94	3.87	3.84	3.81	3.77	3.74	3.70
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51	3.44	3.41	3.38	3.34	3.30	3.27
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22	3.15	3.12	3.08	3.04	3.01	2.97
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01	2.94	2.90	2.86	2.83	2.79	2.75
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85	2.77	2.74	2.70	2.66	2.62	2.58
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72	2.65	2.61	2.57	2.53	2.49	2.45
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62	2.54	2.51	2.47	2.43	2.38	2.34
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53	2.46	2.42	2.38	2.34	2.30	2.25
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46	2.39	2.35	2.31	2.27	2.22	2.18
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40	2.33	2.29	2.25	2.20	2.16	2.11
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35	2.28	2.24	2.19	2.15	2.11	2.06
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31	2.23	2.19	2.15	2.10	2.06	2.01
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27	2.19	2.15	2.11	2.06	2.02	1.97
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23	2.16	2.11	2.07	2.03	1.98	1.93
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.22	2.20	2.12	2.08	2.04	1.99	1.95	1.90
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.22	2.20	2.18	2.10	2.05	2.01	1.96	1.92	1.87
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15	2.07	2.03	1.98	1.94	1.89	1.84
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20	2.18	2.15	2.13	2.05	2.01	1.96	1.91	1.86	1.81
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.15	2.13	2.11	2.03	1.98	1.94	1.89	1.84	1.79
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16	2.14	2.11	2.09	2.01	1.96	1.92	1.87	1.82	1.77
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.12	2.09	2.07	1.99	1.95	1.90	1.85	1.80	1.75
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.17	2.13	2.10	2.08	2.06	1.97	1.93	1.88	1.84	1.79	1.73
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	2.09	2.06	2.04	1.96	1.91	1.87	1.82	1.77	1.71
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.08	2.05	2.03	1.94	1.90	1.85	1.81	1.75	1.70
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01	1.93	1.89	1.84	1.79	1.74	1.68
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92	1.84	1.79	1.74	1.69	1.64	1.58
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.89	1.86	1.84	1.75	1.70	1.65	1.59	1.53	1.47
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83	1.80	1.78	1.75	1.66	1.61	1.55	1.50	1.43	1.35

Right Tailed, D/N = df in denominator = down the rows, df in numerator = across the columns

Note: Table is for an alpha of 0.05

Table of Probabilities for F Distribution

F-TABLE: VALUES OF F OF THE F-DISTRIBUTION (ALPHA = 0.01)
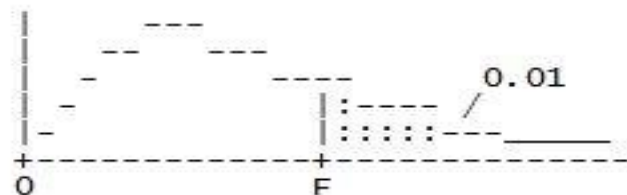
DEGREE OF NUMERATOR (V1)

V2	1	2	3	4	5	6	7	8	9	10
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98

SSA $[\sum (t2/n)] - (T2/N)$

$t^2 A = 1521$

$t^2 B = 961$ $T^2 = 9801$

$t^2 C = 841$

SSA = [1521/8 +961/8 +841/8] - 9801/24

SSA = 415.375 408.375 = 7

SSW = $(\sum X2) - [\sum (t2/n)]$

SSW = 431 - 415.375 = 15.625

$$\text{MSA} = \frac{\text{SSA}}{k-1} = \frac{7}{3-1} = 3.5$$

$$\text{MSW} = \frac{\text{SSW}}{N-k} = \frac{15.625}{24-3} = 0.744$$

$$\text{VR} = \frac{\text{MSA}}{\text{MSW}} = \frac{3.5}{0.744} = 4.704$$

Calculated F > tabulated F → P < 0.05,

so we reject Ho and accept HA

Variance ratio (F) more than <u>**one**</u> indicates the variation is higher among study groups than within the grand sample readings, and thus the presence of grouping affects the value of each reading.

A larger value of V.R resulted from the fact that among groups mean squares was considerably larger than the within groups mean squares .

Since the among groups mean square is based on the dispersion of the sample means about their mean, this quantity will be large when

there is a large discrepancy among the sizes of the sample

means .

Because of this , then a significant value of V.R tell us to reject Ho that all population means are equal.

Limitations of the One -Way ANOVA
A one -way ANOVA will tell you that at least two groups were different from each other.

But it won't tell you what groups were different. If your test returns a significant f-statistic, you may need to run an ad hoc test (like the Least Significant Difference test) to tell you exactly which groups had a difference in means.

Two Way ANOVA

A Two- Way ANOVA is an extension of the One- Way ANOVA. With a One Way, you have one independent variable affecting a dependent variable.

With a Two- Way ANOVA, there are two independents. Use a two- way ANOVA when you have one measurement variable (i.e. a quantitative variable) and two nominal variables.

 In other words, if your experiment has a quantitative outcome and you have two categorical explanatory variables, a two -way ANOVA is appropriate.

For example, you might want to find out if there is an interaction between income and gender for anxiety level at job interviews.

The anxiety level is the outcome, or the variable that can be measured. Gender and Income are the two [categorical variables](#).

These categorical variables are also the independent variables, which are called factors in a Two -Way ANOVA.

The factors can be split into levels. In the above example, income level could be split into three levels: low, middle and high income. Gender could be split into two levels: male, female. Treatment groups are all possible combinations of the factors. In this example there would be 3 x 2= 6treatment groups.

Main Effect and Interaction Effect

The results from a Two -Way ANOVA will calculate a main effect and an interaction effect.

The main effect is like a One- Way ANOVA: each factor's effect is considered separately.

With the interaction effect, all factors are considered at the same time. Interaction effects between factors are easier to test if there is more than one observation in each cell.

Two null hypotheses are tested if you are placing one observation in each cell. For this example, those hypotheses would be:

H_{01}: All the income groups have equal mean stress.

H_{02}: All the gender groups have equal mean stress.

For multiple observations in cells, you would also be testing a third hypothesis:

H_{03}: The factors are independent, *or* the interaction effect does not exist.

An F-statistic is computed for each hypothesis you are testing.

B) RANDOMIZED COMPLETE BLOCK DESIGN: TWO WAY- ANOVA:

The randomized complete block design is a design in which the treatments units (called <u>experimental units</u>) to which the are applied are subdivided into homogeneous groups called <u>blocks</u> .

The treatment are then assigned at random to the experiment units within each block.

It should be emphasized that each treatment appears in every block, and each block receive every treatment.

When blocking is used homogeneous blocks of experimental units.

Objective: is to isolate and remove from the error term variation attributable to the blocks, while assuring that the treatment means will be free of block effects.

The effectiveness of the design depends on the ability to form homogeneous blocks of experimental units.

When blocking is used effectively , the error

mean   square in the ANOVA table will be

reduced, the V.R will be increased , and the

chance of rejecting the null hypothesis will be improved.

Advantages: one of the advantages of the RCBD is that it is easily understood .

Certain complications that may arise in the course of an experiment are easily handled when this design is employed .

The paired comparisons analysis is a special case of the RCBD .

VARIANCE RATIO (VR):

$$VR = \frac{\text{Variance among Treatment groups}}{\text{Variance Residual}} = \frac{MSA}{MSE} = \frac{\dfrac{SSTr}{d.f\,Tr}}{\dfrac{SSE}{d.f\,E}} =$$

$$\frac{\dfrac{[\sum (t2/n)] - (T^2/N)}{SST - SS\,Bl - SSTr}}{(n-1)(K-1)}$$

d. f T r = K – 1 [treatment -1] [numerator df]

Denominator df [(n-1)x (k-1)]

Where n= [no. in each block]

Where:

SST=total sum of squares for the RCBD=

$$\sum [\mathbf{x} - \text{ Mean of sum of means of groups}]^2$$

S ST r = treatment = no. of blocks [mean of each group −

Mean of sum of means of groups]^2

SSB=blocks= no. of groups $\sum$ [mean of block

− Mean of sum of means of groups]^2

SSE=error (residual)=SST- SSBl - S ST r

a physical therapist wished to compare three methods for teaching patients how to use a certain prosthetic device. He felt that the rate of learningwould be different for patients of different ages and wished to design an experiment in which the influence of age could be taken into account.

SOLUTION: The randomized complete block design is the appropriate design

Data: three patients in each of five age groups were selected to participate in the experiment, and one patient in each age group was randomly assigned to each of the teaching methods. The methods of instruction (A, B & C) constitute our three treatments and the five age groups are the blocks.

Time in days required to learn the use of a certain prosthetic device Teaching Method

Age group	A	B	C	TOTAL	Mean
Under 20	7	9	10	26	8.67
20-29	8	9	10	27	9.00
30-39	9	9	12	30	10.00
40-49	10	9	12	31	10.33
50 & over	11	12	14	37	12.33
Total	45	48	58	151	
mean	**9.0**	**9.6**	**11.6**		**10.07**

$$SST = (7-10.07)^2 + (8-10.07)^2 + \ldots\ldots\ldots + (14-10.07) = 46.9335$$

$$SSB = 3 \,[(8.67-10.07)^2 + (9-10.07)^2 + \ldots\ldots + (12.33-10.07)^2 = 24.855$$

$$S\,ST\,r = 5\,[\,(9-10.07)^2 + (9.6-10.07)^2 + (11.6-10.07)^2 = 18.5335$$

$$SSE = SST - SSB - SS\,T\,r = 46.9333 + 24.855 + 18.5333 = 3.545$$

$$MSA = \frac{S\,S\,T\,r}{d.\,f\,T\,r} = \frac{18.5333}{2} = 9.2666$$

tab F df (5-1) (3-1)= 8

Denominator df[(n-1)x (k-1)]

tab F 4.46

[treatment -1] [numerator df]

$$MSE = \frac{SSE}{d.f_E} = \frac{3.545}{8} = 0.443$$

$$VR = \frac{MSA}{MSE} = 20.91$$

Measuring Effect Size in anova

- **Most common technique is "r^2"**
  - Tells you what percent of the variance is due to the treatment

  - r^2 = SS between groups/SS total

ANOVA vs. T Test

A Student's t-test will tell you if there is a significant variation between groups.

A t-test compares means, while the ANOVA compares variances between populations.

You *could* technically perform a series of t-tests on your data. However, as the groups grow in number, you may end up with a *lot* of pair comparisons that you need to run. ANOVA will give you a single number (the f-statistic) and one p-value to help you support or reject the null hypothesis.