Statistics

Post-graduates / year 1

Confidence interval

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- **Level of significance**

The level of significance (α) is the maximum allowed probability of committing a **Type I error**.

The smaller the value of α, the lower the risk of committing a type error.

Common values for α are 0.05 and 0.01 indicating 5% and 1%, respectively.

- **Power**

The probability of not committing a Type II error is called the power of a hypothesis test. Power = 1-β

The statistical power of a study is the power (or ability) of the study to detect a difference if a difference really exists.

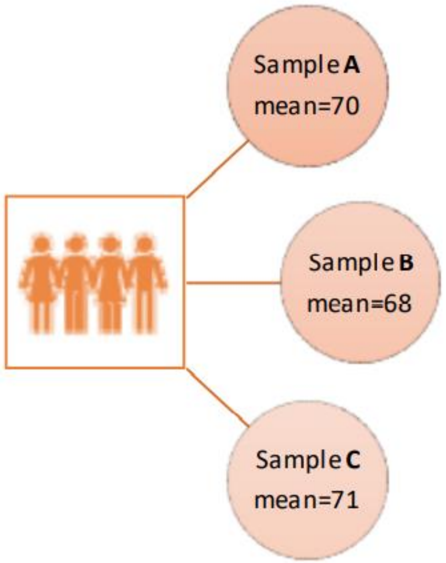
- In practice, β is usually set at 0.2.
- This provides a power value of 0.8 (80%).
- If there is a difference, then the probability of the test detecting it is 80%

- **P-value**

The 'P ' stands for probability. We use this p-value to make a decision about the null hypothesis by comparing its value to the level of significance (α).

- P-value is a probability and therefore lies between 0 and 1.
- It expresses the weight of evidence in favor of or against the stated null hypothesis.
- 0.05 or 5% is commonly used as a cut-off (significance level).
- If the observed p-value is less than this ($p<0.05$), we consider that there is good evidence that the null hypothesis is not true. So, we reject the null hypothesis (and accept the alternative hypothesis).
- $P<0.05$ is described as statistically significant and $P \geq 0.05$ is described as not statistically significant

- **Confidence interval**



A Confidence Interval (CI) is a range of values we are fairly sure the true value lies in

The confidence interval is usually expressed as **95%** CI, but we can see a 99% CI or 90% CI or any other percentage confidence interval.

$$CI = \bar{x} \pm z\frac{s}{\sqrt{n}}$$

A more scientifically accurate interpretation is that if we repeat the experiment a large number of times or infinite times, 95% of all confidence intervals constructed using this procedure should contain the true population mean.

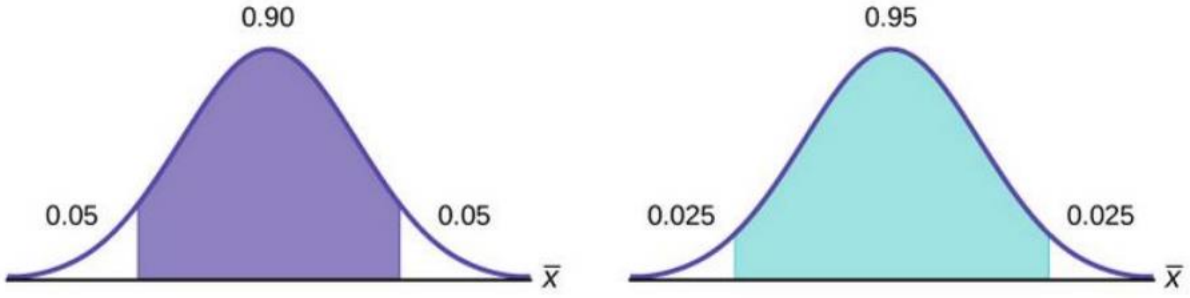
1- Confidence Interval and sample size

If we take a sample of 500 students instead of 50, we expect that the mean of this sample to be more accurate in presenting the whole population than the smaller sample and the 95% CI will be nearer to the mean we calculated (which means it is narrower). If the sample mean is 70kg in both samples, the 95% CI for the small sample may be 67 and 73 kg, but for the large sample, it will be 69 and 71.

The larger the sample size, the narrower the confidence interval

2- Confidence Interval and confidence level

If we are interested only in 90% CI, this allows an error of 10% compared to an error of 5% only in the 95% CI.



90% CI   95% CI

Standard error

- Calculation of the confidence interval depends on the standard error.
- If we take enough samples from a population, the means will be arranged into a distribution around the true population mean.
- The standard deviation of this distribution, i.e. the standard deviation of sample means, is called the standard error.

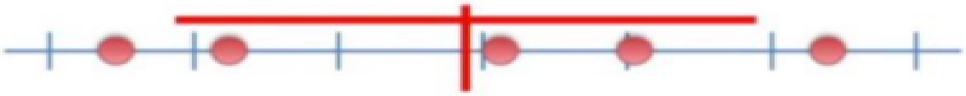
$$\mathrm{SE} = \frac{\sigma}{\sqrt{n}}$$

- **The standard error tells us how accurate the mean of any sample is likely to be compared to the true population mean.**

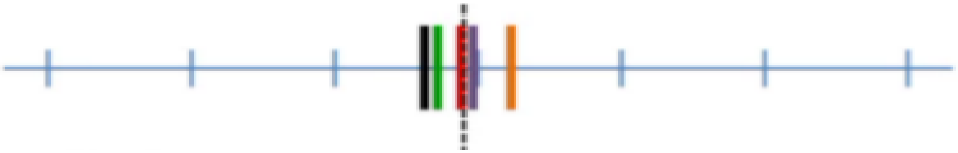
- **When the standard error increases, i.e. the means are more spread out, it becomes more likely that any given mean is an inaccurate representation of the true population mean.**

Standard Deviation and Standard Error:

Standard deviation quantifies the variation within a sample



Standard error quantifies the variation in the means from multiple samples



For calculating the standard error for the mean we divide the standard deviation by the square root of the sample size:

$$SE = \frac{\sigma}{\sqrt{n}}$$

- Standard error increases when standard deviation, i.e. the variance of the population, increases.
- Standard error decreases when sample size increases – as the sample size gets closer to the true size of the population, the sample means cluster more and more around the true population mean.