

CLINICAL PHARMACOKINETIC EQUATIONS AND CALCULATIONS

1- Intravenous Bolus Equation

$$C = (D/V)e^{-k_e t}$$

t = is the time after the intravenous bolus was given

C = is the concentration at time = t

V = is the volume of distribution

k_e = is the elimination rate constant

$$t_{1/2} = 0.693/k_e$$

C_0 = concentration at time = 0

$$k_e = -(\ln C_1 - \ln C_2)/(t_1 - t_2)$$

$$V = D/C_0$$

$$C_0 = C/e^{-k_e t}$$

C_0 = concentration at time = 0

2-Continuous and Intermittent Intravenous Infusion Equations

$$C = (k_0/Cl)(1 - e^{-k_e t}) = [k_0/(k_e V)](1 - e^{-k_e t})$$

k_0 = the drug infusion rate (in amount per unit time, such as mg/h or $\mu\text{g}/\text{min}$).

Cl = is the drug clearance.

[Since $Cl = keV$, this substitution was made in the second version of the equation]

ke = is the elimination rate constant

t = is the time that the infusion has been running.

*If the infusion is allowed to continue until steady state is achieved, the steady-state concentration (C_{ss}) can be calculated easily:

$$C_{ss} = k_0 / Cl = k_0 / (keV).$$

*If the infusion is stopped, post infusion serum concentrations ($C_{postinfusion}$) can be computed

$$C_{postinfusion} = C_{end} e^{-ke t_{postinfusion}}$$

$$ke = -(\ln C_1 - \ln C_2) / (t_1 - t_2)$$

Where t_1 and C_1 are the first time/concentration pair and t_2 and C_2 are the second time/concentration pair;

$$V = \frac{k_0 (1 - e^{-ke t'})}{k_e [C_{max} - (C_{predose} e^{-ke t'})]}$$

- where k_0 is the infusion rate
- ke is the elimination rate constant
- t' = infusion time
- C_{max} is the maximum concentration at the end of infusion
- $C_{predose}$ is the predose concentration.

3- Extravascular Equation

$$C = \{(Fk_a D) / [V(k_a - k_e)]\} (e^{-k_e t} - e^{-k_a t})$$

Where t is the time after the extravascular dose was given ($t = 0$ at the time the dose was administered)

C = is the concentration at time = t

F = is the bioavailability fraction

k_a = is the absorption rate constant

D = is the dose

V= is the volume of distribution

k_e = is the elimination rate constant.

* When only postabsorption, postdistribution serum concentrations are obtained for a drug that is administered extravascularly, the equation simplifies to:

$$C = [(FD)/V]e^{-k_e t}$$

Where C is the concentration at any postabsorption, postdistribution time

F =is the bioavailability fraction

D= is the dose

V= is the volume of distribution

k_e = is the elimination rate constant

t= is any postabsorption, postdistribution time.

$$k_e = - (\ln C_1 - \ln C_2) / (t_1 - t_2),$$

$$t_{1/2} = 0.693/k_e$$

$$C_0 = C/e^{-k_e t},$$

$$V/F = D/C_0$$

Where (V/F) volume of distribution/bioavailability constant

4- Multiple-Dose and Steady-State Equations

In order to change a single dose equation to the multiple dose versions, it is necessary to multiply each exponential term in the equation by the multiple dosing factors:

$$(1 - e^{-nk_i\tau}) / (1 - e^{-k_i\tau})$$

Where n is the number of doses administered

k_i =is the rate constant found in the exponential of the single dose equation

τ =is the dosage interval.

$$C = (D/V)[e^{-k_e t} / (1 - e^{-k_e \tau})]$$

Where C is the steady state concentration at any postdose time (t) after the dose (D) is given

V =is the volume of distribution

k_e = is the elimination rate constant

τ =is the dosage interval.

TABLE 2-1 Single-Dose, Multiple-Dose, and Steady-State One-Compartment Model Equations

ROUTE OF ADMINISTRATION	SINGLE DOSE	MULTIPLE DOSE	STEADY STATE
Intravenous bolus	$C = (D/V)e^{-k_e t}$	$C = (D/V)e^{-k_e t} [(1 - e^{-nk_e \tau}) / (1 - e^{-k_e \tau})]$	$C = (D/V)[e^{-k_e t} / (1 - e^{-k_e \tau})]$
Continuous intravenous infusion	$C = [k_0 / (k_e V)](1 - e^{-k_e t})$	N/A	$C_{ss} = k_0 / Cl = k_0 / (k_e V)$
Intermittent intravenous infusion	$C = [k_0 / (k_e V)](1 - e^{-k_e t})$	$C = [k_0 / (k_e V)](1 - e^{-k_e t}) [(1 - e^{-nk_e \tau}) / (1 - e^{-k_e \tau})]$	$C = [k_0 / (k_e V)](1 - e^{-k_e t}) / (1 - e^{-k_e \tau})$
Extravascular (postabsorption, postdistribution)	$C = [(FD)/V]e^{-k_e t}$	$C = [(FD)/V]e^{-k_e t} [(1 - e^{-nk_e \tau}) / (1 - e^{-k_e \tau})]$	$C = (FD/V)[e^{-k_e t} / (1 - e^{-k_e \tau})]$
Average steady-state concentration (any route of administration)	N/A	N/A	$C_{ss} = [F(D/\tau)] / Cl$

Symbol key: C is drug serum concentration at time = t, D is dose, V is volume of distribution, k_e is the elimination rate constant, n is the number of administered doses, τ is the dosage interval, k_0 is the infusion rate, Cl is clearance, t' is infusion time, N/A is not applicable.

TABLE 2-2 Single-Dose, Multiple-Dose, and Steady-State Pharmacokinetic Constant Computations Utilizing a One Compartment Model

ROUTE OF ADMINISTRATION	SINGLE DOSE	MULTIPLE DOSE	STEADY STATE
Intravenous bolus	$k_e = -(\ln C_1 - \ln C_2) / (t_1 - t_2)$ $t_{1/2} = 0.693/k_e$ $V = D/C_0$ $Cl = k_e V$	$k_e = -(\ln C_1 - \ln C_2) / (t_1 - t_2)$ $t_{1/2} = 0.693/k_e$ $V = D / (C_0 - C_{pre-dose})$ $Cl = k_e V$	$k_e = -(\ln C_1 - \ln C_2) / (t_1 - t_2)$ $t_{1/2} = 0.693/k_e$ $V = D / (C_0 - C_{pre-dose})$ $Cl = k_e V$
Continuous intravenous infusion	N/A	N/A	$Cl = k_0 / C_{ss}$
Intermittent intravenous infusion	$k_e = -(\ln C_1 - \ln C_2) / (t_1 - t_2)$ $t_{1/2} = 0.693/k_e$ $V = [k_0(1 - e^{-k_e t'})] / [k_e[C_{max} - (C_{pre-dose}e^{-k_e t'})]]$ $Cl = k_e V$	$k_e = -(\ln C_1 - \ln C_2) / (t_1 - t_2)$ $t_{1/2} = 0.693/k_e$ $V = [k_0(1 - e^{-k_e t'})] / [k_e[C_{max} - (C_{pre-dose}e^{-k_e t'})]]$ $Cl = k_e V$	$k_e = -(\ln C_1 - \ln C_2) / (t_1 - t_2)$ $t_{1/2} = 0.693/k_e$ $V = [k_0(1 - e^{-k_e t'})] / [k_e[C_{max} - (C_{pre-dose}e^{-k_e t'})]]$ $Cl = k_e V$
Extravascular (postabsorption, postdistribution)	$k_e = -(\ln C_1 - \ln C_2) / (t_1 - t_2)$ $t_{1/2} = 0.693/k_e$ $V/F = D/C_0$ $Cl/F = k_e(V/F)$	$k_e = -(\ln C_1 - \ln C_2) / (t_1 - t_2)$ $t_{1/2} = 0.693/k_e$ $V/F = D / (C_0 - C_{pre-dose})$ $Cl/F = k_e(V/F)$	$k_e = -(\ln C_1 - \ln C_2) / (t_1 - t_2)$ $t_{1/2} = 0.693/k_e$ $V/F = D / (C_0 - C_{pre-dose})$ $Cl/F = k_e(V/F)$
Average steady-state concentration (any route of administration)	N/A	N/A	$Cl/F = (D/\tau) / C_{ss}$

Symbol key: C_1 is drug serum concentration at time = t_1 , C_2 is drug serum concentration at time = t_2 , k_e is the elimination rate constant, $t_{1/2}$ is the half-life, V is the volume of distribution, k_0 is the continuous infusion rate, t' is the infusion time, V/F is the hybrid constant volume of distribution/bioavailability fraction, D is dose, C_0 is the concentration at time = 0, Cl is drug clearance, Cl/F is the hybrid constant clearance/bioavailability fraction, $C_{pre-dose}$ is the predose concentration, C_{ss} is the steady-state concentration, N/A is not applicable.

5- Average Steady-State Concentration Equation

$$C_{ss} = [F(D/\tau)] / Cl$$

Where F is the bioavailability fraction

D = is the dose

τ = is the dosage interval

Cl = is the drug clearance

$$Cl/F = (D/\tau)/C_{ss}$$

Where D is dose and τ is the dosage interval

6- DESIGNING INDIVIDUALIZED DOSAGE REGIMENS USING ONE COMPARTMENT MODEL EQUATIONS

TABLE 2-3 Equations to Compute Individualized Dosage Regimens for Various Routes of Administration

ROUTE OF ADMINISTRATION	DOSAGE INTERVAL (τ), MAINTENANCE DOSE (D OR k_0), AND LOADING DOSE (LD) EQUATIONS
Intravenous bolus	$\tau = (\ln C_{ss_{max}} - \ln C_{ss_{min}})/k_e$ $D = C_{ss_{max}} V(1 - e^{-k_e\tau})$ $LD = C_{ss_{max}} V$
Continuous intravenous infusion	$k_0 = C_{ss} Cl = C_{ss} k_e V$ $LD = C_{ss} V$
Intermittent intravenous infusion	$\tau = [(\ln C_{ss_{max}} - \ln C_{ss_{min}})/k_e] + t'$ $k_0 = C_{ss_{max}} k_e V[(1 - e^{-k_e\tau})/(1 - e^{-k_e t'})]$ $LD = k_0/(1 - e^{-k_e\tau})$
Extravascular (postabsorption, postdistribution)	$\tau = [(\ln C_{ss_{max}} - \ln C_{ss_{min}})/k_e] + T_{max}$ $D = [(C_{ss_{max}} V)/F][(1 - e^{-k_e\tau})/e^{-k_e T_{max}}]$ $LD = (C_{ss_{max}} V)/F$
Average steady-state concentration (any route of administration)	$D = (C_{ss} Cl \tau)/F = (C_{ss} k_e V \tau)/F$ $LD = (C_{ss} V)/F$

Symbol key: $C_{ss_{max}}$ and $C_{ss_{min}}$ are the maximum and minimum steady-state concentrations, k_e is the elimination rate constant, V is the volume of distribution, C_{ss} is the steady-state concentration, k_0 is the continuous infusion rate, t' is the infusion time, T_{max} is the time that $C_{ss_{max}}$ occurs, F is the bioavailability fraction.

7- MULTICOMPARTMENT MODELS

The equation that describes a two compartment model after an intravenous bolus is:

$$[V_1(\alpha - \beta)]\{e^{-\alpha t} + \{[D(k_{21} - \beta)] / [V_1(\alpha - \beta)]\}e^{-\beta t}$$

Where C is the drug serum concentration,

D is the intravenous bolus dose

k_{21} is the rate constant that describes the transfer of drug from compartment 2 to compartment 1

α is the distribution rate constant

β is the elimination rate constant

V_1 is the volume of distribution for compartment 1

t is the time after the dose was administered.

8-MICHAELIS-MENTEN EQUATIONS FOR SATURABLE PHARMACOKINETICS

$$D = (V_{\max} \cdot C_{ss}) / (K_m + C_{ss})$$

Where D is the dose

C_{ss} is the steady-state drug concentration

V_{\max} is the maximum rate of drug metabolism

K_m is the concentration where the rate of metabolism equals $V_{\max}/2$.

$$D = V_{\max} - [K_m(D/C_{ss})]$$

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