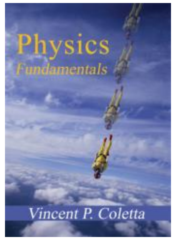
Medical Physics Class
Temperature and Kinetic Theory of Gases Part-1



Physics Fundamentals by Vincent P. Coletta

Learning Goals

Looking forward at …

- **Temperature**
- **Temperature Measurement**
- **Ideal Gas Law**

Temperature

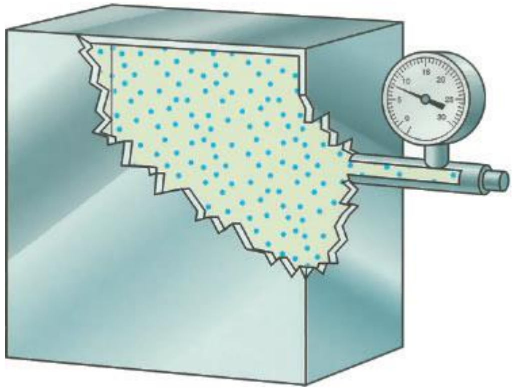
- We can feel the blazing heat of the summer sun or the biting cold of a winter blizzard.
- Our bodies are sensitive even to small changes in the temperature of our surroundings.
- We respond to these changes with several adaptive mechanisms, like sweating or shivering, to maintain a nearly constant internal body temperature.
- This sensitivity to the thermal environment is the basis for our concepts of hot and cold, out of which the scientific definition of temperature evolved.
- Temperature is a quantitative measure of how hot or cold something is.
- In this class we shall see how various kinds of thermometers are used to measure temperature and how temperature can be interpreted as a measure of molecular kinetic energy.

Temperature Measurement

Thermometers

- Temperature is measured with a thermometer.
- Galileo invented the first thermometer, which made use of air's property of expanding as it is heated. The air's volume indicated the temperature.
- Today there are various kinds of thermometers, each appropriate for the range of temperatures and the system to be measured.
- For example, in addition to the common mercury thermometer, there are different kinds of thermometers.



Temperature Measurement

Thermometers

- Each thermometer depends on the existence of some thermometric property of matter.
- For example, the expansion or contraction of mercury in a fever thermometer correlates with the body's sensation of hot and cold.
- A person who has a high fever both is hot to the touch and will register a higher than normal temperature on a mercury thermometer.
- The length of the mercury column in the glass stem of the thermometer gives us a quantitative measure of temperature.

Temperature Measurement

Temperature Scales

- To assign a numerical value to the temperature of a body, we need a temperature scale.
- The two most common scales in everyday use are the Celsius scale (formerly known as the centigrade scale) and the Fahrenheit scale.
- In establishing a temperature scale, one could use any kind of thermometer and any thermometric property.
- For example, the Celsius scale was based on the expansion of a column of liquid such as mercury in a thin glass tube.
- The **absolute**, or **Kelvin**, scale is now the standard in terms of which all other scales, such as Celsius and Fahrenheit, are defined.

Temperature Measurement

Temperature Scales

- The Kelvin scale is chosen as the standard for important reasons.
- First, various laws of physics are most simply expressed in terms of this scale. We shall see an example of this in the ideal gas law, described in the next section.
- Second, zero on the absolute scale has fundamental significance. It is the lowest possible temperature a body can approach.

Temperature Measurement

Temperature Scales

- when the Celsius scale was established, the normal freezing and boiling points of water were assigned respective values of $0°C$ (zero degrees Celsius) and $100°C$. A mercury thermometer was brought to thermal equilibrium with water at each of these temperatures, and the level of the mercury column was marked as $0°C$ for ice water and $100°C$ for boiling water. The mercury column between these two marks was then divided into 100 equal intervals, corresponding to temperature intervals of 1 Celsius degree $(1°C)$. Today the Celsius temperature scale is defined in terms of the Kelvin scale. Celsius temperature T_C is now defined by the equation

$$T_C = T - 273.15$$

Temperature Measurement

Temperature Scales

$$T_C = T - 273.15$$

- The normal freezing point of water open to the air at one atmosphere of pressure is $273.15K$, or 0.00°C.
- The normal boiling point of water is $373.15K$, or 100.00°C.
- This definition of the Celsius scale conforms to the earlier definition based on the freezing and boiling points of water.
- Temperature intervals on the Celsius and Kelvin scales are the same.
- For example, the difference in temperature between the boiling point of water and its freezing point is 100 Celsius degrees (°C), or 100 kelvins.

Temperature Measurement

Temperature Scales

- Fahrenheit temperature T_F, measured in degrees Fahrenheit (°F), is defined relative to the Celsius temperature T_C by the

$$T_F = 32 + \frac{9}{5} T_C$$

- The normal freezing and boiling points of water on the Fahrenheit scale are 32 °F and 212 °F respectively. The interval between these points is 180 °F. There are only 100 °C between these same points. Thus Celsius degrees are bigger than Fahrenheit degrees: 1°C is $\frac{180}{100}$, or $\frac{9}{5}$, times 1°F .

Temperature Measurement

Measuring a Fever on the Celsius Scale

Normal internal body temperature is 98.6°F. A temperature of 106°F is considered a high fever. Find the corresponding temperatures on the Celsius scale.

$$T_C = \tfrac{5}{9}(T_F - 32.0)$$

A temperature of 98.6° F corresponds to

$$T_C = \tfrac{5}{9}(98.6 - 32.0) = 37.0°\ C$$

and a temperature of 106° F corresponds to

$$T_C = \tfrac{5}{9}(106 - 32.0) = 41.1°\ C$$

Ideal Gas Law

- The pressure of a gas can be changed in several ways. One way to increase the pressure of a gas confined to a fixed volume is to increase the number of gas molecules in the volume.
- You do this, for example, when you pump air into a bicycle tire or an automobile tire.
- Another way to change the pressure of a gas is to change its temperature. For example, when the air in an automobile tire heats up, its pressure increases significantly.
- A third way to change gas pressure is to change the volume containing the gas; decreasing volume causes an increase in pressure.

Ideal Gas Law

- For low-density gases, there is a simple, universal relationship between the gas $pressure\, P$, $volume\, V$, $Kelvin\, temperature\, T$, and $number\, of\, gas\, molecules\, N$. The product of P and V is proportional to the product of N and T:

$$PV = NkT$$

- This equation is called the **ideal gas law.** The constant *k* is known as "Boltzmann's constant" and is found from experiment to have the value

$$k = 1.380 \times 10^{-23}\ \mathrm{J/K}$$

- In applying the ideal gas law, temperature must be expressed in kelvins, not in °C or °F.

Ideal Gas Law

- Special cases of the gas law are found when one considers the variation of two of the variables P, V, N, and T, while the other two variables are held constant.
- For example, if N and T are fixed, the ideal gas law implies that the product PV is constant:

$$PV = \text{constant}$$

- This result is known as Boyle's law.
- Boyle's law implies that if the volume of a gas is reduced to half its original value the pressure of the gas is doubled.

Ideal Gas Law

- If P and N are fixed, the ideal gas law implies that the volume of the gas is directly proportional to its temperature:

$$V \propto T \qquad \text{(for constant } N \text{ and } P\text{)}$$

- If V and N are fixed, the ideal gas law implies that

$$P \propto T \qquad \text{(for constant } N \text{ and } V\text{)}$$

- The very definition of temperature on the Kelvin scale requires that this relationship be satisfied, at least in the limit of a very low-density gas.

Ideal Gas Law

EXAMPLE 2 The Temperature of an Ideal Gas After Compression

An ideal gas initially has a volume of 1.0 liter (L), a pressure of 1.0 atmosphere (atm), and a temperature of 27° C. The pressure is raised to 2.0 atm, compressing the volume of the gas to 0.60 L. Find the final temperature of the gas.

SOLUTION We are given the following initial and final values of P, V, and T:

$$P_i = 1.0 \text{ atm} \qquad P_f = 2.0 \text{ atm}$$
$$V_i = 1.0 \text{ L} \qquad V_f = 0.60 \text{ L}$$
$$T_{Ci} = 27° \text{ C} \qquad T_{Cf} = ?$$

The number of molecules, N, is constant. The problem is to find the final temperature T_{Cf}. We can do this simply by first writing the ideal gas law for the initial state of the gas and again for the final state and then taking the ratio of the two expressions:

$$P_iV_i = NkT_i$$
$$P_fV_f = NkT_f$$
$$\frac{T_f}{T_i} = \frac{P_fV_f}{P_iV_i}$$

Since this equation involves ratios of pressures and volumes, we may insert these quantities in the units in which they are given, that is, atmospheres and liters, rather than converting to standard units of Pa and m^3. The conversion to standard units would simply introduce identical factors for both initial and final values, and these factors would cancel. We must be careful, however, to convert temperature from degrees Celsius to kelvins, even when a ratio is used, as it is here, since this change in units involves an additive term, rather than a multiplicative factor. Thus we must use $T_i = 27 + 273 = 300$ K. Substituting values into the preceding equation, we obtain

$$\frac{T_f}{300 \text{ K}} = \frac{(2.0 \text{ atm})(0.60 \text{ L})}{(1.0 \text{ atm})(1.0 \text{ L})} = 1.2$$

or

$$T_f = (1.2)(300 \text{ K}) = 360 \text{ K}$$

This corresponds to a final Celsius temperature T_{Cf} of

$$T_{Cf} = T_f - 273 = 360 - 273$$
$$= 87° \text{ C}$$

Ideal Gas Law

EXAMPLE 3 The Number of Air Molecules in a Hot-air Balloon

The air inside a hot-air balloon (Fig. 12–3) is at a temperature of 100.0° C, while the temperature of the surrounding air in the atmosphere is 20.0° C. Find the ratio of the number of air molecules inside the balloon to the number of air molecules contained in an equal volume of air outside the balloon. Assume that the air pressure is the same inside and outside.

SOLUTION We first use the ideal gas law (Eq. 12–5) to obtain an expression for the number of air molecules in a volume V.

$$N = \frac{PV}{kT}$$

Using subscripts i and o to denote air inside and outside the balloon, we obtain the ratio of the number of air molecules inside to the number outside:

$$\frac{N_i}{N_o} = \frac{P_i V_i / kT_i}{P_o V_o / kT_o}$$

Since pressures P_i and P_o are equal and volumes V_i and V_o are equal, this reduces to

$$\frac{N_i}{N_o} = \frac{T_o}{T_i}$$

Fig. 12–3 Hot-air balloon.

Converting temperatures to kelvins, we find

$$\frac{N_i}{N_o} = \frac{273 + 20}{273 + 100} = \frac{293\ \mathrm{K}}{373\ \mathrm{K}} = 0.786$$

According to Archimedes' principle, the hot-air balloon will experience a buoyant force equal to the weight of the displaced air. Since there is less air inside the balloon than in the volume of atmosphere displaced, the buoyant force is greater than the weight of the hot air, and the balloon will rise if the weight carried by the balloon is not too great. (Problem 50 asks you to calculate the volume of a hot-air balloon required to support 5000 N.)

Ideal Gas Law

- **Atomic Mass**
- It is often convenient to express the ideal gas law in a slightly different form, known as the "molar form." To accomplish this, we first define atomic mass and the mole.
- **Atomic mass is the mass of an atom relative to other atoms, using a scale in which the most common type of carbon atom is defined to have a mass of exactly 12.**
- A hydrogen atom has about $\frac{1}{12}$ the mass of a carbon atom and so has an atomic mass of approximately 1.
- A helium atom has about $\frac{4}{12}$ the mass of a carbon atom and so has an atomic mass of approximately 4.
- The periodic table of the elements shows the atomic masses of all the elements.
- The atomic masses listed there are actually averages over the different types of atoms naturally occurring for each element.
- For example, the atomic mass of carbon is given as 12.01, rather than exactly 12, because roughly 1% of all naturally occurring carbon atoms have a mass of 13.

Ideal Gas Law

- **Atomic Mass**
- The molecular mass of a molecule is the sum of the atomic masses of the atoms making up the molecule. For example, the molecular mass of the H2O molecule equals the atomic mass of oxygen plus twice the atomic mass of hydrogen, that is, approximately $16 + 2(1) = 18$.
- The unit of mass on the atomic mass scale is called the **atomic mass unit,** denoted by u. We can relate this unit to the gram. Experiment shows that

$$1\ \mathrm{u} = 1.6606 \times 10^{-24}\ \mathrm{g}$$

Ideal Gas Law

- **Mole**
- Even very small quantities of matter consist of an enormously large number of molecules, and so it is convenient to express the quantity of matter in terms of a large unit, called the "mole."
- A mole is defined as a certain number of atoms or molecules, called Avogadro's number, denoted by N_a. The value of Avogadro's number is such that one mole of a substance, consisting of any kind of atom or molecule, **has a mass numerically equal to the atomic or molecular mass of that substance expressed in grams**.
- For example, one mole of carbon-12 atoms has a mass of 12 g, and one mole of H_2O molecules has a mass of 18 g.

Ideal Gas Law

- **Mole**
- One can compute Avogadro's number by dividing the mass of 1 mole of carbon 12 (12 g) by the mass of a single carbon-12 atom, equal to 12 atomic mass units, where the atomic mass unit is related to the gram by the preceding equation.

$$N_A = \frac{12\ \text{g}}{12\ \text{u}} = \frac{12\ \text{g}}{12(1.6606 \times 10^{-24}\ \text{g})}$$

$$\boxed{N_A = 6.022 \times 10^{23}}$$

- We may express the number of molecules, N, of a substance as the product of Avogadro's number, N_A, and the number of moles, denoted by n:

$$N = nN_A$$

Ideal Gas Law

EXAMPLE 5 Number of Atoms in a Nail

Find the number of atoms in an iron nail of mass 5.00 g.

SOLUTION First we inspect the periodic table (shown on the inside back cover) and find that the atomic mass of iron (Fe) is 55.847. This means that 1 mole of naturally occurring iron has a mass of 55.847 g. We can now calculate the number of moles of iron in the nail, which we denote by n:

$$n = (5.00\ \text{g})\left(\frac{1\ \text{mole}}{55.847\ \text{g}}\right) = 8.95 \times 10^{-2}\ \text{mole}$$

Since 1 mole contains Avogadro's number of atoms, the nail contains a number of atoms equal to the number of moles times N_A:

$$N = nN_A = (8.95 \times 10^{-2}\ \text{mole})\left(\frac{6.022 \times 10^{23}\ \text{atoms}}{1\ \text{mole}}\right)$$
$$= 5.39 \times 10^{22}\ \text{atoms}$$

Ideal Gas Law

- **Molar Form of the Ideal Gas Law**
- To obtain the molar form of the ideal gas law, we substitute $N = nN_A$ into our original form of the gas law

$$PV = NkT = nN_AkT$$

- The product N_Ak is called the ideal gas constant, denoted by R.

$$R = N_Ak$$

$$= (6.022 \times 10^{23})(1.380 \times 10^{-23}\ \mathrm{J/K})$$

$$R = 8.31\ \mathrm{J/K}$$

- Substituting R for N_Ak in the ideal gas law, we obtain the molar form of the gas law.

$$PV = nRT$$

Ideal Gas Law

EXAMPLE 6 Finding the Mass of a Volume of Air

Find the mass of air in a room with dimensions 5.00 m × 4.00 m × 3.00 m, if the air pressure is 1.00 atm and the temperature is 27.0° C.

SOLUTION First we apply the ideal gas law (Eq. 12–13) to find the number of moles, using the Kelvin temperature ($T = 273 + 27.0 = 300$ K) and expressing pressure in Pa (Eq. 11–5: 1.00 atm = 1.01×10^5 Pa).

$$n = \frac{PV}{RT}$$

$$= \frac{(1.00\,\text{atm})(1.01 \times 10^5\,\text{Pa/atm})(5.00\,\text{m} \times 4.00\,\text{m} \times 3.00\,\text{m})}{(8.31\,\text{J/K})(300\,\text{K})}$$

$$= 2.43 \times 10^3\ \text{moles}$$

Since 1 mole has a mass equal to the molecular mass in grams, the mass of air equals the product of the number of moles times the molecular mass in grams. A nitrogen molecule N_2 has a molecular mass of $2(14) = 28$, and an oxygen molecule O_2 has a molecular mass of $2(16) = 32$. Air consists of approximately 80% nitrogen and 20% oxygen, and so the average molecular mass is $0.8 \times 28 + 0.2 \times 32 = 28.8$ Thus

$$m = (2.43 \times 10^3\ \text{moles})(28.8\ \text{g/mole}) = 7.00 \times 10^4\ \text{g}$$

$$= 70.0\ \text{kg}$$

The End Part-1