Mathematics and Biostatistics

Probability

1st Semester 2021

Lecture 6

Review

- Things we look for in a distribution
  1. Central Tendency
     a. MEAN -- average
     b. MEDIAN -- middle value
     c. MODE -- most frequently observed value.
  2. Variability of values and their spread
     a. Quartiles -- Median
     b. Standard deviation -- Mean
  3. Shape of the distribution.
     a. Normal Curve
     b. Normal distribution (Gaussian distribution)
     c. The 68–95–99.7 rule
  4. Gaps and clumping of values

Probability

- Probability can be defined as a measure of how likely an event was to occur.
- A probability of 0 indicates the event would not occur.
- A probability of 1 indicates the event was sure to occur.
- Values between 0 and 1 indicated occurrence somewhere in between the extremes.
- We will denote events with capital letters: A, B, C etc. These can be viewed as generic forms of an outcome in a statistical analysis.
- If we want to write the probability of an event A we will use the notation P(A).
- Read as the probability of A.

Probability

- Determine a single event with a single outcome.
- Identify the total number of outcomes that can occur.
- Divide the number of events by the number of possible outcomes.

- Example When a coin is tossed, there are two possible outcomes: heads (H) or tails (T)
- We say that the probability of the coin landing H is

$$P(H)= \frac{\text{the number of heads (H)}}{\text{the number of possible outcomes}}=\frac{1}{2}$$

- And the probability of the coin landing T is

$$P(T)= \frac{\text{the number of tails (T)}}{\text{the number of possible outcomes}}=\frac{1}{2}$$

Probability

- The probabilities over the entire sample space must sum to 1.
- The first of these is that if we have an event A in some sample space. Then

$$P(A) + P(\text{Not } A) = 1$$

- This rule becomes useful in situations where you cannot directly determine P(A) but you can compute the probability of not A events.

  Then

$$P(A) = 1 - P(\text{Not } A)$$

- Note (Not A) $\leftrightarrow$ ($\neg A$), read as not A.

Probability

- A simple example is a sample space of two events: A and B.

  Then

  $$P(A) + P(B) = 1, \quad P(B) = 1-P(A)$$

  and likewise, $P(A) = 1-P(B)$.

Types of Events

- The term mutually exclusive is used to describe events which cannot simultaneously occur.
- The occurrence of one rules out the occurrence of the other.
- A clear example is the set of outcomes of a single coin toss, which can result in either heads or tails, but not both.

- P(A and B) = 0.
- If the events are not mutually exclusive, then this rule does not hold.

Types of Events

- A joint event is the simultaneous occurrence of two or more events.
- Joint probability is the probability of event A occurring while event B occurs.
- The events must not be able to influence each other.

$$P(A \text{ and } B) = P(A) \times P(B).$$

Types of Events

- Example: rolling two six-sided dice in a fair.
- Event “A” = The probability of rolling a 6 in the first roll is

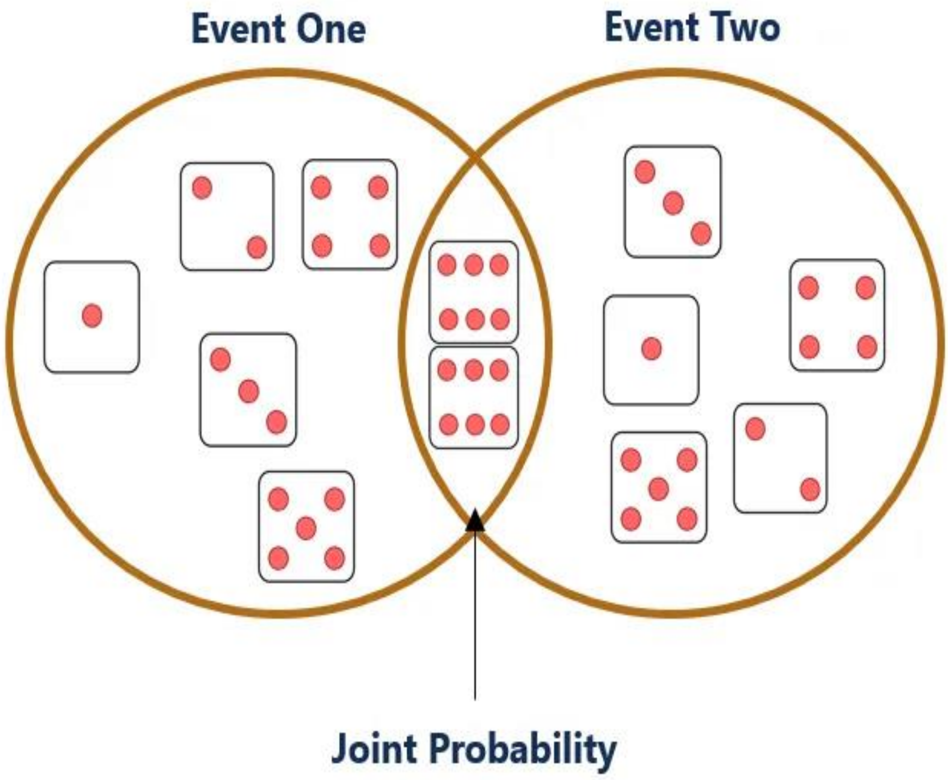
$$P(A)=1/6 = 0.1666.$$

- Event “B” = The probability of rolling a 6 in the second roll is

$$P(B)=1/6 = 0.1666.$$

- Therefore, the joint probability of event “A” and “B” is

$$P(A \text{ and } B)=P(A) \times P(B) = 0.02777 = 2.8\%$$



Types of Events

- Conditional events are the possibility of an events or outcomes occurring based on the occurrence of a preceding events or outcomes.
- This probability is written P(B|A), notation for the probability of B given A.
- Conditional probability is derived by multiplying the preceding event's probability by the updated likelihood of the subsequent, or conditional, occurrence.

$$P(B|A)= P(A \text{ and } B)/P(A)$$

Types of Events

- For example:
- Event A is that an individual applying for college will be accepted. There is an 80% chance that this individual will be accepted to college.
- Event B is that this individual will be given free scholarships. Scholarships will only be provided for 60% of all of the accepted students.

P(B|A)= P(A and B)/P(A)

- P(Accepted and Scholarships)=P (Scholarships|Accepted) x P (Accepted)

  = (0.60)*(0.80) = 0.48.

Types of Events

- Two events A and B are said to be independent or not associated if and only if:

$$P(A \text{ and } B) = P(A)*P(B)$$

- In the case where events A and B are independent (where event A has no effect on the probability of event B), the conditional probability of event B given event A is simply the probability of event B, that is P(B).

$$P(B|A) = P(A \text{ and } B)/P(A)$$

Types of Events

- Example of independence.

The data Yield:

- P(D) = 13/100
- P(E) = 50/100
- P(D and E) = 10/100

- from these we can determine if disease and exposure are independent.
- If they are independent, then 0.10 should be close to the produce of 0.13 and 0.50 0.13*0.50 = 0.065

Disease ▶ Exposure ▼	Yes	NO	Total
Yes	10	40	50
No	3	47	50
Total	13	87	100

The end of lecture