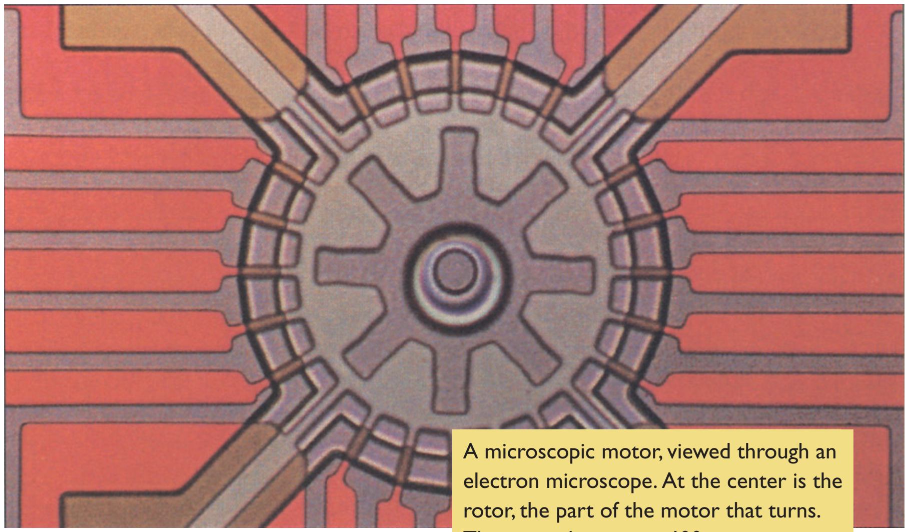
Return to Table of Contents

Measurement and Units



A microscopic motor, viewed through an electron microscope. At the center is the rotor, the part of the motor that turns. The rotor diameter is 100 microns, or 0.1 millimeters, about the thickness of a human hair.

Use of Units

Suppose you are driving with a friend and your gas tank is nearly empty. When you ask your friend how far it is to the nearest gas station, she responds, "About 3," with no indication of whether she means 3 blocks or 3 miles or perhaps some other measure of distance. Such a response is not helpful. Your friend gives you no information at all unless she gives the numerical value of the distance and the unit of length in terms of which that distance is measured—blocks, miles, whatever. As this example illustrates, it is important to express the numerical value of a quantity in terms of some unit. Distance or length may be expressed in units such as meters, feet, miles, or kilometers. Time may be expressed in units of seconds, hours, days, or years. Speed may be expressed in units of miles per hour, kilometers per hour, meters per second, and so on. Always remember to **include the unit in expressing the numerical value of any physical quantity**. Without the unit the number is meaningless.

Fundamental Quantities

All physical quantities can be defined in terms of a very small number of **fundamental physical quantities.** All the quantities studied in the first three chapters of this book can be defined in terms of just two fundamental quantities: **length** and **time.** For example, the speed of an object is defined as the distance traveled by the object divided by the elapsed time. In Chapter 4 we shall need to introduce a third fundamental quantity, **mass,** which is measured in units such as kilograms or grams.

We define the fundamental quantities by defining how we measure them. For instance, the length of an object is *defined* by comparing the object with multiples of some standard length, say, a meter. When we say that a basketball player is 2 meters tall, we mean that 2 vertical meter sticks, one on top of the other, will just reach from the floor to the top of the player's head. The time of any event is *defined* by measurement of the event's time on a clock, using standard units of time—hours, minutes, and seconds. We shall return to certain subtle questions concerning measurement of time in Chapter 27 when we study Einstein's theory of relativity.

Base and Derived Units

The units used to express fundamental quantities are called **base units,** and the units used to express all other quantities are called **derived units.** Thus meters, feet, seconds, and hours are all base units, since they are used to measure the two fundamental quantities length and time. The units miles per hour and meters per second are examples of derived units.

Base units are further characterized as being either primary or secondary. For each fundamental quantity, one base unit is designated the primary unit and all other units for that quantity are secondary. For measuring time, the second is the primary base unit, and minutes, hours, days, and so on are all secondary base units.

SI System of Units

For consistency and reproducibility of experimental results, it is important that all scientists use a standard system of units. The **Système International (SI)** is the system now used in most scientific work throughout the world. This system uses primary base units of meters, kilograms, and seconds for measurements of length, mass, and time respectively. In this book we shall use SI units primarily. Occasionally, however, in the early chapters, we shall also use the British system, in which length is measured in feet and force in pounds, since some of these units will be more familiar to you than the corresponding SI units.

Definition of the Second

The primary SI unit of time is the **second** (abbreviated s). Before 1960 the second was defined as a certain fraction of a day; that is, the second was defined as a fraction of the time required for one rotation of the earth. According to this definition, there are 86,400 seconds in a day.* A difficulty with this standard is that the earth's rate of rotation is not constant, and so a day is not a constant, reproducible standard of time. The earth's rotational rate experiences small random fluctuations from day to day. In addition there are seasonal variations and a gradual slowing down over the years. To take these facts into account, the second was defined, before 1960, as 1/86,400 of an average day during the year 1900.

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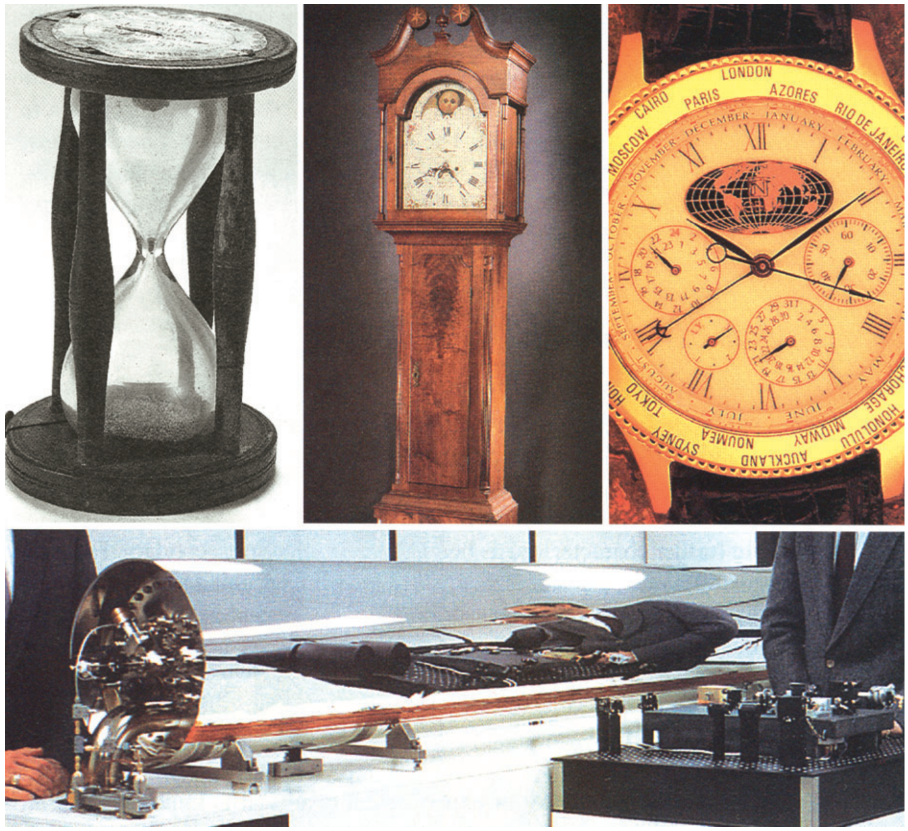
*1 day = 24 hours, 1 hour = 60 minutes, and 1 minute = 60 s. So 1 day = (24) (60) (60) s = 86,400 s.



Like the other devices shown here, an atomic clock (bottom) is used to measure time.

We now use a more precise and reproducible standard to define the second—a standard consistent with the earlier definition. The second is now defined in terms of the radiation emitted by cesium atoms. Radiation is a periodic wave phenomenon. The time per cycle, called the period, is characteristic of the radiation's source. **The second is defined to be 9,192,631,770 periods of radiation emitted by cesium atoms under certain conditions.** The device used to measure time with cesium radiation is a large and elaborate device called an atomic clock. Atomic clocks are extremely accurate. Two of them will agree with one another to within 1 part in 1013. An atomic clock is maintained by the National Institute of Standards and Technology.

Secondary units of time, such as minutes (min) and hours (h), are defined in terms of the second (1 min = 60 s, 1 h = 60 min = 3600 s).

Definition of the Meter

The primary SI unit of length is the **meter** (abbreviated m). Originally the meter was defined as one ten-millionth (10^{-7}) of the distance from the earth's equator to the North Pole. Later the meter was redefined to be the distance between two lines engraved on a certain bar made of a platinum-iridium alloy and carefully preserved in a French laboratory. The distance between the engraved lines was consistent with the older, less precise definition. Copies of the standard meter bar were distributed throughout the world.

In 1960 the meter was redefined as a certain multiple of the wavelength of the orange light emitted by krypton atoms under certain conditions. This newer atomic definition was again made consistent with the older definitions.

The most recent definition of the meter was made in 1983. By that time measurements of the speed of light had become so precise that their accuracy was limited by the precision of the krypton standard meter. The speed of light in a vacuum is a fundamental constant of nature. And since time could be measured on an atomic clock with much greater precision than distance could be measured, it made sense to turn the definition of the unit of length around, defining it in terms of speed and time. As of 1983 **the meter is defined to be the distance traveled by light in a vacuum during a time interval of 1/299,792,458 second.** So now, by definition, the speed of light is 299,792,458 m/s.

Names of Units

Some derived units in the SI system are given no special name. An example is the unit of speed, m/s. Other derived units are given special names. An example is the SI unit of force, the newton (abbreviated N), defined as 1 kg-m/s^2. The names of some SI derived units and their definitions are given on the inside front cover of this book.

Powers of Ten

Units that are powers-of-ten multiples of other units are often convenient to use, and so we use certain prefixes to denote those multiples. For example, *centi-* means a factor of 10^{-2}, *milli-* means a factor of 10^{-3}, and *kilo-* means a factor of 10^{+3}. Thus 1 centimeter (cm) = 10^{-2} m, 1 millimeter (mm) = 10^{-3} m, and 1 kilometer (km) = 10^{+3} m. The most commonly used powers-of-ten prefixes are listed on the inside front cover of this book.

Conversion of Units

It is often necessary to convert units from one system to another. For example, you may need to convert a distance given in miles to units of meters. To do this, you can use the conversion factor 1 mile = 1609 meters. A table of useful conversion factors is given on the inside front cover of this book.

EXAMPLE 1 Astronomical Distance

The star Sirius is about 8 light-years from earth. A light-year (abbreviated LY) is a unit of distance—the distance light travels in 1 year. One LY equals approximately 10^{16} m. Express the distance to Sirius in meters.

SOLUTION Since the ratio $\dfrac{10^{16}\text{ m}}{1\text{ LY}}$ equals 1, we can multiply the equation expressing distance in light-years by this factor without changing the equation. We then cancel units of light-years and find the distance in meters.

$$\text{distance} = 8\,\cancel{\text{LY}}\left(\frac{10^{16}\text{ m}}{1\,\cancel{\text{LY}}}\right) = 8 \times 10^{16}\text{ m}$$

EXAMPLE 2 Speed in SI Units

Express a speed of 60 miles per hour (mi/h) in meters per second.

SOLUTION We use the conversion factors 1 mi = 1609 m and 1 h = 3600 s, expressed as ratios, in such a way that when we multiply by these ratios we can cancel out miles and hours, leaving units of meters per second.

$$\text{speed} = \left(60\ \frac{\cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(\frac{1609\ \text{m}}{1\ \cancel{\text{mi}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)$$

$$= 27\ \frac{\text{m}}{\text{s}}$$

Consistency of Units

In solving problems, we shall often use equations expressing relationships between various physical quantities. Algebraic symbols such as x and t are used to represent the physical quantities. Whenever we solve for the value of one quantity by substituting numerical values for other quantities in an equation, it is important to include the units along with the numerical values. The units are carried along in the calculation and treated as algebraic quantities. We then obtain from the calculation both the numerical answer and the correct units.

Using units in this way will alert you when you make certain common errors. For example, consider the following equation from Chapter 2:

$$x = \tfrac{1}{2}\, at^2$$

where x represents distance, a represents acceleration, and t represents time. Suppose we wish to calculate x at time $t = 5$ s, given an acceleration $a = 4\ \text{m/s}^2$. We substitute these values into the equation and find:

$$x = \frac{1}{2}\left(4\ \frac{\text{m}}{\text{s}^2}\right)(5\ \text{s})^2$$

$$= \frac{1}{2}(4)(5)^2\left(\frac{\text{m}}{\text{s}^2}\right)(\text{s}^2)$$

$$= 50\ \text{m}$$

We obtain our answer in meters, a correct unit for distance.

Suppose we had mistakenly written the equation as $x = \tfrac{1}{2}at$, forgetting the exponent 2 on the t. When we substitute values into this incorrect equation, we obtain units for x of

$$\left(\frac{\text{m}}{\text{s}^2}\right)(\text{s}) = \frac{\text{m}}{\text{s}}$$

which are clearly incorrect units for a distance. These incorrect units reveal that we have made some kind of mistake—either we have used an incorrect equation or we have substituted a quantity with incorrect units into a correct equation.

Of course, getting the units to come out correctly is no guarantee that you have not made some other kind of error. But at least it allows you to eliminate some kinds of errors.

Significant Figures

When you measure any physical quantity, there is always some uncertainty in the measured value. For example, if you measure the dimensions of a desk with a meter stick marked with smallest divisions of millimeters, your measurements may be accurate to the nearest millimeter but not to the nearest tenth of a millimeter. When you state the dimensions, you could explicitly indicate the uncertainty in your measurements. For example, you might measure the length of a desk to the nearest millimeter (or tenth of a centimeter) and express the desk's length as 98.6 ± 0.1 cm. This means that you believe the length to be between 98.5 cm and 98.7 cm.

In this text, we shall not explicitly indicate the uncertainty in a measured value. We shall, however, imply this uncertainty by the way we express a value. Saying that the length of the desk is 98.6 cm means that we have some confidence in the three measured digits, in other words, confidence that the true length differs from this number by no more than 0.1 cm. We say that there are three **significant figures** in this measurement. If you say that the length is 98.60 cm, giving four significant figures, you are implying that your measurement is accurate to four significant figures, in other words, that the uncertainty is no more than 0.01 cm. Since your measurement does not have this degree of precision, it would be misleading to state the result in this way. The measured length is 98.6 cm, *not* 98.60 cm.

Sometimes the number of significant figures is unclear. For example, if we say a certain distance is 400 m, are the two zeroes significant figures or are they included just to indicate the location of the decimal point? Do we mean that the uncertainty in distance is 1 meter? Using powers-of-ten notation avoids such ambiguity. For example, if we say the distance is 4.00×10^2 m, we give three significant figures, meaning that the uncertainty in distance is 0.01×10^2 m, or 1 m. But if we say the distance is 4.0×10^2 m, we give two significant figures, meaning that the uncertainty is 0.1×10^2 m, or 10 m.

Often it is tempting to state a result you have calculated with too many significant figures, simply because this is the way the numerical value appears on your calculator. For example, suppose you wish to calculate the area of a desk top. You measure a length of 98.6 cm and a width of 55.2 cm. You compute the rectangular area by multiplying these two numbers. Your calculator reads 5442.72. But each of your measurements is accurate to only three significant figures, and so the product is also accurate to only three significant figures. Thus you should round off the calculator reading and state the area as 5440 cm^2 or, better yet, 5.44×10^3 cm^2.

Now suppose you had measured the width of the desk with less precision than the length—measuring only to the nearest centimeter, so that the result is 55 cm. The length is known to three significant figures, but the width is known to only two significant figures. The less precise measurement is the limiting factor in the precision with which you can calculate the area. **When two or more numbers are multiplied or divided, the final answer should be given to a number of significant figures equal to the smallest number of significant figures in any of the numbers used in the calculation.** So, when you multiply a length of 98.6 cm times a width of 55 cm, do *not* state the area as 5423 cm^2, as indicated on your calculator. Rather, round off to two significant figures and state the area as 5400 cm^2 or as 5.4×10^3 cm^2.

When numbers are added or subtracted, the uncertainty in the calculated value is limited by the number having the greatest uncertainty. Thus, **when you add or subtract, the number of decimal places retained in the answer should equal the smallest number of decimal places in any of the quantities you add or subtract.** For example, the sum 12.25 m + 0.6 m + 44 m should *not* be written as 56.85 m but rather should be rounded off to 57 m.

Often numbers that appear in equations do not represent measured values and so are not subject to the rules for significant figures. For example, the numerical factor $\frac{1}{2}$ appears in the equation $x = \frac{1}{2}at^2$. This number is exact. There is no uncertainty in its value, and it places no limitation on the number of significant figures to which x can be calculated. If, for example, values for a and t are known to 3 significant figures, x may be calculated to 3 significant figures.

Order-of-Magnitude Estimates

It is often useful to estimate a number to the nearest power of ten. Such an estimate is called an **order-of-magnitude estimate**. Estimating is appropriate either when the available data do not permit any greater accuracy or when you don't need to know the number with any greater accuracy. Estimates can also be useful in checking the results of a more careful calculation, simply because it is so easy to calculate when you are working only with powers of ten.

For example, suppose we want to estimate the number of high schools in the United States. We could begin by estimating the country's high school age population. Since high school takes 4 years and an average lifetime is roughly 80 years, we might estimate that one person in 20 of the 200 million or so people in the country is of high school age. This gives a high-school-aged population of about 10 million, or 10^7. Of course this estimate is not very accurate, since not all age groups are equally represented in the population, and certainly not everyone of high school age is in high school. But 10^7 high school students is a reasonable order-of-magnitude estimate. Next we estimate that the average high school has an enrollment on the order of 10^3. This means that the number of high schools in the United States is on the order of $10^7/10^3 = 10^4$.

Problems

Standards of Length and Time

1 Two atomic clocks keep almost exactly the same time, but one runs faster than the other by 1 part in 10^{13}. How long would you have to wait before the clocks' readings differed by 1 s?

2 Before 1799 the legal standard of length in France was the foot of King Louis XIV. Since the king could not personally measure the length of everything with his foot, what was needed to make this standard unit of measure at all useful?

3 In 1983 the meter was redefined as the distance traveled by light in a vacuum during a time interval of 1/299,792,458 s, so that now the speed of light is exactly 299,792,458 m/s. Why wasn't the number rounded off so that the speed could be exactly 300,000,000 m/s?

Unit Conversion

4 How many picoseconds are in 1 h?

5 How many volts are in 30 megavolts?

6 How many milliamps are in 0.2 amp?

7 A picture has dimensions of 20 cm by 30 cm. Find the area in m^2.

8 A rectangular metal plate has dimensions of 8 cm by 5 cm by 3 mm. Find the plate's volume in m^3.

9 A football field is 100 yards long. Express this length in meters.

10 A room has dimensions of 5 m by 4 m. How many square yards of carpet are required to carpet the room?

11 Express the speed of light in units of mi/s.

12 You are driving on the Autobahn in Germany at a speed of 180 km/h. Express your speed in mi/h.

Consistency of Units

13 In the following equations, t is time in s, v is velocity in m/s, and a is acceleration in m/s^2:

$$v = at \qquad v = \frac{a^2}{t} \qquad v = at^2$$

Which of these equations is consistent with the units?

14 In the following equations, t is time in s, x is distance in m, v is velocity in m/s, and a is acceleration in m/s^2:

$$v^2 = 2ax \qquad v = \frac{a^2t^2}{x} \qquad v = \sqrt{xat}$$

Which of the equations is consistent with the units?

15 If you calculate v^2/a, where v is in m/s and a is in m/s^2, what units will your answer have?

Significant Figures

16 How many significant figures are in each of the following numbers: (a) 25.673; (b) 2200; (c) 2.200×10^3; (d) 3005; (e) 0.0043; (f) 4.30×10^{-3}?

17 How many significant figures are in each of the following numbers: (a) 165; (b) 500; (c) 5.00×10^2; (d) 40,001; (e) 0.0070; (f) 7.000×10^{-3}?

18 Round off each number in Problem 16 to two significant figures.

19 Round off each of the following quantities to three significant figures: (a) 5782 m; (b) 2.4751×10^5 s; (c) 3.822×10^{-3} kg; (d) 0.06231 m.

20 A nickel has a radius of 1.05 cm and a thickness of 1.5 mm. Find its volume in m^3.

21 A rectangular plot of land has dimensions of 865 m by 2234 m. How many acres is this? (1 acre = 43,560 square feet.)

22 Find the sum of the following distances: 4.65 m, 31.5 cm, 52.7 m.

23 Find the sum of the following masses: 21.6 kg, 230 kg, 55 g.

Order-of-Magnitude Estimates

24 Estimate the total volume of water on earth.

25 Estimate the total volume of the earth's atmosphere.

26 Estimate the number of heartbeats in an average lifetime.

27 Estimate the total time you will spend during your lifetime waiting for traffic lights to change from red to green.

28 Estimate the total time you will spend during your lifetime waiting in line at the grocery store.

29 Estimate the total number of pediatricians in the United States.

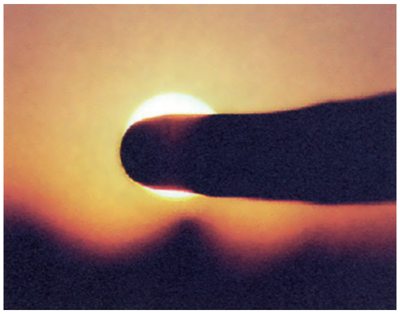
30 Estimate the total number of teachers of college English composition courses in the United States.

31 Estimate the surface area of a water reservoir that is 10 m deep and big enough to supply the water needs of the Los Angeles area for 1 year.

32 (a) Estimate the maximum traffic capacity in one direction on an interstate highway in units of cars per minute. (b) Estimate how long it would take to evacuate a city of 1 million on one interstate highway.

33 Estimate the number of M&Ms needed to fill a 1-liter (10^3 cm^3) bottle.

34 Suppose you are a visitor on another planet and observe the setting sun. You notice that your little finger, which is 1 cm wide, just covers the sun when you extend your arm out and hold your finger 1 m away from your eyes. The bottom edge of the sun begins to dip below the horizon, and 5 minutes later the sun completely disappears. Estimate the length of a day on the planet.



N-Rays, Polywater, and Cold Fusion

Most non-scientists believe that science advances inexorably—facts leading to theory, leading to further facts that allow refinement of the theory, and so on. It may appear that simple adherence to a cookbook like "scientific method" is a surefire route to success—good scientific law, like a good cake, coming from careful measurement and mixing of the right ingredients under just the right conditions. The history of science, however, teaches otherwise. Sometimes a scientist's insight leads to a brilliant and unexpected discovery, for example, Einstein's theory of relativity. And sometimes what appears at first to be a revolutionary discovery turns out to be a disappointing mistake.

Such mistakes are not always easy to recognize. An erroneous claim of an important discovery can arise from a desire to interpret facts in a way that confirms a scientist's own theories. It can be difficult for one to refute such a claim by attempting to reproduce an experiment because no two experiments are ever performed under exactly the same conditions. The scientist who claims to have observed a new effect can always say to another scientist who has repeated the experiment and not seen the effect: "But you have not carried out my experiment in exactly the way I did. You did not use reagents from the same source; you did not observe long enough or carefully enough; your instruments were faulty…."

Our first example occurred long ago, in 1903—before, you might think, we knew as much as we know today. In that year, the French scientist Rene Blondlot, who had been working with the newly discovered X-rays, claimed to have found another new type of radiation. Blondlot named his rays "N-rays" after Nancy, the city in which he was working. He turned out paper after paper on the subject and became quite famous, drawing many well-known scientists into the ranks of his supporters and giving birth to an entire school of N-ray research.

Reports of failures to confirm Blondlot's findings began to appear, but defenders of Blondlot offered critiques of the challengers' experimental methods and powers of observation. Finally, in late 1904, Blondlot was visited by the American Robert Wood, professor of physics at Johns Hopkins University. Through some clever sleight of hand, Wood fooled Blondlot into claiming to observe N-rays under conditions in which, given Blondlot's own theories, they could not possibly have appeared. When Wood described his trick in a letter to *Nature*, a prestigious scientific journal, it was the beginning of the end for N-rays.

Our second example dates back only to the late 1960s. In those years, a Soviet scientist studying the properties of water discovered what he claimed to be a polymerized form of water. This so-called polywater, which had been produced by long and repeated heating of the water in an elaborate glassware apparatus, was observed to be a clear, plastic-like material. The discovery sent shock waves through the scientific community and was given apocalyptic coverage in the press, which claimed with some scientific support that the clumping of the water molecules into this undrinkable form could, if not contained, eventually spread to all the water in the world. The supposed polywater, after careful measurement and analysis, was finally unmasked as ordinary water containing certain impurities. This simple conclusion took nearly 7 years to reach! That much time was required before all the researchers who had failed to reproduce the experiment were finally believed.

Our third example is of still more recent vintage. In April 1989, chemists Stanley Pons and Martin Fleischmann called a press conference to announce the discovery of an amazing new process called "cold fusion." Fusion is the combining of light atomic nuclei to form heavier nuclei and is the process by which the sun generates energy. The process was believed to require sun-like conditions of extremely high temperature in order to overcome the electrical repulsion between the positively charged nuclei and get them close enough together so that they could fuse and release energy. Huge research efforts around the world have been devoted to controlling fusion in enormously large and expensive machines, producing high temperatures (Fig. A). Although steady progress has been made, these machines are not yet energy efficient; that is, they do not generate as much energy as is needed to operate them.

Pons and Fleischmann claimed to have bypassed the usual way of creating fusion at high temperatures by producing cold fusion in a small beaker containing heavy (deuterium-rich) water and a palladium coil carrying electricity (Fig. B). This process, if it had actually worked, would have provided virtually limitless low-cost energy and would have solved the world's energy problems.

Not surprisingly, government and industry were anxious to invest in this work, and other scientists around the world were eager to reproduce the cold fusion effect.

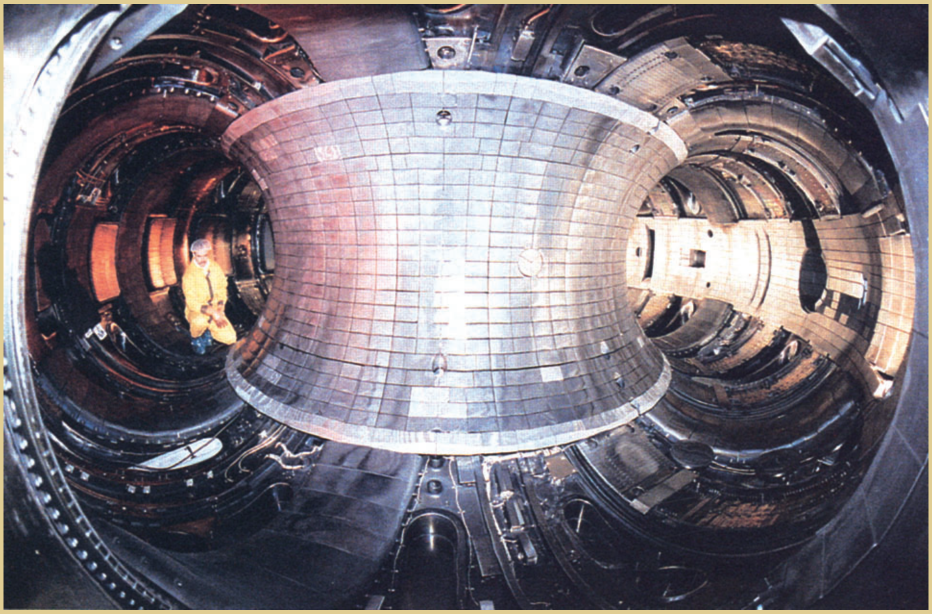


Fig. A Inside the Princeton fusion reactor.

Fig. B Cold fusion apparatus.

However, very few of the many labs that repeated the "cold fusion" experiment found any evidence that fusion was taking place. Pons and Fleischmann attacked their critics and maintained their claim. Lack of positive experimental results by others eventually took its toll. The general consensus today is that, whatever Pons and Fleischmann may have observed, it was not fusion. Like the proponents of N-rays and polywater before them, Pons and Fleischmann had been too caught up in visions of a grand discovery, and had abandoned the openness required to successfully probe the secrets of nature.

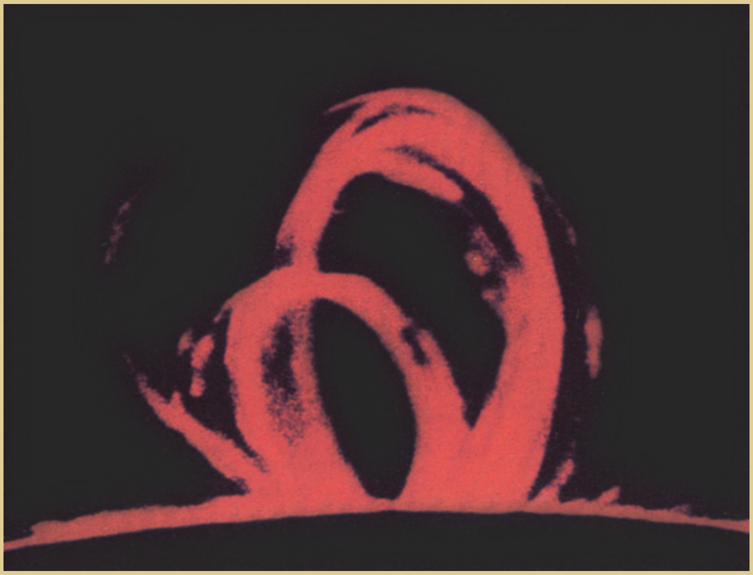


Fig. C Enormous energy is displayed in a solar flare, seen during a solar eclipse.