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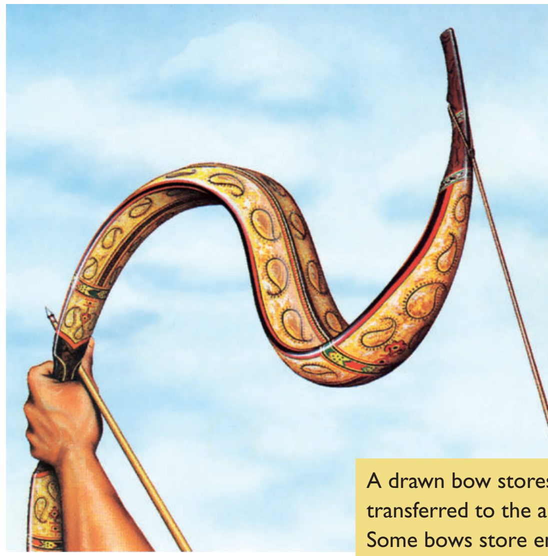
CHAPTER 7 Energy



A drawn bow stores energy, which is transferred to the arrow as it is shot. Some bows store enough energy to shoot an arrow half a mile. *(McEwen E et al: Sci Am 264:76 [cover], June 1991.)*

The Concept of Energy

How fast must a spacecraft move to escape the earth? How much electric power can be generated using water from a certain waterfall? How many calories do you burn riding a bicycle uphill? To answer questions such as these we shall use a fundamental law of nature—**the law of conservation of energy.** This law states that **there exists a numerical quantity called "energy" that remains fixed in any process that occurs in nature.** We express the law more concisely by saying that **"energy is conserved."** The law of energy conservation applies without exception to all systems. If a certain isolated system has, say, 50 units of energy initially, that system will continue to have 50 units of energy, no matter what changes the system undergoes. It is possible for a system to lose energy only if that system is not isolated. Then the energy lost shows up as energy gained by some other system with which the first system has interacted.

Energy comes in many forms. Electrical energy, chemical energy, nuclear energy, and thermal energy are some forms of energy we shall study in later chapters. In this chapter we shall study only **mechanical energy,** which consists of two distinct types: (1) **kinetic energy,** associated with the motion of a body, and (2) **potential energy,** associated with the position of a body and a particular kind of mechanical force.

In general, the law of conservation of energy applies to the numerical sum of all forms of energy. If we add up mechanical energy, electrical energy, chemical energy, and so forth, the total energy of an isolated system is always constant. In this chapter we shall see that, under certain special circumstances, a system's mechanical energy alone is conserved. We shall show how this principle of conservation of mechanical energy follows from Newton's laws of motion. Rather than use Newton's laws directly to analyze the forces acting on a system, it is often easier to apply energy principles. For example, we shall use conservation of energy to calculate how fast a spacecraft must move to escape the earth (Ex. 7), to find the electric power generated in a certain Bavarian home using water from a small waterfall (Problem 59), and to estimate the energy needed to ride a bicycle uphill (Ex. 15).

7–1 Work and Kinetic Energy

Definition of Work Done by a Constant Force in One Dimension

In this section we shall define work and kinetic energy and then show how they are related through the work-energy theorem. The full significance of work and kinetic energy can be appreciated only after you see how they are connected through this important theorem.

When a force acts through a distance, we say, "The force does work." More precisely, the **work** W done by a constant force **F** acting on a body moving in a straight line (Fig. 7–1) is defined to be the product of the force component F_x in the direction of motion times the distance Δx the body moves:

$$W = F_x \Delta x \qquad \text{(constant force; linear motion)} \qquad (7\text{–}1)$$

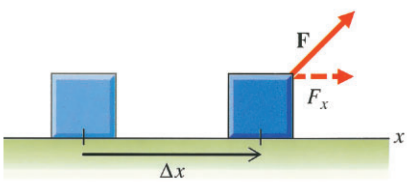


Fig. 7–1 As a block moves a distance Δx, the force **F** does work $F_x \Delta x$.

If a body does not move, $\Delta x = 0$, and so, even though forces may act on the body, no work is done by those forces (Fig. 7–2a), and no work is done on a moving body by any force that is perpendicular to the direction of the body's motion (Fig.7–2b), since such a force has a zero component in the direction of motion.

"Work" is a word commonly used to mean human effort. No such meaning is implied by the definition of work used in physics. For example, as you sit studying physics, you may be making an enormous effort, but there is no work being done, according to our definition of work as force acting through a distance. On the other hand, little or no effort is required to fall onto your bed. And yet work is done by the force you exert on your mattress and springs as they are being compressed. It is important not to confuse the physical concept of work with effort or with any other meaning attached to the word "work" in everyday language.

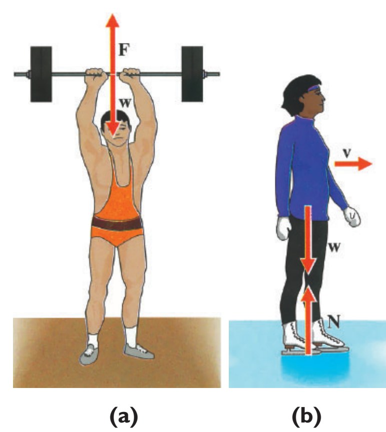


Fig. 7–2 **(a)** No work is done on a stationary barbell either by the barbell's weight **w** or by the force **F** exerted by the weight lifter. **(b)** No work is done by forces **N** and **w** acting on a skater, since neither force has a component in the direction of motion.

Units

The unit of work is the unit of force times the unit of distance—the N-m in SI. This unit is given the name "joule" (abbreviated J), in honor of James Joule, who demonstrated by numerous experiments in the nineteenth century that heat is a form of energy:

$$1 \text{ joule} = 1 \text{ N-m} = 1 \text{ kg-m}^2/\text{s}^2 \qquad (7\text{–}2)$$

In the cgs system the unit of work is the *erg,* defined as a dyne-cm. Since $1 \text{ N} = 10^5$ dyne and $1 \text{ m} = 10^2$ cm, $1 \text{ N-m} = 10^7$ dyne-cm or

$$1 \text{ J} = 10^7 \text{ erg} \qquad (7\text{–}3)$$

In the British system the unit of work is not given a separate name; it is simply called a "foot-pound," ft-lb. From the relationships between N and lb and between m and ft, it is easy to relate J to ft-lb:

$$1 \text{ J} = 0.738 \text{ ft-lb} \qquad (7\text{–}4)$$

EXAMPLE 1 Pulling a Suitcase

An airline passenger pulls his suitcase a horizontal distance of 40.0 m, exerting a force **F** of magnitude 25.0 N, directed 30.0° above the horizontal (Fig. 7–3). Find the work done by the force **F**.

SOLUTION To find the work we apply the definition (Eq. 7–1), using the component of force in the forward direction, the direction of motion.

$$W = F_x \Delta x = F \cos 30.0° \, \Delta x$$
$$= (25.0 \text{ N})(\cos 30.0°)(40.0 \text{ m})$$
$$= 866 \text{ J}$$

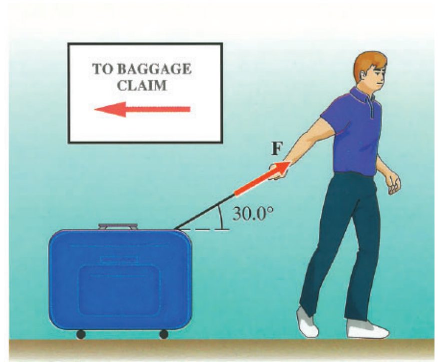


Fig. 7–3

Net Work

We define the **net work** on a body, W_{net}, to be the sum of the work done by all the forces acting on the body:

$$W_{net} = \Sigma W \qquad (7\text{–}5)$$

Net work is important because the effect of work on the energy of a body depends only on the net work, as we shall see when we discuss the work-energy theorem.

EXAMPLE 2 Lifting a Box

A woman slowly lifts a box weighing 40.0 N from the floor to a shelf 1.50 m above (Fig. 7–4). (a) Find the work done by the force **F** the woman exerts on the box. (b) Find the work done on the box by its weight **w**. (c) Find the net work done on the box.

SOLUTION **(a)** Since the box is lifted slowly, we assume that acceleration is negligible and therefore no net force acts on the box. This means that the woman exerts an upward force **F** of magnitude 40.0 N, balancing the box's weight. The force **F** acts in the direction of motion, and so the force component used in calculating the work W_F done by **F** is the full force of 40.0 N.

$$W_F = F_x \Delta x = F \, \Delta x = (40.0 \text{ N})(1.50 \text{ m})$$
$$= 60.0 \text{ J}$$

(b) The box's weight **w** acts opposite the direction of motion, and so its component in the direction of motion is negative ($-w$). Thus the work W_w done by **w** is negative.

$$W_w = w_x \Delta x = -w \, \Delta x = -(40.0 \text{ N})(1.50 \text{ m})$$
$$= -60.0 \text{ J}$$

(c) The net work done on the box is the sum of the work done by each of the forces acting on the box. Net work equals zero:

$$W_{net} = \Sigma W = W_F + W_w = +60 \text{ J} - 60 \text{ J} = 0$$

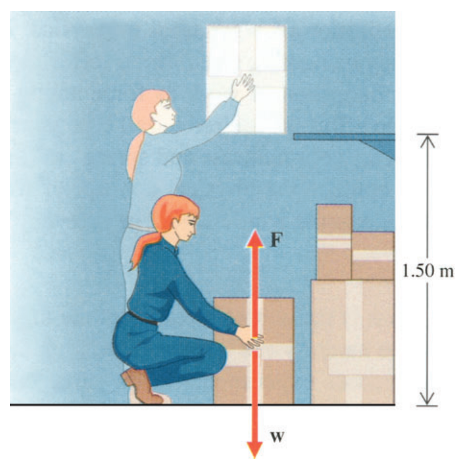


Fig. 7–4

Another way to find net work is to calculate the work done by the net force. Here the net force is zero, and so the work done by the net force must also be zero. Thus we get the same answer for W_{net} as we found by adding the work done by each force. It is easy to show that net work always equals the work done by the net force:

$$W_{net} = \Sigma W = \Sigma (F_x \Delta x) = (\Sigma F_x)(\Delta x) = F_{net} \Delta x$$

Kinetic Energy

A body's **kinetic energy** K is defined to be half its mass m times the square of its speed v.

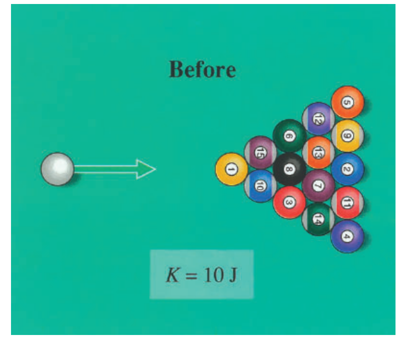
$$K = \tfrac{1}{2}mv^2 \qquad (7\text{–}6)$$

From its definition, kinetic energy must have units equal to mass units times velocity units squared—SI units of kg-(m/s)^2. Since 1 N = 1 kg-m/s^2, the SI unit of kinetic energy is N-m, or J, the same as the unit of work.

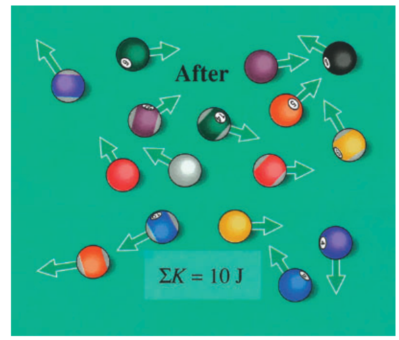
Sometimes kinetic energy is a conserved quantity. The simplest case of this is when a body moves at constant speed. Since both mass m and speed v are constant, the body's kinetic energy $\frac{1}{2}mv^2$ is also constant. Kinetic energy is conserved. A more interesting example of conservation of kinetic energy occurs in the game of pool. Suppose a cue ball is shot into a rack of balls (Fig. 7–5). If the cue ball has a mass of 0.2 kg and is initially moving at 10 m/s, its initial kinetic energy

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.2 \text{ kg})(10 \text{ m/s})^2 = 10 \text{ J}$$

The other balls are initially at rest and so have no kinetic energy. Just after the collision, the kinetic energy of 10 J is shared among all balls. That is, if we add up the kinetic energies of all the balls just after the collision, the total is approximately 10 J. Kinetic energy is approximately* conserved in the collision of pool balls.



(a)



(b)

Fig. 7–5 The total kinetic energy of pool balls just after the "break" equals the kinetic energy of the cue ball before the break. Kinetic energy is conserved.

Work-Energy Theorem

Suppose a body moves along the x-axis and is subject to a number of constant forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \ldots$, whose resultant $\Sigma\,\mathbf{F}$ is a constant force directed along the x-axis (Fig. 7–6). According to Newton's second law, the body experiences an acceleration a_x given by

$$a_x = \frac{\Sigma\, F_x}{m} \qquad (7\text{–}7)$$

where m is the body's mass. The body is accelerated, and so, as it moves through the distance $\Delta x = x - x_0$, its velocity changes. The final velocity v_x is related to the initial velocity v_{x0}, to the distance Δx, and to the acceleration a_x by the kinematic equation (from Chapter 2) $v_x^2 = v_{x0}^2 + 2a_x\,\Delta x$. There is only one component of velocity, $v^2 = v_x^2$, and so we may write the kinematic equation as

$$v^2 = v_0^2 + 2a_x\,\Delta x$$

Substituting for a_x from Newton's second law (Eq. 7–7), we obtain

$$v^2 = v_0^2 + 2\left(\frac{\Sigma\, F_x}{m}\right)(\Delta x)$$

Multiplying this equation by $m/2$ and rearranging, we can express this result

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = \Sigma\,(F_x\,\Delta x) \qquad (7\text{–}8)$$

Thus we equate the change in kinetic energy ($\Delta K = K - K_0 = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$) to the net work [$W_{\text{net}} = \Sigma\, W = \Sigma\,(F_x\,\Delta x)$].

$$\Delta K = W_{\text{net}} \qquad (7\text{–}9)$$

This result is known as the **work-energy theorem.**

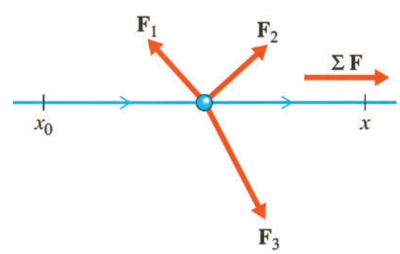


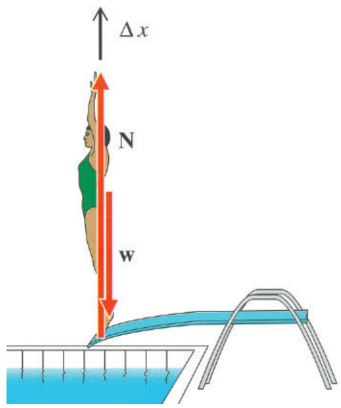
Fig. 7–6 A body moves from x_0 to x, and the forces acting on the body do work.

*A small part of the initial kinetic energy is converted to thermal energy and sound energy during the collision, and so kinetic energy is not exactly conserved.

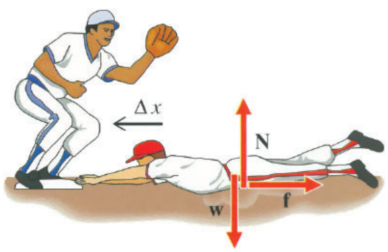
According to the work-energy theorem, when there is no net work done on an object, the object's change in kinetic energy is zero, or, in other words, kinetic energy is conserved. In Fig. 7–2a no work is done on the barbell, and so the barbell's kinetic energy remains constant—equal to zero. The skater in Fig. 7–2b has nonzero kinetic energy that remains constant, assuming that forces **F** and **w** are the only forces, since neither of these forces does work.

Fig. 7–7 shows two examples in which kinetic energy is not conserved. In Fig. 7–7a positive work is done by the normal force on a diver and negative work is done by the diver's weight, as the diver springs upward. Since the normal force is greater than the weight, the net work is positive. So, according to the work-energy theorem, the diver's kinetic energy increases ($\Delta K > 0$); in other words, the diver's speed increases. We can also see this from Newton's second law: the resultant force produces an upward acceleration.

In Fig. 7–7b only the force of friction does work on a baseball player sliding into second base. The other two forces have no component in the direction of motion and therefore do no work. The work done by friction is negative, since this force has a negative component along the line of motion. Thus the net work on the sliding player is negative, and, from the work-energy theorem, the player loses kinetic energy ($\Delta K < 0$); in other words, the player slows down. We can also predict this by applying Newton's second law: the resultant force is the frictional force, which produces an acceleration opposite the direction of motion and therefore slows the player.



(a) $\Delta K = W_{net} > 0$



(b) $\Delta K = W_{net} < 0$

Fig. 7–7 **(a)** A diver's kinetic energy increases as she springs upward because positive net work is done on her, since $N > w$. **(b)** A baseball player's kinetic energy decreases as he slides into second because friction does negative work on him.

EXAMPLE 3 **Final Speed of a Sled**

A child and sled having a combined weight of 335 N start from rest and slide 25.0 m down a 15.0° slope. Find the speed of the sled at the bottom of the slope, assuming negligible air resistance and a constant force of kinetic friction of 20.0 N.

SOLUTION The speed at the bottom of the slope may be calculated once the kinetic energy at that point is found from the work-energy theorem. Since the sled is initially at rest, $K_i = 0$. So the change in kinetic energy, which according to the work-energy theorem equals the net work done by the forces acting on the sled (shown in Fig. 7–8), is

$$\Delta K = K_f - 0 = W_{net} = W_N + W_w + W_f$$

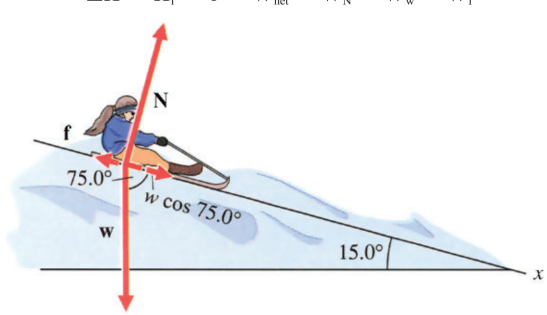


Fig. 7–8

The normal force does no work, since it is perpendicular to the motion:

$$W_N = 0$$

The weight does positive work, since there is a positive component w_x in the direction of motion:

$$W_w = w_x\,\Delta x = (335\text{ N})(\cos 75.0°)(25.0\text{ m}) = 2170\text{ J}$$

The work done by friction is negative since **f** opposes the motion:

$$W_f = f_x\,\Delta x = (-20.0\text{ N})(25.0\text{ m}) = -500\text{ J}$$

Adding the various work terms, we obtain

$$K_f = 0 + 2170\text{ J} - 500\text{ J} = 1670\text{ J}$$

Since $K_f = \frac{1}{2}mv_f^2$, v_f may be expressed

$$v_f = \sqrt{\frac{2K_f}{m}}$$

Since the mass $m = w/g = (335\text{ N})/(9.80\text{ m/s}^2) = 34.2$ kg, we find

$$v_f = \sqrt{\frac{2(1670\text{ J})}{34.2\text{ kg}}} = 9.88\text{ m/s}$$

Variable Force in Three Dimensions

We could have solved the previous example by first calculating the resultant force on the sled, then using Newton's second law to find the sled's acceleration, and finally applying the kinematic equation relating the velocity to the acceleration and distance. So the work-energy theorem has merely provided an alternative method for solving this kind of problem. However, the work-energy theorem may be generalized to deal with problems in which the forces are not constant and for which the path may not be linear. Direct solution of such problems from Newton's second law is much more difficult, since acceleration is not constant and the kinematic equations derived in Chapter 2 are not valid. The energy method then offers a significant advantage.

Consider a particle moving along a curved path and subject to a single variable force **F**, as shown in Fig. 7–9a. Let the path be divided into small intervals of length Δs, each of which is approximately linear and over each of which **F** is approximately constant in magnitude and direction, with a component F_s along the path (Fig. 7–9b). For each small interval, the change in kinetic energy is approximately equal to $F_s\,\Delta s$, and therefore the total change in kinetic energy from i to f is approximately equal to the sum of the $F_s\,\Delta s$ terms:

$$\Delta K = K_f - K_i \approx \Sigma\,(F_s\,\Delta s) \qquad (7\text{–}10)$$

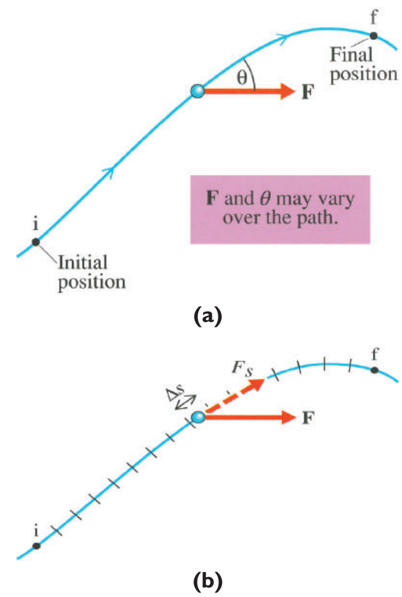


Fig. 7–9 **(a)** Force **F** acts on a particle as it moves along a curved path from i to f. **(b)** The total work done by **F** on the particle is the sum of the work done over small subintervals of length Δs.

The smaller the intervals, the better the approximation becomes, since then the intervals are more nearly linear and the force more nearly constant over each interval. Eq. 7–10 leads us to generalize our definition of work as follows. **The total work done by a force acting on a particle as the particle moves from position i to position f is a sum of terms $F_s\,\Delta s$:**

$$W = \Sigma\,(F_s\,\Delta s) \qquad (7\text{–}11)$$

The intervals of length Δs used in this definition of work must be small enough that F_s is nearly constant over each interval.

Combining the two preceding equations, we may write

$$\Delta K = W$$

When two or more forces act on a particle moving along a curved path, it is the sum of the work done by all the forces that equals the change in kinetic energy. In other words, the change in kinetic energy equals the net work:

$$\Delta K = W_{net} \qquad \text{(work-energy theorem)} \qquad (7\text{–}12)$$

Graphical Interpretation of Work

Next we shall interpret our definition of work graphically, a technique that is useful in evaluating the work done by a varying force. Suppose we graph F_s as a function of s—in other words, graph the component of force acting on a particle as a function of the distance the particle moves over some interval i to f. Such a graph is shown in Fig. 7–10 for an arbitrary force. The interval i to f is divided into small subintervals of length Δs over which F_s is nearly constant. The product $F_s\,\Delta s$ is the area of a single rectangle. According to our definition of work (Eq. 7–11), the work is equal to a sum of terms $F_s\,\Delta s$ for very short intervals. Since $F_s\,\Delta s$ is the area of a rectangle, the work is equal to the sum of the areas of all the rectangles. But this is very nearly just the area under the curve, shaded blue.* Thus we see that the **work done by a force is the area under the F_s versus s curve between the initial and final points**.

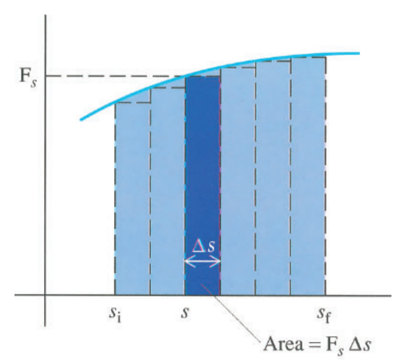


Fig. 7–10 The work done by the force **F** on a particle as it moves from i to f equals the area under the graph of F_s versus s (shaded blue).

*The difference between the area of the rectangles and the area under the curve disappears as we make the rectangles narrower.

EXAMPLE 4 Speed of an Arrow as it Leaves a Bowstring

The force exerted by a certain bow on an arrow decreases linearly after the arrow is released by the archer, starting at a value $F_s = 275$ N when the bow is fully drawn and decreasing to $F_s = 0$ as the arrow leaves the bowstring. The tail of the arrow moves from $s = 0$ to $s = 0.500$ m as the arrow is shot (Fig. 7–11a). Find the final speed of the arrow, which has a mass of 3.00×10^{-2} kg.

SOLUTION After we find the net work, we may use the work-energy theorem to find the final kinetic energy and the final velocity. Only the force **F** does work. This work equals the shaded area under the curve in Fig. 7–11b—the area of the triangle of base 0.500 m and height 275 N:

$$W_{net} = \tfrac{1}{2}(275\ \text{N})(0.500\ \text{m}) = 68.8\ \text{J}$$

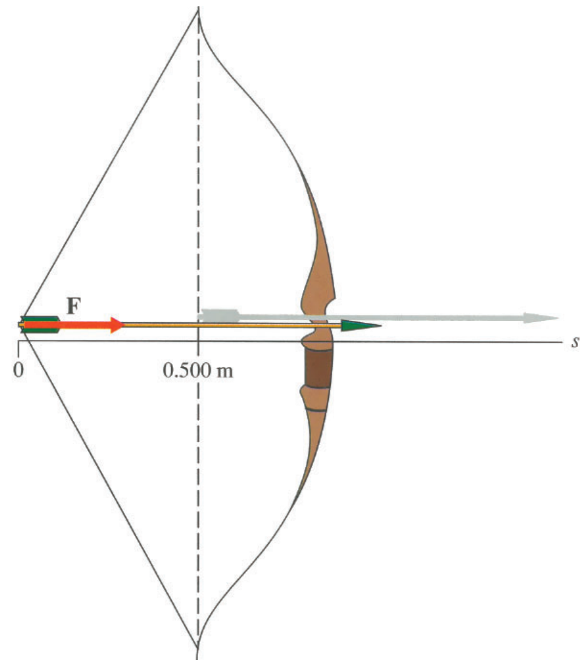
We apply the work-energy theorem, setting the initial kinetic energy equal to zero, since the arrow is initially at rest:

$$\Delta K = K_f - 0 = W_{net} = 68.8\ \text{J}$$

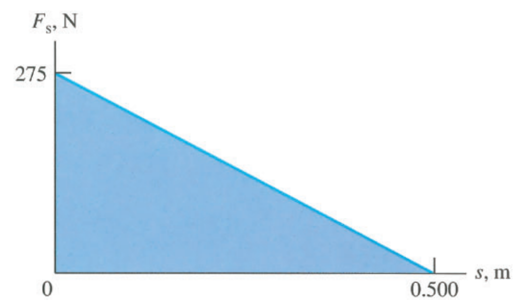
$$K_f = \tfrac{1}{2}mv_f^2 = 68.8\ \text{J}$$

Solving for v_f, we find

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(68.8\ \text{J})}{3.00 \times 10^{-2}\ \text{kg}}} = 67.7\ \text{m/s}$$



(a)



(b)

Fig. 7–11

In the next two sections we shall see how work done by certain forces is related to another form of energy, called "potential energy." Under certain conditions the sum of a system's kinetic energy plus potential energy is conserved.

7–2 Gravitational Potential Energy; Constant Gravitational Force

In this section we shall find a simple general expression for the work done on a body on or near the earth's surface by the constant force of gravity. We shall find that this work always equals the decrease in a quantity called "gravitational potential energy," which depends on the body's elevation. We shall see that when gravity is the only force doing work on a body, the sum of the body's kinetic energy plus its gravitational potential energy is conserved.

Work Done by a Constant Gravitational Force

Suppose a roller coaster starts from rest and accelerates down a curving track, falling through a vertical distance $y_i - y_f$, as it moves from point i to point f (Fig. 7–12). We shall obtain an expression for the work done on the roller coaster by its weight. Rather than use the actual path from i to f, we use an alternative path (path I in Fig. 7–12) to derive an expression for work, since it is much easier to derive the work for this alternative path than for the actual path. The expression we obtain, however, will apply to *any* path between points i and f, as shown at the end of this section.

Path I consists of a vertical displacement followed by a horizontal displacement. Work is done by the gravitational force only along the vertical part, for which there is a constant force $m\mathbf{g}$ along the direction of motion (Fig. 7–12). The work W_G equals the product of this force and the distance $y_i - y_f$:

$$W_G = mg(y_i - y_f)$$
$$W_G = mgy_i - mgy_f \qquad (7\text{–}13)$$

There is no work done along the horizontal part of path I because the force $m\mathbf{g}$ has no component along the direction of motion.

We have derived Eq. 7–13 by considering the work done by gravity on a roller coaster for a specific path. However, this equation applies to the work done on *any* body of mass m by its weight $m\mathbf{g}$, as the body moves from initial elevation y_i to final elevation y_f along *any* path. According to Eq. 7–13, the work equals the difference in the values of the quantity mgy, which we call **gravitational potential energy** and denote by U_G:

$$U_G = mgy \qquad (7\text{–}14)$$

Thus the work equals the decrease in gravitational potential energy—the initial value $U_{G,i}$ minus the final value $U_{G,f}$:

$$W_G = U_{G,i} - U_{G,f} \qquad (7\text{–}15)$$

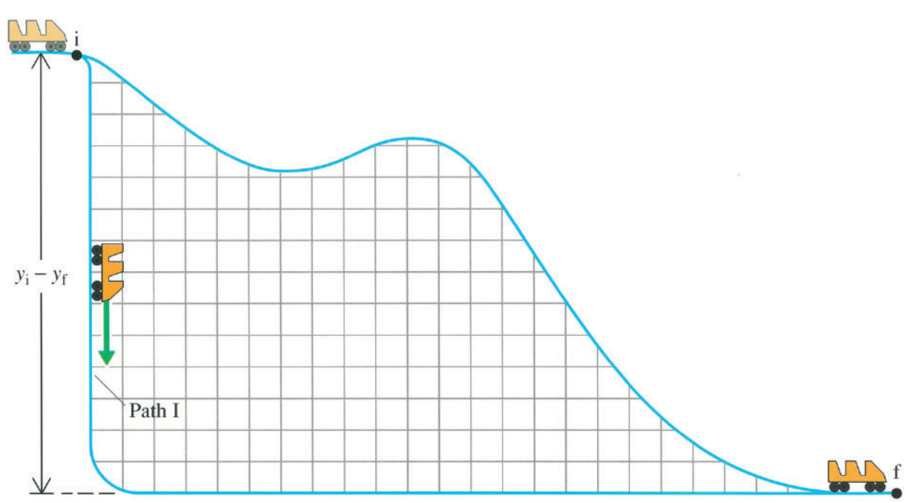


Fig. 7–12 A roller coaster moves along a curved track from point i to point f. Path I is an alternate path between the same points.

For example, suppose a roller coaster weighing 10^4 N starts at an elevation of 40 m, where its potential energy $mgy = 4 \times 10^5$ J, and falls to an elevation of 10 m, where its potential energy $mgy = 10^5$ J. No matter what path the roller coaster follows, the gravitational force does work on it equal to its decrease in potential energy of 3×10^5 J.

Conservation of Energy

Suppose the gravitational force alone does work. Then

$$W_{net} = W_G = U_{G,i} - U_{G,f}$$

From the work-energy theorem, however, we also know that

$$W_{net} = \Delta K = K_f - K_i$$

Equating these two expressions for the net work, we obtain

$$K_f - K_i = U_{G,i} - U_{G,f}$$

Thus, for example, in the case of the roller coaster, if there is negligible work done by friction or any other force except gravity, the roller coaster will gain kinetic energy equal to its lost potential energy of 3×10^5 J.

Rearranging terms in the equation above, we can express our result:

$$K_f + U_{G,f} = K_i + U_{G,i} \tag{7–16}$$

We define the total mechanical energy E to be the sum of the kinetic and gravitational potential energies:

$$E = K + U_G \tag{7–17}$$

Then Eq. 7–16 may be written

$$E_f = E_i \tag{7–18}$$

When gravity is the only force doing work on a body, the sum of the body's kinetic energy plus its gravitational potential energy—the total mechanical energy—is conserved.

As a simple example of conservation of mechanical energy, consider a body in free fall. As a body falls, its speed increases. Its kinetic energy increases while its potential energy decreases, so that the sum of the two—the total mechanical energy—remains constant. This is illustrated in Fig. 7–13 for a 1 kg body falling from rest through a distance of 1 m.

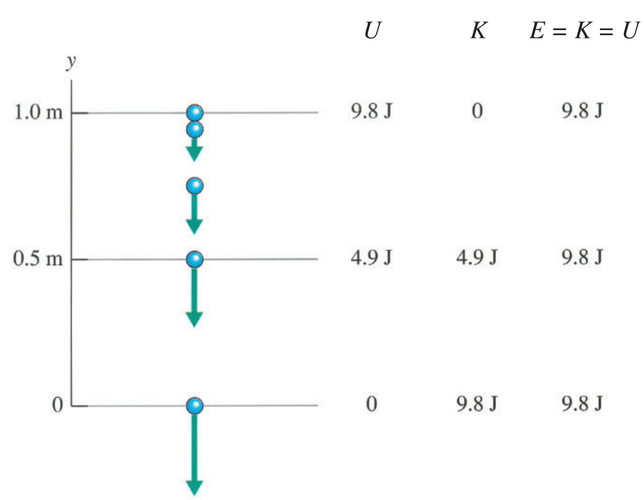


Fig. 7–13 The sum of this falling body's kinetic energy and its gravitational potential energy is a constant 9.8 J. The body's total mechanical energy is conserved.

EXAMPLE 5 Energy of a Thrown Ball

A ball of mass 0.200 kg is thrown vertically upward with an initial velocity of 10.0 m/s. Find (a) the total mechanical energy of the ball, (b) its maximum height, and (c) its speed as it returns to its original level. Neglect air resistance.

SOLUTION **(a)** The total mechanical energy is the sum of the kinetic energy plus the gravitational potential energy:

$$E = K + U_G = \frac{1}{2}mv^2 + mgy$$

We take the origin of the y-axis to be the initial position; then the initial energy E_i is purely kinetic:

$$E_i = \frac{1}{2}mv^2 + 0 = \frac{1}{2}(0.200\text{ kg})(10.0\text{ m/s})^2 = 10.0\text{ J}$$

Since the gravitational force is the only force doing work on the ball (with air resistance being neglected), E will remain equal to 10.0 J throughout the motion of the ball.

(b) When the height is maximum, the ball is momentarily at rest and $K = 0$. The total mechanical energy E is purely potential energy at this point:

$$E = mgy$$

Solving for y, we obtain

$$y = \frac{E}{mg} = \frac{10.0\text{ J}}{(0.200\text{ kg})(9.80\text{ m/s}^2)} = 5.10\text{ m}$$

(c) When the ball returns to its initial height, y again equals 0, and the potential energy is zero. Then the total energy is again purely kinetic, and the kinetic energy therefore equals 10.0 J. This is the same as the initial value of kinetic energy, and so the speed of the ball must also be the same.

$$E = K = 10.0\text{ J}$$
$$v = 10.0\text{ m/s}$$

It is a general characteristic of projectile motion that, in the absence of air resistance, the projectile has the same speed for points at the same elevation (Fig. 7–14). This follows from the fact that the potential energy will be the same at such points, and conservation of total mechanical energy then implies that the kinetic energy will also be the same.

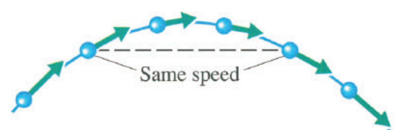
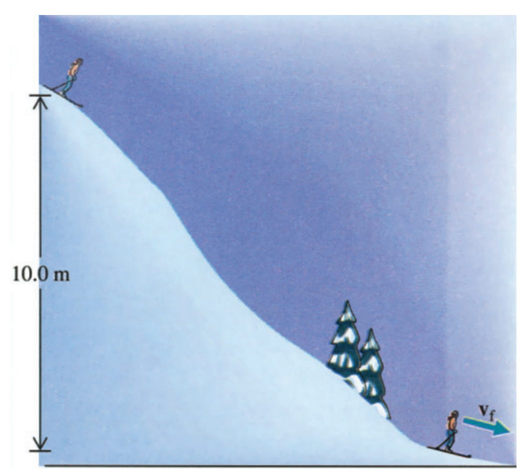


Fig. 7–14 A projectile has the same speed at points with the same elevation, if air resistance is negligible.

EXAMPLE 6 Speed of a Skier at the Bottom of a Hill

A skier starts from rest at the top of a ski slope and skis downhill (Fig. 7–15a). Find the skier's speed after her elevation decreases by 10.0 m, assuming no work is done by friction or air resistance.

Continued on next page.



(a)

Fig. 7–15

EXAMPLE 6 Speed of a Skier at the Bottom of a Hill—continued

SOLUTION The forces acting on the skier are shown in Fig. 7–15b. Since the normal force is perpendicular to the motion, it does no work. Only the weight does work. Therefore the total mechanical energy is conserved.

$$E_f = E_i$$

$$K_f + U_{G,f} = K_i + U_{G,i}$$

$$\tfrac{1}{2}mv_f^2 + 0 = 0 + mgy_i$$

Notice that we have arbitrarily chosen the origin so that $y_f = 0$. Solving for v_f, we find

$$v_f = \sqrt{2gy_i} = \sqrt{2(9.80\ \text{m/s}^2)(10.0\ \text{m})}$$
$$= 14.0\ \text{m/s}$$

(b)

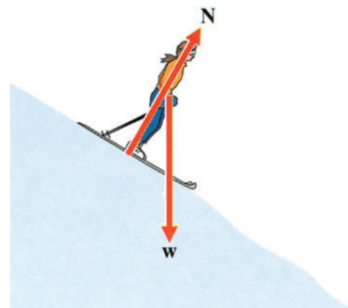


Fig. 7–15, continued

The skier would have this same final speed if she had fallen straight down through a vertical distance of 10.0 m, since her decrease in gravitational potential energy is determined solely by her vertical drop.

Proof That Work Done by Gravity Is Path-Independent

We shall now show that the work done by the gravitational force on a body is independent of the path the body travels from its initial position to its final position. Fig. 7–16 shows an arbitrary path between points i and f at respective elevations y_i and y_f. (To be specific you can think of a roller coaster moving along *any* path between points i and f.) Since the motion is not linear, we must use the general expression for work (Eq. 7–11):

$$W_G = \Sigma\,(F_s\,\Delta s)$$

Fig. 7–16 shows a blowup of a short, approximately linear, segment of the path. The component of the force $m\mathbf{g}$ along the path is $mg\cos\theta$, and so the work done by gravity over the interval Δs is

$$F_s\,\Delta s = (mg\cos\theta)(\Delta s) = mg\,(\Delta s\cos\theta)$$

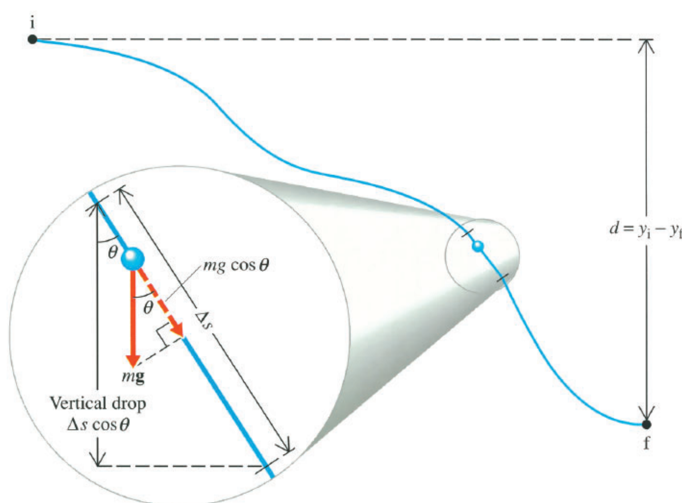


Fig. 7–16 Finding the work done by gravity as a body moves over an arbitrary path from i to f. The body's elevation decreases by $y_i - y_f$.

But $\Delta s \cos\theta$ equals the vertical drop, as indicated in the figure; thus

$$F_s \, \Delta s = mg(\text{vertical drop})$$

We obtain the total work over the entire path by adding the contributions arising from all vertical drops in the interval from i to f:

$$\begin{aligned} W_G &= mg[\Sigma\,(\text{vertical drops})] = mg(\text{net vertical drop}) \\ &= mg(y_i - y_f) \\ &= mgy_i - mgy_f \end{aligned}$$

This is the same result we obtained in Eq. 7–13, thus completing our proof that the work done by gravity is independent of path.

7–3 Gravitational Potential Energy; Variable Gravitational Force

In the last section we obtained an expression for the gravitational potential energy of a body on or near the earth's surface, where the gravitational force is constant. In this section we shall consider problems in which the gravitational force varies. Suppose a mass M (a planet, for example) exerts a gravitational force **F** on a smaller mass m (such as an approaching spacecraft). This force does work as m moves from an initial position i to a final position f (Fig. 7–17). If m moves over a significant distance compared to the separation of the two masses, the gravitational force **F** is not constant. Then one finds the work done by this force by breaking the path up into short intervals over which the force is nearly constant and calculating the sum:

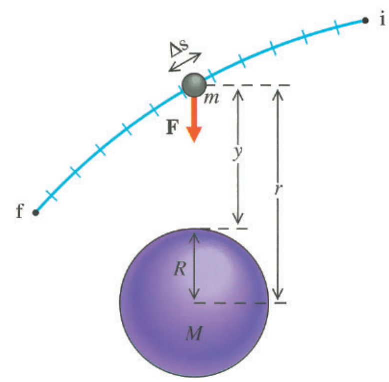


Fig. 7–17 Finding the work done by the gravitational force exerted by a mass M (a planet, for example) on a smaller mass m, as m moves from i to f.

$$W = \Sigma\,(F_s \, \Delta s)$$

Evaluation of this sum requires the use of integral calculus, and so we will not evaluate it here. But the result turns out to be quite simple. The work done by gravity equals the decrease in the gravitational potential energy, U_G:

$$W_G = U_{G,i} - U_{G,f}$$

where the potential energy depends on the distance r between the centers of the two masses, as given by the equation

$$U_G = -\frac{GmM}{r} \tag{7–19}$$

According to this equation, gravitational potential energy is always a negative quantity for all finite values of r. At $r = \infty$ the potential energy equals zero, whereas at $r = R$ (the radius of the larger body), $U_G = -GmM/R$. Suppose, for example, a spacecraft of mass m approaches the earth from a very great distance, so that its potential energy starts out equal to zero. The spacecraft's potential energy steadily decreases to a minimum value of $-GmM/R$ at the surface of the earth. The potential energy decreases by GmM/R.

Since the work done by gravity equals the difference in two values of the potential energy, we can always add an arbitrary constant to the potential energy at every point to provide a more convenient reference level of zero potential energy. Since the same constant is added to both the initial and final values, the difference in potential energy is unchanged. For example, if we add the constant GmM/R to the expression for potential energy given in Eq. 7–19, we obtain a second equally valid potential energy function U_G':

$$U_G' = -\frac{GmM}{r} + \frac{GmM}{R} \tag{7–20}$$

The zero of potential energy in this case occurs when $r = R$:

$$U_G' = -\frac{GmM}{R} + \frac{GmM}{R} = 0$$

whereas at $r = \infty$, we have

$$U_G' = \frac{GmM}{R}$$

The decrease in potential energy of a spacecraft of mass m, approaching the earth from a great distance, equals

$$U_{G,i}' - U_{G,f}' = \frac{GmM}{R} - 0 = \frac{GmM}{R}$$

the same decrease we calculated using the function U_G.

Most terrestrial bodies are always at nearly the same distance from the center of the earth. For example, when a batter hits a home run, the distance of the baseball from the center of the earth varies little over its trajectory. For such bodies the difference in gravitational potential energy between any two points can be calculated using one of the expressions just given (Eq. 7–19 or 7–20), or using Eq. 7–14 ($U_G = mgy$), which is valid when the gravitational force is constant. Problem 72 outlines a proof that these different equations for gravitational potential energy are consistent.

EXAMPLE 7 Speed of a Meteoroid Entering the Atmosphere

Find the speed of a meteoroid (Fig. 7–18) as it first enters the earth's atmosphere, if, when it is very far from the earth, it is moving relatively slowly, so that its initial kinetic energy is negligible.

SOLUTION Since the earth's gravitational force is the only force acting on the body, its mechanical energy is conserved:

$$E_f = E_i$$

or

$$K_f + U_{G,f} = K_i + U_{G,i}$$

Since the body is initially very far from the earth, its initial potential energy is approximately zero.

$$U_{G,i} \approx 0$$

And we are given that its initial kinetic energy is approximately zero.

$$K_i \approx 0$$

Substituting into the energy conservation equation, we obtain

$$K_f + U_{G,f} = 0$$

or

$$\tfrac{1}{2}mv_f^2 - \frac{GmM}{r_f} = 0$$

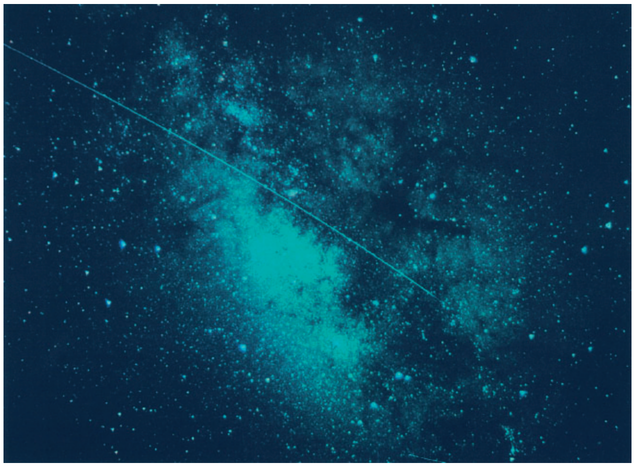
where m is the meteoroid's mass, M is the earth's mass, and r_f is approximately the earth's radius. Solving for v_f, we find

$$v_f = \sqrt{\frac{2GM}{r_f}}$$

$$= \sqrt{\frac{2\left(6.67 \times 10^{-11}\ \frac{\text{N}-\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})}{6.38 \times 10^6\ \text{m}}}$$

$$= 1.12 \times 10^4\ \text{m/s} = 11.2\ \text{km/s}$$

Most meteoroids have higher speeds as they enter the atmosphere (typically 13 to 70 km/s), since they usually have a significant kinetic energy when they are far from the earth.



(a)

(b)

Fig. 7–18 (a) A meteor. **(b)** A 15-ton meteorite. A meteor, or "shooting star," is the bright streak of light that occurs when a solid particle (a "meteoroid") from space enters the earth's atmosphere and is heated by friction. Billions of meteoroids hit the earth each day. You can usually see several meteors per hour on a clear, moonless night. Fortunately, most meteoroids are no larger than a small pebble and vaporize before they reach the ground. Occasionally, a very large meteoroid strikes the earth (the fallen body is called a "meteorite"). For example, one such body formed the great meteor crater in Arizona over 5000 years ago (see p. 87). Another weighing about 10^5 tons destroyed hundreds of square miles of forest in Siberia in 1908. Most meteoroids originate from bodies that are already within the solar system.

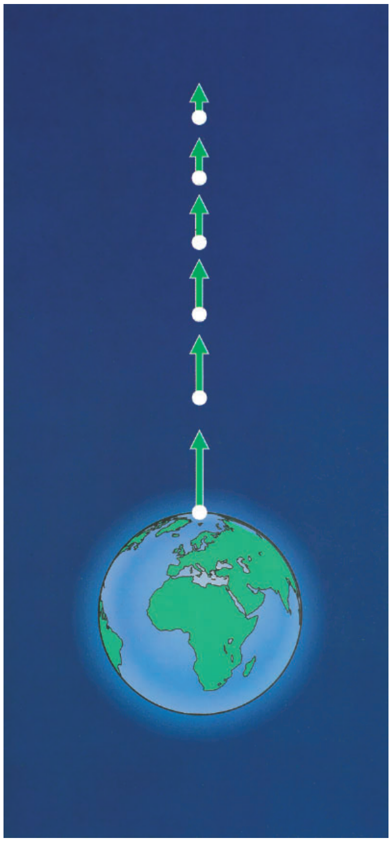


Fig. 7–19 A ball leaving the earth with an extremely large initial velocity would escape the earth. As it moves away from the earth, the ball's velocity at first decreases, but eventually its velocity is nearly constant.

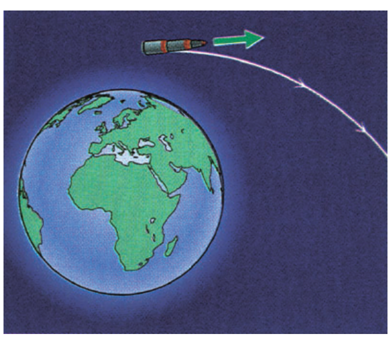


Fig. 7–20 If the initial speed is greater than or equal to the escape velocity ($v_E = \sqrt{2GM/R}$), the spacecraft will escape the planet.

Escape Velocity

What goes up must come down. If you throw a ball vertically upward, it always comes down again. But suppose you could give a ball an extremely large initial velocity, say, 50,000 km/h. The ball would escape the earth, never to return (if we assume negligible air resistance*). Initially, as the ball rose, it would decelerate at the rate of 9.80 m/s^2. However, because of the large initial velocity, it would rise to great heights, and the gravitational force and gravitational acceleration (which vary as $1/r^2$) would decrease as it rose. The ball could then rise still higher with less gravitational acceleration. Eventually, when the ball was far from the earth, the earth's gravity would no longer produce a significant effect. The ball would then continue with nearly constant velocity (Fig. 7–19).

Of course it is not possible to simply throw a ball with such a large initial velocity. However, rocket engines have given spacecraft large enough velocities to leave the earth's surface and explore the solar system. Some spacecraft are even able to escape the solar system.

Suppose that a spacecraft blasts off from a planet and reaches a large velocity while still close to the planet's surface. The rocket engines are then turned off. From that point on, the planet's gravitational force is the only force acting on the spacecraft, and its mechanical energy is therefore conserved.

$$E_i = E_f$$

$$\tfrac{1}{2}mv_i^2 - \frac{GmM}{r_i} = \tfrac{1}{2}mv_f^2 - \frac{GmM}{r_f}$$

The minimum initial velocity necessary to escape the planet is called the **escape velocity,** denoted by v_E. This is the value of the initial velocity that results in a final velocity v_f approaching zero as the distance r_f approaches infinity. We insert $v_f = 0$, $r_f = \infty$, $v_i = v_E$ and set r_i equal to the planet's radius R, and the energy conservation equation becomes

$$\tfrac{1}{2}mv_E^2 - \frac{GmM}{R} = 0$$

Solving for v_E, we obtain

$$v_E = \sqrt{\frac{2GM}{R}}$$

In deriving this equation, we did not need to assume any particular direction for the spacecraft's initial velocity vector, since kinetic energy is a scalar quantity, involving only the magnitude of velocity ($K = \tfrac{1}{2}mv^2$). So our conclusion is valid for a spacecraft moving away from the earth in any direction. If the initial speed exceeds v_E, the spacecraft escapes, never to return (unless acted upon by some other force) (Fig. 7–20).

*Air resistance would actually be a very large force on the ball at such a large velocity. However, our assumption of negligible air resistance is correct if we take the initial velocity to be the velocity of the ball at an elevation of about 100 km, above which the atmosphere is very thin.

EXAMPLE 8 Earth's Escape Velocity

Find the value of the escape velocity on earth.

SOLUTION Inserting values for the earth's mass and radius into Eq. 7–21, we find

$$v_E = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{6.38 \times 10^6\ \text{m}}}$$

$$= 1.12 \times 10^4\ \text{m/s} = 11.2\ \text{km/s}$$

Any object on earth with a velocity of at least 11.2 km/s in any direction away from the earth will leave the earth and never return, unless it is acted upon by forces other than just the earth's gravitational force.

7–4 Spring Potential Energy; Conservation of Energy

There are other kinds of potential energy besides gravitational; that is, there are other forces for which the work can be expressed as a decrease in some kind of potential energy. One of these is spring potential energy. For example, the compressed spring that launches the ball in a pinball machine stores spring potential energy (Fig. 7–21). We shall see that under certain circumstances spring potential energy can be converted into kinetic energy (for example, the kinetic energy of the ball in a pinball machine).

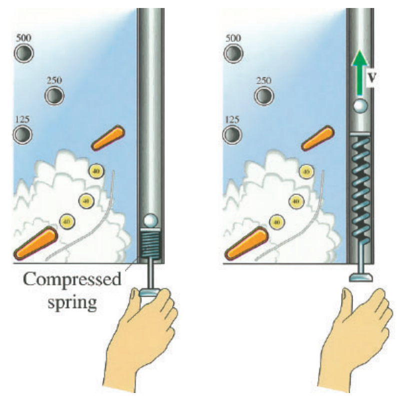


Fig. 7–21 When a ball is shot from a pinball machine, its kinetic energy comes from the potential energy of a compressed spring.

Work Done by a Spring Force

Suppose a body is attached to a spring that can be either stretched or compressed (Fig. 7–22). As discussed in Chapter 4, the force that the spring exerts on the body has an x component given by

$$F_x = -kx \tag{7–22}$$

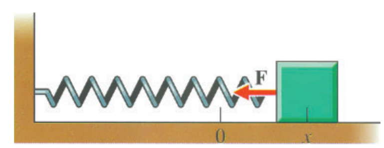


Fig. 7–22 The force **F** exerted on a body by a spring does work on the body as it moves.

where x is the displacement from the equilibrium position. Since this force varies with position, we compute the work by applying the general definition of work (Eq. 7–11), expressing the work as a sum:

$$W = \Sigma\,(F_x\,\Delta x)$$

As shown in Section 7–1, the work equals the area under the force versus displacement curve, between the initial and final points. The shaded area in Fig. 7–23 gives the work done by the spring force on a body in contact with the spring, as the body moves from x_i to x_f. This area is counted as negative because the force F_x is negative over the interval, the displacement is positive, and so each of the products in the sum ($F_x\,\Delta x$) is negative. The shaded area can be computed as the difference in the areas of two triangles. The larger triangle has a base extending from 0 to x_f and an area of $\frac{1}{2}(-kx_f)(x_f) = -\frac{1}{2}kx_f^2$. The smaller triangle has a base extending from 0 to x_i, and an area of $\frac{1}{2}(-kx_i)(x_i) = -\frac{1}{2}kx_i^2$. The work W_s done by the spring is the difference in these two areas:

$$W_s = \text{Area of larger triangle} - \text{Area of smaller triangle}$$

$$= -\tfrac{1}{2}kx_f^2 - (-\tfrac{1}{2}kx_i^2)$$

$$= \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2$$

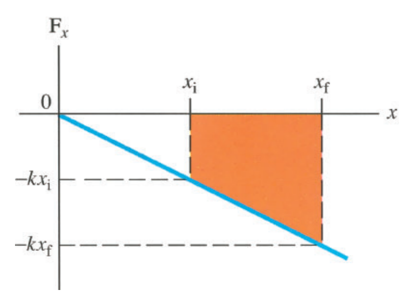


Fig. 7–23 Finding the work done by the spring force.

We see that the work is expressed as the difference in the values of the function $\frac{1}{2}kx^2$, evaluated at the two points x_i and x_f. This function we call the **spring potential energy** and denote by U_s.

$$U_s = \frac{1}{2}kx^2 \tag{7–23}$$

Now we can express the work W_s done by the spring as the decrease in spring potential energy.

$$W_s = U_{s,i} - U_{s,f} \tag{7–24}$$

For example, the work done by a spring of force constant 10^3 N/m on an attached mass moving from $x = 0$ to $x = 0.1$ m is

$$\begin{aligned} W_s &= \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 = 0 - \tfrac{1}{2}(10^3\ \text{N/m})(0.1\ \text{m})^2 \\ &= -5\ \text{J} \end{aligned}$$

The spring does −5 J of work on the attached mass, meaning that the kinetic energy of the mass will decrease by 5 J, if no other forces act on it.

Conservation of Energy

If the spring force is the only force that does work on the body, then $W_{net} = W_s = U_{s,i} - U_{s,f}$, and applying the work-energy theorem, we find

$$\Delta K = W_{net}$$

$$K_f - K_i = U_{s,i} - U_{s,f}$$

or

$$K_f + U_{s,f} = K_i + U_{s,i} \tag{7–25}$$

In this case we define the total mechanical energy E to be the sum of the kinetic energy and the spring potential energy:

$$E = K + U_s \quad \text{(when only the spring force does work)} \tag{7–26}$$

and Eq. 7–25 may be written

$$E_f = E_i \tag{7–27}$$

Total mechanical energy is conserved when the spring force is the only force that does work. This is the same result obtained for the gravitational force, except that a different kind of potential energy is used here in defining the mechanical energy. Both the spring force and the gravitational force are called **conservative forces.**

EXAMPLE 9 Energy of a Bow and Arrow

When an archer pulls an arrow back in a bow, potential energy is stored in the stretched bow. Suppose the force required to draw the bowstring back in a certain bow varies linearly with the displacement of the center of the string, so that the bow behaves as a stretched spring. A force of 275 N is required to draw the string back 50.0 cm. (a) Find the potential energy stored in the bow when fully drawn. (b) Find the speed of an arrow of mass 3.00×10^{-2} kg as it leaves the bow, assuming that the arrow receives all the mechanical energy initially stored in the bow.

SOLUTION **(a)** First we find the force constant, using Eq. 7–22 ($F_x = -kx$):

$$k = \frac{-F_x}{x}$$

A force of 275 N is exerted by the string on the arrow in the forward direction when the string is displaced 50.0 cm backwards. Thus

$$k = \frac{-275\ \text{N}}{-0.500\ \text{m}} = 550\ \text{N/m}$$

Now we can apply Eq. 7–23 to find the potential energy.

$$U_S = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(550\ \text{N/m})(-0.500\ \text{m})^2$$
$$= 68.8\ \text{J}$$

(b) Here we assume that no other forces do work, and so mechanical energy is conserved. As the bow leaves the string, the system's energy is the kinetic energy of the arrow.

$$E_f = E_i$$
$$K_f + U_{S,f} = K_i + U_{S,i}$$
$$\tfrac{1}{2}mv_f^2 + 0 = 0 + U_{S,i}$$
$$v_f = \sqrt{\frac{2U_{S,i}}{m}} = \sqrt{\frac{2(68.8\ \text{J})}{3.00 \times 10^{-2}\ \text{kg}}}$$
$$= 67.7\ \text{m/s}$$

Work Done by Both a Spring Force and a Gravitational Force

Suppose that both a spring force and a gravitational force do work on a body and that these are the only forces doing work. The net work is then the sum of the work done by the two forces.

$$\Sigma\, W = W_G + W_S$$

The work done by each force can still be expressed as a decrease in potential energy of the respective type.

$$\Sigma\, W = U_{G,i} - U_{G,f} + U_{S,i} - U_{S,f}$$

or

$$\Sigma\, W = (U_{G,i} + U_{S,i}) - (U_{G,f} + U_{S,f}) \qquad (7\text{–}28)$$

We shall find that a generalization of our definition of mechanical energy will allow us to maintain the principle of conservation of mechanical energy. We first define the total potential energy to be the sum of the gravitational potential energy and the spring potential energy.

$$U = U_G + U_S \qquad (7\text{–}29)$$

Then Eq. 7–28 may be written

$$\Sigma\, W = U_i - U_f$$

But according to the work-energy theorem, $\Sigma\, W = K_f - K_i$. Therefore

$$K_f - K_i = U_i - U_f$$

or

$$K_f + U_f = K_i + U_i$$

If we define the total mechanical energy E to be the sum of the kinetic energy and the *total* potential energy, we find once again that the total mechanical energy is conserved.

$$E_f = E_i \tag{7–30}$$

$$E = K + U \tag{7–31}$$

EXAMPLE 10 Maximum Height of an Arrow

Suppose the arrow described in the preceding example is shot vertically upward. Find the maximum height the arrow rises before falling back to the ground. Neglect air resistance.

SOLUTION The forces acting on the arrow are the spring-like force of the bowstring (as the arrow is shot) and the gravitational force. There are no other forces that do work on the arrow (neglecting friction and air resistance). Therefore the total mechanical energy (the sum of kinetic energy, spring potential energy, and gravitational potential energy) is conserved. To find the maximum height the arrow rises, we equate the initial energy (when the bow is drawn and the arrow is at rest) to the final energy (at the top of the flight).

$$E_f = E_i$$
$$K_f + U_{G,f} + U_{S,f} = K_i + U_{G,i} + U_{S,i}$$

Both the initial and final kinetic energies equal zero. We choose the origin of our y-axis at the arrow's starting point, so that the initial gravitational potential energy is zero. The final spring potential energy is zero because the bow is no longer stretched. Thus the conservation of energy equation becomes

$$U_{G,f} = U_{S,i}$$
$$mgy_f = U_{S,i}$$

Using the potential energy found in Ex. 9, we solve for y_f.

$$y_f = \frac{U_{S,i}}{mg} = \frac{68.8 \text{ J}}{(3.00 \times 10^{-2} \text{ kg})(9.80 \text{ m/s}^2)}$$
$$= 234 \text{ m}$$

In solving this problem we did not need to calculate the arrow's speed as it left the bow. Since mechanical energy is conserved throughout, we simply equated the energy when the bow was drawn to the energy when the arrow reached its highest point.

Fig. 7–24 The work done by friction on a skier moving from A to B depends on the skier's path between these points. Friction is a nonconservative force.

7–5 Conservative and Nonconservative Forces

Conservation of Mechanical Energy

The principle of conservation of total mechanical energy is satisfied when any number of forces act, so long as the work done by each of the forces can be expressed as a decrease in some kind of potential energy. When only such forces, called **conservative forces,** act on a body, the body's mechanical energy is conserved—the mechanical energy being defined as kinetic energy plus the sum of the potential energies corresponding to each of the conservative forces. As we have seen, both the gravitational force and the spring force are conservative. Another example of a conservative force is the electric force. In Chapter 18 we introduce electrical potential energy. Friction is an example of a force that is not conservative. There is no potential energy associated with friction, and so the mechanical energy of a body is not conserved when friction does work on it.

For example, suppose a skier skis down a slope from point A to point B (Fig. 7–24). The work done by friction depends on the path the skier chooses between points A and B. Little work is done by friction if the path is direct. But if the skier turns back and forth, there is considerable negative work done by friction, which tends to cancel the positive work done by gravity and to keep the skier's kinetic energy more or less constant.

Nonconservation of Mechanical Energy

In general both conservative and nonconservative forces act on a body. If we label the sum of the work done by all the conservative forces ΣW_c and the sum of the work done by all the nonconservative forces ΣW_{nc}, then the net work is the sum of these two terms.

$$W_{net} = \Sigma W_c + \Sigma W_{nc}$$

But ΣW_c equals the decrease in the total potential energy, $U_i - U_f$, and W_{net} equals the increase in kinetic energy, $K_f - K_i$, and so we find

$$K_f - K_i = U_i - U_f + \Sigma W_{nc}$$

or

$$K_f + U_f = K_i + U_i + \Sigma W_{nc}$$

Using the definition of the total mechanical energy ($E = K + U$), we may write this as

$$E_f = E_i + \Sigma W_{nc} \tag{7–32}$$

Using ΔE to denote the change in mechanical energy, $E_f - E_i$, we may express this result as

$$\Sigma W_{nc} = \Delta E \tag{7–33}$$

The nonconservative work may be either positive or negative. If it is positive, E increases, and if it is negative, E decreases. For example, friction is a nonconservative force that, since it always opposes the motion of a body, always does negative work on the body. Therefore, when friction is the only nonconservative force acting on a body, the body's mechanical energy decreases: $\Delta E = W_f < 0$.

EXAMPLE 11 Increasing the Energy of a Barbell by Lifting It

A weight lifter lifts a 1.00×10^3 N (225 lb) weight a vertical distance of 2.00 m (Fig. 7–25). (a) Find the increase in the total mechanical energy of the weight, assuming that there is little or no increase in the weight's kinetic energy. (b) Find the work done by the force **F** exerted on the weight by the weight lifter.

SOLUTION **(a)** The weight's change in mechanical energy, ΔE, equals its increase in gravitational potential energy.

$$\begin{aligned}\Delta E = \Delta U_G &= mgy_f - mgy_i \\ &= mg(y_f - y_i) = (1.00 \times 10^3\ \text{N})(2.00\ \text{m}) \\ &= 2.00 \times 10^3\ \text{J}\end{aligned}$$

(b) The contact force **F** exerted on the weight by the weight lifter is a nonconservative force. The work W_F done by this force equals the total nonconservative work done on the weight and, according to Eq. 7–33, equals the weight's increase in mechanical energy.

$$W_F = \Sigma W_{nc} = \Delta E = 2.00 \times 10^3\ \text{J}$$

It is easy to verify this result by directly computing the work done by **F** as the product of the force times the distance. Since the force **F** balances the weight, it has a magnitude of 1.00×10^3 N. We can compute the work as

$$F_x\,\Delta x = (1.00 \times 10^3\ \text{N})(2.00\ \text{m}) = 2.00 \times 10^3\ \text{J}$$

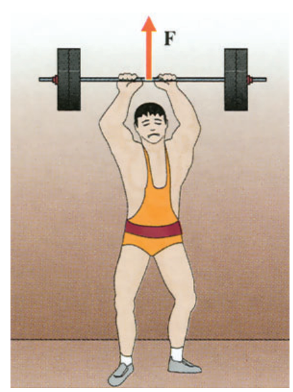


Fig. 7–25 A weight lifter provides a nonconservative force **F**, doing positive work and increasing the weight's mechanical energy.

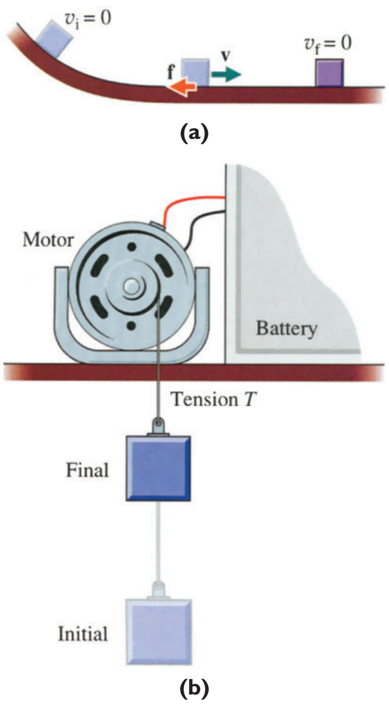


Fig. 7–26 **(a)** A block slides down an incline and then comes to rest. The mechanical energy of the block decreases. **(b)** An electric motor, powered by a battery, raises a weight. The mechanical energy of the weight increases.

Other Forms of Energy

Eq. 7–32 ($E_f = E_i + W_{nc}$) might *seem* to imply that the principle of conservation of energy is not always valid. However, this equation implies only that *mechanical* energy is not always conserved. It is always possible to identify a change in energy of some system that exactly balances a change in the mechanical energy of a body, though this compensating energy may be some other form of energy. To have a universal law of conservation of energy, we must enlarge the definition of energy to include more than just mechanical energy.

Fig. 7–26 shows examples of nonconservation of mechanical energy. In Fig. 7–26a the friction force on a block sliding down an incline causes the block to come to rest. Friction does negative work on the block, which accounts for the block's loss of mechanical energy ($\Delta E = W_f < 0$). But there is an increase in the temperature and the "thermal energy" of both the block and the surface. It turns out that the block's loss of mechanical energy is exactly balanced by the increase of thermal energy (to be studied in Chapter 13). In Fig. 7–26b an electric motor, powered by a battery, raises a weight. The mechanical energy of the weight increases as the result of the positive work done by the nonconservative force exerted on the weight by the tension **T** in the line ($\Delta E = W_T > 0$). It turns out that the weight's gain in mechanical energy is balanced by a loss in the battery's chemical energy (if we assume there is negligible friction and electrical resistance). The chemical energy of batteries is discussed in Chapter 19.

7–6 Power

The rate at which work is performed by a force is defined to be the power output of the force. The average power, denoted by $\overline{P}$, is the work divided by the time Δt over which the work is performed.

$$\overline{P} = \frac{W}{\Delta t} \qquad \text{(average power)} \quad (7\text{–}34)$$

The instantaneous power, P, is the limiting value of this ratio, for a time interval approaching zero.

$$P = \operatorname*{limit}_{\Delta t \to 0} \frac{W}{\Delta t} \qquad \text{(instantaneous power)} \quad (7\text{–}35)$$

In our previous discussion of work and energy, we did not consider the time during which work is performed, or the rate at which work is performed. But this is an important consideration in many applications. For example, suppose you carry a heavy piece of furniture up a flight of stairs. The work required is the same, whether you walk or run up the stairs. However, you may find it difficult or impossible to run with such a heavy load. The key difference is that if you run, the rate at which work is done is much greater; that is, the power required is much greater. And you may not be capable of producing that much power.

Units

The SI unit of power is the J/s, which is called the "watt" (abbreviated W), in honor of James Watt, the inventor of the steam engine.

$$1\ \text{W} = 1\ \text{J/s} \tag{7–36}$$

In the British system, the unit of power is the ft-lb/s. The horsepower, abbreviated hp, is a larger, more commonly used unit.

$$1\ \text{hp} = 550\ \text{ft-lb/s} \tag{7–37}$$

This definition was introduced by Watt, based on his estimate of the maximum average power that could be delivered by a typical horse, over a period of a work day. One horsepower equals approximately three fourths of a kilowatt, or more precisely

$$1\ \text{hp} = 746\ \text{W} \tag{7–38}$$

A convenient unit of work or energy is the kilowatt-hour, abbreviated kWh. It is defined as the work or energy delivered at the rate of 1 kilowatt for a period of 1 hour. Since $W = \overline{P}\,\Delta t$,

$$1\ \text{kWh} = (1\ \text{kW})(1\ \text{h}) = (10^3\ \text{W})(3.60 \times 10^3\ \text{s})$$

$$1\ \text{kWh} = 3.60 \times 10^6\ \text{J} \tag{7–39}$$

The kilowatt-hour is commonly used by utility companies to measure the use of electrical energy. For example, if you are using electrical energy at the rate of 2 kW for a period of 10 hours, your energy consumption is 20 kWh.

EXAMPLE 12 The Power Required to Lift a Chamber From the Ocean Floor

A deep sea, underwater observation chamber is raised from the bottom of the ocean, 1 mile below the surface, by means of a steel cable. The chamber moves upward at constant velocity, reaching the surface in 5.00 minutes. The cable is under a constant tension of 2.00×10^3 lb. Find the power output required of the electric motor that pulls the cable in.

SOLUTION The power output of the motor is the rate of production of work by the tension force it supplies.

$$\overline{P} = \frac{W}{\Delta t} = \frac{F_x\,\Delta x}{\Delta t} = \frac{(2.00 \times 10^3\ \text{lb})(5280\ \text{ft})}{(5.00\ \text{min})(60\ \text{s/min})}$$

$$= (3.52 \times 10^4\ \text{ft-lb/s})\left(\frac{1\ \text{hp}}{550\ \text{ft-lb/s}}\right)$$

$$= 64.0\ \text{hp}$$

It is sometimes convenient to express the instantaneous power in terms of the instantaneous speed of the body on which work is performed. Since the work done by force **F** during a displacement Δs is $F_s\,\Delta s$, the power P may be written

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \lim_{\Delta t \to 0} \frac{F_s\,\Delta s}{\Delta t}$$

or, since speed $v = \lim_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t}$,

$$\boxed{P = F_s v} \tag{7–40}$$

EXAMPLE 13 The Power Required to Drive Uphill or to Accelerate

(a) A car weighing 1.00×10^4 N travels at a constant velocity of 20.0 m/s up a 5.00° incline. Find the power that must be supplied by the force moving the car up the hill, assuming negligible friction and air resistance. (b) Find the power that must be supplied to accelerate the car at 0.100g on level ground when its speed is 10.0 m/s.

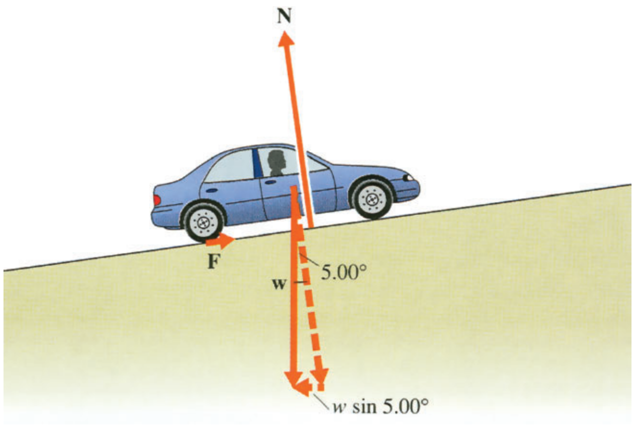


Fig. 7–27

SOLUTION (a) Since the car's velocity is constant, the road must exert a force **F** that balances the component of weight down the incline (Fig. 7–27).

$$F = w \sin 5.00°$$

The power delivered by this force is found by applying Eq. 7–40.

$$P = F_s v = (w \sin 5.00°)v$$
$$= (1.00 \times 10^4 \text{ N})(\sin 5.00°)(20.0 \text{ m/s})$$
$$= 1.74 \times 10^4 \text{ W}$$

or, since 1 hp = 746 W,

$$P = (1.74 \times 10^4 \text{ W})\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 23.4 \text{ hp}$$

(b) From Newton's second law, we know that the force **F** accelerating the car has magnitude

$$F = ma = m(0.100g) = 0.100w$$

The power supplied by this force is

$$P = F_s v = (0.100w)v$$
$$= (0.100)(1.00 \times 10^4 \text{ N})(10.0 \text{ m/s})$$
$$= 1.00 \times 10^4 \text{ W} \quad \text{(or 13.4 hp)}$$

A Closer Look

The Energy to Run

Why is it so much harder to run than to ride a bicycle at the same speed? When you ride a bicycle, it is after all your own body that produces your motion, just as when you run. And yet cycling requires much less effort than running. After 30 minutes or an hour of running along a level road at a moderate pace, even a well-conditioned runner may tire, whereas a cyclist can keep the same pace with little effort (Fig. 7–A). In everyday language, we say that "running burns calories" or that "running uses a lot of energy." To understand the physical basis of such expressions, to see why running requires so much energy and is so much less energy efficient than bicycle riding, we shall apply concepts of work and energy to the human body. In the next (optional) section, we shall show in detail how to extend concepts of work and energy to systems of particles such as human bodies and machines. Here we shall simply describe in a general way how energy is used by the body when muscles contract and specifically how that energy is used in running and cycling.

The following are some general properties of work and energy associated with muscular exertion:

Work Done by Muscles

Muscles consist of bundles of muscle fibers. Under tension, these fibers can shorten, or "contract," as protein filaments within the fibers slide over each other. (See Chapter 10, Fig. 10–4 for a detailed description of the mechanism of muscle contraction.) Contraction of a muscle fiber means that a force (the tension in the muscle fiber) acts through a distance (the distance the fiber contracts). Hence work is done by contracting muscle fibers. The direct effect of a muscle's contraction may be to move one of the body's limbs. The moving limb, in turn, may exert a force on the surroundings and do work on the surroundings. For example, if you hold a weight in your hand and contract the biceps muscle in your arm, your hand and forearm swing upward, raising the weight. The work done by your biceps muscle is approximately equal to the work done by the force your hand exerts on the weight. The effect of this work is to increase the weight's gravitational potential energy. (See Chapter 10, Example 3 and Problem 36 for details.)

Heat Generated by the Body When Muscles Contract

Heat, a disordered form of energy, is generated whenever muscles do work. Typically the quantity of heat generated when muscles contract is about three times as great as the work done by the muscles. When your muscles do very much work, you can usually feel the heat generated by your body. You may begin to sweat, which is a way the body gets rid of excess heat.

Internal Energy of the Body

The body's internal energy is the total energy of all the particles within the body. Chemical reactions within the body provide the energy necessary to produce muscle contraction. The energy released by these chemical reactions produces the work and heat associated with muscle contraction. The body thereby loses some of its internal energy. Conservation of energy implies that the body's loss of internal energy equals the sum of the work and heat generated.

Loss of internal energy =
Work done by muscles +
Heat generated

When your body loses much internal energy in a short time interval, you tend to feel tired. Your body's internal energy is replenished by the consumption of food.

Now we can use these basic concepts of work and energy to understand why cycling requires less energy than running. A good bicycle is an exceptionally efficient means of using the body's internal energy to produce motion. Suppose you ride a high-tech bicycle with thin, well-inflated tires and very little friction in its moving parts. Riding such a bike over flat, level pavement at, say, 10 km/h, requires little effort. Once moving, both the kinetic energy and the gravitational potential energy of the bicycle and your body stay constant with just a little pedaling required.

Fig. 7–A An exhausted runner and a still fresh cyclist have traveled the same distance at the same speed.

Continued.

At such a low bike speed, there is not much air resistance. Consequently, only a little work needs to be done by your legs as they push against the pedals and your body loses little internal energy in producing this small amount of work. The work that is done by your legs is needed to compensate for the small negative work done by friction and air resistance. If you did not pedal at all, your bike would gradually slow down.

If you were to ride a bike uphill or at a much higher speed, or if your tires were not well inflated, or if there were much friction in your wheel bearings, you would have to do considerably more work.

In contrast to riding a bike, when you run on a flat, level surface, your kinetic energy and gravitational potential energy can never be exactly constant. Watch a runner and you will see that the runner's head moves up and down somewhat, an indication of some change in elevation of the runner's center of mass. This means that the runner's gravitational potential energy is not constant. Some of that energy is lost each time the runner's body moves downward, and this energy must then be supplied as the body moves upward again. More efficient runners, especially champion marathoners, bob up and down less than average runners do and thereby use less energy.

A runner's center-of-mass kinetic energy also necessarily varies somewhat, again in contrast to that of a cyclist. Although this effect is more difficult to see, a runner's center of mass continually alternates between speeding up and slowing down with each stride. Although the variation in center-of-mass speed is slight, it does require a significant amount of work for the legs to increase the center-of-mass kinetic energy from the minimum value to the maximum value during each stride.

Fig. 7–B The runner's center of mass moves from point P to point P′, increasing elevation by a distance *h*.

Fig. 7–C The road exerts a backward force on a runner's foot as it hits the surface. This force tends to reduce the runner's speed.

To obtain a more detailed understanding of just how runners use energy, experimental studies have been performed using force platforms and high-speed photography (see Chapter 4, Example 11). Such studies* indicate that the body's internal work is used in four ways:

1 To raise the body's center of mass a few centimeters each step, increasing **gravitational potential energy**. Fig. 7–B indicates how running tends to produce an up-and-down motion. Gravitational potential energy is lost as the center of mass falls to its original level, with the energy being converted first into kinetic energy and then lost as the foot strikes the ground. So even though the ground you run on may be level, you are in a sense always going uphill. Of course, if you do actually run up a hill, your leg muscles must do more work to provide extra gravitational potential energy. Running downhill can reduce the work your legs do.

2 To increase slightly the body's center-of-mass speed and hence its **center-of-mass kinetic energy** at the beginning of each stride. This is necessary because as each stride is completed a backward force must be exerted on the forward foot by the road to stop the foot's forward motion and begin its backward motion. The entire body then experiences a backward external force (Fig. 7–C). This force slightly decelerates the runner's center of mass, which then must be accelerated again to its maximum speed. This is accomplished by the leg muscles doing work, pushing the foot backward against the road, so that the road now pushes the runner forward.

*See Alexander R McN: *Biomechanics*, 1975, Halstead Press, New York, pp 28–30.

3 To provide the kinetic energy of the legs as they swing back and forth. This energy, called **rotational kinetic energy,** is lost as the foot strikes the ground and must be provided by the leg muscles that do work as they push the legs back and forth relative to the body's center of mass. (Rotational kinetic energy will be discussed in Chapter 9, Section 9–4.)

4 To compensate for the negative work done by **air resistance**. If you run at a slow or moderate pace with no wind, air resistance is not a very significant factor—certainly less significant than the other three. However, if you run directly into a strong wind, air resistance can become very significant, sometimes requiring more work than any of the other forms of energy used in running.

In Section 9–6, we shall use a rough mechanical model of running to estimate the power that must be provided by a runner's legs for each of the four types of energy use we have just described. There, for a running speed of 3.0 m/s (9 minutes per mile), we obtain the following estimates:

1 Center-of-mass potential energy, 94 W

2 Center-of-mass kinetic energy, 77 W

3 Rotational kinetic energy of legs, 58 W

4 Air resistance, 11 W

This gives a total power estimate of 240 W, or about one third horsepower! Although this is just a crude estimate,* it does give some idea of the considerable energy we use when we run.

*Studies of the power output of muscular forces have been made, using situations where this power is easily measured directly. For example, in the case of cycling, the power output of the muscular forces is very nearly the power delivered to the bike's pedals, and this power can be measured. The maximum power output that the human body can produce depends very much on the strength and conditioning of the individual. Exceptional athletes can maintain a power output of $\frac{1}{3}$ hp for about 1 hour, or 1 hp for about 1 minute, or 2 hp for about 6 seconds.

*7–7 Energy of a System of Particles

When a gymnast performs the "iron cross" (Fig. 7–28), his body is stationary and therefore no work is done by any force he exerts on his surroundings. And yet the gymnast's muscles tire after a few seconds of performing this difficult feat. Energy is used because the muscle fibers are under tension and are continually contracting and relaxing. Work is done by these internal tension forces, and energy must be supplied by the gymnast's body to do this work. In an example such as this, energy does not belong to a single particle. Instead, we have to regard the total energy as being distributed over the system of particles in the gymnast's body. We need to extend our treatment of energy to systems of particles so that we will be able to introduce some of the most interesting applications of energy concepts, such as work done by internal forces within a human body or a machine.

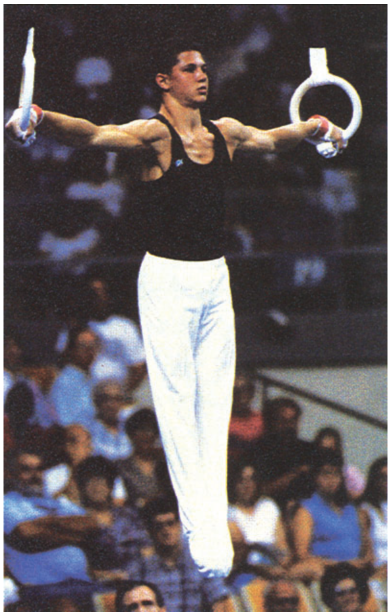


Fig. 7–28 A gymnast performs the iron cross.

Eq. 7–33 ($\Sigma\, W_{\text{nc}} = \Delta E$) applies to each particle in a system of n particles; that is, a particle's increase in mechanical energy equals the net work done by internal or external forces acting on that particle. If we write out this equation for each particle of the system and add the equations, we obtain a useful equation, applicable to the system as a whole.

$$\Sigma\, W_{\text{nc},1} = \Delta E_1 \quad \text{(for particle 1)}$$
$$\Sigma\, W_{\text{nc},2} = \Delta E_2 \quad \text{(for particle 2)}$$
$$\vdots$$
$$\Sigma\, W_{\text{nc},1} + \Sigma\, W_{\text{nc},2} + \cdots = \Delta E_1 + \Delta E_2 + \cdots = \Delta(E_1 + E_2 + \cdots) \qquad (7\text{–}41)$$

We define the system's energy E to be the **sum of the single particle energies.**

$$E = E_1 + E_2 + \cdots \tag{7–42}$$

If we now let $\Sigma\, W_{nc}$ denote the **sum of the work done on all particles** of the system, we can express Eq. 7–41 as

$$\Sigma\, W_{nc} = \Delta E \tag{7–43}$$

This equation looks identical to Eq. 7–33 for a single particle. Here, however, we interpret E as the energy of a system and $\Sigma\, W_{nc}$ as the total work done on the system.

Sometimes internal forces do work, for example, forces within the muscles of the gymnast in Fig. 7–28, or forces on the pistons in the cylinders of an automobile engine. However, internal work is not present in a rigid body, a body in which there is no relative motion of the body's particles. This follows from the fact that the internal forces occur in oppositely directed, action-reaction pairs. Interacting particles within a rigid body experience the same displacement. Therefore the work done on one particle by an internal force **F** is the negative of the work done on the other interacting particle by the reaction force −**F**. The net work is then zero.

The nonconservative forces acting on a body deliver average power, $\Sigma\, \overline{P}_{nc}$, which is the net work per unit time performed by these forces.

$$\Sigma\, \overline{P}_{nc} = \Sigma\, \frac{W_{nc}}{\Delta t} = \frac{\Sigma\, W_{nc}}{\Delta t}$$

Using Eq. 7–43 ($\Sigma\, W_{nc} = \Delta E$), this may be expressed

$$\Sigma\, \overline{P}_{nc} = \frac{\Delta E}{\Delta t} \tag{7–44}$$

EXAMPLE 14 The Power Required to Run Up a Flight of Stairs

Compute the mechanical power provided by internal forces within the body of a person of mass 80.0 kg who runs up a flight of stairs, rising a vertical distance of 3.00 m in 3.00 s.

SOLUTION The only nonconservative forces doing work on the body are internal forces within the body. We find the average power output of these forces, $\overline{P}_{int}$, by applying Eq. 7–44.

$$\overline{P}_{int} = \Sigma\, \overline{P}_{nc} = \frac{\Delta E}{\Delta t}$$

If the body's kinetic energy is approximately constant during the climb, the only change in energy is the increase in gravitational potential energy.

$$\Delta E = \Delta U_G = mgy_f - mgy_i = mg(y_f - y_i)$$

Inserting this into the expression for power, we obtain

$$\overline{P}_{int} = \frac{mg(y_f - y_i)}{\Delta t} = \frac{(80.0\ \text{kg})(9.80\ \text{m/s}^2)(3.00\ \text{m})}{3.00\ \text{s}}$$

$$= 784\ \text{W} \quad (\text{or } 1.05\ \text{hp})$$

EXAMPLE 15 Energy and Power Required for Hill Climbing on a Bicycle

A cyclist rides up Latigo Canyon, rising from sea level to a final elevation of 869 m (2850 ft) (Fig. 7–29). The combined weight of the cyclist and the bicycle is 825 N (185 lb).

(a) Find the increase in mechanical energy in the system of the cyclist and the bicycle.

(b) What is the minimum work done by internal forces in the cyclist's body?

(c) How much power is supplied by the cyclist if he reaches the top in 1 hour?

Fig. 7–29 The cyclist is ascending Latigo Canyon in Malibu, California. Beginning at sea level the road rises 2850 ft over a distance of 7 miles.

SOLUTION **(a)** Kinetic energy is approximately constant, and so the increase in mechanical energy equals the increase in the cyclist's gravitational potential energy, which is quite large.

$$\Delta E = \Delta U_G = mg(\Delta y) = (825\text{ N})(869\text{ m})$$
$$= 7.17 \times 10^5\text{ J}$$

(b) There are several nonconservative forces acting on and within the system. The external forces of air resistance and road friction both do negative work on the system. In addition there is some internal friction in the bicycle wheel bearings and crank; these forces also do negative work. It is the positive work done by tension forces in the cyclist's muscles that is responsible for the increase in mechanical energy. In the most ideal case of negligible friction and air resistance, the work W_{int} done by the cyclist's muscles equals the net nonconservative work. According to Eq. 7–43, this equals the system's increase in mechanical energy:

$$W_{int} = \Sigma W_{nc} = \Delta E = 7.17 \times 10^5\text{ J}$$

More realistically, the internal work of the muscles must be somewhat greater, since part of it is used to balance the negative work done by friction and air resistance.

The source of energy here is stored "internal energy" in the cyclist's body. We shall study internal energy in Chapter 14 on thermodynamics. Here we simply note that the human body is at best only about 25% efficient; that is, the work performed by the muscles equals about 25% of the internal energy used by the body (the other 75% is converted to heat). Thus the loss in internal energy equals $4(7.17 \times 10^5\text{ J}) = 2.87 \times 10^6$ J, or 685 Calories (about the number of Calories provided by a half-pint of Häagen-Dazs ice cream).

(c) The power is found when we divide the work by the time interval.

$$P = \frac{W_{int}}{\Delta t} = \frac{7.17 \times 10^5\text{ J}}{3600\text{ s}} = 199\text{ W} \quad \text{(or 0.27 hp)}$$

CHAPTER 7 SUMMARY

The kinetic energy of a particle of mass m moving at speed v is given by

$$K = \tfrac{1}{2}mv^2$$

The work done on a particle is given by

$$W = \Sigma\,(F_s\,\Delta s)$$

where the sum is over short intervals of length Δs, and F_s is the component of the force along the interval. If the path is linear and the force constant,

$$W = F_x\,\Delta x$$

Work and kinetic energy are related through the work-energy theorem, which states that a particle's increase in kinetic energy equals the net work done by the force acting on the particle.

$$\Delta K = W_{\text{net}}$$

A force is called conservative if the work done by it can be expressed as a decrease in some kind of potential energy. The gravitational force and the spring force are both conservative. Their respective potential energies are:

$$U_G = -\frac{GmM}{r}$$

or

$$U_G = mgy \qquad \text{(if the distance from the center of the earth is essentially constant)}$$

and

$$U_S = \tfrac{1}{2}kx^2$$

Friction is a nonconservative force.

The total mechanical energy is defined to be the sum of the kinetic energy and the various potential energies.

$$E = K + U = K + U_G + U_S + \cdots$$

If only conservative forces do work on a particle, its total mechanical energy is conserved.

$$E_f = E_i \qquad \text{(conservative forces only)}$$

More generally, a particle's mechanical energy may increase or decrease, if positive or negative work is done by nonconservative forces.

$$\Sigma\,W_{nc} = \Delta E = E_f - E_i$$

This equation may also be applied to a system of particles if the energy E is interpreted as the sum of single particle energies.

$$E = E_1 + E_2 + \cdots + E_n$$

Power is the rate at which work is done.

Average power $$\overline{P} = \frac{W}{\Delta t}$$

Instantaneous power $$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t}$$

The instantaneous power provided by a force **F** to a body moving at speed v may be expressed

$$P = F_s v$$

The net power provided by nonconservative forces may be expressed

$$\Sigma\,\overline{P}_{nc} = \frac{\Delta E}{\Delta t}$$

Questions

1 Can the kinetic energy of a body ever be negative?

2 You drive your car along a curving road at constant speed.
(a) Does your kinetic energy change?
(b) Is the work done by the force accelerating your car positive, negative, or zero?

3 (a) Does the kinetic energy of a body depend on the reference frame of the observer?
(b) Does the work done on a body depend on the reference frame of the observer?

4 A child on a skateboard grabs the rear bumper of a car and is towed up a hill. The speed of the car is 20 mph at the bottom and 10 mph at the top of the hill. Is the work done on the child and skateboard by the following forces positive, negative, or zero: (a) force of the bumper on the child; (b) force of gravity on the child; (c) resultant force on the child?

5 Fig. 7–15 shows a skier skiing down a hill. Could the skier have chosen another path between the same two points such that (a) the work done by gravity was greater; (b) the work done by friction was greater?

6 Two objects are simultaneously released from the same height. One falls straight down, and the other slides without friction down a long inclined plane.
(a) Do both have the same acceleration?
(b) Do both have the same final speed?
(c) Do both take the same time to descend?

7 A satellite goes from a low circular earth orbit to a higher circular earth orbit.
(a) Does the gravitational force on the satellite increase, decrease, or remain the same?
(b) Does the satellite's gravitational potential energy increase, decrease, or remain the same?

8 Can the work done by a force always be expressed as a decrease in potential energy?

9 A boy rides a bicycle along level ground at *approximately* constant velocity, without pedaling. Is mechanical energy approximately conserved?

10 A boy rides a bicycle along level ground at constant velocity, pedaling at a steady rate.
(a) Is mechanical energy conserved?
(b) Is there any work done by individual nonconservative forces?
(c) Is there any net work done by nonconservative forces?

11 Suppose you run up a hill at constant speed.
(a) Is there net work done on your body?
(b) Is there a net nonconservative work done on your body?
(c) Is your mechanical energy conserved?

12 (a) As you drive a car from the top of a mountain to its base, is the car's mechanical energy conserved? Explain.
(b) Would you expect to get better gas mileage than on a flat road?

13 A fountain of water shoots high in the air. The water then falls back into a surrounding pool.
(a) Describe the transformation of energy the water undergoes, beginning with the water shooting upward at the base of the fountain.
(b) Is the mechanical energy of the water conserved as the water completes its cycle?
(c) What provides the nonconservative force that does positive work on the water?

14 A worker raises a load of bricks from the ground to a platform. The worker can lift one brick at a time or all the bricks together.
(a) In which case is the work greater, or is it the same in either case? Neglect the work done in raising the body each time he bends over.
(b) Take into account the work done in raising the body. In which case is the work greater?
(c) In which case is the required power greater?

15 Given that an automobile can develop only a limited amount of power, does this put a limit on the maximum slope of a mountain road that a given automobile can drive up at a given speed?

16 Weight lifters find that the greatest gains in strength (and the greatest muscle soreness) occur as a result of doing "negatives," that is, doing negative work on very large weights. Is negative work done by raising or lowering a weight?

Answers to Odd-Numbered Questions

1 no; **3** (a) yes; (b) yes; **5** (a) no; (b) yes; **7** (a) decrease; (b) increase; **9** yes; **11** (a) no; (b) yes; (c) no; **13** (a) kinetic energy to gravitational potential energy to kinetic energy to heat; (b) yes; (c) the water pump; **15** yes

Problems (listed by section)

7–1 Work and Kinetic Energy

1 Find the kinetic energy of (a) a bullet of mass 5.00 g traveling at a speed of 300 m/s; (b) a woman of mass 50.0 kg running at a speed of 9.00 m/s; (c) a car of mass 1.00×10^3 kg moving at a speed of 20.0 m/s.

2 The moon has a mass of 7.36×10^{22} kg and moves about the earth in a circular orbit of radius 3.80×10^8 m with a period of 27.3 days.
(a) Find the moon's kinetic energy as observed on earth.
(b) Would the moon's kinetic energy be the same from the sun's reference frame?

3 A man of mass 80.0 kg walks down the aisle of an airplane at a speed of 1.00 m/s in the forward direction while the plane moves at a speed of 300 m/s relative to the earth. Find the man's kinetic energy relative to (a) the plane; (b) the earth.

4 Suppose you carry a bag of groceries weighing 125 N from your car to your kitchen, a distance of 50 m, without raising or lowering the bag.
(a) What is the work done by the force you exert on the bag?
(b) Would the work be different if your kitchen were in an upstairs apartment?

5 Tarzan, who weighs 875 N, swings from a vine through the jungle. How much work is done by the tension in the vine as he drops through a vertical distance of 4.00 m?

6 One boat tows another boat by means of a tow line, which is under a constant tension of 500 N. The boats move at a constant speed of 5.00 m/s. How much work is done by the tension in 1.00 min?

7 You lift a box weighing 200 N from the floor to a shelf 1.50 m above.
(a) What is the minimum work done by the force you exert on the box?
(b) When would the work be greater than this minimum?

8 A weight lifter raises a 900 N weight a vertical distance of 2.00 m. Compute the work done by the force exerted on the weight by the weight lifter.

9 You are loading a refrigerator weighing 2250 N onto a truck, using a wheeled cart. The refrigerator is raised 1.00 m to the truck bed when it is rolled up a ramp. Calculate the minimum work that must be done by the force you apply and the magnitude of the force if the ramp is at an angle with the horizontal of (a) 45.0°; (b) 10.0°.

10 A man drags a table 4.00 m across the floor, exerting a constant force of 50.0 N, directed 30.0° above the horizontal.
(a) Find the work done by the applied force.
(b) How much work is done by friction? Assume the table's velocity is constant.

11 The driver of a 1500 kg car, initially traveling at 10.0 m/s, applies the brakes, bringing the car to rest in a distance of 20.0 m.
(a) Find the net work done on the car.
(b) Find the magnitude and direction of the force that does this work. (Assume this force is constant.)

12 A child on a sled is initially at rest on an icy horizontal surface. The sled is pushed until it reaches a final velocity of 6.00 m/s in a distance of 15.0 m. The coefficient of friction between the ice and runners of the sled is 0.200, and the weight of the child and the sled is 350 N. Find the work done by the force pushing the sled.

***13** Air bags are used in cars to decelerate the occupants slowly when a car is suddenly decelerated in a crash.
(a) Compute the work done by the decelerating force acting on a 55.0 kg driver if the car is brought to rest from an initial speed of 20.0 m/s.
(b) Find the minimum thickness of the air bag if the average decelerating force is not to exceed 8900 N (2000 lb), and the center of the car moves forward 0.800 m during impact.

***14** A particle moves along the x-axis from $x = 0$ to $x = 4$ m while acted upon by a force whose x component is given in Fig. 7–30. Estimate the work done by the force.

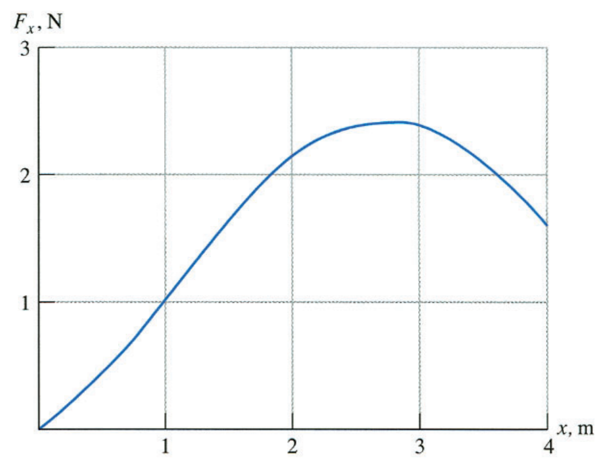


Fig. 7–30

7–2 Gravitational Potential Energy; Constant Gravitational Force

15 Suppose you are driving in the High Sierras from Mammoth Mountain to the Owens Valley 1500 m below. If the mechanical energy of your car were conserved, what would be your approximate final speed?

16 A small weight is suspended from a string of negligible weight and given an initial horizontal velocity of 2.00 m/s, with the string initially vertical. Find the maximum angle θ that the string makes with the vertical if the string is 1.00 m long.

17 You ski straight down a 45.0° slope, starting from rest and traveling a distance of 10.0 m along the slope. Find your final velocity, assuming negligible air resistance and friction.

18 A skier of mass 70.0 kg rides a ski lift to the top, which is 500 m higher than the base of the lift.
(a) Find the increase in the skier's gravitational potential energy.
(b) Find the minimum work done by the force exerted on the skier by the lift.
(c) When the skier skis down the run, what would her final velocity be if no force other than gravity did work? What other forces do work?

19 A cyclist coasts up a 10.0° slope, traveling 20.0 m along the road to the top of the hill. If the cyclist's initial speed is 9.00 m/s, what is the final speed? Ignore friction and air resistance.

***20** On a ski jump a skier accelerates down a ramp that curves upward at the end, so that the skier is launched through the air like a projectile. The objective is to attain the maximum distance down the hill. Prove that the vertical drop h must equal at least half the horizontal range R (Fig. 7–31).

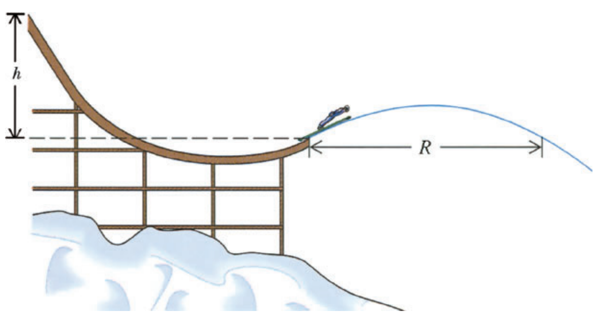


Fig. 7–31

21 Find the minimum work required to carry a truckload of furniture weighing 2.00×10^4 N to a third-story apartment, 20.0 m above the truck.

22 A ladder 2.50 m long, weighing 225 N, initially lies flat on the ground. The ladder is raised to a vertical position. Compute the work done by the force lifting the ladder. The ladder's center of gravity is at its geometric center.

23 What is the average force exerted on the diver in Fig. 7–7a by the diving board, if she weighs 700 N and accelerates from rest to a speed of 4.00 m/s while moving 0.300 m upward.

7–3 Gravitational Potential Energy; Variable Gravitational Force

24 Compute the escape velocity on Jupiter, which has a radius of 7.14×10^7 m and mass 318 times the earth's mass.

25 Find the minimum initial speed of a projectile in order for it to reach a height of 2000 km above the surface of the earth.

26 Suppose a rocket is at an elevation of 100 km and has an initial velocity of 1.00×10^4 m/s, directed vertically upward. If the rocket engines do not burn and no force other than the earth's gravity acts on the rocket, how far does it go?

27 If a space probe has a speed of 2.00×10^4 m/s as it leaves the earth's atmosphere, what is its speed when it is far from the earth?

***28** The Little Prince is a fictional character who lives on a very small planet (Fig. 7–32). Suppose that the planet has a mass of 2.00×10^{13} kg and a radius of 1.00×10^3 m.
(a) How long would it take for an object to fall from rest a vertical distance of 1.00 m?
(b) Suppose the Little Prince throws a ball vertically upward, giving it an initial velocity of 1.00 m/s. What would be the maximum height reached by the ball? (HINT: Don't assume g to be constant.)
(c) At what speed could a ball be thrown horizontally so that it would travel in a circular orbit just above the surface of the planet?

Fig. 7–32 The Little Prince.

*29 A satellite of mass m is in a circular earth orbit of radius r.
(a) Find an expression for the satellite's mechanical energy.
(b) Calculate the satellite's energy and speed if $m = 1.00 \times 10^4$ kg and $r = 1.00 \times 10^7$ m.

*30 Halley's comet is in an elongated elliptical orbit around the sun and has a period of about 76 years. Last seen in 1986, it will again be close to the sun and the earth in 2061. The comet's maximum distance from the sun is 5.3×10^{12} m, at which point (called "aphelion") its speed is 910 m/s.
(a) Find its speed when it is at its point of closest approach (perihelion), 8.8×10^{10} m from the sun, which has a mass of 2.0×10^{30} kg.
(b) The radius of the earth's orbit is 1.5×10^{11} m, a distance defined as an astronomical unit. Estimate the time required for Halley's comet to travel a distance equal to one astronomical unit, when it is near perihelion.

7–4 Spring Potential Energy; Conservation of Energy

31 A spring with a force constant of 1500 N/m is compressed 10.0 cm. Find the work done by the force compressing the spring.

32 When an archer pulls an arrow back in his bow, he is storing potential energy in the stretched bow.
(a) Compute the potential energy stored in the bow, if the arrow of mass 5.00×10^{-2} kg leaves the bow with a speed of 40.0 m/s. Assume that mechanical energy is conserved.
(b) What average force must the archer exert in stretching the bow if he pulls the string back a distance of 30.0 cm?

33 A toy consists of a plastic head attached to a spring of negligible mass. The spring is compressed a distance of 2.00 cm against the floor, and then the toy is released. The toy has a mass of 100 g and rises to a height of 60.0 cm above the floor. What is the spring constant?

34 An elevator car of mass 800 kg falls from rest 3.00 m, hits a buffer spring, and then travels an additional 0.400 m, as it compresses the spring by a maximum of 0.400 m. What is the force constant of the spring?

35 A 4.00 kg block starts from rest and slides down a frictionless incline, dropping a vertical distance of 3.00 m, before compressing a spring of force constant 2.40×10^4 N/m. Find the maximum compression of the spring.

36 A mass of 0.250 kg is attached to the end of a massless spring of unknown spring constant. The mass is dropped from rest at point A, with the spring initially unstretched. As the mass falls, the spring stretches. At point B the mass is as shown in Fig. 7–33.
(a) Find the force constant of the spring.
(b) Find the magnitude of the acceleration of the mass at point B.

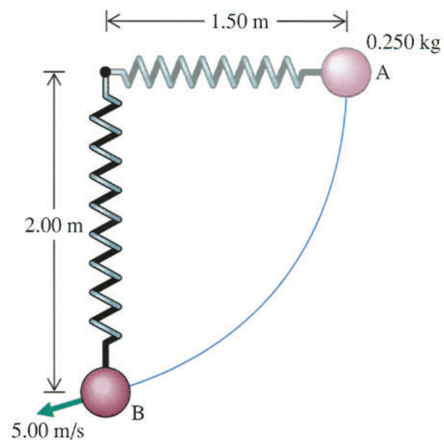


Fig. 7–33

37 A pole-vaulter begins a jump with a running start. He then plants one end of the pole and rotates his body about the other end, thereby rising upward (Fig. 7–34). At the beginning of the vault the pole bends, storing potential energy somewhat in the manner of a compressed spring. Near the top of the arc, the pole unbends, releasing its potential energy and pushing the pole-vaulter higher. With what speed must the pole-vaulter approach the bar if he is to raise his center of mass 5.00 m? Assume that mechanical energy is conserved. The world record in the pole vault is 6.0 m, and the fastest speed achieved by a runner is about 10 m/s.

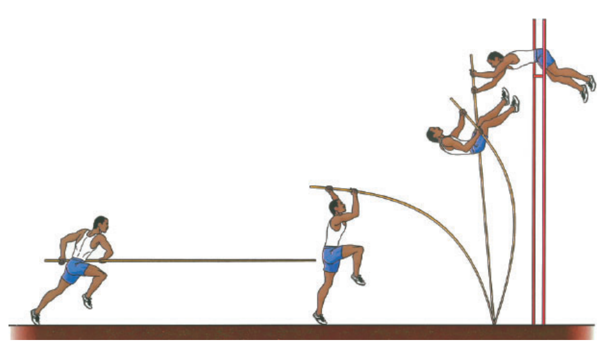


Fig. 7–34

7–5 Conservative and Nonconservative Forces

38 A 1.00 kg block starts from rest at the top of a 20.0 m long 30.0° incline. Its kinetic energy at the bottom of the incline is 98.0 J. How much work is done by friction?

39 A 1.00 kg block slides down a 20.0 m long 30.0° incline at constant velocity. How much work is done by friction?

40 A ball of mass 0.300 kg is thrown upward, rising 10.0 m above the point at which it was released. Compute the average force exerted on the ball by the hand, if the hand moves through a distance of 20.0 cm as the ball is accelerated.

41 A person jumps from a burning building onto a fireman's net 15.0 m below. If the average force exerted by the net on the person is not to exceed 20 times the body weight, by how much must the center of the net drop as the person comes to rest?

***42** A skier skis down a steep slope, maintaining a constant speed by making turns back and forth across the slope as indicated in Fig. 7–35. The side edge of the skis cuts into the snow so that there is no chance of sliding directly down the slope; that is, there is a large static frictional force perpendicular to the length of the skis. There is a much smaller kinetic friction along the length of the skis. If the coefficient of kinetic friction is 0.10, what must be the total length of the skier's tracks as he drops a vertical distance of 100 m down a 40° slope at constant speed?

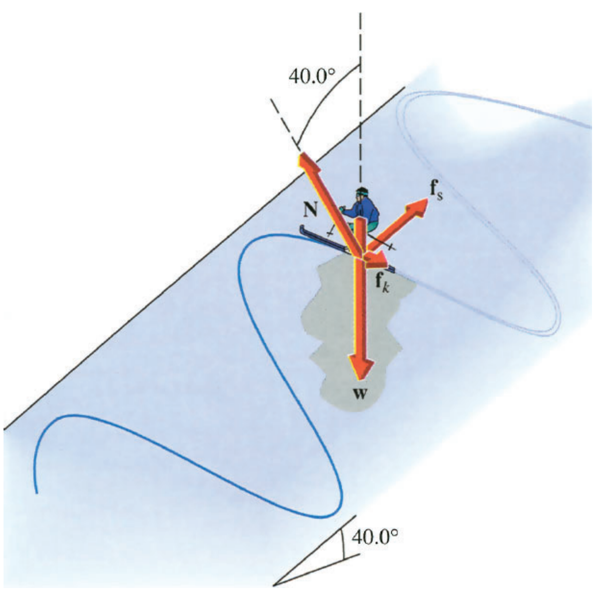


Fig. 7–35

43 A cyclist competing in the Tour de France coasts down a hill, dropping through a vertical distance of 30.0 m. The cyclist has an initial speed of 8.00 m/s and a final speed of 20.0 m/s. What fraction of the cyclist's initial mechanical energy is lost? What nonconservative forces cause this?

7–6 Power

44 How much work could be performed by a 746 W (1 hp) motor in 1 hour?

45 How much mechanical power must be supplied by a car to pull a boat on a trailer at a speed of 20.0 m/s if the force exerted by the car on the trailer is 2000 N?

46 A weight lifter raises a 1000 N weight a vertical distance of 2.00 m in a time interval of 2.00 s. Compute the power provided by the weight lifter's force.

47 Find the weight that could be lifted vertically at the constant rate of 10.0 ft/s, using the mechanical power provided by a 3.00 hp motor.

48 Ten boxes, each 20.0 cm high and weighing 200 N, initially are all side by side on the floor. The boxes are lifted and placed in a vertical stack 2.00 m high in a time interval of 5.00 s. Compute the power necessary to stack the boxes.

***49** Compute the minimum power necessary to operate a ski lift that carries skiers along a 45.0° slope. The lift carries 100 skiers of average weight 700 N at any one time, at a constant speed of 5.00 m/s.

50 (a) Compute the electrical energy used by a household whose monthly electrical bill is $30, computed at the rate of $0.06 per kWh.
(b) How long could this energy be used to burn ten 100 W light bulbs?
(c) If this energy were used to raise a car of mass 2000 kg, to what height would the car be raised?

51 The total annual use of energy in the United States is approximately 10^{19} J. Solar energy provides 1400 W per square meter of area in direct sunlight if this area is perpendicular to the sun's rays. Suppose that solar energy could be used with 100% efficiency. Find the total area of solar energy collectors needed to provide the nation's energy needs if on an average day these collectors could be used for 8.0 hours.

*7–7 Energy of a System of Particles

52 A hiker weighing 575 N carries a 175 N pack up Mt. Whitney (elevation, 4420 m), increasing her elevation by 3000 m.

(a) Find the minimum internal work done by the hiker's muscles.

(b) If she is capable of producing up to 746 W (1.0 hp) for an extended time, what is the minimum time for her to ascend?

53 Find the minimum internal work that must be done by the muscles of a shot-putter to impart an initial velocity to a 16.0 lb shot sufficient to give it a horizontal range of 60.0 ft.

54 A pole-vaulter of mass 80.0 kg is initially at rest before beginning his approach to the bar. He is instantaneously at rest at the high point of his jump, having raised his center of mass 5.00 m. Find the minimum internal work done by the vaulter's muscles.

55 Two blocks of mass 100 g each are initially at rest on a frictionless horizontal surface. The blocks are in contact with opposite ends of a spring of force constant 500 N/m, which is compressed 20.0 cm. Find the final speed of each block, after the spring is allowed to expand.

***56** A car's engine develops mechanical power at the rate of 30.0 hp while moving along a level road at a speed of 60.0 mi/h. Half the mechanical energy developed by the engine is delivered to the wheels, with the remainder being wasted because of internal friction. Find the mechanical power developed by the engine in order for the 2500 lb car to travel at the same speed up a 6.00% grade ($\sin \theta = 6.00 \times 10^{-2}$).

Fig. 7–36 This house in the Bavarian forest near Munich derives its electric power from a small generator, powered by water from a canal.

***57** A small canal diverts water from the Perlbach, a river in Bavaria. Water flowing through the canal drops through a small distance and turns a waterwheel, which powers an electric generator, providing electricity for the house shown in Fig. 7–36. Calculate the maximum electric power that can be generated if water moves through the canal at the rate of 1.00×10^3 kg/s and drops through a vertical distance of 2.00 m. (Only about 1 kW is used by the household, the remainder being sold to the local power company.)

Additional Problems

***58** A pendulum swings through an arc of 90.0° (45.0° on either side of the vertical). The mass of the bob is 3.00 kg and the length of the suspending cord is 2.00 m. Find (a) the tension in the cord at the end points of the swing; (b) the velocity of the bob as it passes its lowest point and the tension in the cord at this point.

***59** Tarzan grabs a vine, which is initially horizontal, and attempts to swing to the ground (Fig. 7–37). Tarzan weighs 890 N, and the breaking strength of the vine he knows to be 1780 N. As Tarzan is swinging, he is surprised to find that the vine breaks at a certain angle θ. Find θ.

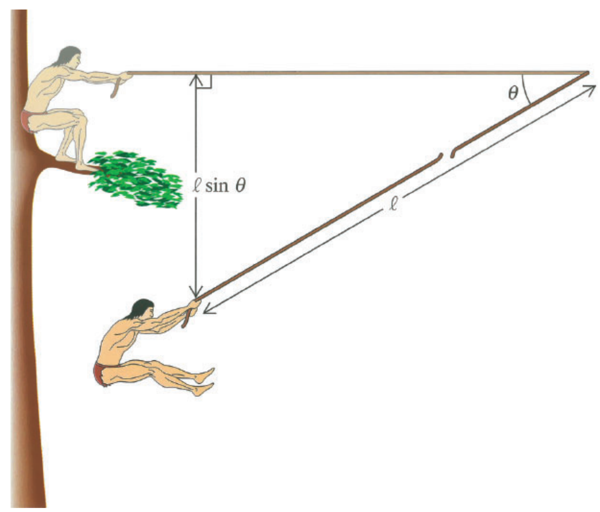


Fig. 7–37

***60** Find the minimum initial height h of the roller coaster in Fig. 7–38 if the roller coaster is to complete the 20.0 m diameter loop. Neglect friction.

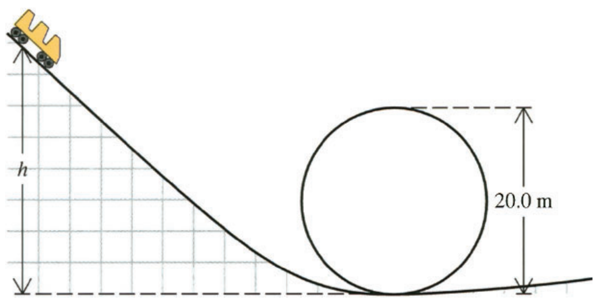


Fig. 7–38

61 The roller coaster in Fig. 7–39 has an initial speed of 7.00 m/s at point A. Find the apparent weight of a 450 N (100 lb) passenger at points B and C. Neglect friction.

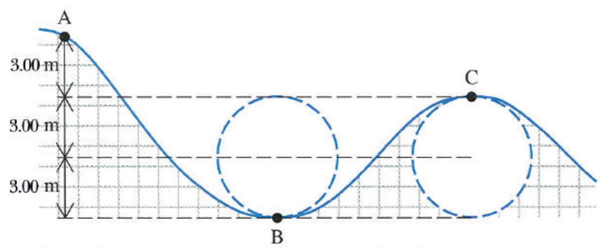


Fig. 7–39

***62** In his novel *From the Earth to the Moon,* published in 1865, Jules Verne first suggested that it might be possible to travel to the moon by firing a very high velocity projectile at the moon. Find the minimum initial velocity of such a projectile as it leaves the earth's atmosphere. Take into account the moon's gravitational force.

***63** In the novel *The Moon Is a Harsh Mistress,* a colony on the moon threatens the earth with bombardment by heavy stones. These stones are relatively easy to propel from the moon, with its low gravity, and yet reach a very high velocity as they strike the earth.

(a) Compute the minimum initial velocity necessary for a projectile at the surface of the moon in order for it to reach the earth.

(b) Find the velocity of the projectile as it enters the earth's atmosphere.

(c) Calculate the ratio of the projectile's final kinetic energy to its initial kinetic energy.

***64** It is estimated that artificial earth satellites have produced approximately 40,000 pieces of debris larger than a pea. Suppose that one of the larger pieces of debris of mass 100 kg is in a circular orbit at two earth radii from the center of the earth and that the mass strikes a satellite in the same orbit, traveling in the opposite direction. Calculate the kinetic energy of the mass relative to the satellite. For comparison, the energy released by 1 million tons of TNT (or a 1 megaton nuclear bomb) equals 4.18×10^9 J.

65 A person weighing 170 lb produces mechanical power of 0.10 hp in walking on a horizontal surface. Suppose that the person can provide a maximum mechanical power of 0.20 hp for 5 hours. What is the maximum height the person could climb up a mountain in this length of time? Assume that the extra 0.10 hp is used to provide gravitational potential energy.

****66** Two blocks are attached to opposite ends of a string that passes over a massless, frictionless pulley (Fig. 7–40). Block A of mass 10.0 kg lies on a 60.0° incline with a coefficient of friction of 0.500, and block B of mass 1.00 kg is attached to a vertical spring of force constant 200 N/m. The blocks are initially at rest with the spring at equilibrium. Find the maximum height that block B rises.

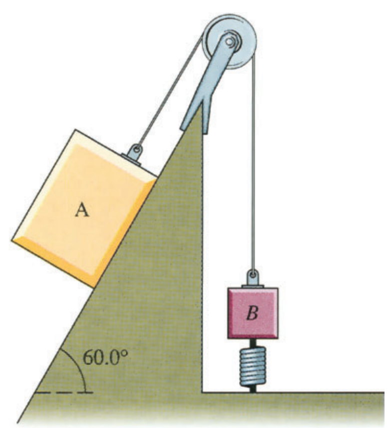
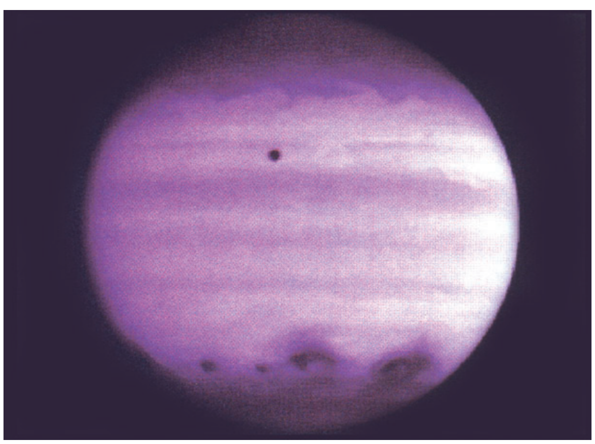


Fig. 7–40

****67** A football of mass 0.500 kg is thrown by a quarterback, who accelerates the ball over a path of length 40.0 cm, releasing the ball with an initial velocity at an angle of 45.0° above the horizontal. The horizontal range of the football is 55.0 m. Find the average force exerted on the ball by the quarterback's hand. Ignore air resistance.

***68** An athlete who weighs 800 N is able to raise his center of mass 0.500 m in a vertical jump.
(a) Compute the internal work done by the athlete's leg muscles as he pushes off from the ground.
(b) Find the athlete's speed as the feet leave the ground.
(c) Find the time during which the feet are in contact with the ground and the body accelerates upward, assuming that the center of mass moves through a distance of 0.400 m at constant acceleration.
(d) Calculate the average mechanical power produced.

***69** Only a few hundred comets have been observed. (Halley's comet is the most spectacular of these.) However, it is now believed that there are perhaps 10^{12} comets, composing what is called the Oort cloud, with orbits much larger than the planetary orbits (Fig. 7–41). It may be perturbation of these comets' orbits by other bodies (a nearby star, for example) that occasionally sends one of them into a new orbit much closer to the sun, so that then, like Halley's comet, it becomes visible on earth. Some of these comets would very likely strike the earth, with devastating effects.* Suppose that a comet from the Oort cloud is slowed by a passing star, so that it falls toward the sun ($m = 1.99 \times 10^{30}$ kg) and strikes the earth. Find the comet's speed when it reaches earth, 1 astronomical unit (Au), or 1.49×10^{11} m, from the sun, if initially the comet is 50,000 Au from the sun and is moving at a speed of only a few m/s. Ignore the earth's gravitational effect, which is relatively small.



Collision of a comet with Jupiter, July 16–23, 1994, as seen in ultraviolet light.

*It has been proposed that a shower of new comets might rain down on the planets every 30 million years or so, as the solar system passes through a heavily populated part of our galaxy. This would account for increased geological activity on earth about every 30 million years. It might even account for the sudden mass extinction of dinosaurs and many other species, which occurred about 65 million years ago, about the same time that there was deposited on the earth's crust a thin sedimentary layer rich in iridium, which is otherwise rare on earth.

****70** (a) When the comet in the last problem collides with the earth, an enormous cloud of dust is thrown into the atmosphere. Estimate the mass of the dust, assuming that the comet's mass is 1.0×10^{14} kg (the approximate mass of Halley's comet), that half the comet's energy is converted to gravitational potential energy of the dust cloud, and that the dust is uniformly spread in a layer 20 km thick (where most of the earth's atmosphere is concentrated).
(b) Calculate the density of the dust and compare with the density of air at the surface of the earth (1.2 kg/m^3). The dust would likely remain for months or years, would cut off solar radiation, shrouding the earth in darkness, and could well result in extinction of many species, including the human species. A similar outcome has been predicted for a nuclear war; the scenario in this case is referred to as "nuclear winter."

****71** Repeat Problem 69, this time taking into account the earth's gravity.

72 Use Eq. 7–20 to express the gravitational potential energy U_G' of a body of mass m at a distance r from the center of the earth, where r is the sum of the earth's radius R and the distance y of the mass above the earth's surface. Show that when $y << R$, this expression reduces to $U_G' \approx mgy$. (The symbol $<<$ means 'much less than'.)

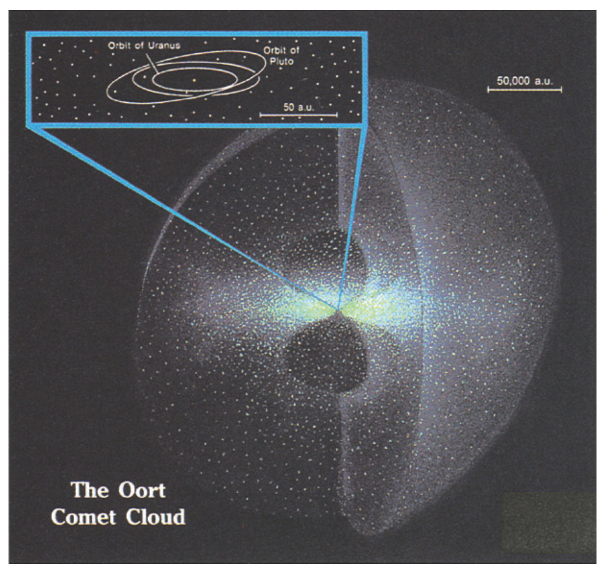


Fig. 7–41 (Diagram by Steven Simpson: *Sky and Telescope* 73:239, March 1987.)