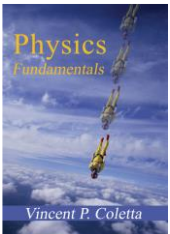


# Medical Physics Class

## Temperature



Physics Fundamentals by Vincent P. Coletta

# Learning Goals

---

*Looking forward at ...*

- **Temperature**
- **Temperature Measurement**
- **Ideal Gas Law**

# Temperature

---

- We can feel the blazing heat of the summer sun or the biting cold of a winter blizzard.
- Our bodies are sensitive even to small changes in the temperature of our surroundings.
- We respond to these changes with several adaptive mechanisms, like sweating or shivering, to maintain a nearly constant internal body temperature.
- This sensitivity to the thermal environment is the basis for our concepts of hot and cold, out of which the scientific definition of temperature evolved.
- **Temperature** is a quantitative measure of how hot or cold something is.
- In this class we shall see how various kinds of thermometers are used to measure temperature and how temperature can be interpreted as a measure of molecular kinetic energy.

# Temperature Measurement

## Thermometers

- Temperature is measured with a **thermometer**.
- Galileo invented the first thermometer, which made use of air's property of expanding as it is heated. **The air's volume indicated the temperature.**
- Today there are various kinds of thermometers, each appropriate for the range of temperatures and the system to be measured.
- For example, in addition to the common mercury thermometer, there are different kinds of thermometers.



# Temperature Measurement

---

## Thermometers

- Each thermometer depends on the existence of some **thermometric property of matter**.
- For example, the expansion or contraction of mercury in a **fever thermometer** correlates with the body's sensation of hot and cold.
- A person who has a high fever will register a higher than normal temperature on a mercury thermometer.
- The length of the mercury column in the glass stem of the thermometer gives us a quantitative measure of temperature.



# Temperature Measurement

---

## Temperature Scales

- To assign a numerical value to the temperature of a body, we need a **temperature scale**.
- The two most common scales in everyday use are the **Celsius scale** (formerly known as the **centigrade** scale) and the **Fahrenheit scale**.
- In establishing a temperature scale, one could use any kind of thermometer and any thermometric property.
- For example, the Celsius scale was based on the expansion of a column of liquid such as mercury in a thin glass tube.
- The **absolute**, or **Kelvin**, scale is **now** the standard in terms of which all other scales, such as Celsius and Fahrenheit, are defined.

# Temperature Measurement

---

## Temperature Scales

- The Kelvin scale is chosen as the standard for important reasons.
- **First**, various laws of physics are most simply expressed in terms of this scale. We shall see an example of this in the ideal gas law, described in the next section.
- **Second**, zero on the absolute scale has fundamental significance. It is the lowest possible temperature a body can approach.

# Temperature Measurement

---

## Temperature Scales

- when the Celsius scale was established, the **normal freezing and boiling points of water** were assigned respective values of 0°C (zero degrees Celsius) and 100°C. A mercury thermometer was brought to thermal equilibrium with water at each of these temperatures, and the level of the mercury column was marked as 0°C for ice water and 100°C for boiling water. The mercury column between these two marks was then divided into 100 equal intervals, corresponding to temperature intervals of 1 Celsius degree (1°C). **Today** the Celsius temperature scale is defined in terms of the Kelvin scale. Celsius temperature  $T_C$  is now defined by the equation

$$T_C = T - 273.15$$



# Temperature Measurement

---

## Temperature Scales

$$T_c = T - 273.15$$

- The normal freezing point of water open to the air at one atmosphere of pressure is  $273.15K$ , or  $0.00^{\circ}C$ .
- The normal boiling point of water is  $373.15K$ , or  $100.00^{\circ}C$ .
- This definition of the Celsius scale conforms to the earlier definition based on the freezing and boiling points of water.
- **Temperature intervals** on the Celsius and Kelvin scales are the same.
- For example, the difference in temperature between the boiling point of water and its freezing point is 100 Celsius degrees ( $^{\circ}C$ ), or 100 kelvins.

# Temperature Measurement

---

## Temperature Scales

- Fahrenheit temperature  $T_F$ , measured in degrees Fahrenheit ( $^{\circ}\text{F}$ ), is defined relative to the Celsius temperature  $T_C$  by the

$$T_F = 32 + \frac{9}{5}T_C$$

- The normal freezing and boiling points of water on the Fahrenheit scale are  $32^{\circ}\text{F}$  and  $212^{\circ}\text{F}$  respectively. The interval between these points is  $180^{\circ}\text{F}$ . There are only  $100^{\circ}\text{C}$  between these same points. Thus Celsius degrees are bigger than Fahrenheit degrees:  $1^{\circ}\text{C}$  is  $\frac{180}{100}$ , or  $\frac{9}{5}$ , times  $1^{\circ}\text{F}$ .

# Temperature Measurement

---

## Measuring a Fever on the Celsius Scale

Normal internal body temperature is 98.6°F. A temperature of 106°F is considered a high fever. Find the corresponding temperatures on the Celsius scale.

$$T_C = \frac{5}{9}(T_F - 32.0)$$

A temperature of 98.6° F corresponds to

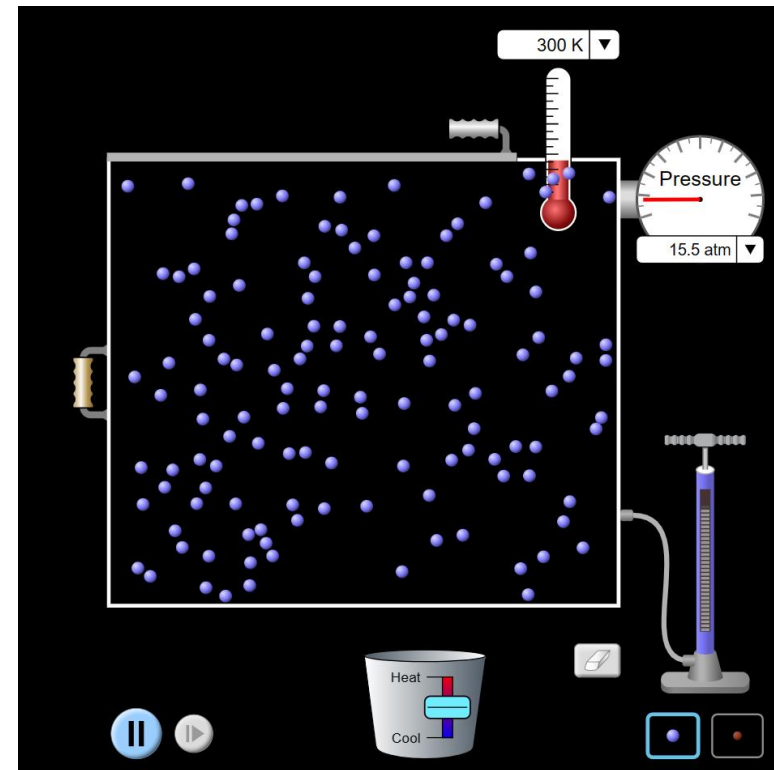
$$T_C = \frac{5}{9}(98.6 - 32.0) = 37.0^\circ \text{C}$$

and a temperature of 106° F corresponds to

$$T_C = \frac{5}{9}(106 - 32.0) = 41.1^\circ \text{C}$$

# Ideal Gas Law

- **An ideal gas** is a theoretical gas composed of many randomly moving point particles that are not subject to interparticle interactions.
- *pressure  $P$ ,*
- *volume  $V$ ,*
- *Kelvin temperature  $T$ ,*
- *number of gas molecules  $N$*



# Ideal Gas Law

---

- The pressure of a gas can be changed in several ways. **One way** to increase the pressure of a gas confined to a fixed volume is to increase the number of gas molecules in the volume.
- You do this, for example, when you pump air into a bicycle tire or an automobile tire.
- **Another way** to change the pressure of a gas is to change its temperature. For example, when the air in an automobile tire heats up, its pressure increases significantly.
- **A third way** to change gas pressure is to change the volume containing the gas; decreasing volume causes an increase in pressure.

# Ideal Gas Law

---

- For low-density gases, there is a simple, **universal relationship** between the gas *pressure*  $P$ , *volume*  $V$ , *Kelvin temperature*  $T$ , and *number of gas molecules*  $N$ . The product of  $P$  and  $V$  is proportional to the product of  $N$  and  $T$ :

$$PV = NkT$$

- This equation is called the **ideal gas law**. The constant  $k$  is known as “**Boltzmann’s constant**” and is found from experiment to have the value

$$k = 1.380 \times 10^{-23} \text{ J/K}$$

- In applying the ideal gas law, temperature must be expressed in kelvins, not in °C or °F.

# Ideal Gas Law

---

- Special cases of the gas law are found when one considers the variation of two of the variables  $P$ ,  $V$ ,  $N$ , and  $T$ , while the other two variables are held constant.
- For example, if  $N$  and  $T$  are fixed, the ideal gas law implies that the product  $PV$  is constant:

$$PV = \text{constant}$$

- This result is known as Boyle's law.
- Boyle's law implies that if the volume of a gas is reduced to half its original value the pressure of the gas is doubled.

# Ideal Gas Law

---

- If  $P$  and  $N$  are fixed, the ideal gas law implies that the volume of the gas is directly proportional to its temperature:

$$V \propto T$$

(for constant  $N$  and  $P$ )

- If  $V$  and  $N$  are fixed, the ideal gas law implies that

$$P \propto T$$

(for constant  $N$  and  $V$ )

- The very definition of temperature on the Kelvin scale requires that this relationship be satisfied, at least in the limit of a very low-density gas.



# Ideal Gas Law

## EXAMPLE 2 The Temperature of an Ideal Gas After Compression

An ideal gas initially has a volume of 1.0 liter (L), a pressure of 1.0 atmosphere (atm), and a temperature of 27° C. The pressure is raised to 2.0 atm, compressing the volume of the gas to 0.60 L. Find the final temperature of the gas.

**SOLUTION** We are given the following initial and final values of  $P$ ,  $V$ , and  $T$ :

$$P_i = 1.0 \text{ atm} \quad P_f = 2.0 \text{ atm}$$

$$V_i = 1.0 \text{ L} \quad V_f = 0.60 \text{ L}$$

$$T_{ci} = 27^\circ \text{ C} \quad T_{cf} = ?$$

The number of molecules,  $N$ , is constant. The problem is to find the final temperature  $T_{cf}$ . We can do this simply by first writing the ideal gas law for the initial state of the gas and again for the final state and then taking the ratio of the two expressions:

$$P_i V_i = NkT_i$$

$$P_f V_f = NkT_f$$

$$\frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i}$$

Since this equation involves ratios of pressures and volumes, we may insert these quantities in the units in which they are given, that is, atmospheres and liters, rather than converting to standard units of Pa and m<sup>3</sup>. The conversion to standard units would simply introduce identical factors for both initial and final values, and these factors would cancel. We must be careful, however, to convert temperature from degrees Celsius to kelvins, even when a ratio is used, as it is here, since this change in units involves an additive term, rather than a multiplicative factor. Thus we must use  $T_i = 27 + 273 = 300 \text{ K}$ . Substituting values into the preceding equation, we obtain

$$\frac{T_f}{300 \text{ K}} = \frac{(2.0 \text{ atm})(0.60 \text{ L})}{(1.0 \text{ atm})(1.0 \text{ L})} = 1.2$$

or

$$T_f = (1.2)(300 \text{ K}) = 360 \text{ K}$$

This corresponds to a final Celsius temperature  $T_{cf}$  of

$$\begin{aligned} T_{cf} &= T_f - 273 = 360 - 273 \\ &= 87^\circ \text{ C} \end{aligned}$$

# Ideal Gas Law

## EXAMPLE 3 The Number of Air Molecules in a Hot-air Balloon

The air inside a hot-air balloon (Fig. 12–3) is at a temperature of  $100.0^\circ\text{C}$ , while the temperature of the surrounding air in the atmosphere is  $20.0^\circ\text{C}$ . Find the ratio of the number of air molecules inside the balloon to the number of air molecules contained in an equal volume of air outside the balloon. Assume that the air pressure is the same inside and outside.

**SOLUTION** We first use the ideal gas law (Eq. 12–5) to obtain an expression for the number of air molecules in a volume  $V$ .

$$N = \frac{PV}{kT}$$

Using subscripts  $i$  and  $o$  to denote air inside and outside the balloon, we obtain the ratio of the number of air molecules inside to the number outside:

$$\frac{N_i}{N_o} = \frac{P_i V_i / kT_i}{P_o V_o / kT_o}$$

Since pressures  $P_i$  and  $P_o$  are equal and volumes  $V_i$  and  $V_o$  are equal, this reduces to

$$\frac{N_i}{N_o} = \frac{T_o}{T_i}$$



**Fig. 12–3** Hot-air balloon.

Converting temperatures to kelvins, we find

$$\frac{N_i}{N_o} = \frac{273 + 20}{273 + 100} = \frac{293\text{ K}}{373\text{ K}} = 0.786$$

According to Archimedes' principle, the hot-air balloon will experience a buoyant force equal to the weight of the displaced air. Since there is less air inside the balloon than in the volume of atmosphere displaced, the buoyant force is greater than the weight of the hot air, and the balloon will rise if the weight carried by the balloon is not too great. (Problem 50 asks you to calculate the volume of a hot-air balloon required to support 5000 N.)

# Ideal Gas Law

---

- **Mole**

- One can compute Avogadro's number by dividing the mass of 1 mole of carbon 12 (12 g) by the mass of a single carbon-12 atom, equal to 12 atomic mass units, where the atomic mass unit is related to the gram by the preceding equation.

$$N_A = \frac{12 \text{ g}}{12 \text{ u}} = \frac{12 \text{ g}}{12(1.6606 \times 10^{-24} \text{ g})}$$

$$N_A = 6.022 \times 10^{23}$$

- We may express the number of molecules,  $N$ , of a substance as the product of Avogadro's number,  $N_A$ , and the number of moles, denoted by  $n$ :

$$N = nN_A$$

# Ideal Gas Law

---

## EXAMPLE 6 Finding the Mass of a Volume of Air

Find the mass of air in a room with dimensions  $5.00 \text{ m} \times 4.00 \text{ m} \times 3.00 \text{ m}$ , if the air pressure is  $1.00 \text{ atm}$  and the temperature is  $27.0^\circ \text{ C}$ .

**SOLUTION** First we apply the ideal gas law (Eq. 12–13) to find the number of moles, using the Kelvin temperature ( $T = 273 + 27.0 = 300 \text{ K}$ ) and expressing pressure in Pa (Eq. 11–5:  $1.00 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$ ).

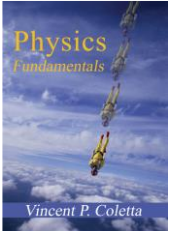
$$\begin{aligned}n &= \frac{PV}{RT} \\&= \frac{(1.00 \text{ atm})(1.01 \times 10^5 \text{ Pa/atm})(5.00 \text{ m} \times 4.00 \text{ m} \times 3.00 \text{ m})}{(8.31 \text{ J/K})(300 \text{ K})} \\&= 2.43 \times 10^3 \text{ moles}\end{aligned}$$

Since 1 mole has a mass equal to the molecular mass in grams, the mass of air equals the product of the number of moles times the molecular mass in grams. A nitrogen molecule  $\text{N}_2$  has a molecular mass of  $2(14) = 28$ , and an oxygen molecule  $\text{O}_2$  has a molecular mass of  $2(16) = 32$ . Air consists of approximately 80% nitrogen and 20% oxygen, and so the average molecular mass is  $0.8 \times 28 + 0.2 \times 32 = 28.8$ . Thus

$$\begin{aligned}m &= (2.43 \times 10^3 \text{ moles})(28.8 \text{ g/mole}) = 7.00 \times 10^4 \text{ g} \\&= 70.0 \text{ kg}\end{aligned}$$

# Medical Physics Class

## Kinetic Theory of Gases



Physics Fundamentals by Vincent P. Coletta

# Learning Goals

---

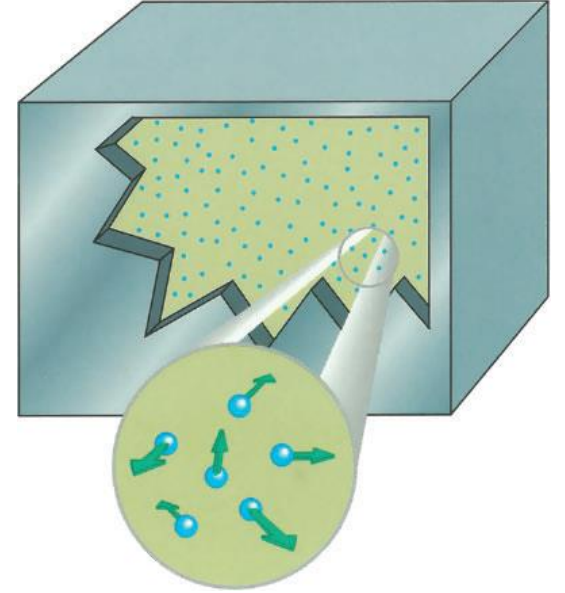
*Looking forward at ...*

- **Kinetic Theory Model of an Ideal Gas**
- **Vapor Pressure and Humidity**
- **Boiling**
- **Thermal Expansion**
- **Thermal Expansion of water**



# Kinetic Theory; Model of an Ideal Gas

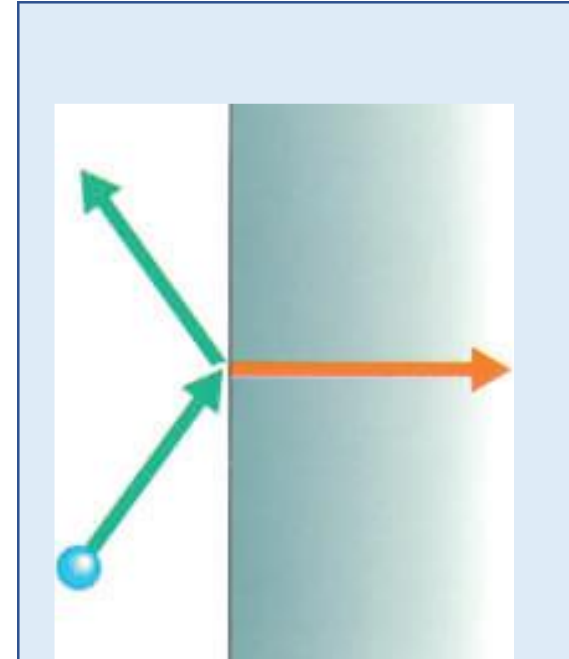
- **Kinetic theory** is an area of physics that was developed in the late nineteenth century
- **Kinetic theory** provides an explanation for the behavior of a macroscopic system in terms of its microscopic components—atoms or molecules, which obey dynamical laws.
- In this section we shall use kinetic theory to provide an explanation for the pressure of an ideal gas in terms of a molecular model.



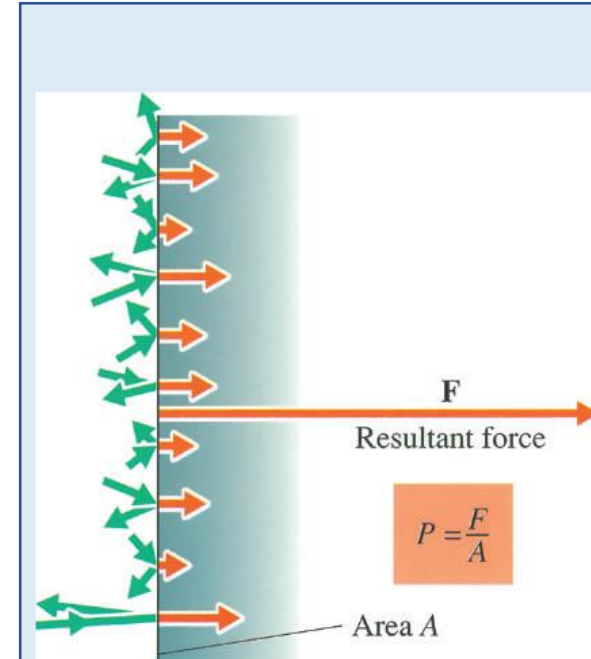
A container of gas consists of a large number of molecules moving randomly and colliding with the walls of the container.

# Kinetic Theory

- A gas contained in a volume of macroscopic dimensions consists of an enormously large number of molecules.
- These molecules move in a random, chaotic way throughout the volume of the container.
- When a molecule strikes a surface, it bounces off, exerting a small force on the surface.
- At any instant there will be many molecules colliding with the surface.
- The effect of these collisions is to produce a resultant force, which may be quite large.



A single molecule collides with a wall of the container; during the collision the molecule exerts a small force perpendicular to the surface.

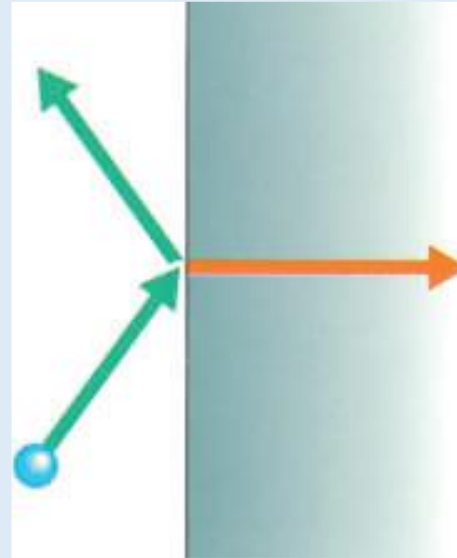


At any instant a large number of molecules collide with the surface. The effect of all these collisions is a resultant force  $F$  and pressure  $P$ , which are effectively constant.

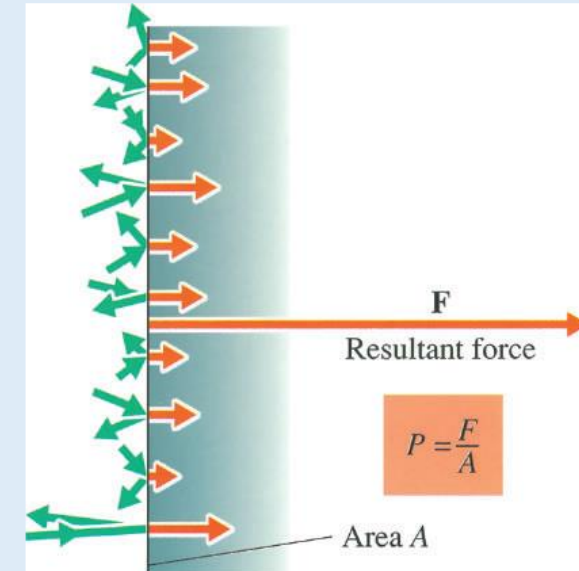


# Kinetic Theory

- The resultant force is not steady but fluctuates rapidly, depending on the number of molecules striking the surface at any instant.
- But, for a surface of macroscopic size, the number of molecules involved is so large that fluctuations in the net force are negligibly small.
- Thus the molecules exert a pressure on the container walls that is effectively constant.



A single molecule collides with a wall of the container; during the collision the molecule exerts a small force perpendicular to the surface.



At any instant a large number of molecules collide with the surface. The effect of all these collisions is a resultant force  $F$  and pressure  $P$ , which are effectively constant.

# Kinetic Theory

---

## Kinetic Interpretation of Temperature

- In this section we shall derive the ideal gas law, using the more realistic model of molecules moving randomly in all directions.
- This derivation, first presented by James Clerk Maxwell in 1859, shows that **the average kinetic energy of an ideal gas molecule equals  $\frac{1}{2} kT$** .
- A molecule of mass  $m$  traveling at speed  $v$  has kinetic energy

$K = \frac{1}{2} m v^2$ . Denoting the average values of  $K$  and  $v^2$  by  $\bar{K}$  and  $\overline{v^2}$ , we may express Maxwell's result as:

$$\bar{K} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$

# Kinetic Theory

---

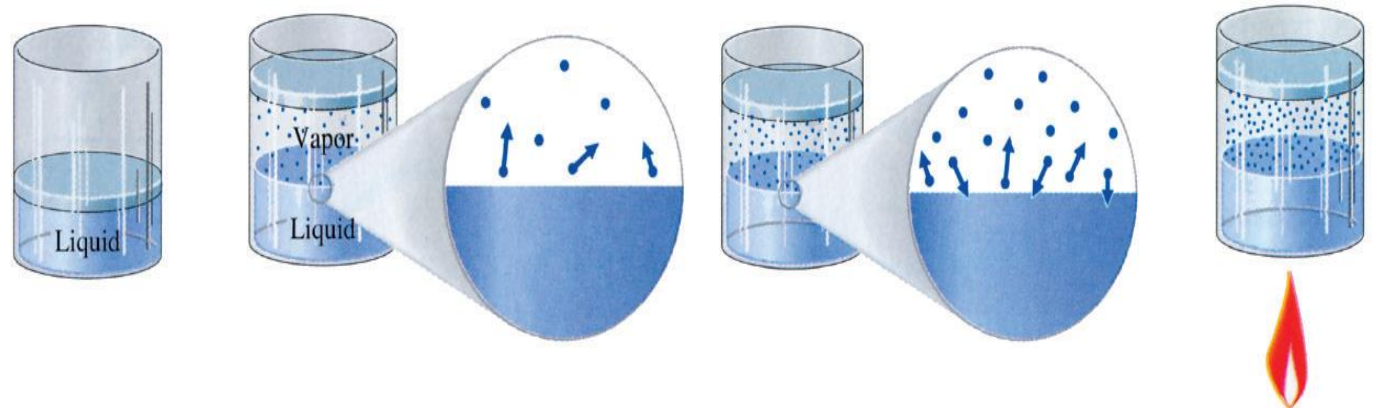
## Kinetic Interpretation of Temperature

- This equation shows that the temperature of an ideal gas is a measure of the average kinetic energy of its molecules.
- The phenomenon of **thermal equilibrium** is easy to understand. Two systems of gas that are initially at different temperatures have different values of average kinetic energy per molecule.
- When the systems are placed in thermal contact, the system with the higher temperature will lose energy as the system with the lower temperature gains energy.
- This process continues until the average molecular kinetic energies and hence also the temperatures of the two systems are the same.
- Thus thermal equilibrium is simply a consequence of the equal sharing of kinetic energy among the molecules of both systems.
- Although this equation is derived only for an ideal gas, it applies to any system, including **liquids and solids**.
- The average translational kinetic energy of the molecules in a body at absolute temperature  $T$  equals  $\frac{3}{2}kT$ .

$$\bar{K} = \frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT$$

# Vapor Pressure and Humidity

- The molecules in a liquid have a distribution of velocities, similar to the Maxwell-Boltzmann distribution of molecular velocities in a gas.
- Although intermolecular forces bind most of the molecules close together in the liquid, some of the molecules move fast enough to leave the surface of the liquid, like a rocket with a velocity greater than escape velocity leaving the earth.
- Since the molecules that evaporate are those with the **greatest velocity** and **kinetic energy**, the average kinetic energy of the molecules remaining in the liquid decreases, and so the temperature of the liquid decreases.

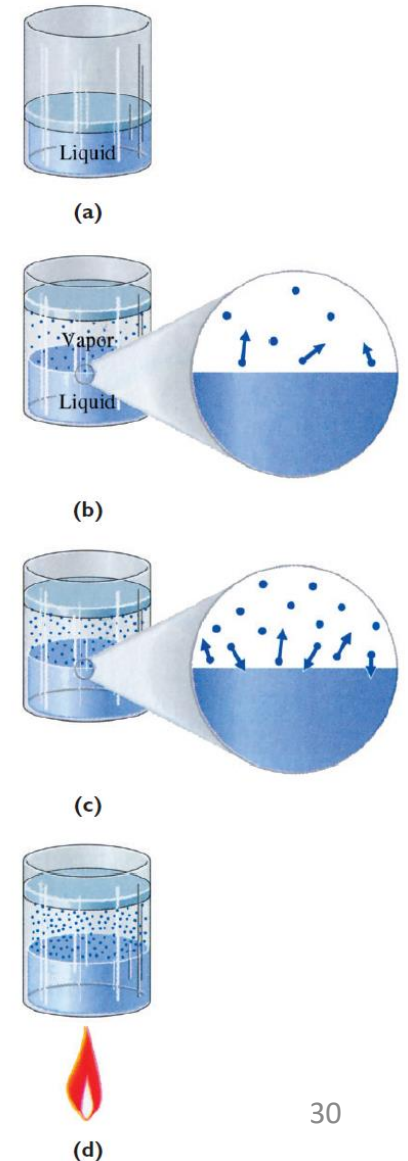


# Vapor Pressure and Humidity

- You can feel the cooling effect of evaporation when you step out of a shower and water evaporates from your skin.
- The effect is more dramatic when the air is very dry. If you step out of a swimming pool in the desert, even though the air may be quite hot, you can be chilled by water evaporating from your skin.

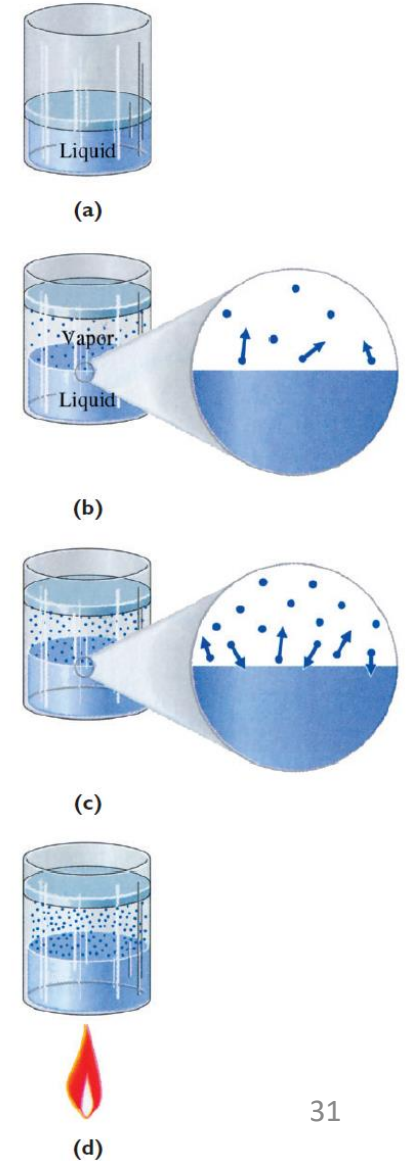
# Vapor Pressure and Humidity

- To better understand the process of evaporation, consider a liquid in a closed container with a piston (Fig. a).
- If the piston is raised, evaporation begins. Vapor fills the space above the liquid (Fig. b) and creates a pressure, called **vapor pressure**.
- As more and more molecules enter the vapor, some molecules begin to go from the vapor back into the liquid.
- Initially more molecules leave the liquid than enter it, and both the density and pressure of the vapor increases. There is soon reached an **equilibrium state**, in which as many molecules enter the liquid as leave it.



# Vapor Pressure and Humidity

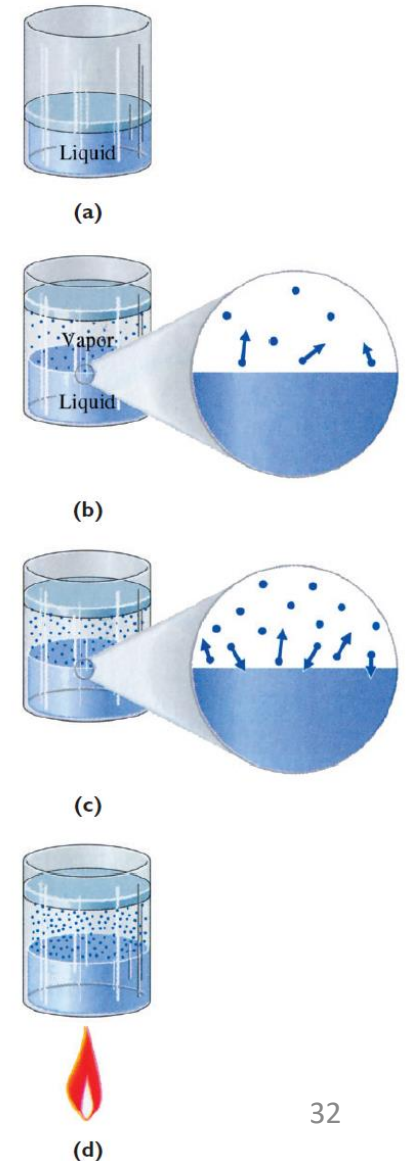
- We say the vapor is then “**saturated**,” since there can be no further increase in the number of molecules in the vapor (Fig. c).
- In this equilibrium state, vapor pressure reaches its maximum value, called saturated vapor pressure.
- If the piston is raised higher, more molecules enter the vapor phase until the vapor pressure again reaches the same saturated vapor pressure.





# Vapor Pressure and Humidity

- As the temperature of a liquid increases, more of its molecules have sufficient kinetic energy to escape the liquid.
- Thus, as temperature increases, the rate at which molecules leave the surface of the liquid increases; that is, the rate of evaporation increases.
- An equilibrium state is not reached until the density of the vapor phase increases enough that the rate at which molecules reenter the liquid matches the new higher rate at which molecules leave the liquid (Fig. d).
- The higher-density equilibrium state is one of higher pressure. Thus, as temperature increases, saturated vapor pressure increases.





# Boiling

---

- If the temperature of a liquid is raised enough so that the liquid's vapor pressure equals the pressure of the surrounding air, the liquid begins to boil.
- That is, bubbles of vapor form within the bulk liquid. These vapor bubbles push outward against the liquid, which is at approximately the same pressure as the air.
- Water boils at a temperature of  $100^{\circ}\text{C}$  when the surrounding air is at a pressure of  $1.00 \text{ atm}$  because the saturated vapor pressure of air at  $100^{\circ}\text{C}$  is  $1.00 \text{ atm}$ .
- If the surrounding air is at a lower pressure, water will boil at a lower temperature.
- **For example**, on a mountain at an elevation of  $3000 \text{ m}$ , where atmospheric pressure is only  $0.7 \text{ atm}$ , water boils at a temperature of  $90^{\circ}\text{C}$ , since its saturated vapor pressure at that temperature equals  $0.7 \text{ atm}$ .

# Thermal Expansion

---

Nearly all solids and liquids expand as they are heated.

The fractional increase in volume,  $\Delta V/V$ , is often found to be directly proportional to the increase in temperature,  $\Delta T$ .

The constant of proportionality is called the “**volume coefficient of expansion,**” denoted by  $\beta$ . Thus

$$\frac{\Delta V}{V} = \beta \Delta T$$

The change in volume  $\Delta V$  is proportional to the original volume  $V$ , as well as to the temperature change  $\Delta T$ .

$$\Delta V = \beta V \Delta T$$

# Thermal Expansion

---

Thus, for example, if 1 *liter* ( $1000 \text{ cm}^3$ ) of water is heated from  $20^\circ\text{C}$  to  $25^\circ\text{C}$ , its volume increases by only about  $1 \text{ cm}^3$ . But if the water in a swimming pool of volume  $1000 \text{ m}^3$  is heated over the same temperature interval, the water increases in volume by  $1 \text{ m}^3$ , or  $10^6 \text{ cm}^3$ . In both cases the ratio  $\Delta V/V$  is  $10^{-3}$ .

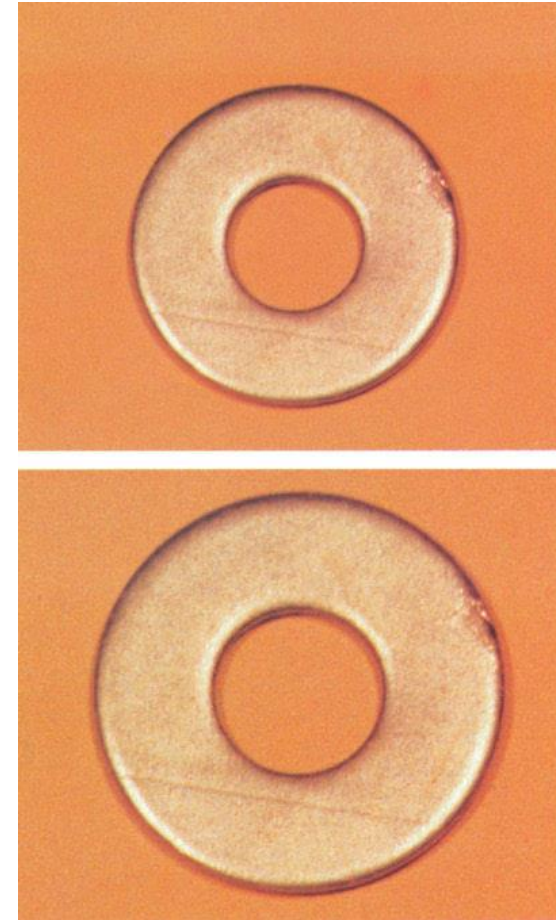
$$\frac{\Delta V}{V} = \beta \Delta T$$

$$\Delta V = \beta V \Delta T$$

# Thermal Expansion

---

- In the case of solids, the volume expansion is accomplished by an increase in all linear dimensions.
- As a solid is heated, the distance between any two points in the solid increases.
- The fractional increase in length is normally the same in all directions.
- Thus, if a block of metal expands thermally by 0.1% in length, the block's height and width will each also increase by 0.1%.
- The increase in size is like a photographic enlargement the Figure.
- As a washer is heated from temperature  $T$  to temperature  $T + \Delta T$ , all its linear dimensions get bigger.
- The actual expansion, however, is much smaller than indicated here.
- The expansion shown here is 20%, which is approximately 100 times greater than the expansion of aluminum heated  $100^{\circ}\text{C}$ .



# Thermal Expansion

---

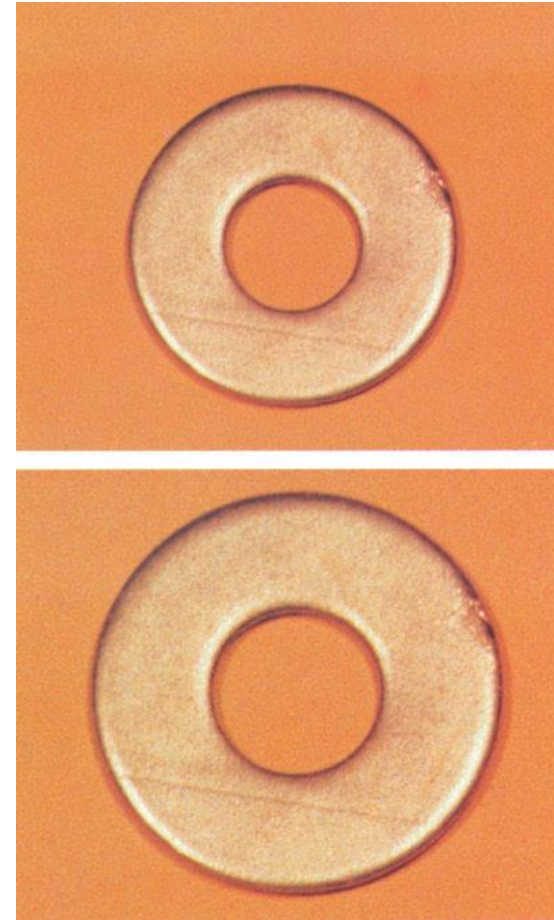
- Instead of using a volume coefficient of expansion for solids, we normally use a linear expansion coefficient  $\alpha$ , which is a measure of the fractional change in the linear dimensions of the solid.
- For a temperature change  $\Delta T$ , a length  $l$  changes by  $\Delta l$ , where

$$\frac{\Delta l}{l} = \alpha \Delta T$$

$$\Delta l = \alpha l \Delta T$$

- It is possible to show that the volume coefficient of expansion for a solid equals 3 times its linear coefficient:

$$\beta = 3\alpha$$



# Thermal Expansion

- Coefficients of expansion for various materials are given in Table.
- For liquids there is no measure of linear expansion, since liquids must conform to the shapes of their containers.

**Table 12-2** Coefficients of thermal expansion

Solids	$\alpha, (\text{C}^\circ)^{-1}$	Liquids and gases	$\beta, (\text{C}^\circ)^{-1}$
Aluminum	$2.4 \times 10^{-5}$	Ethyl alcohol	$1.1 \times 10^{-3}$
Brass	$1.9 \times 10^{-5}$	Mercury	$1.8 \times 10^{-4}$
Brick	$1 \times 10^{-5}$	Olive oil	$7.2 \times 10^{-4}$
Concrete	$1.2 \times 10^{-5}$	Water	$2.1 \times 10^{-4}$
Copper	$1.7 \times 10^{-5}$	Air	$3.67 \times 10^{-3}$
Diamond	$1.2 \times 10^{-6}$	Hydrogen	$3.66 \times 10^{-3}$
Glass	$4 \times 10^{-6}$ to $1 \times 10^{-5}$	Nitrogen	$3.67 \times 10^{-3}$
Gold	$1.4 \times 10^{-5}$		
Graphite	$2 \times 10^{-6}$		
Ice	$5.1 \times 10^{-5}$		
Paraffin	$1 \times 10^{-4}$		
Steel	$1.2 \times 10^{-5}$		
Wood, parallel to fiber	$5 \times 10^{-6}$ to $1 \times 10^{-5}$		
Wood, across fiber	$3 \times 10^{-5}$ to $6 \times 10^{-5}$		

# Thermal Expansion

---

## EXAMPLE : Overflow of an Expanding Liquid

A glass container that has a volume of 1.0 liter and is filled with alcohol is initially refrigerated at a temperature of 7° C. How much of the alcohol will overflow if the container is placed in a warm room where the temperature is 27° C? The linear coefficient of expansion for the glass is  $1.0 \times 10^{-5} (\text{C}^\circ)^{-1}$ .

**SOLUTION** The volume coefficient of expansion for the glass is 3 times its linear coefficient, or  $3.0 \times 10^{-5} (\text{C}^\circ)^{-1}$ . Thus

$$\begin{aligned}\Delta V_{\text{glass}} &= \beta_{\text{glass}} V \Delta T \\ &= [3.0 \times 10^{-5} (\text{C}^\circ)^{-1}](1.0 \times 10^3 \text{ cm}^3)(20 \text{ C}^\circ) \\ &= 0.60 \text{ cm}^3\end{aligned}$$

From Table 12–1 we find that the volume coefficient of expansion for alcohol is  $1.1 \times 10^{-3}$ —much larger than the coefficient for glass. Therefore the alcohol will expand more than the glass.

$$\begin{aligned}\Delta V_{\text{alc}} &= \beta_{\text{alc}} V \Delta T \\ &= [1.1 \times 10^{-3} (\text{C}^\circ)^{-1}](1.0 \times 10^3 \text{ cm}^3)(20 \text{ C}^\circ) \\ &= 22 \text{ cm}^3\end{aligned}$$

Since the volume of the glass container increases by only about 1 cm<sup>3</sup>, about 21 cm<sup>3</sup> of the alcohol will overflow.

# The End

---

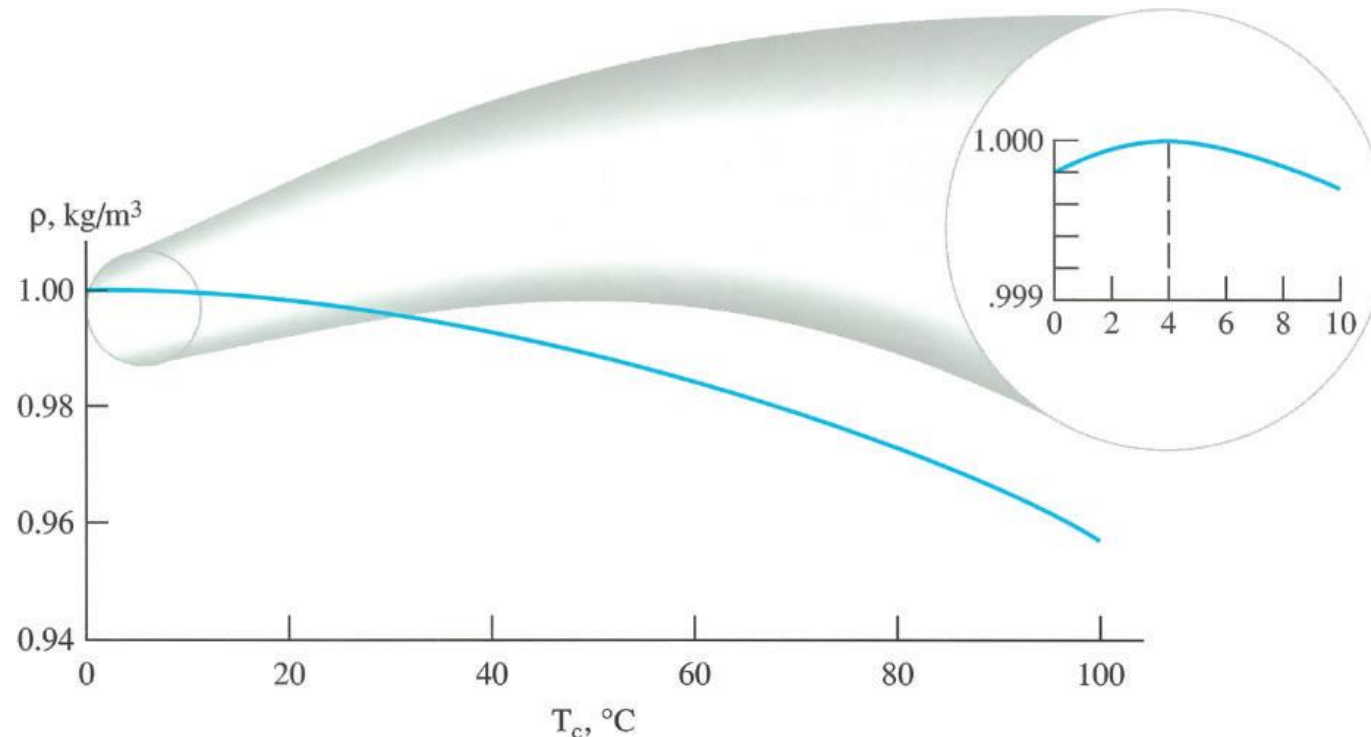


**Do not forget to do  
your homework**



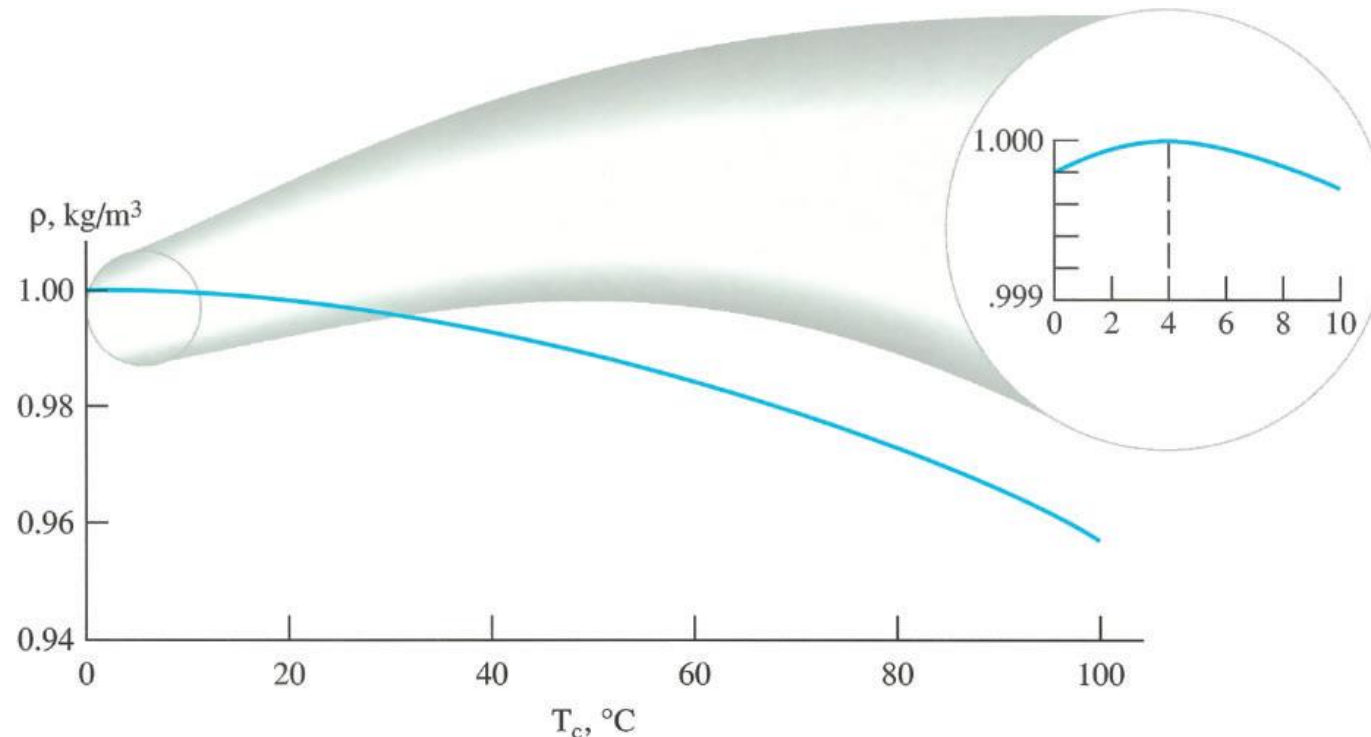
# Thermal Expansion of Water

In many cases the simple linear dependence of  $\Delta V$  on  $\Delta T$  expressed by Eq. 12–20 is valid over all temperature ranges commonly encountered, with a constant value for  $\beta$ . However, for some substances the variation of volume with temperature is more complicated. Water is such a substance. Figure below shows the density of water as a function of temperature. Notice that at most temperatures the density of water decreases as its temperature increases; that is, water expands as it is heated. But in the temperature range from  $0^\circ\text{C}$  to  $4^\circ\text{C}$  water contracts as it is heated.



# Thermal Expansion of Water

But in the temperature range from 0°C to 4°C water contracts as it is heated. Water is one of the few materials that have this property. This has an important effect on the rate at which lakes freeze. As air temperatures drop, the temperature of the water in a lake drops also, with the cooling occurring first at the surface of the lake. For temperatures above 4°C this cooling proceeds very efficiently. As water cools, it becomes more dense and sinks to a lower level in the lake, as warmer, less dense water rises to take its place.



# Thermal Expansion of Water

Thus there is a natural mixing of warmer and colder water, causing rapid cooling of water beneath the surface. However, when the water reaches a uniform temperature of 4°C, the process changes. Cooling of the surface water below 4°C decreases its density. Thus it stays at the surface, and further cooling of the water beneath proceeds more slowly. The surface of the lake may freeze. But in even the coldest weather, large lakes do not freeze solid. The water at the bottom of the lake remains at 4°C, enabling the marine life there to survive.

