

# 12 Temperature and Kinetic Theory



We can feel the blazing heat of the summer sun or the biting cold of a winter blizzard. Our bodies are sensitive even to small changes in the temperature of our surroundings. We respond to these changes with several adaptive mechanisms, like sweating or shivering, to maintain a nearly constant internal body temperature. This sensitivity to the thermal environment is the basis for our concepts of hot and cold, out of which the scientific definition of temperature evolved. Temperature is a quantitative measure of how hot or cold something is. In this chapter we shall see how various kinds of thermometers are used to measure temperature and how temperature can be interpreted as a measure of molecular kinetic energy.

## 12-1 Temperature Measurement

### Thermometers

Temperature is measured with a thermometer. Galileo invented the first thermometer, which made use of air's property of expanding as it is heated. The air's volume indicated the temperature. Today there are various kinds of thermometers, each appropriate for the range of temperatures and the system to be measured. For example, in addition



**Fig. 12-1** Thermometers. The length of a column of mercury indicates temperature in a mercury thermometer **(a)**. Electrical properties of a probe are used to measure temperature in a thermistor thermometer **(b)** and a thermocouple thermometer **(c)**. In an optical pyrometer **(d)**, used to measure very high temperatures, light emitted by the hot body is compared with light produced by an electrically heated filament inside the pyrometer. **(e)** Gas pressure indicates temperature in a constant-volume gas thermometer.

to the common mercury thermometer, there are electrical resistance thermometers, thermistor thermometers, and optical pyrometers (Fig. 12-1).

Each thermometer depends on the existence of some **thermometric property** of matter. For example, the expansion or contraction of mercury in a fever thermometer correlates with the body's sensation of hot and cold. A person who has a high fever both is hot to the touch and will register a higher than normal temperature on a mercury thermometer. The length of the mercury column in the glass stem of the thermometer gives us a quantitative measure of temperature.

The constant volume gas thermometer maintains a quantity of gas in a fixed volume (Fig. 12-1e). As the gas is heated, its pressure increases, as indicated by a pressure gauge. Pressure is the thermometric property in this thermometer. Pressure readings are used to indicate temperature.

Suppose a mercury thermometer is used to measure the temperature of a hot liquid. When the thermometer is first immersed in the liquid, the length of the mercury column increases. A short time later it reaches an equilibrium level and remains constant. This final equilibrium level indicates the common temperature of the liquid and the thermometer. The interaction between the thermometer and the system, during which the mercury expands, is called "thermal interaction." The final state, in which the length of the mercury column no longer changes, is called a state of **thermal equilibrium**. The length of the mercury column then indicates the temperature of both the thermometer and the liquid. When two bodies are in thermal equilibrium, they are at the same temperature.

### Temperature Scales

To assign a numerical value to the temperature of a body, we need a temperature scale. The two most common scales in everyday use are the Fahrenheit scale and the Celsius scale (formerly known as the centigrade scale). In establishing a temperature scale, one could use any kind of thermometer and any thermometric property. For example, the Celsius scale was based on the expansion of a column of liquid such as mercury in a thin glass tube. One could just as well base a temperature scale on the volume of a fixed quantity of air at atmospheric pressure, as in Galileo's first thermometer. There is no point in having many independently defined temperature scales. The **absolute**, or **Kelvin**, scale is now the standard in terms of which all other scales, such as Celsius and Fahrenheit, are defined.

The Kelvin scale is chosen as the standard for important reasons. First, various laws of physics are most simply expressed in terms of this scale. We shall see an example of this in the ideal gas law, described in the next section. Second, zero on the absolute scale has fundamental significance. It is the lowest possible temperature a body can approach.

The definition of the Kelvin scale is based on a constant-volume gas thermometer and uses the thermal properties of water to establish a reference temperature. Liquid water can coexist with both water vapor and ice in thermal equilibrium at only one temperature. This state, called the "triple point," is achieved when a vacuum-sealed container of liquid water is cooled until it is in equilibrium with both water vapor and ice. The triple-point temperature  $T_{tr}$  is defined to be 273.16 kelvin (abbreviated K):

$$T_{tr} = 273.16 \text{ K} \quad (12-1)$$

The triple point is very close to the normal freezing point of water at atmospheric pressure (273.15 K). The reason for assigning the particular value 273.16 to the triple point is so that the Kelvin scale will be related to the older Celsius scale in a simple way, as we shall see.

Temperature on the Kelvin scale is defined according to the following prescription. A container of low-density gas with a fixed volume (a constant-volume gas thermometer) is allowed to reach thermal equilibrium with water at the triple point. The pressure of the gas at the triple point,  $P_{tr}$ , is measured. Then the gas thermometer is placed in thermal contact with the body whose temperature is to be determined, and when thermal equilibrium is reached, the gas pressure  $P$  is again measured. The ratio  $P/P_{tr}$  defines the temperature of the body according to the equation

$$T = (273.16 \text{ K}) \frac{P}{P_{tr}} \quad (12-2)$$

For example, when the pressure of the gas in the thermometer is exactly double its pressure at the triple point, the temperature of the body by definition is  $273.16 \text{ K} \times 2 = 546.32 \text{ K}$ . The gas used in this thermometer can be virtually any gas, so long as the density of the gas is low enough. All gases give the same value for the temperature of a given system, in the limit as the gas density approaches zero.

Historically, when the **Celsius** scale was established, the normal freezing and boiling points of water were assigned respective values of  $0^\circ \text{C}$  (zero degrees Celsius) and  $100^\circ \text{C}$ . A mercury thermometer was brought to thermal equilibrium with water at each of these temperatures, and the level of the mercury column was marked as  $0^\circ \text{C}$  for ice water and  $100^\circ \text{C}$  for boiling water. The mercury column between these two marks was then divided into 100 equal intervals, corresponding to temperature intervals of 1 Celsius degree ( $1 \text{ C}^\circ$ ). Today the Celsius temperature scale is defined in terms of the Kelvin scale. Celsius temperature  $T_c$  is now defined by the equation

$$T_c = T - 273.15 \quad (12-3)$$

On the Celsius scale, the triple point of water is  $273.16 - 273.15$ , or  $0.01^\circ \text{C}$ . The normal freezing point of water open to the air at one atmosphere of pressure is  $273.15 \text{ K}$ , or  $0.00^\circ \text{C}$ . The normal boiling point of water is  $373.15 \text{ K}$ , or  $100.00^\circ \text{C}$ . This definition of the Celsius scale conforms to the earlier definition based on the freezing and boiling points of water. Temperature intervals on the Celsius and Kelvin scales are the same. For example, the difference in temperature between the boiling point of water and its freezing point is 100 Celsius degrees ( $\text{C}^\circ$ ), or 100 kelvins.

**Table 12-I** The Kelvin, Celsius, and Fahrenheit temperature scales

	Kelvin scale		Celsius scale		Fahrenheit scale	
Sun's surface temperature	6000 K		$6000^\circ \text{C}$		$10,000^\circ \text{F}$	
Gold melts	1336 K		$1063^\circ \text{C}$		$1945^\circ \text{F}$	
Water boils	373 K	—	$100^\circ \text{C}$	—	$212^\circ \text{F}$	—
Human body temperature	310 K	100 K	$37^\circ \text{C}$	100 $\text{C}^\circ$	$99^\circ \text{F}$	180 $\text{F}^\circ$
Typical room temperature	293 K		$20^\circ \text{C}$		$68^\circ \text{F}$	
Water freezes	273 K	—	$0^\circ \text{C}$	—	$32^\circ \text{F}$	—
Mercury freezes	234 K		$-39^\circ \text{C}$		$-38^\circ \text{F}$	
Ethyl alcohol freezes	143 K		$-130^\circ \text{C}$		$-202^\circ \text{F}$	
Nitrogen liquifies	77 K		$-196^\circ \text{C}$		$-321^\circ \text{F}$	
Helium liquifies	4.2 K		$-269^\circ \text{C}$		$-452^\circ \text{F}$	
Absolute zero	0 K		$-273^\circ \text{C}$		$-460^\circ \text{F}$	

**Fahrenheit** temperature  $T_F$ , measured in degrees Fahrenheit ( $^{\circ}\text{F}$ ), is defined relative to the Celsius temperature  $T_C$  by the equation

$$T_F = 32 + \frac{9}{5}T_C \quad (12-4)$$

The normal freezing and boiling points of water on the Fahrenheit scale are  $32^{\circ}\text{F}$  and  $212^{\circ}\text{F}$  respectively. The interval between these points is  $180^{\circ}\text{F}$ . There are only  $100^{\circ}\text{C}$  between these same points. Thus Celsius degrees are bigger than Fahrenheit degrees:  $1^{\circ}\text{C}$  is  $\frac{180}{100}$ , or  $\frac{9}{5}$ , times  $1^{\circ}\text{F}$ .

### EXAMPLE 1 Measuring a Fever on the Celsius Scale

Normal internal body temperature is  $98.6^{\circ}\text{F}$ . A temperature of  $106^{\circ}\text{F}$  is considered a high fever. Find the corresponding temperatures on the Celsius scale.

**SOLUTION** Solving Eq. 12-4 for  $T_C$ , we obtain

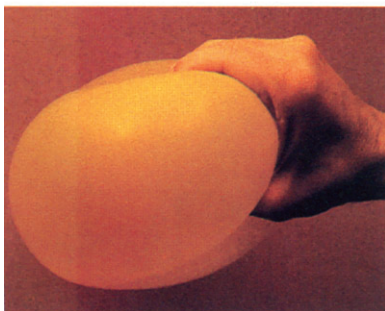
$$T_C = \frac{5}{9}(T_F - 32.0)$$

A temperature of  $98.6^{\circ}\text{F}$  corresponds to

$$T_C = \frac{5}{9}(98.6 - 32.0) = 37.0^{\circ}\text{C}$$

and a temperature of  $106^{\circ}\text{F}$  corresponds to

$$T_C = \frac{5}{9}(106 - 32.0) = 41.1^{\circ}\text{C}$$



**Fig. 12-2** If you squeeze one end of a balloon, you force the air inside the balloon into a smaller volume, and the pressure of the air increases.

## 12-2 Ideal Gas Law

The pressure of a gas can be changed in several ways. One way to increase the pressure of a gas confined to a fixed volume is to increase the number of gas molecules in the volume. You do this, for example, when you pump air into a bicycle tire or an automobile tire. Another way to change the pressure of a gas is to change its temperature. For example, when the air in an automobile tire heats up, its pressure increases significantly. A third way to change gas pressure is to change the volume containing the gas; decreasing volume causes an increase in pressure (Fig. 12-2).

For low-density gases, there is a simple, universal relationship between the gas pressure  $P$ , volume  $V$ , Kelvin temperature  $T$ , and number of gas molecules  $N$ . The product of  $P$  and  $V$  is proportional to the product of  $N$  and  $T$ :

$$PV \propto NT$$

or

$$PV = NkT \quad (12-5)$$

This equation is called the **ideal gas law**. The constant  $k$  is known as “Boltzmann’s constant” and is found from experiment to have the value

$$k = 1.380 \times 10^{-23} \text{ J/K} \quad (12-6)$$

The ideal gas law is most accurate in describing noble gases like neon or helium at low densities. But the ideal gas law provides a good approximate description of the behavior of other gases, so long as they are not close to the liquid state. In applying the ideal gas law, temperature must be expressed in kelvins, not in  $^{\circ}\text{C}$  or  $^{\circ}\text{F}$ .

Special cases of the gas law are found when one considers the variation of two of the variables  $P$ ,  $V$ ,  $N$ , and  $T$ , while the other two variables are held constant. For example, if  $N$  and  $T$  are fixed, the ideal gas law implies that the product  $PV$  is constant:

$$PV = \text{constant} \quad (\text{for constant } N \text{ and } T) \quad (12-7)$$

This result is known as Boyle's law, in honor of Robert Boyle, who discovered it in 1660. Boyle's law implies that if the volume of a gas is reduced to half its original value the pressure of the gas is doubled.

If  $P$  and  $N$  are fixed, the ideal gas law implies that the volume of the gas is directly proportional to its temperature:

$$V \propto T \quad (\text{for constant } N \text{ and } P) \quad (12-8)$$

This result was discovered by Joseph Gay-Lussac in 1802.

If  $V$  and  $N$  are fixed, the ideal gas law implies that

$$P \propto T \quad (\text{for constant } N \text{ and } V) \quad (12-9)$$

The very definition of temperature on the Kelvin scale requires that this relationship be satisfied, at least in the limit of a very low-density gas.

### EXAMPLE 2 The Temperature of an Ideal Gas After Compression

An ideal gas initially has a volume of 1.0 liter (L), a pressure of 1.0 atmosphere (atm), and a temperature of  $27^\circ\text{C}$ . The pressure is raised to 2.0 atm, compressing the volume of the gas to 0.60 L. Find the final temperature of the gas.

**SOLUTION** We are given the following initial and final values of  $P$ ,  $V$ , and  $T$ :

$$\begin{array}{ll} P_i = 1.0 \text{ atm} & P_f = 2.0 \text{ atm} \\ V_i = 1.0 \text{ L} & V_f = 0.60 \text{ L} \\ T_{\text{Ci}} = 27^\circ\text{C} & T_{\text{Cf}} = ? \end{array}$$

The number of molecules,  $N$ , is constant. The problem is to find the final temperature  $T_{\text{Cf}}$ . We can do this simply by first writing the ideal gas law for the initial state of the gas and again for the final state and then taking the ratio of the two expressions:

$$\begin{aligned} P_i V_i &= NkT_i \\ P_f V_f &= NkT_f \\ \frac{T_f}{T_i} &= \frac{P_i V_i}{P_f V_f} \end{aligned}$$

Since this equation involves ratios of pressures and volumes, we may insert these quantities in the units in which they are given, that is, atmospheres and liters, rather than converting to standard units of Pa and  $\text{m}^3$ . The conversion to standard units would simply introduce identical factors for both initial and final values, and these factors would cancel. We must be careful, however, to convert temperature from degrees Celsius to kelvins, even when a ratio is used, as it is here, since this change in units involves an additive term, rather than a multiplicative factor. Thus we must use  $T_i = 27 + 273 = 300\text{ K}$ . Substituting values into the preceding equation, we obtain

$$\frac{T_f}{300 \text{ K}} = \frac{(2.0 \text{ atm})(0.60 \text{ L})}{(1.0 \text{ atm})(1.0 \text{ L})} = 1.2$$

or

$$T_f = (1.2)(300 \text{ K}) = 360 \text{ K}$$

This corresponds to a final Celsius temperature  $T_{\text{Cf}}$  of

$$\begin{aligned} T_{\text{Cf}} &= T_f - 273 = 360 - 273 \\ &= 87^\circ\text{C} \end{aligned}$$

### Dalton's Law of Partial Pressures

The ideal gas law may also be applied to a mixture of noninteracting ideal gases. Suppose that  $N_1$  molecules of a single ideal gas contained in a volume  $V$  produce a pressure  $P_1$  at temperature  $T$  when this is the only gas in the volume, and suppose that  $N_2$  molecules of a second ideal gas contained in a volume  $V$  produce a pressure  $P_2$  at temperature  $T$  when the second gas is the only one contained in the volume. According to **Dalton's law of partial pressures**, a mixture of these two ideal gases produces a pressure  $P$  that is the sum of  $P_1$  and  $P_2$  (if there is no chemical interaction between the two gases):

$$P = P_1 + P_2$$

Using the ideal gas law to express  $P_1$  and  $P_2$  in terms of  $T$ ,  $V$ , and the respective numbers of molecules,  $N_1$  and  $N_2$ , we find

$$\begin{aligned} P &= \frac{N_1 kT}{V} + \frac{N_2 kT}{V} \\ &= \frac{(N_1 + N_2)kT}{V} \end{aligned}$$

Letting  $N$  denote the total number of molecules of both gases, that is,  $N = N_1 + N_2$ , we obtain

$$P = \frac{NkT}{V}$$

Dalton's law may be generalized to apply to a mixture of any number of noninteracting ideal gases. Thus the ideal gas law may be applied to air, with  $N$  representing the total number of molecules of any type—nitrogen, oxygen, and so forth.

#### EXAMPLE 3 The Number of Air Molecules in a Hot-air Balloon

The air inside a hot-air balloon (Fig. 12–3) is at a temperature of  $100.0^\circ\text{C}$ , while the temperature of the surrounding air in the atmosphere is  $20.0^\circ\text{C}$ . Find the ratio of the number of air molecules inside the balloon to the number of air molecules contained in an equal volume of air outside the balloon. Assume that the air pressure is the same inside and outside.

**SOLUTION** We first use the ideal gas law (Eq. 12–5) to obtain an expression for the number of air molecules in a volume  $V$ .

$$N = \frac{PV}{kT}$$

Using subscripts  $i$  and  $o$  to denote air inside and outside the balloon, we obtain the ratio of the number of air molecules inside to the number outside:

$$\frac{N_i}{N_o} = \frac{P_i V_i / kT_i}{P_o V_o / kT_o}$$

Since pressures  $P_i$  and  $P_o$  are equal and volumes  $V_i$  and  $V_o$  are equal, this reduces to

$$\frac{N_i}{N_o} = \frac{T_o}{T_i}$$



Fig. 12–3 Hot-air balloon.

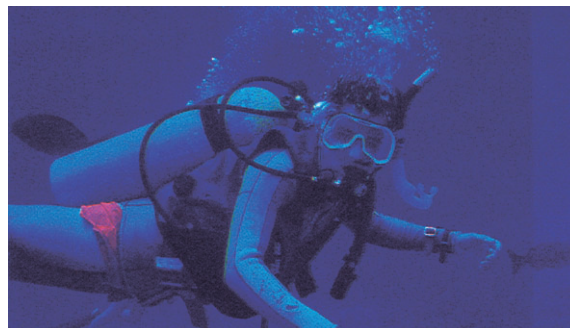
Converting temperatures to kelvins, we find

$$\frac{N_i}{N_o} = \frac{273 + 20}{273 + 100} = \frac{293\text{ K}}{373\text{ K}} = 0.786$$

According to Archimedes' principle, the hot-air balloon will experience a buoyant force equal to the weight of the displaced air. Since there is less air inside the balloon than in the volume of atmosphere displaced, the buoyant force is greater than the weight of the hot air, and the balloon will rise if the weight carried by the balloon is not too great. (Problem 50 asks you to calculate the volume of a hot-air balloon required to support 5000 N.)

**EXAMPLE 4 A Deep Dive Must be Short**

A Scuba diver breathes air stored in a tank carried on the back. At the beginning of a dive air pressure in the tank is about 200 atm, or 3000 lb/in<sup>2</sup>. Air pressure inside the tank gradually decreases as the diver uses the air. A pressure regulator adjusts the pressure of the air coming out of the tank so that when the diver breathes it the air is at the same pressure as the surrounding water. A certain diver has sufficient air to stay 60 min at a depth of 10 m below the ocean's surface, where the pressure is 2.0 atm (Fig. 12-4). How long would this diver's air supply last at a depth of 30 m, where the pressure is 4.0 atm? The diver breathes the same volume of air per unit time at any depth. Assume a constant temperature.



**Fig. 12-4** Scuba diver.

**SOLUTION** From the ideal gas law, we know that the number of air molecules in a volume  $V$  at pressure  $P$  and temperature  $T$  is given by

$$N = \frac{PV}{kT}$$

This equation shows that the number of air molecules in a given volume of air breathed by the diver is directly proportional to the air's pressure. Thus, as the diver descends to greater depths where the air pressure must be greater, a given volume of inhaled air contains more air molecules, and the air supply is therefore consumed more quickly. Since the volume of air consumed per unit time is constant, the volume consumed in time  $t$  is proportional to  $t$ .

$$V \propto t$$

Since  $N$  is proportional to  $PV$ , it follows that

$$N \propto Pt$$

A fixed number of air molecules are available to the diver. Thus the product of the air pressure and the time to consume the air has the same value at any depth.

$$P't' = Pt$$

Inserting  $P = 2.0$  atm and  $t = 60$  min for a depth of 10 m and  $P' = 4.0$  atm for a depth of 30 m, we find that the air supply at this depth lasts a time  $t'$ , where

$$(4.0 \text{ atm})t' = (2.0 \text{ atm})(60 \text{ min})$$

$$t' = 30 \text{ min}$$

Because the pressure is doubled, the air lasts only half as long.

## Atomic Mass

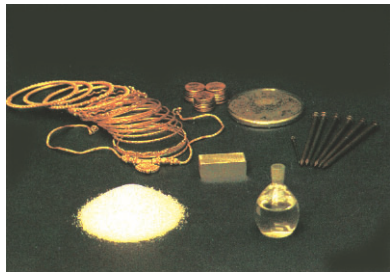
It is often convenient to express the ideal gas law in a slightly different form, known as the "molar form." To accomplish this, we first define atomic mass and the mole.

**Atomic mass is the mass of an atom relative to other atoms, using a scale in which the most common type of carbon atom is defined to have a mass of exactly 12.** A hydrogen atom has about  $\frac{1}{12}$  the mass of a carbon atom and so has an atomic mass of approximately 1. A helium atom has about  $\frac{4}{12}$  the mass of a carbon atom and so has an atomic mass of approximately 4. The periodic table of the elements shows the atomic masses of all the elements. The atomic masses listed there are actually averages over the different types of atoms naturally occurring for each element. For example, the atomic mass of carbon is given as 12.01, rather than exactly 12, because roughly 1% of all naturally occurring carbon atoms have a mass of 13.

The molecular mass of a molecule is the sum of the atomic masses of the atoms making up the molecule. For example, the molecular mass of the  $\text{H}_2\text{O}$  molecule equals the atomic mass of oxygen plus twice the atomic mass of hydrogen, that is, approximately  $16 + 2(1) = 18$ .

The unit of mass on the atomic mass scale is called the **atomic mass unit**, denoted by  $u$ . We can relate this unit to the gram. Experiment shows that

$$1 u = 1.6606 \times 10^{-24} \text{ g}$$



**Fig. 12-5** One mole of several substances: gold, copper, mercury, aluminum, iron, salt, and water. Each sample contains  $6.022 \times 10^{23}$  molecules. The masses of the samples vary in proportion to the molecular masses: from 18 g of water up to 201 g of mercury.

## Mole

Even very small quantities of matter consist of an enormously large number of molecules, and so it is convenient to express the quantity of matter in terms of a large unit, called the “mole.” **A mole is defined as a certain number of atoms or molecules, called Avogadro’s number, denoted by  $N_A$ .** The value of Avogadro’s number is such that **one mole** of a substance, consisting of any kind of atom or molecule, **has a mass numerically equal to the atomic or molecular mass of that substance expressed in grams.** For example, one mole of carbon-12 atoms has a mass of 12 g, and one mole of  $H_2O$  molecules has a mass of 18 g (Fig. 12–5).

One can compute Avogadro’s number by dividing the mass of 1 mole of carbon 12 (12 g) by the mass of a single carbon-12 atom, equal to 12 atomic mass units, where the atomic mass unit is related to the gram by the preceding equation.

$$N_A = \frac{12 \text{ g}}{12 \text{ u}} = \frac{12 \text{ g}}{12(1.6606 \times 10^{-24} \text{ g})}$$

$$N_A = 6.022 \times 10^{23} \quad (12-10)$$

We may express the number of molecules,  $N$ , of a substance as the product of Avogadro’s number,  $N_A$ , and the number of moles, denoted by  $n$ :

$$N = nN_A \quad (12-11)$$

### EXAMPLE 5 Number of Atoms in a Nail

Find the number of atoms in an iron nail of mass 5.00 g.

**SOLUTION** First we inspect the periodic table (shown on the inside back cover) and find that the atomic mass of iron (Fe) is 55.847. This means that 1 mole of naturally occurring iron has a mass of 55.847 g. We can now calculate the number of moles of iron in the nail, which we denote by  $n$ :

$$n = (5.00 \text{ g}) \left( \frac{1 \text{ mole}}{55.847 \text{ g}} \right) = 8.95 \times 10^{-2} \text{ mole}$$

Since 1 mole contains Avogadro’s number of atoms, the nail contains a number of atoms equal to the number of moles times  $N_A$ :

$$\begin{aligned} N &= nN_A = (8.95 \times 10^{-2} \text{ mole}) \left( \frac{6.022 \times 10^{23} \text{ atoms}}{1 \text{ mole}} \right) \\ &= 5.39 \times 10^{22} \text{ atoms} \end{aligned}$$

## Molar Form of the Ideal Gas Law

To obtain the molar form of the ideal gas law, we substitute  $N = nN_A$  (Eq. 12–11) into our original form of the gas law (Eq. 12–5):

$$PV = NkT = nN_A kT$$

The product  $N_A k$  is called the ideal gas constant, denoted by  $R$ .

$$\begin{aligned} R &= N_A k \\ &= (6.022 \times 10^{23})(1.380 \times 10^{-23} \text{ J/K}) \\ R &= 8.31 \text{ J/K} \end{aligned} \quad (12-12)$$

Substituting  $R$  for  $N_A k$  in the ideal gas law, we obtain the molar form of the gas law.

$$PV = nRT \quad (12-13)$$



**EXAMPLE 6 Finding the Mass of a Volume of Air**

Find the mass of air in a room with dimensions  $5.00\text{ m} \times 4.00\text{ m} \times 3.00\text{ m}$ , if the air pressure is  $1.00\text{ atm}$  and the temperature is  $27.0^\circ\text{C}$ .

**SOLUTION** First we apply the ideal gas law (Eq. 12-13) to find the number of moles, using the Kelvin temperature ( $T = 273 + 27.0 = 300\text{ K}$ ) and expressing pressure in Pa (Eq. 11-5:  $1.00\text{ atm} = 1.01 \times 10^5\text{ Pa}$ ).

$$\begin{aligned} n &= \frac{PV}{RT} \\ &= \frac{(1.00\text{ atm})(1.01 \times 10^5\text{ Pa/atm})(5.00\text{ m} \times 4.00\text{ m} \times 3.00\text{ m})}{(8.31\text{ J/K})(300\text{ K})} \\ &= 2.43 \times 10^3\text{ moles} \end{aligned}$$

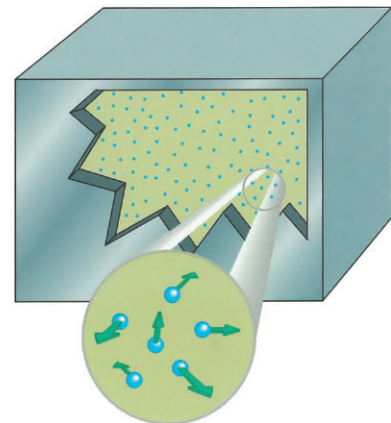
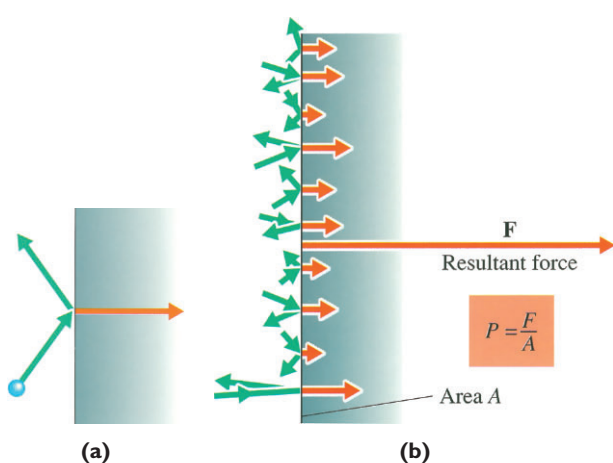
Since 1 mole has a mass equal to the molecular mass in grams, the mass of air equals the product of the number of moles times the molecular mass in grams. A nitrogen molecule  $\text{N}_2$  has a molecular mass of  $2(14) = 28$ , and an oxygen molecule  $\text{O}_2$  has a molecular mass of  $2(16) = 32$ . Air consists of approximately 80% nitrogen and 20% oxygen, and so the average molecular mass is  $0.8 \times 28 + 0.2 \times 32 = 28.8$ . Thus

$$\begin{aligned} m &= (2.43 \times 10^3\text{ moles})(28.8\text{ g/mole}) = 7.00 \times 10^4\text{ g} \\ &= 70.0\text{ kg} \end{aligned}$$

**12-3 Kinetic Theory; Model of an Ideal Gas****Kinetic Theory**

**Kinetic theory** is an area of physics that was developed in the late nineteenth century by Rudolph Clausius, James Clerk Maxwell, Ludwig Boltzmann, and others. Kinetic theory provides an explanation for the behavior of a macroscopic system in terms of its microscopic components—atoms or molecules, which obey dynamical laws. In this section we shall use kinetic theory to provide an explanation for the pressure of an ideal gas in terms of a molecular model.

A gas contained in a volume of macroscopic dimensions consists of an enormously large number of molecules. These molecules move in a random, chaotic way throughout the volume of the container (Fig. 12-6). When a molecule strikes a surface, it bounces off, exerting a small force on the surface (Fig. 12-7a). At any instant there will be many molecules colliding with the surface. The effect of these collisions is to produce a resultant force, which may be quite large (Fig. 12-7b).



**Fig. 12-6** A container of gas consists of a large number of molecules moving randomly and colliding with the walls of the container.

**Fig. 12-7 (a)** A single molecule collides with a wall of the container; during the collision the molecule exerts a small force perpendicular to the surface. **(b)** At any instant a large number of molecules collide with the surface. The effect of all these collisions is a resultant force  $\mathbf{F}$  and pressure  $P$ , which are effectively constant.

The resultant force is not steady but fluctuates rapidly, depending on the number of molecules striking the surface at any instant. But, for a surface of macroscopic size, the number of molecules involved is so large that fluctuations in the net force are negligibly small. Thus the molecules exert a pressure on the container walls that is effectively constant.

By way of analogy, consider the pressure that a strong, fine spray of water from a shower head produces on your back. There are many individual collisions with the back, resulting in a fairly steady pressure. In the case of gas pressure, each molecule exerts an extremely small force on the surface with which it collides. And yet there is such an enormously large number of collisions that the net effect is a pressure that is much greater and much more steady than that produced by a stream of water from a shower. Indeed atmospheric pressure is  $1.01 \times 10^5$  Pa, or 14.7 lb/in<sup>2</sup>, an indication that one's back is constantly subject to a total force amounting to several hundred pounds because of air pressure. Of course in normal circumstances atmospheric pressure is the same on all surfaces of the body, and so it gives rise to no resultant force.

### Kinetic Interpretation of Temperature

Rudolf Clausius first derived the ideal gas law from kinetic theory in 1857. He used a very oversimplified model of a gas, in which the molecules were assumed to move in three orderly lines, perpendicular to the sides of the box. In the next section we shall derive the ideal gas law, using the more realistic model of molecules moving randomly in all directions. This derivation, first presented by James Clerk Maxwell in 1859, shows that **the average kinetic energy of an ideal gas molecule equals  $\frac{1}{2}kT$** . A molecule of mass  $m$  traveling at speed  $v$  has kinetic energy  $K = \frac{1}{2}mv^2$ . Denoting the average values of  $K$  and  $v^2$  by  $\bar{K}$  and  $\bar{v}^2$ , we may express Maxwell's result as:

$$\bar{K} = \frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT \quad (12-14)$$

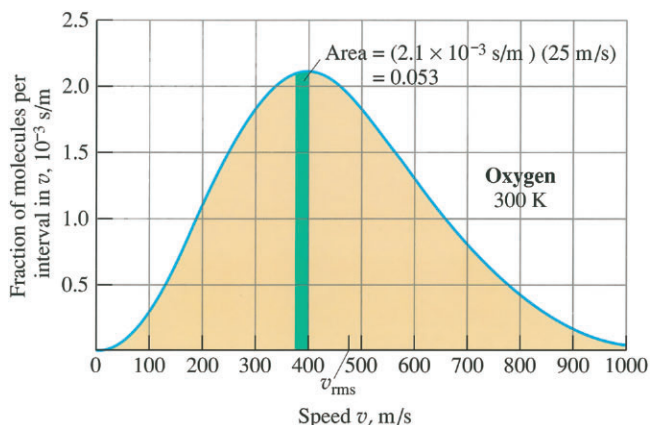
This equation shows that **the temperature of an ideal gas is a measure of the average kinetic energy of its molecules**. In light of this result, the phenomenon of thermal equilibrium is easy to understand. Two systems of gas that are initially at different temperatures have different values of average kinetic energy per molecule. When the systems are placed in thermal contact, the system with the higher temperature will lose energy as the system with the lower temperature gains energy. This process continues until the average molecular kinetic energies and hence also the temperatures of the two systems are the same. Thus thermal equilibrium is simply a consequence of the equal sharing of kinetic energy among the molecules of both systems.

Although Eq. 12-14 is easily derived only for an ideal gas, it applies to any system, including liquids and solids. The average translational kinetic energy of the molecules in a body at absolute temperature  $T$  equals  $\frac{3}{2}kT$ .

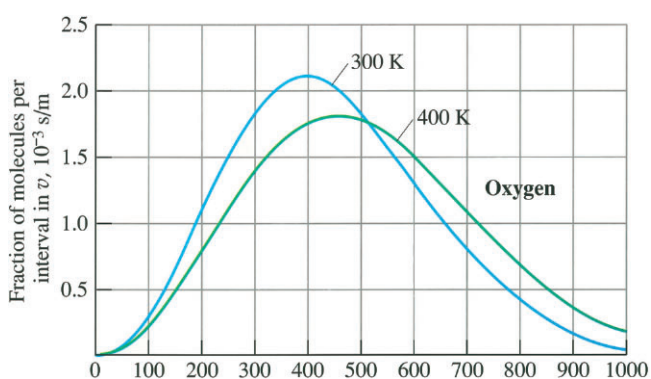
### Maxwell Distribution

Maxwell carried his analysis a step farther and derived an expression for the distribution of molecular speeds. Maxwell's theoretical prediction is shown graphically in Fig. 12-8 for oxygen at a temperature of 300 K.

Fig. 12-9 indicates how the distribution changes with temperature. As  $T$  increases, the distribution becomes broader and shifts to the right, toward higher values of  $v$ . The Maxwell distribution has been amply verified by experiments.



**Fig. 12-8** Distribution of molecular speeds for oxygen at 300 K. The area under the curve between any two values of  $v$  gives the fraction of the molecules having speed somewhere in the interval between the two values of  $v$ . For example, the cross-hatched area is 0.053, meaning that 5.3% of the molecules have values of  $v$  between 375 m/s and 400 m/s.



**Fig. 12-9** Distribution of molecular speeds for oxygen at 300 K and at 400 K.

### Root Mean Square Speed

It is convenient to be able to characterize the Maxwell distribution by a single number, typical of molecular speeds at a given temperature. For this purpose one may use either the average speed or the root mean square (rms) speed, defined as the square root of the average squared speed and denoted by  $v_{\text{rms}}$ .

$$v_{\text{rms}} = \sqrt{\overline{v^2}} \quad (12-15)$$

For the Maxwell distribution  $v_{\text{rms}}$  is approximately equal to the average speed.

Since we have already derived an expression for the average molecular kinetic energy (Eq. 12-14), it is easy to obtain an expression for  $v_{\text{rms}}$ .

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$

Thus

$$\overline{v^2} = \frac{3kT}{m}$$

and

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \quad (12-16)$$

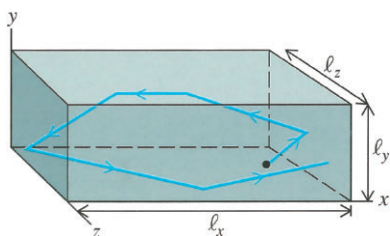
**EXAMPLE 7** RMS Speed of Oxygen Molecules at Room Temperature

Calculate  $v_{\text{rms}}$  for oxygen molecules at a temperature of 300 K.

**SOLUTION** The molecular mass of oxygen is 32, which means that an oxygen molecule has a mass of 32 atomic mass units (32 u), where  $u = 1.66 \times 10^{-27}$  kg. Applying Eq. 12-16, we find

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT}{32u}} \\ &= \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{32(1.66 \times 10^{-27} \text{ kg})}} \\ &= 484 \text{ m/s} \end{aligned}$$

This speed, faster than a speeding bullet, is near the center of the molecular speed distribution shown in Fig. 12-8.

**\*12-4** Derivation of the Ideal Gas Law

**Fig. 12-10** The path of a single gas molecule in a box.

We shall now derive the ideal gas law, using the kinetic theory model of an ideal gas described in the last section. We shall do this by first obtaining an expression for the pressure exerted by the molecules on a surface. This expression turns out to be a function of the number of molecules per unit volume and the molecules' average kinetic energy. By relating temperature to molecular kinetic energy, we shall obtain the ideal gas law. This derivation serves two purposes: (1) to understand how the empirical ideal gas law is the result of molecular dynamics and (2) to give a mechanical interpretation of temperature as a measure of molecular kinetic energy.

To derive the ideal gas law, we shall find it necessary to make certain assumptions:

1. **It is valid to apply Newton's laws to gas molecules**, just as we would to billiard balls or other macroscopic bodies.\*
2. **Collisions of molecules with the walls of the container are elastic, and the walls are smooth.** It follows that when a molecule strikes a wall, the molecule rebounds, with its component of velocity perpendicular to the wall reversed, as shown in Problem 32, Chapter 8.
3. **The number of molecules is large, and their motion is random.**
4. **The time during which molecules are in contact either with the walls or with each other is negligible** compared to the time during which they are moving freely. This requires of course that the molecules not be too tightly packed, that is, that the density of the gas be sufficiently low.

Let the container for the gas be a rectangular box with sides of length  $l_x$ ,  $l_y$ , and  $l_z$ , aligned with the  $x$ ,  $y$ ,  $z$  coordinate axes (Fig. 12-10). First we shall derive an expression for the pressure on the right side of the container, based on the assumption of no collisions between molecules. After deriving this result, we shall consider the effect of intermolecular collisions.

\*Many features of atoms and molecules can be described only by use of the more general (and difficult) methods of quantum theory, rather than Newton's laws, as we shall see in Chapter 29. It is correct to use classical mechanics whenever the results obtained by classical mechanics are a good approximation to quantum predictions, as is the case here.

Consider the path of a single molecule, beginning at the instant just before it strikes the right side of the box. The effect of the collision with the right side is to reverse the  $x$  component of the molecule's velocity:  $v_x$  changes to  $-v_x$ . If this molecule does not collide with any other molecule, it will strike the left side of the box a time  $\ell_x/v_x$  later, bounce off the left wall with its  $x$  component of the velocity reversed again ( $-v_x$  changes to  $+v_x$ ), and return to the right side a time  $\ell_x/v_x$  after this collision. Thus the complete time  $\Delta t$  for the molecule to traverse the length of the box and return to the right side is given by

$$\Delta t = \frac{2\ell_x}{v_x}$$

Of course, the molecule may strike other sides of the box between collisions with the right and left ends, as shown in Fig. 12-10. These collisions do not change  $v_x$ , and so they do not affect the time interval between successive collisions with the right side of the box.

Each time the molecule collides with the right surface, the molecule exerts a force  $F_x$  to the right. This is just the reaction to the force  $F'_x$  the surface exerts on the molecule, and by Newton's third law  $F_x = -F'_x$ . According to Newton's second law, the average value of the force on the molecule,  $\bar{F}_x$ , is the product of the molecule's mass and its average  $x$  component of acceleration,  $\bar{a}_x$ . Thus

$$\begin{aligned}\bar{F}_x &= -\bar{F}'_x = -m\bar{a}_x \\ &= -m \frac{\Delta v_x}{\Delta t}\end{aligned}$$

For each collision with the wall, the molecule undergoes a change in  $v_x$  given by  $\Delta v_x = (-v_x) - v_x = -2v_x$ . And since the time interval  $\Delta t$  between collisions is  $2\ell_x/v_x$ , the average force may be expressed

$$\bar{F}_x = \frac{-m(-2v_x)}{2\ell_x/v_x} = \frac{mv_x^2}{\ell_x}$$

Within the box there are  $N$  molecules, each having its own value of  $v_x^2$ . The resultant force  $\Sigma \bar{F}_x$  is just the sum of the forces exerted by the  $N$  molecules.

$$\Sigma \bar{F}_x = \Sigma \frac{mv_x^2}{\ell_x} = \frac{m}{\ell_x} \Sigma v_x^2$$

We may express  $\Sigma v_x^2$  in terms of the average value of  $v_x^2$ , denoted by  $\overline{v_x^2}$ , and defined as  $\overline{v_x^2} = \frac{1}{N} \Sigma v_x^2$ .

$$\Sigma \bar{F}_x = \frac{mN\overline{v_x^2}}{\ell_x}$$

Dividing  $\Sigma \bar{F}_x$  by the area of the surface  $\ell_y\ell_z$ , we obtain the pressure  $P$ .

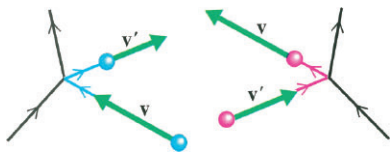
$$P = \frac{\Sigma \bar{F}_x}{\ell_y\ell_z} = \frac{Nm\overline{v_x^2}}{\ell_x\ell_y\ell_z}$$

Since the volume  $V = \ell_x\ell_y\ell_z$ , we have

$$P = \frac{Nm\overline{v_x^2}}{V}$$

or

$$PV = Nm\overline{v_x^2} \quad (12-17)$$



**Fig. 12–11** A molecule changes its velocity from  $\mathbf{v}$  to  $\mathbf{v}'$  because of a collision. A second molecule, at approximately the same location, undergoes a collision that changes its velocity from  $\mathbf{v}'$  to  $\mathbf{v}$ .

The model we have given so far is a reasonable model for a very low-density gas. However, it is a terribly unrealistic picture of a gas such as air at normal density, since we have neglected collisions between molecules. An air molecule may undergo something on the order of  $10^5$  collisions for every cm it travels. These collisions will certainly destroy the regular path we have assumed for a single molecule moving across the length of the box. We shall see, however, that the very large number of molecules and the randomness of their motion cause the result to be the same as though there were no collisions. At this point assumption 3 must be used in a very specific way: for every molecule that has its velocity changed from  $\mathbf{v}$  to  $\mathbf{v}'$  by a collision, we assume that there is another molecule at approximately the same location that undergoes a collision that changes its velocity from  $\mathbf{v}'$  to  $\mathbf{v}$ . See Fig. 12–11. Whenever a molecule undergoes a collision, it is replaced by another molecule at essentially the same position and moving with the same velocity, and so the preceding calculation of pressure is unaffected.

To show that this assumption is plausible, we note that at typical gas densities of  $10^{25}$  molecules per cubic meter a tiny cube of edge length  $10^{-2}$  mm, or  $10^{-5}$  m, and volume  $10^{-15}$  m<sup>3</sup> contains on the order of  $10^{10}$  molecules. Although only a small fraction of these molecules undergo collisions at any one instant (perhaps one out of  $10^3$  or  $10^4$ ), still the total number in the cube undergoing simultaneous collisions is large (about  $10^6$  or  $10^7$ ), and the collisions are completely random. So it is very reasonable to assume that with this large number of random collisions the distribution of molecular velocities remains unchanged.

Finally, if the preceding calculation of pressure is to remain unchanged by the effects of intermolecular collisions, we must assume that the time of intermolecular contact is small compared to the time between collisions (assumption 4), and so our calculation of  $\Delta t$  is unaffected.

It is convenient to express Eq. 12–17 in terms of the average value of  $v^2$ , rather than the average value of the component  $v_x^2$ . Since  $v^2 = v_x^2 + v_y^2 + v_z^2$ , the average value of  $v^2$  is the sum of the averages of  $v_x^2$ ,  $v_y^2$ , and  $v_z^2$ .

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$$

But there is no reason for the average squared components of velocity along the  $x$ ,  $y$ , and  $z$  axes to differ, and so  $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$ , and we can express the equation above as

$$\overline{v^2} = 3\overline{v_x^2}$$

or

$$\overline{v_x^2} = \frac{1}{3}\overline{v^2}$$

Substituting this result into Eq. 12–17, we obtain

$$PV = Nm\left(\frac{1}{3}\overline{v^2}\right) = N\left(\frac{1}{3}\overline{v^2}\right)$$

We can complete our derivation of the ideal gas law ( $PV = NkT$ ) if we identify  $\frac{1}{3}m\overline{v^2}$  in the expression above as  $kT$ . Thus in the process of deriving the gas law, we arrive at a mechanical interpretation of the temperature of an ideal gas.

$$\frac{1}{3}m\overline{v^2} = kT$$

or

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT \quad (12-18)$$

We have used an atomic model of a gas to derive the ideal gas law. Although today this seems a very natural way to explain gas pressure, it was not always so. In the early 1900s the very existence of atoms was not universally accepted, since there was no direct evidence of them. Ludwig Boltzmann was one of the primary advocates of the atomic theory and used the atomic hypothesis in his analysis of macroscopic

systems. But Boltzmann's work was severely criticized by other famous physicists, including Ernst Mach, on the grounds that physical theory should not be based on hypothetical entities such as atoms. Boltzmann committed suicide in 1906, just 2 years before direct confirmation of the atomic theory and universal acceptance of his methods.

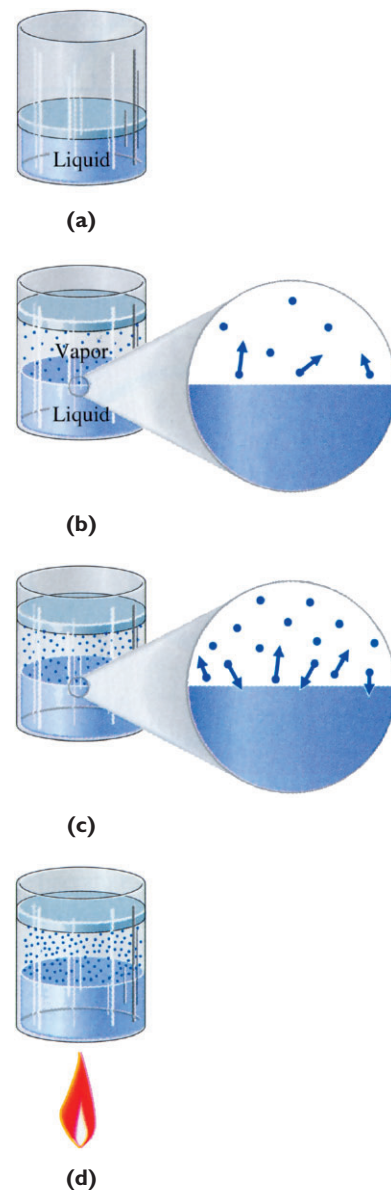
### \*12-5 Vapor Pressure and Humidity

If you fill a glass with water and leave it, within a day or two the water level in the glass will drop noticeably, unless the surrounding air is very damp. The water "evaporates." This means that water molecules leave the liquid and form a vapor or gas that mixes with the air.

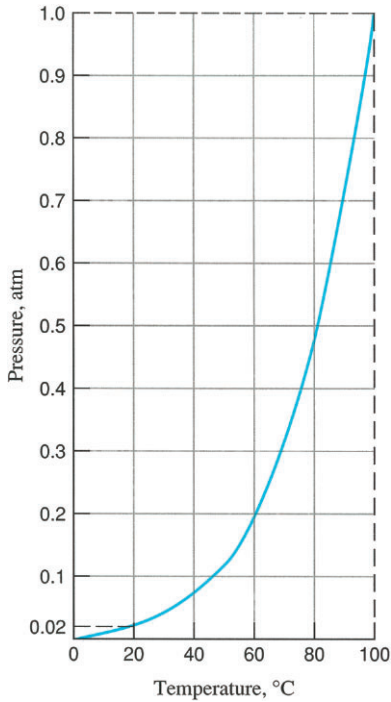
We can understand evaporation on a molecular level, using concepts of kinetic theory. The molecules in a liquid have a distribution of velocities, similar to the Maxwell-Boltzmann distribution of molecular velocities in a gas. Although intermolecular forces bind most of the molecules close together in the liquid, some of the molecules move fast enough to leave the surface of the liquid, like a rocket with a velocity greater than escape velocity leaving the earth. Since the molecules that evaporate are those with the greatest velocity and kinetic energy, the average kinetic energy of the molecules remaining in the liquid decreases, and so the temperature of the liquid decreases. You can feel the cooling effect of evaporation when you step out of a shower and water evaporates from your skin. The effect is more dramatic when the air is very dry. If you step out of a swimming pool in the desert, even though the air may be quite hot, you can be chilled by water evaporating from your skin.

To better understand the process of evaporation, consider a liquid in a closed container with a piston (Fig. 12-12a). If the piston is raised, evaporation begins. Vapor fills the space above the liquid (Fig. 12-12b) and creates a pressure, called **vapor pressure**. As more and more molecules enter the vapor, some molecules begin to go from the vapor back into the liquid. Initially more molecules leave the liquid than enter it, and both the density and pressure of the vapor increases. There is soon reached an equilibrium state, in which as many molecules enter the liquid as leave it. We say the vapor is then "saturated," since there can be no further increase in the number of molecules in the vapor (Fig. 12-12c). In this equilibrium state, vapor pressure reaches its maximum value, called **saturated vapor pressure**. If the piston is raised higher, more molecules enter the vapor phase until the vapor pressure again reaches the same saturated vapor pressure.

As the temperature of a liquid increases, more of its molecules have sufficient kinetic energy to escape the liquid. Thus, as temperature increases, the rate at which molecules leave the surface of the liquid increases; that is, the rate of evaporation increases. An equilibrium state is not reached until the density of the vapor phase increases enough that the rate at which molecules reenter the liquid matches the new higher rate at which molecules leave the liquid (Fig. 12-12d). The higher-density equilibrium state is one of higher pressure. Thus, as temperature increases, saturated vapor pressure increases.



**Fig. 12-12** (a) A liquid in a closed container with a movable piston. (b) Molecules begin to leave the liquid and form a vapor when the piston is raised. (c) Saturated vapor in equilibrium with the liquid. Equal numbers of molecules enter and leave the liquid. (d) Saturated vapor at a higher temperature has greater density and pressure.



**Fig. 12-13** Saturated vapor pressure of water as a function of temperature.

Fig. 12-13 shows a graph of saturated vapor pressure versus temperature for water. At 20° C water's saturated vapor pressure is only 0.02 atm, at 30° C it doubles to 0.04 atm, and at 100° C it increases to 1.0 atm—50 times greater than at 20° C.

When water is exposed to the open air, evaporation proceeds in the same way as when no air is present. According to Dalton's law of partial pressures, a mixture of gases produces a pressure that is the sum of the pressures produced by each of the separate gases. Thus nitrogen, oxygen, and water vapor each contribute their own partial pressures to the total pressure of our atmosphere. At a temperature of 20° C and a pressure of 1 atm, the partial pressure of water vapor in the air can be no greater than 2% of the total pressure, or 0.02 atm, since the saturated vapor pressure of water is 0.02 atm at 20° C.

We refer to the water content of air as its **humidity**. Humidity is at a maximum at a given temperature when the partial pressure of water in the air equals the saturated vapor pressure of water at that temperature. Since saturated vapor pressure increases rapidly with increasing temperature, the maximum humidity of warm air is much greater than the maximum humidity of cooler air. Typically air contains water vapor at a partial pressure below its saturated value. The **relative humidity** of air is defined as the ratio of the partial pressure of the water vapor in the air to the saturated vapor pressure of water at that temperature.

$$\text{Relative humidity} = \frac{\text{Partial pressure of water vapor}}{\text{Saturated vapor pressure of water}} \times 100 \quad (12-19)$$

For example, if the partial pressure of water vapor in air at 20° C is 0.01 atm, since the saturated vapor pressure of water is 0.02 atm at 20° C, the relative humidity of the air is

$$\frac{0.01 \text{ atm}}{0.02 \text{ atm}} \times 100 = 50\%.$$

### Dew, Fog, and Rain

When air containing water vapor is cooled sufficiently, some of the vapor condenses to the liquid state. This condensation begins when the air drops below the temperature at which the water vapor's partial pressure equals the saturated vapor pressure, in other words, when it drops below the temperature at which relative humidity is 100%. This temperature is called the **dew point**. Such condensation often occurs at night when dew forms on the ground, as the temperature of the ground falls and cools the surrounding air. For example, suppose air temperature during the day is 30° C with a relative humidity of 75%, meaning that the partial pressure of water vapor in the air is 75% of the saturated vapor pressure of 0.04 atm at 30° C, or 0.03 atm. Then, if the temperature of the air near the ground falls to 20° C at night, since the saturated vapor pressure of water at that temperature is only 0.02 atm, the partial pressure of water vapor in the air must drop by one third from 0.03 atm to 0.02 atm, meaning that one third of the water vapor must condense.

Fog results when a humid warm air mass mixes with cooler air and tiny droplets of water form as the temperature of the warm air drops below the dew point. Clouds form when air rises and is cooled below the dew point. When a cloud cools suddenly, condensation is more rapid, water droplets increase in size, and then come together to form larger drops, which fall as rain.



## Boiling

If the temperature of a liquid is raised enough so that the liquid's vapor pressure equals the pressure of the surrounding air, the liquid begins to boil. That is, bubbles of vapor form within the bulk liquid. These vapor bubbles push outward against the liquid, which is at approximately the same pressure as the air. Water boils at a temperature of  $100^\circ\text{C}$  when the surrounding air is at a pressure of 1.00 atm because the saturated vapor pressure of air at  $100^\circ\text{C}$  is 1.00 atm. If the surrounding air is at a lower pressure, water will boil at a lower temperature. For example, on a mountain at an elevation of 3000 m, where atmospheric pressure is only 0.7 atm, water boils at a temperature of  $90^\circ\text{C}$ , since its saturated vapor pressure at that temperature equals 0.7 atm.

## 12-6 Thermal Expansion

Nearly all solids and liquids expand as they are heated. The fractional increase in volume,  $\Delta V/V$ , is often found to be directly proportional to the increase in temperature,  $\Delta T$ . The constant of proportionality is called the "volume coefficient of expansion," denoted by  $\beta$ . Thus

$$\frac{\Delta V}{V} = \beta \Delta T$$

or

$$\Delta V = \beta V \Delta T \quad (12-20)$$

The change in volume  $\Delta V$  is proportional to the original volume  $V$ , as well as to the temperature change  $\Delta T$ . Thus, for example, if 1 liter ( $1000\text{ cm}^3$ ) of water is heated from  $20^\circ\text{C}$  to  $25^\circ\text{C}$ , its volume increases by only about  $1\text{ cm}^3$ . But if the water in a swimming pool of volume  $1000\text{ m}^3$  is heated over the same temperature interval, the water increases in volume by  $1\text{ m}^3$ , or  $10^6\text{ cm}^3$ . In both cases the ratio  $\Delta V/V$  is  $10^{-3}$ .

In the case of solids, the volume expansion is accomplished by an increase in all linear dimensions. As a solid is heated, the distance between any two points in the solid increases. The fractional increase in length is normally the same in all directions. Thus, if a block of marble expands thermally by 0.1% in length, the block's height and width will each also increase by 0.1%. The increase in size is like a photographic enlargement (Fig. 12-14).

Instead of using a volume coefficient of expansion for solids, we normally use a linear expansion coefficient  $\alpha$ , which is a measure of the fractional change in the linear dimensions of the solid. For a temperature change  $\Delta T$ , a length  $\ell$  changes by  $\Delta\ell$ , where

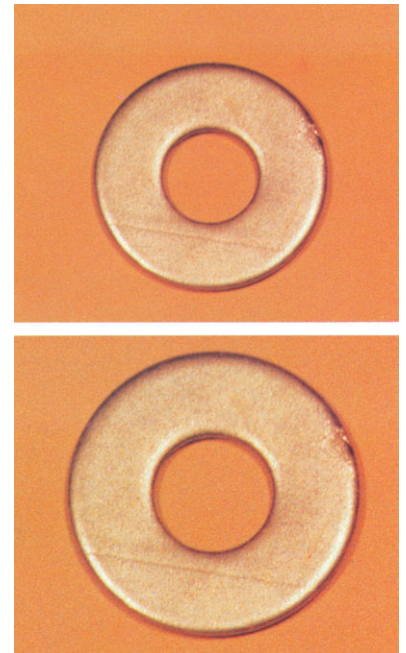
$$\frac{\Delta\ell}{\ell} = \alpha \Delta T$$

or

$$\Delta\ell = \alpha \ell \Delta T \quad (12-21)$$

It is possible to show that the volume coefficient of expansion for a solid equals 3 times its linear coefficient:

$$\beta = 3\alpha \quad (\text{for solids}) \quad (12-22)$$



**Fig. 12-14** As a washer is heated from temperature  $T$  to temperature  $T + \Delta T$ , all its linear dimensions get bigger: Even the hole gets bigger. The actual expansion, however, is much smaller than indicated here. The expansion shown here is 20%, which is approximately 100 times greater than the expansion of aluminum heated  $100^\circ\text{C}$ .

A proof of this result is outlined in Problem 42. Coefficients of expansion for various materials are given in Table 12–2. For liquids there is no measure of linear expansion, since liquids must conform to the shapes of their containers.

**Table 12–2** Coefficients of thermal expansion

Solids	$\alpha, (\text{C}^\circ)^{-1}$	Liquids and gases	$\beta, (\text{C}^\circ)^{-1}$
Aluminum	$2.4 \times 10^{-5}$	Ethyl alcohol	$1.1 \times 10^{-3}$
Brass	$1.9 \times 10^{-5}$	Mercury	$1.8 \times 10^{-4}$
Brick	$1 \times 10^{-5}$	Olive oil	$7.2 \times 10^{-4}$
Concrete	$1.2 \times 10^{-5}$	Water	$2.1 \times 10^{-4}$
Copper	$1.7 \times 10^{-5}$	Air	$3.67 \times 10^{-3}$
Diamond	$1.2 \times 10^{-6}$	Hydrogen	$3.66 \times 10^{-3}$
Glass	$4 \times 10^{-6}$ to $1 \times 10^{-5}$	Nitrogen	$3.67 \times 10^{-3}$
Gold	$1.4 \times 10^{-5}$		
Graphite	$2 \times 10^{-6}$		
Ice	$5.1 \times 10^{-5}$		
Paraffin	$1 \times 10^{-4}$		
Steel	$1.2 \times 10^{-5}$		
Wood, parallel to fiber	$5 \times 10^{-6}$ to $1 \times 10^{-5}$		
Wood, across fiber	$3 \times 10^{-5}$ to $6 \times 10^{-5}$		

### EXAMPLE 8 Expansion of the Golden Gate Bridge

Find the change in the total length of the 2700 m long Golden Gate Bridge, as the temperature of the bridge increases from  $5^\circ\text{C}$  to  $25^\circ\text{C}$ . The bridge is constructed of steel and concrete.

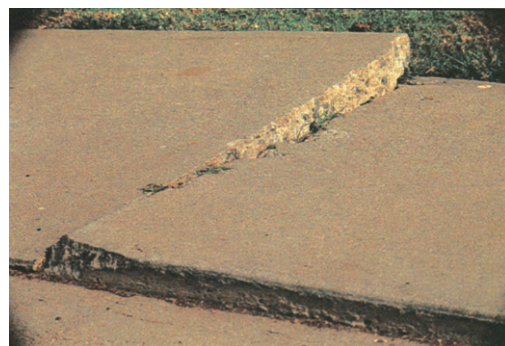
**SOLUTION** From Table 12–1 we find that steel and concrete have the same coefficient of thermal expansion:  $1.2 \times 10^{-5} (\text{C}^\circ)^{-1}$ . Thus both materials expand equally. We apply Eq. 12–21 to find the change in the bridge's length for the  $20.0\text{C}^\circ$  increase in temperature.

$$\begin{aligned}\Delta\ell &= \alpha\ell\Delta T = [1.2 \times 10^{-5} (\text{C}^\circ)^{-1}](2700\text{ m})(20.0\text{C}^\circ) \\ &= 0.65\text{ m} = 65\text{ cm}\end{aligned}$$

Bridges must be constructed with expansion joints to allow for such thermal expansion (Fig. 12–15); otherwise they might buckle. Sidewalks can also buckle. If a concrete sidewalk is poured at a cool temperature with no allowance for thermal expansion, the sidewalk can buckle when it gets hot (Fig. 12–16).



**Fig. 12–15** An expansion joint on a bridge.



**Fig. 12–16** This sidewalk buckled because of thermal expansion.

**EXAMPLE 9 Overflow of an Expanding Liquid**

A glass container that has a volume of 1.0 liter and is filled with alcohol is initially refrigerated at a temperature of 7° C. How much of the alcohol will overflow if the container is placed in a warm room where the temperature is 27° C? The linear coefficient of expansion for the glass is  $1.0 \times 10^{-5} (\text{C}^\circ)^{-1}$ .

**SOLUTION** The volume coefficient of expansion for the glass is 3 times its linear coefficient, or  $3.0 \times 10^{-5} (\text{C}^\circ)^{-1}$ . Thus

$$\begin{aligned}\Delta V_{\text{glass}} &= \beta_{\text{glass}} V \Delta T \\ &= [3.0 \times 10^{-5} (\text{C}^\circ)^{-1}](1.0 \times 10^3 \text{ cm}^3)(20 \text{ C}^\circ) \\ &= 0.60 \text{ cm}^3\end{aligned}$$

From Table 12-1 we find that the volume coefficient of expansion for alcohol is  $1.1 \times 10^{-3}$ —much larger than the coefficient for glass. Therefore the alcohol will expand more than the glass.

$$\begin{aligned}\Delta V_{\text{alc}} &= \beta_{\text{alc}} V \Delta T \\ &= [1.1 \times 10^{-3} (\text{C}^\circ)^{-1}](1.0 \times 10^3 \text{ cm}^3)(20 \text{ C}^\circ) \\ &= 22 \text{ cm}^3\end{aligned}$$

Since the volume of the glass container increases by only about 1 cm<sup>3</sup>, about 21 cm<sup>3</sup> of the alcohol will overflow.

**EXAMPLE 10 Thermal Stress**

A 2.0 m long aluminum window frame is mounted snugly in a brick wall at a temperature of 10° C. Find the stress in the frame when the temperature rises to 30° C. Young's modulus for aluminum is  $6.9 \times 10^{10} \text{ N/m}^2$ .

**SOLUTION** From Table 12-1 we find that the coefficients of thermal expansion for aluminum and brick are  $2.4 \times 10^{-5} (\text{C}^\circ)^{-1}$  and  $1 \times 10^{-5} (\text{C}^\circ)^{-1}$  respectively. Thus, if both were free to expand as the temperature increases, the aluminum would expand more. However, the expansion of the aluminum is limited by the surrounding brick. The window can expand no more than the opening, which expands in the same way as the brick itself. The brick walls exert stress on the aluminum, in effect compressing it from the dimensions it would have if it were free to expand to the dimensions of the expanded opening.

First we apply Eq. 12-21 to compute the increase in the length of the opening,  $\Delta \ell_{\text{opening}}$ , for a 20 C° temperature increase, using the thermal expansion coefficient for brick.

$$\begin{aligned}\Delta \ell_{\text{opening}} &= \alpha \ell \Delta T = [1 \times 10^{-5} (\text{C}^\circ)^{-1}](2.0 \text{ m})(20 \text{ C}^\circ) \\ &= 4 \times 10^{-4} \text{ m} = 0.4 \text{ mm}\end{aligned}$$

Next we compute the increase in length the aluminum *would* have if it were free to expand thermally.

$$\begin{aligned}\Delta \ell_{\text{Al}} &= \alpha \ell \Delta T = [2.4 \times 10^{-5} (\text{C}^\circ)^{-1}](2.0 \text{ m})(20 \text{ C}^\circ) \\ &= 9.6 \times 10^{-4} \text{ m} = 0.96 \text{ mm}\end{aligned}$$

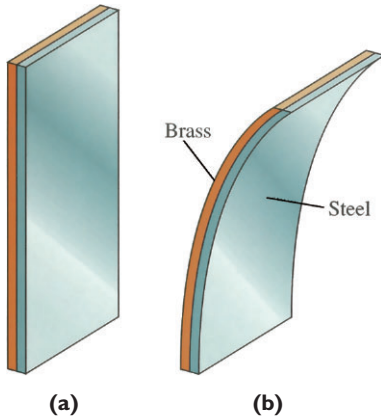
The brick's effect on the window frame is to reduce its expansion from 0.96 mm to 0.4 mm. That is, the brick in effect compresses the aluminum frame by  $0.96 \text{ mm} - 0.4 \text{ mm} = 0.6 \text{ mm}$ . This means that the brick exerts a stress  $\sigma$ , which we can compute using Eqs. 10-6 ( $\epsilon = \Delta \ell / \ell$ ) and 10-8 ( $Y = \sigma / \epsilon$ )

$$\begin{aligned}\sigma &= Y\epsilon = Y \frac{\Delta \ell}{\ell} \\ &= (6.9 \times 10^{10} \text{ N/m}^2) \frac{0.6 \times 10^{-3} \text{ m}}{2.0 \text{ m}} \\ &= 2 \times 10^7 \text{ N/m}^2\end{aligned}$$

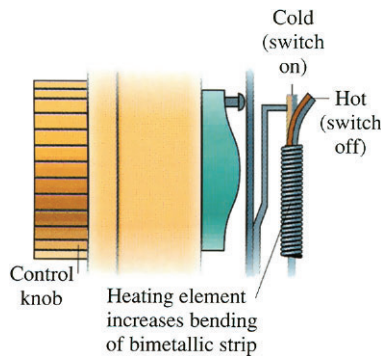
This means that every 1 cm<sup>2</sup> section of the window frame adjacent to the brick is subjected to a force

$$F = \sigma A = (2 \times 10^7 \text{ N/m}^2)(10^{-2} \text{ m})^2 = 2000 \text{ N}$$

Such a large force is likely to bend the frame and break the window glass.



**Fig. 12-17** (a) A bimetallic strip. Both metals initially have the same length. (b) When heated, the brass expands more than the steel.



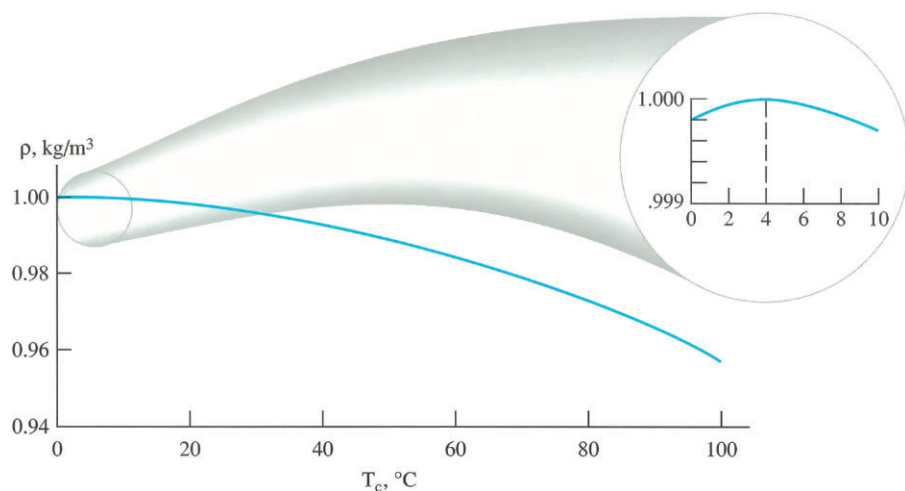
**Fig. 12-18** The thermostat switch opens when the temperature increases enough to bend the bimetallic strip to the right.

### Bimetallic Strip

Thermal expansion is applied in a bimetallic strip, which can be used to make a thermometer or a thermostat. The bimetallic strip is formed by welding or riveting together two thin strips of metal with different  $\alpha$ 's (Fig. 12-17). For example, a bimetallic strip may consist of brass and steel, with thermal expansion coefficients of  $1.9 \times 10^{-5} (\text{C}^\circ)^{-1}$  and  $1.2 \times 10^{-5} (\text{C}^\circ)^{-1}$  respectively. When heated, the brass tends to expand more than the steel, since  $\alpha$  is greater for brass than for steel. But along the surface where the two metals are bonded together, their expansion must be the same. The result is that the two strips bend toward the steel, and so the average separation between molecules in the brass is greater than the average separation between molecules in the steel. Fig. 12-18 shows how a bimetallic strip can be used in a thermostat, which controls a heating system. The thermostat switch is on when the temperature is low, and the switch turns off when the temperature increases.

### Thermal Expansion of Water

In many cases the simple linear dependence of  $\Delta V$  on  $\Delta T$  expressed by Eq. 12-20 is valid over all temperature ranges commonly encountered, with a constant value for  $\beta$ . However, for some substances the variation of volume with temperature is more complicated. Water is such a substance. Fig. 12-19 shows the density of water as a function of temperature. Notice that at most temperatures the density of water decreases as its temperature increases; that is, water expands as it is heated. But in the temperature range from  $0^\circ \text{C}$  to  $4^\circ \text{C}$  water contracts as it is heated. Water is one of the few materials that have this property. This has an important effect on the rate at which lakes freeze. As air temperatures drop, the temperature of the water in a lake drops also, with the cooling occurring first at the surface of the lake. For temperatures above  $4^\circ \text{C}$  this cooling proceeds very efficiently. As water cools, it becomes more dense and sinks to a lower level in the lake, as warmer, less dense water rises to take its place. Thus there is a natural mixing of warmer and colder water, causing rapid cooling of water beneath the surface. However, when the water reaches a uniform temperature of  $4^\circ \text{C}$ , the process changes. Cooling of the surface water below  $4^\circ \text{C}$  decreases its density. Thus it stays at the surface, and further cooling of the water beneath proceeds more slowly. The surface of the lake may freeze. But in even the coldest weather, large lakes do not freeze solid. The water at the bottom of the lake remains at  $4^\circ \text{C}$ , enabling the marine life there to survive.



**Fig. 12-19** The density of water as a function of temperature.

# CHAPTER 12 SUMMARY

The Kelvin temperature scale is based on measurements of pressure for a low-density gas at constant volume. The temperature 273.16 K is assigned to the triple point of water (the point at which water can coexist in the solid, liquid, and vapor states). The ratio of the gas pressure  $P$  to its value at the triple point,  $P_{tr}$ , defines the temperature.

$$T = (273.16 \text{ K}) \frac{P}{P_{tr}}$$

Celsius and Fahrenheit temperature scales are defined relative to the Kelvin scale.

$$T_c = T - 273.15$$

$$T_f = 32 + \frac{9}{5} T_c$$

The ideal gas law relates the pressure  $P$ , volume  $V$ , number of molecules  $N$ , and temperature  $T$  for an ideal gas:

$$PV = NkT$$

where  $k$  is Boltzmann's constant.

$$k = 1.380 \times 10^{-23} \text{ J/K}$$

One mole of a substance contains Avogadro's number of molecules,  $N_A$ , and has a mass in grams numerically equal to the molecular mass of the substance.

$$N_A = 6.022 \times 10^{23}$$

The ideal gas law may also be expressed in terms of the number of moles,  $n$ .

$$PV = nRT$$

where

$$R = N_A k = 8.31 \text{ J/K}$$

## Questions

- Two bodies, A and B, are initially at the same temperature. If the temperature of A increases by  $1 \text{ C}^\circ$  and the temperature of B increases by  $1 \text{ F}^\circ$ , which has the higher final temperature?
- Will a thermometer in direct sunlight accurately measure the temperature of the surrounding air?
- Air bubbles are expelled by a scuba diver on the ocean floor (Fig. 12–20). Does the size of a bubble change as it rises to the surface?

Dalton's law of partial pressures states that a mixture of noninteracting ideal gases produces a pressure that is the sum of the pressures that would be produced by each of the gases alone.

Kinetic theory uses a molecular model to derive results such as the ideal gas law.

At a given temperature, the distribution of molecular speeds is given by the Maxwell distribution. A typical molecular speed at any given temperature  $T$  is the rms speed:

$$v_{\text{rms}} = \sqrt{v^2} = \sqrt{\frac{3kT}{m}}$$

where  $m$  is the molecular mass.

For many materials, heating produces an increase in volume that is proportional to the increase in temperature,  $\Delta T$ .

$$\Delta V = \beta V \Delta T$$

where  $V$  is the original volume and  $\beta$  is the coefficient of expansion.

For solids there is a linear coefficient of expansion,  $\alpha$ , that determines the change in length  $\Delta \ell$ , corresponding to a temperature change  $\Delta T$ .

$$\Delta \ell = \alpha \ell \Delta T$$

For a solid the volume coefficient is 3 times the linear coefficient:

$$\beta = 3\alpha$$



Fig. 12–20

- 4 What is the average velocity of gas molecules?
- 5 A container is filled with a mixture of hydrogen and helium gases. Compared to helium molecules, do the hydrogen molecules on the average have greater, lesser, or the same: (a) speed; (b) kinetic energy?
- 6 If you increase the temperature of a gas, while holding its volume fixed, will the time interval between a molecule's collisions with one wall of the container increase, decrease, or remain constant?
- 7 Is a molecule of oxygen gas at 300 K more likely to have a speed of 100 m/s or 600 m/s?
- 8 Two drinking glasses are stuck together, one inside the other. It is possible to separate the glasses by immersing the outer glass in water and filling the inner glass with water of a different temperature. Hot tap water and ice water work well. Where should the ice water be?
- 9 Water fills a large industrial cooking pot of height 1 meter. The pot is heated from beneath, and when the water at the bottom of the pot reaches a certain temperature, it begins to boil. Will this temperature be the normal boiling point of 100° C, or will it be at a slightly higher temperature or at a slightly lower temperature?

### Answers to Odd-Numbered Questions

1 A; 3 Yes, volume increases because of a decrease in pressure; 5 (a) greater; (b) same; 7 600 m/s; 9 Slightly higher temperature

### Problems (listed by section)

#### 12-1 Temperature Measurement

- 1 Find the temperature in °F corresponding to the following temperatures on the Celsius scale: 0.00° C, 10.0° C, 20.0° C, 30.0° C, 40.0° C.
- 2 Find the temperature in °C corresponding to the following temperatures on the Fahrenheit scale: 0.00° F, 10.0° F, 20.0° F, 40.0° F, 60.0° F, 80.0° F.
- 3 Find the pressure of a constant-volume gas thermometer at a temperature of 300 K if its pressure at the triple point is 0.100 atm.
- 4 Find the triple point pressure of a constant-volume gas thermometer if its pressure is 0.400 atm at a temperature of 100° C.
- 5 At what temperature are the Celsius and Fahrenheit temperatures the same?

#### 12-2 Ideal Gas Law

- 6 Find the mass in grams of a carbon dioxide molecule.
- 7 Find the number of atoms in a gold ring of mass 4.00 g.
- 8 Find the number of H<sub>2</sub>O molecules in 1.00 liter (1000 cm<sup>3</sup>) of water.
- 9 Find the number of molecules in 1.00 cm<sup>3</sup> of air at a pressure of 1.00 atm and a temperature of 300 K.
- 10 Find the volume occupied by 1.00 mole of ideal gas at standard conditions of pressure and temperature ( $P = 1.00$  atm,  $T = 273$  K).
- 11 A gas consists of 5.00 grams of oxygen and 5.00 grams of nitrogen and occupies a volume of 1.00 liter at a temperature of 350 K.
- Find the number of gas molecules.
  - Find the partial pressure of the oxygen.
  - Find the partial pressure of the nitrogen.
  - Find the total pressure of the gas.
- 12 In deflating an air mattress you press the valve release so that air begins to escape. After some of the air has escaped, the pressure inside the air mattress decreases to atmospheric pressure, and no more air flows out. By squeezing all the air out from the end of the air mattress opposite the valve, you reduce the volume and increase the pressure in the remaining air, so that it will flow out. The air initially occupies a volume of 0.120 m<sup>3</sup>. By how much do you need to reduce its volume in order to increase the pressure to 1.20 atm?
- 13 By how much does the air pressure in a house increase if the house is sealed and the air temperature increases from 10.0° C to 20.0° C? The initial air pressure is 1.00 atmosphere.

- 14 Air is pumped into a bicycle tire. The air initially in the tire has a volume of  $2500 \text{ cm}^3$ , a temperature of  $20.0^\circ \text{C}$ , and a gauge pressure of  $2.00 \text{ atm}$ . How many molecules of air must be pumped into the tire in order to raise the gauge pressure to  $5.00 \text{ atm}$ ? Assume that the volume and temperature of the air inside the tire are approximately constant.
- 15 A driver measures her “cold” tire gauge pressure to be  $1.93 \times 10^5 \text{ Pa}$ . The measurement is made before driving, when the tires are at the temperature of the surroundings,  $20.0^\circ \text{C}$ . After driving several miles, she checks the gauge pressure again and finds that it has increased to  $2.20 \times 10^5 \text{ Pa}$ . Find the final temperature of the air in the tires, assuming negligible change in volume.
- 16 A balloon is filled with  $5.00$  liters of helium at a pressure of  $1.20 \text{ atm}$  and a temperature of  $27.0^\circ \text{C}$ .
- Find the mass of the helium.
  - Find the buoyant force on the balloon if it is placed in the atmosphere with air pressure at  $1.00 \text{ atm}$  and temperature at  $23.0^\circ \text{C}$ .
  - Find the acceleration of the balloon if the rubber has a mass of  $4.00 \text{ g}$ .
- 17 A  $1.00$  liter flask contains a certain quantity of ideal gas at  $300 \text{ K}$ . Then an equal quantity of the same gas is added to the flask, after which the absolute pressure is  $1.50$  times its original value. What is the final temperature?
- 18 A  $1.00$  liter cylinder contains helium and oxygen, with partial pressures of  $0.100 \text{ atm}$  and  $0.200 \text{ atm}$  respectively, at a temperature of  $300 \text{ K}$ . Find the mass density of each gas.
- 21 A volume of nitrogen gas has a density of  $1.00 \text{ kg/m}^3$  and a pressure of  $1.00 \text{ atm}$ . Find the temperature of the gas and the rms speed of its molecules.
- 22 Find the rms speed of (a) a helium molecule at  $300 \text{ K}$ ; (b) an iron atom of mass  $55.9 \text{ u}$  at  $300 \text{ K}$ ; (c) a pollen particle of mass  $5.00 \times 10^{-13} \text{ g}$  suspended in water at a temperature of  $300 \text{ K}$ .
- 23 (a) Find the ratio of the average energy of a neon gas molecule to the average energy of a helium gas molecule if both gases are at the same temperature. (b) Is either kind of molecule more likely than the other to lose energy during molecular collisions?
- 24 Estimate the fraction of oxygen molecules with speeds between  $200 \text{ m/s}$  and  $600 \text{ m/s}$  at  $300 \text{ K}$ . Use Fig. 12–8.

### \*12–5 Vapor Pressure and Humidity

- 25 What air pressure is required for water to boil at a temperature of  $60^\circ \text{C}$ ?
- 26 What is the maximum partial pressure of water vapor in air at a temperature of  $80^\circ \text{C}$ ?
- 27 The water vapor in air at  $20^\circ \text{C}$  has a partial pressure of  $0.0050 \text{ atm}$ . What is the relative humidity?
- 28 When the relative humidity is  $60\%$  and the temperature is  $30^\circ \text{C}$ , what is the partial pressure of water vapor in the air?
- 29 What is the dew point for air that has a relative humidity of  $50\%$  and a temperature of  $30^\circ \text{C}$ ?

### 12–6 Thermal Expansion

#### 12–3 Kinetic Theory; Model of an Ideal Gas

- 19 Suppose that a  $1.00 \text{ cm}^3$  box in the shape of a cube is perfectly evacuated, except for a single particle of mass  $1.00 \times 10^{-3} \text{ g}$ . The particle is initially moving perpendicular to one of the walls of the box at a speed of  $400 \text{ m/s}$ . Assume that the collisions of the particle with the walls are elastic.
- Find the mass density inside the box.
  - Find the average pressure on the walls perpendicular to the particle’s path.
  - Find the average pressure on the other walls.
  - Find the temperature inside the box. Discuss the assumption of elastic collisions, in light of this result.
- 20 Find the rms speed of  $\text{H}_2\text{O}$  molecules in atmospheric water vapor at a temperature of  $20.0^\circ \text{C}$ .
- 30 A brick is initially  $10.00 \text{ cm}$  high and an aluminum can is initially  $10.01 \text{ cm}$  high. By how much must their temperatures be raised in order for the brick and the can to have exactly the same height?
- 31 Thin copper wire of length  $100 \text{ m}$  is wound on a cylindrical copper spool of length  $10 \text{ cm}$ . What is the change in length of the wire when the temperature is raised from  $10^\circ \text{C}$  to  $40^\circ \text{C}$ ?
- 32 A solid wood door  $0.90 \text{ m}$  wide is hung in a metal doorway at  $0.0^\circ \text{C}$  with a clearance of  $2.0 \text{ mm}$  on the side. If the linear coefficient of expansion of the wood is  $6.0 \times 10^{-5} (\text{C}^\circ)^{-1}$ , at what temperature would the door begin to touch the side of the doorway? Assume that the metal doorway does not expand significantly.
- \*33 An aluminum canteen is initially filled with  $600 \text{ cm}^3$  of water at  $0.0^\circ \text{C}$ . If the canteen is not tightly closed, how much of the water will overflow when its temperature rises to  $30^\circ \text{C}$ ?

- 34** The exterior brick surface of a house has a surface area of  $200 \text{ m}^2$  in the winter when the temperature is  $0.0^\circ \text{C}$ . How much does the surface area increase in the summer when the temperature rises to  $30^\circ \text{C}$ ?
- \*35** Two drinking glasses of diameter  $8.0 \text{ cm}$  are stuck together, one inside the other. The glasses can be separated without force if the outer glass is thermally expanded while the inner glass is thermally contracted. Initially both glasses are at  $20^\circ \text{C}$ . How much clearance can one produce by cooling the inner glass to  $10^\circ \text{C}$  by filling it with cold water while heating the outer glass to  $30^\circ \text{C}$  by running hot water over it? The linear expansion coefficient for the glass is  $1.0 \times 10^{-5} (\text{C}^\circ)^{-1}$ .
- \*\*36** Steel wire of radius  $1 \text{ mm}$  is cooled from  $20^\circ \text{C}$  to  $0^\circ \text{C}$ . How much tension would you have to apply to the wire to stretch it back to its original length? Young's modulus for steel is  $2.0 \times 10^{11} \text{ N/m}^2$ .
- 37** By how much will the volume of coolant in an automobile radiator increase when its temperature increases by  $100^\circ \text{C}$  if the initial volume of coolant is  $12 \text{ liters}$  and its thermal coefficient of expansion is  $5.0 \times 10^{-4} (\text{C}^\circ)^{-1}$ ?

### Additional Problems

- \*38** An air bubble is expelled by an underwater diver at a depth of  $10.0 \text{ m}$ . As the bubble rises to the surface, the pressure decreases and the volume of the bubble therefore increases. The temperature of the water is constant. Find the final volume of the bubble if its initial volume is  $10.0 \text{ cm}^3$ .
- \*39** A cylindrical chamber, open at the bottom, is to be submerged in the ocean and used as a diving chamber (Fig. 12–21). Initially air at a pressure of  $1.00 \text{ atm}$  and a temperature of  $300 \text{ K}$  fills the cylinder of height  $3.00 \text{ m}$ . What will be the height  $h$  of the air in the cylinder if it is at a depth of  $10.0 \text{ m}$  where the temperature is  $290 \text{ K}$ ?
- \*\*40** An ideal gas is heated from  $T$  to  $T + \Delta T$ . Show that the volume coefficient of expansion is  $1/T$ .
- \*41** Almost the entire volume of mercury in a thermometer is contained in the bulb. The remainder extends up into a capillary tube. Suppose that the bulb has a volume of  $1.00 \text{ cm}^3$ . Find the inner radius of the capillary tube if the level of the mercury rises  $2.00 \text{ cm}$  per Celsius degree increase in temperature. Thermometer glass has a volume coefficient of expansion much less than that of mercury. Therefore you can ignore the expansion of the glass.
- \*\*42** A cube of edge length  $\ell$  has a volume  $V = \ell^3$ . When the cube's temperature is increased by  $\Delta T$ , the edge length increases to  $\ell + \Delta\ell$  and the volume increases to  $V + \Delta V = (\ell + \Delta\ell)^3$ . Obtain an expression for  $\Delta V/V$  and show that the volume coefficient of expansion  $\beta$  is related to the linear expansion coefficient  $\alpha$  by the equation  $\beta = 3\alpha$ , assuming  $\alpha \Delta T$  is small.
- 43** Two boxes, each having a volume of  $1.00 \text{ liter}$ , contain helium gas. The gas in one box has a mass of  $4.00 \text{ g}$  and is initially at a temperature of  $350 \text{ K}$ , and the gas in the other box has a mass of  $8.00 \text{ g}$  and is initially at a temperature of  $300 \text{ K}$ . The two gases are brought into thermal contact and reach a common final temperature  $T$ .
- Find the initial pressure and rms speed for each gas.
  - Find the final rms speed.
  - Find the final temperature  $T$ .
  - Find the final pressure for each gas.
- \*\*44** A sealed glass jar contains air initially at a pressure of  $1.00 \text{ atm}$  and a temperature of  $0^\circ \text{C}$ . Find the air pressure when the jar is heated to  $400^\circ \text{C}$ . The linear expansion coefficient of the glass is  $1.00 \times 10^{-5} (\text{C}^\circ)^{-1}$ .
- 45** At a temperature of  $0.0^\circ \text{C}$  the steel rectangle shown in Fig. 12–22 has a gap of  $1.0 \text{ mm}$ . What is the change in the gap if the steel is heated to  $1000^\circ \text{C}$ ?

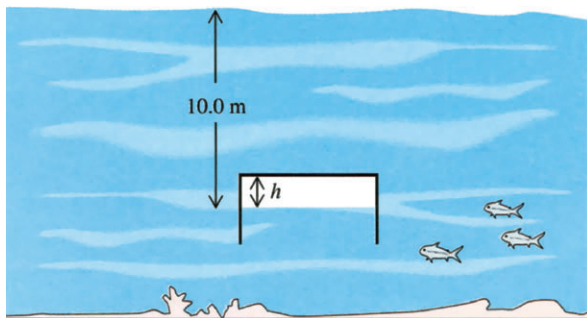


Fig. 12–21

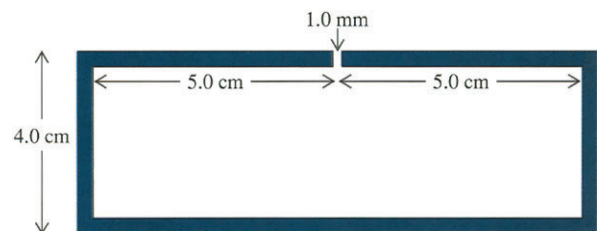


Fig. 12–22



- \*46** Suppose that a steel band encircled the earth. By how much would you have to raise the temperature of the metal to expand it enough so that it would be lifted 100 m away from the surface?
- \*47** Global warming causes the sea level to rise due to both the melting of glaciers and the thermal expansion of the sea. Estimate the rise in sea level from thermal expansion as the earth's mean temperature increased by  $0.5\text{ }^{\circ}\text{C}$  from 1860 to the present.
- \*\*48** Estimate the thickness of the layer of water that would cover the earth if all the water vapor in the earth's atmosphere were to condense. Treat the air as a uniform layer 10 km thick, with a temperature of  $20^{\circ}\text{C}$  and a relative humidity of 50%.
- \*\*49** Wood paneling with an unusually high thermal coefficient of expansion of  $1.0 \times 10^{-4}\text{ }(^{\circ}\text{C})^{-1}$  is installed at a temperature of  $5^{\circ}\text{C}$ , with no space between boards to allow for thermal expansion. The temperature later increases to  $30^{\circ}\text{C}$ , and the paneling buckles because it expands considerably while the surface to which it is nailed does not. Find the average angle at which each piece of paneling protrudes from the wall.
- 50** Find the volume of air inside a hot-air balloon required to support a weight of 5000 N if the temperature of the air inside the balloon is  $100^{\circ}\text{C}$  and the temperature of the surrounding air is  $20^{\circ}\text{C}$ . Use the result of Ex. 3, where we found that the density of the hot air is 78.6% of the cooler air's density ( $1.2\text{ kg/m}^3$ ).