Reinforced concrete

Introduction

Concrete is a mixture of sand, gravel, crushed rock. or other aggregates held together in a rocklike mass with a paste of cement and water, Somtimes one or more admixtures are added to change certain characteristcs of the concrete such as its workability, durability and time of hardening.

As with most rocklike subtances, concrete has a high compressive strength and a very low tensile strength, Reinforced concrete is combination of concrete and steel wherein the steel reinforcement provide the tensile strength lacking in the concrete. Steel reinforcing is also capable of resisting compression forces.

Rienforced concrete may be the most important material available for construction. It is used in one form or another for almost all structures, great or small buildings, bridges, pavements, dams, retaining walls, tunnels, drainage and irrigation, facilities, tanks, and so on.

Types of cement

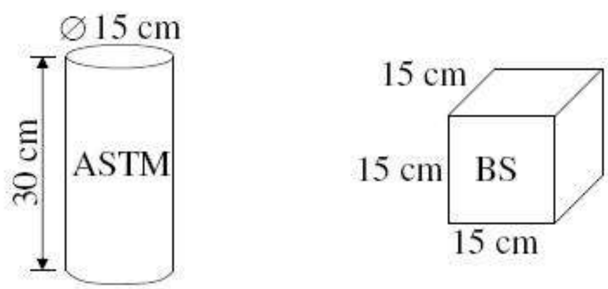
Type I : General purpose.
Type II : lower heat of hydration than type I
Type III : high early strength
Type IV : low heat of hydration
Type V : Sulfate resisting

Properties of Reinforced Concrete

1-Compressive Strength

The compressive strength of concrete (f'_c) is determined by testing to failure 28-day old (300 mm by 150 mm). Concrete cylinders at a specified rate of loading. For the 28 day period the cylinders are usually kept under water or in a room with constant temperature and 100% humidity.

Normal used ; 21, 24, 28 and 32 N/mm^2



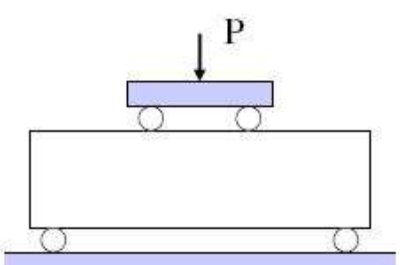
$$(f'_c)_{ASTM} \cong 0.85(f'_c)_{BS}$$

2-Tensile Strength

The tensile strength of concrete varies from about 8 to 15% of its compressive strength. A major reason for this small strength is the fact that concrete is filled with fine cracks.

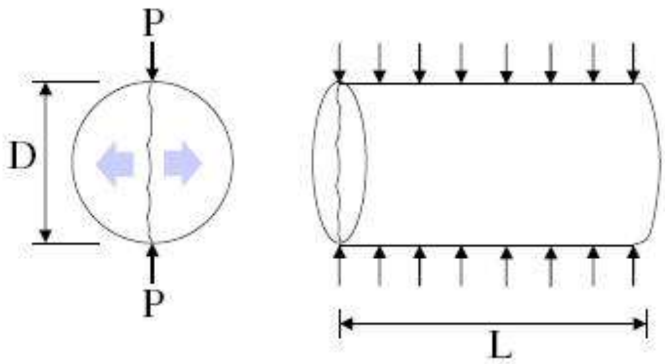
The tensile strength of concrete in flexture is aquite important when considering beam cracks and deflection. For these considerations the tensile strength obtained with the modulus of rupture test have long been used. The modulus of rupture (which is defiened as the flextural tensile strength of concrete) is usually measured by loading a 150mm x 150 mm x 750 mm plain (i.e unreinforced) rectangular beam with simple supports placed 600 mm on center) to failure with equal concentrated load at its one third point.

$$f_r = 0.62\sqrt{f'_c} \quad (\text{MPa})$$



$$f_r = \frac{Mc}{I} = \text{Modulus of rupture}$$

The tensile strength of concrete may also be measured with split-cylinder test.



$$f_{ct} = \frac{2P}{\pi LD}$$

3-Modulus of Elasticity

Es is the modulus of elasticity. This is actually a secant modulus with the line (whose slop equals the modulus) drawn from the origion to a point on the stress-strain curve

$$E_c = w_c^{1.5}(0.043)\sqrt{f'_c}\,MPa \quad \text{for } w_c = 1440 - 2480\,kg/m^3$$

$$E_c = 4700\sqrt{f'_c}\,MPa \quad \text{for normal weight concrete}$$

Reinforcing Steel

The reinforcing used for concrete structures may be in the form of bar or welded wire fabric. Rienforcing bars are referred to as plain or deformed. (deformed bars are used for almost all applications)

Loading

Loads that act on structure can be divided into three broad categories: dead loads, live loads, and enviromental loads.

Dead loads are loads of constant magnitude that remain in one position. They include the weight of the structure under consideration, as well as any fixtures that are permanently attached to it. For reinforced Concrete building, Some dead loads are the frames, walls, floors, Ceilings, Stairways, roofs & pulmping

Live loads are loads that can change in magnitude and postion. They include occupancy loads, warehouse material, construction loads, overhead service cranes, equipment operating loads and many other.

Enviromental loads are loads caused by the enviroment in which the structure is located. They are caused by rain, snow, wind, temperature change and earthquacks.

Flexural Analysis and Design of Beams

The basic assumptions made in flexural design and analyses are:

1- Section perpendicular to the axis of bending that are plane before bending remain plane after bending.

2- A perfect bond exist between the reinforcement and the Concrete such that the strain in the reinforcement is equal to the strain in the concrete at the same level

3- The strain in both the concrete and reinforcement are assumed to be directly proportional to the distance from the neutral axis.

4- Concrete is assumed to fail when the Compressive strain reaches 0.003

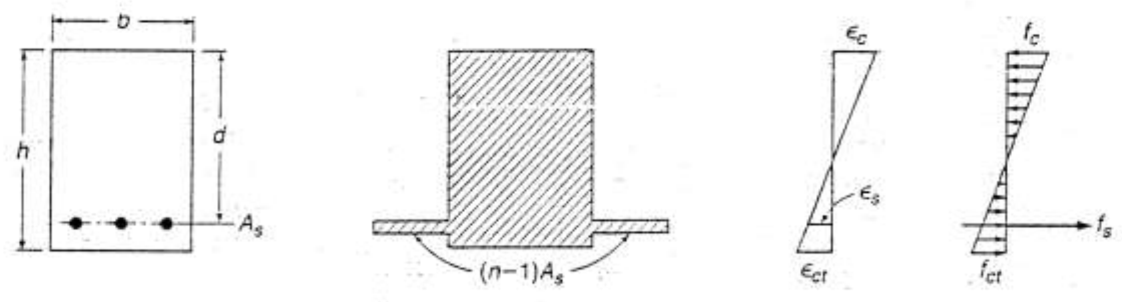
5- The tensile strength of Concrete is neglected

6- The stresses in the Concrete and reinforcement Can be Computed from the strain using stress strain Curves for Concrete and steel, respectively.

Elastic stresses: uncracked section

At small loads when the tensile stresses are less than the modulus of rupture (the bending tensile stress at which the concrete begins to crack). the entire cross section of the beam resist bending with compression on one side and tension on the other.

Transformed area method is replacing the actual area of the reinforcement with an equivalent concrete area equal to nAs located at the level of the steel



$$f = \frac{M.C}{I}$$

$$f_r = 0.62\sqrt{fc'}$$

$$Es = 200\ 000\ MPa$$

$$n = \frac{Es}{Ec}$$

$$Ec = 4700\sqrt{fc'}Mpa$$

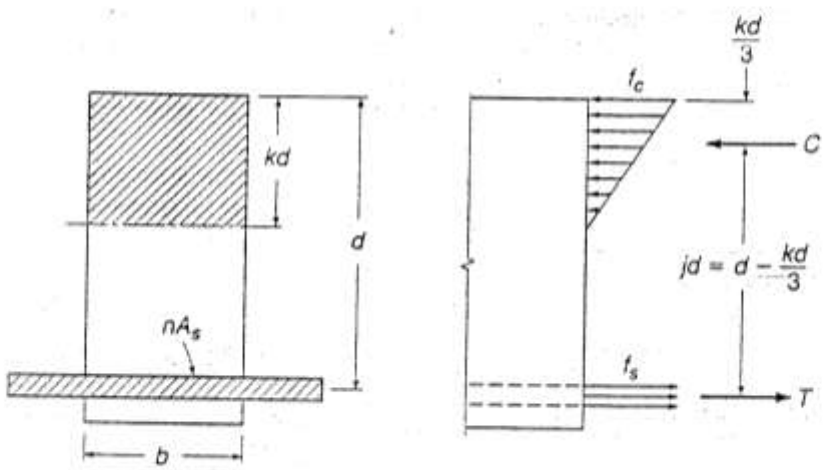
$$\bar{y} = \frac{b.h(h/2) + (n-1)As.d}{b.h + (n-1)As}$$

$$M_{cr} = \frac{f_r.I}{y_b}$$

$$\boldsymbol{I = I_{Concrete} + As(n-1)(d-\bar{y})^2}$$

Elastic stresses: cracked section

When the bending moment is sufficiently large to cause the tensile stress in the extreme fibers to be greater than the modulus of rupture, it is assumed that all of the concrete on the tensile side of the beam is cracked and must be neglected in the flexural calculations.



$$C = \frac{f_c}{2} bkd$$

$$T = A_s . f_s$$

$$T = C$$

$$\frac{f_c}{2} bkd = A_s f_s$$

kd.b(kd/2)=n.A_s(d-kd) ***find*** *kd*

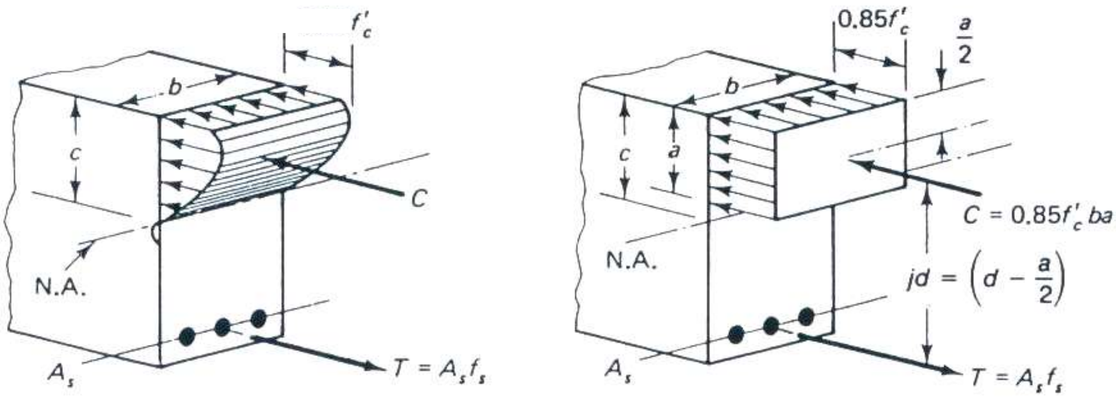
Also, neutral axis location can be determined using:-

$$k = \sqrt{(\rho.n)^2 + 2\rho.n} - \rho.n \qquad \text{where} \qquad \rho = \frac{A_s}{bd}$$

$$j = 1 - \frac{k}{3}$$

Ultimate Strength

The distribution of Concrete Compressive stress at or near ultimate load is modeled with equivalent stress block.



The equivalent rectangular Concrete stress distribution has what is Knc as a β_1 Coefficient is proportion of average stress.

$$a = \beta_1 C$$

$$\beta_1 = 0.85 \qquad for \qquad f'_c \le 30MPa$$

$$\beta_1 = 0.85 - \frac{0.05}{7}(f'_c - 30) \qquad for \qquad f'_c > 30MPa$$

$$0.65 \le \beta_1 \le 0.85$$

Loading

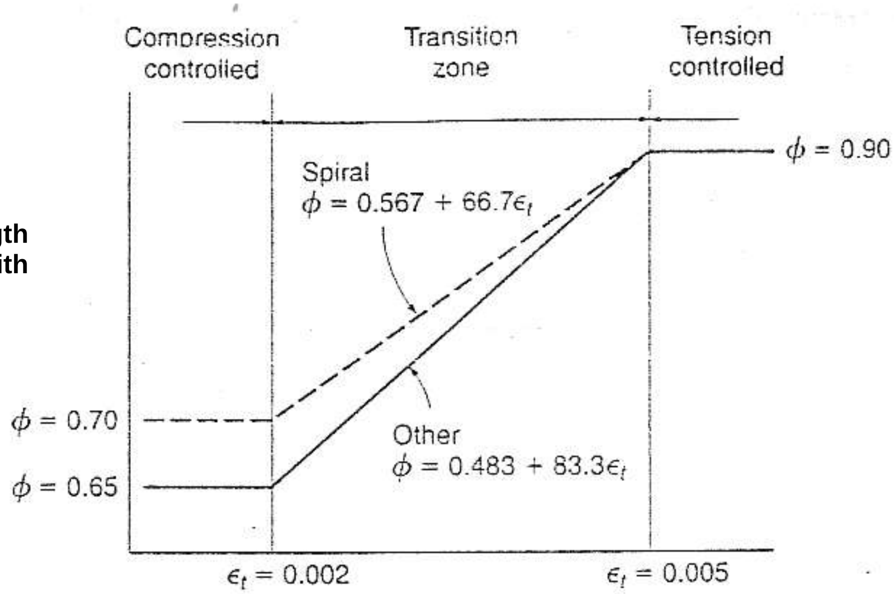
The most general equation for the ultimate load is:

$$U=1.2D+1.6L$$

Strength Reduction Factor (Φ)

- Tension-controlled section $\Phi = 0.9$
- Compression-controlled section
  - Members with spiral reinforcement $\Phi = 0.7$
  - other reinforced members $\Phi = 0.65$
- Shear and Torsion $\Phi = 0.75$
- Bearing on Concrete $\Phi = 0.65$

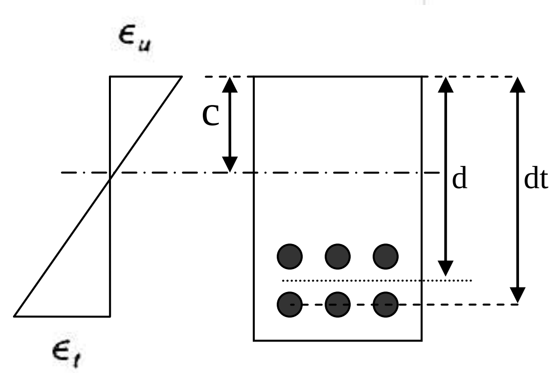
Variation of strength reduction factor with net tensile strain



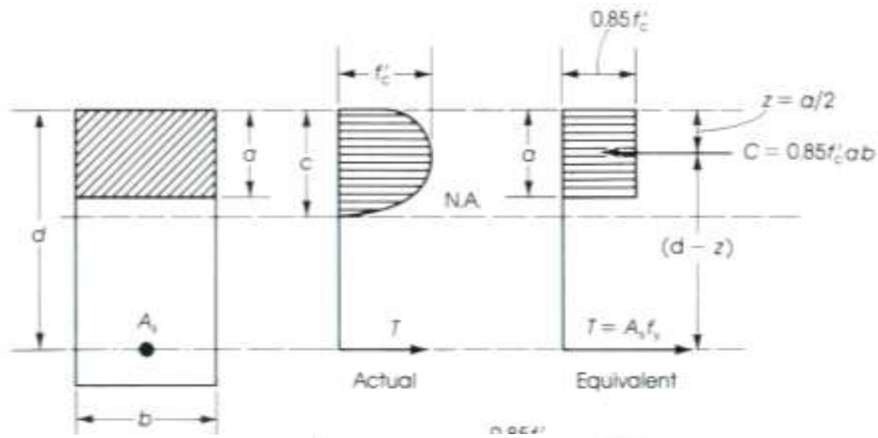
ϵ_t : is the net tensile strain of the reinforcement farthest from the compression face of the concrete at depth d_t

- For beams with single layer of reinforcement $d_t = d$
- For beams with multiple layers of reinforcement $d_t > d$

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c}$$



Flexural behavior of singly rectangular reinforced concrete beam.



$$C = 0.85 f_c^{'}\ a\ b$$

$$T = A_s f_s$$

$$T = C$$

$$A_s f_y = 0.85 f_c^{'}\ a\ b$$

$$a = \frac{A_s f_y}{0.85 f_c^{'}\ b}$$

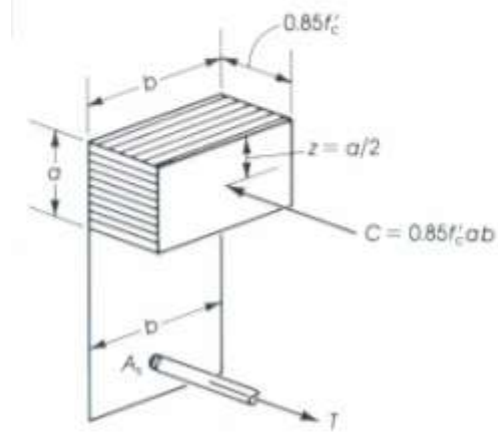
$$a = \frac{\rho\ f_y d}{0.85 f'c}$$

$$M_n = A_s f_y (d - \frac{a}{2})$$

$$M_n = 0.85 f_c^{'}\ a\ b(d - \frac{a}{2})$$

$$M_n = \rho\ f_y b d^2 (1 - 0.59 \rho \frac{f_y}{f_c^{'}})$$

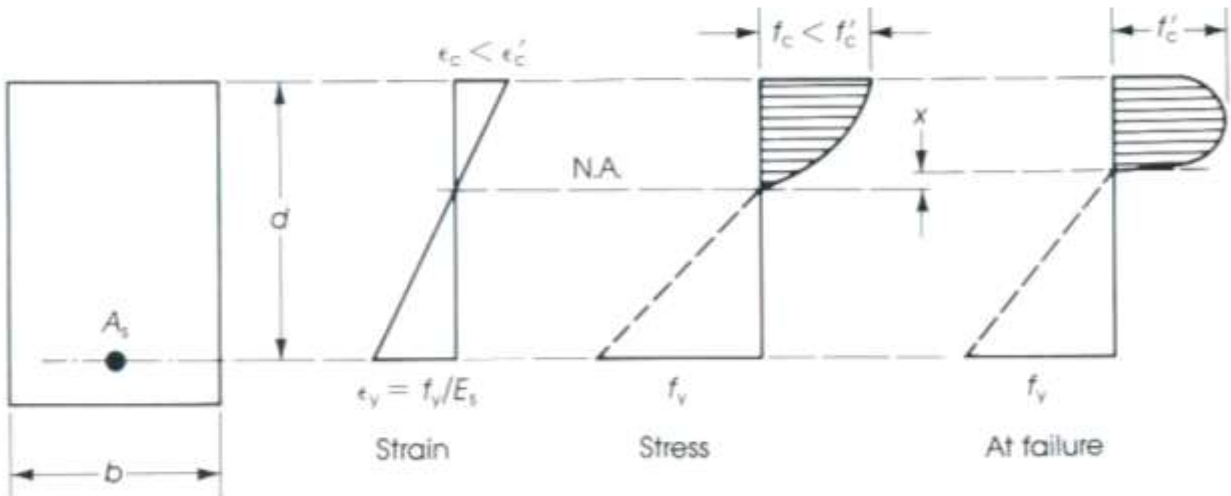
$$M_u \le \phi M_n$$



Types of failure

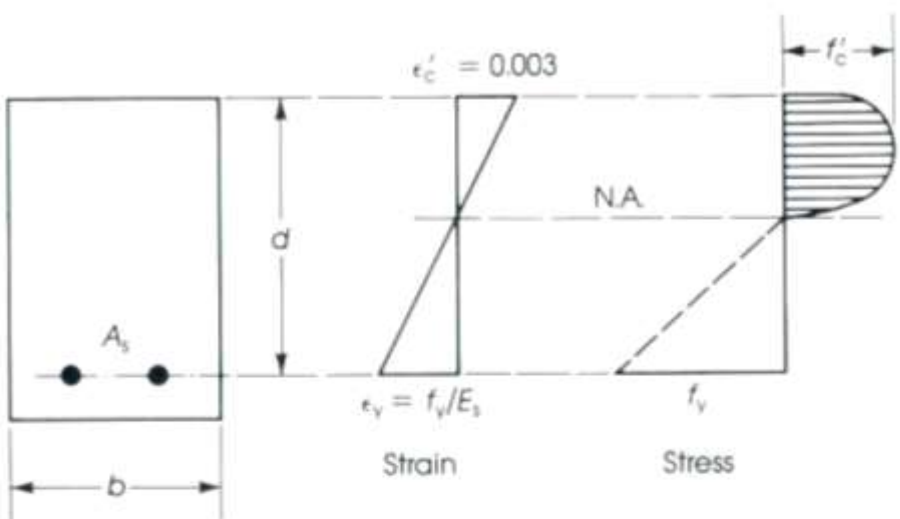
1- Tension Failure

The reinforced yields before the concrete crushes (under-reinforced beam)



2- Balanced Failure

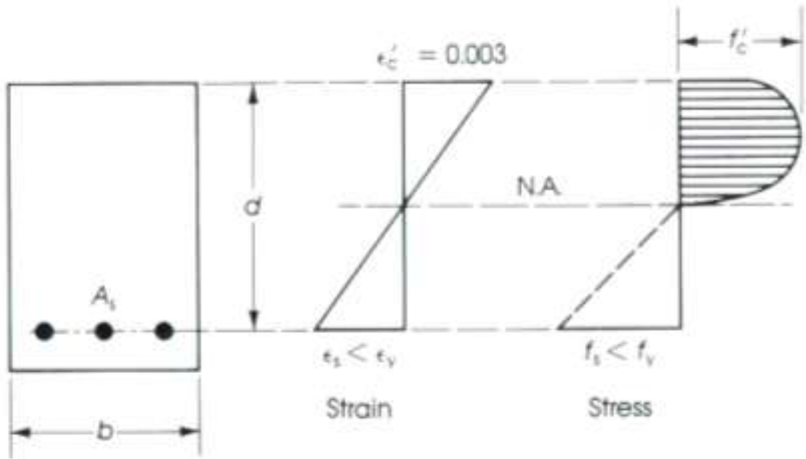
The concrete crushed and the steel yields at the same time (balanced-reinforced beam)



3 – Compression Failure

– The concrete will crush before the steel yields

– This is sudden failure (over-reinforced beam)



The under-reinforced beam is the most desirable

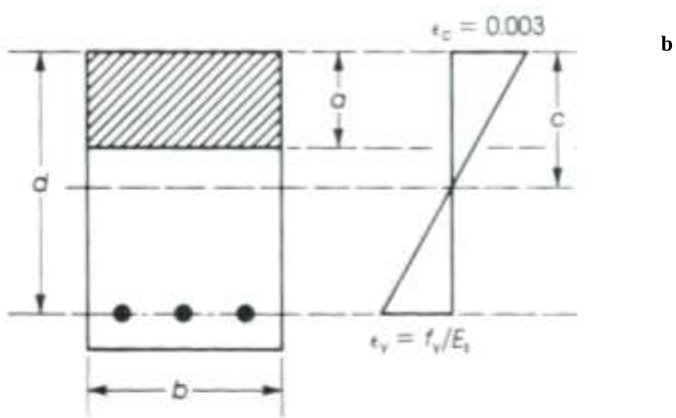
<u>Balanced reinforcement ratio</u> ρ_b

$$\varepsilon_c = \varepsilon_u = 0.003$$

$$\varepsilon_s = \varepsilon_y \Rightarrow f_s = f_y$$

$$\frac{c_b}{d} = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$



or

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y} \frac{600}{600 + \varepsilon_y} \qquad where \qquad \varepsilon_y = \frac{f_y}{200000}$$

Max. reinforcement ratio ρ_{max}

To ensure underreinforced behavior. ACI Code 10.3.5 establishes:-

$\epsilon_t \geq 0.004$

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$$

when $\epsilon_t = 0.004$

$$\rho_{\max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004}$$

when $\epsilon_t = 0.005$ $(\phi = 0.9)$

$$\rho_{t=0.005} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005}$$

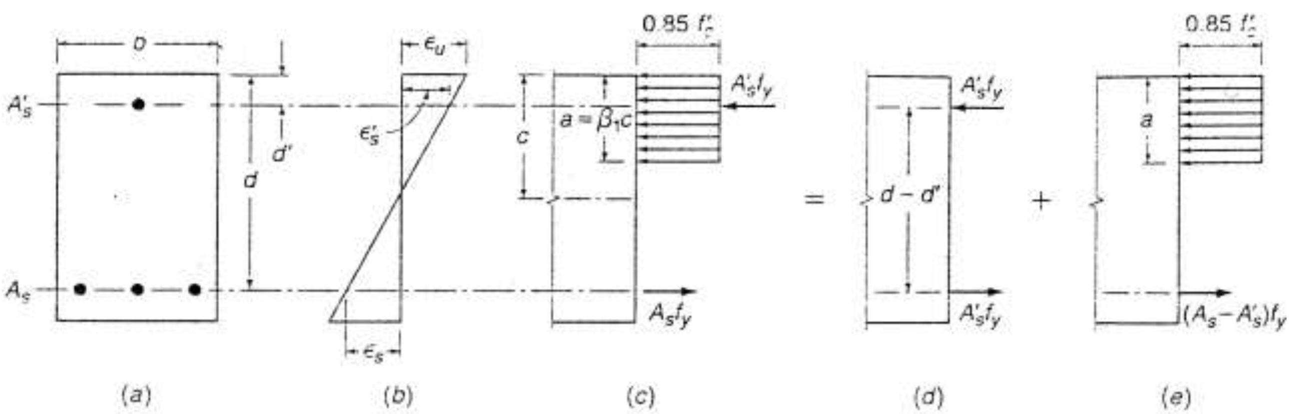
Min. reinforcement ratio ρ_{min}

$$\rho_{\min} = \frac{\sqrt{f_c'}}{4 f_y} \geq \frac{1.4}{f_y}$$

Flexural behavior of Doubly rectangular reinforced concrete beam.

If a beam is limited in cross section because of architectural or other consideration, it may happen that the concrete cannot develop the compression force required to resist the given bending moment, in this case, reinforcement is added in compression zone, resulting in a so-called doubly reinforced beam, i.e. one with compression as well as tension reinforcement.

Tension and compression steel both at yield stress



A rectangular beam cross section is shown with compression steel A'_s placed a distance d' from the compression face and with tensile steel A_s at effective depth d. It is assumed initially that both A'_s and A_s are stressed to f_y at failure. $(f_s = f_y)$, $(f'_s = f_y)$

Part 1

$T_1=C_1$

$(As-As')fy=0.85\ f'c\ a\ b$

$$a=\frac{(As-As')\,fy}{0.85\,fc'\,b} \qquad or \qquad a=\frac{(\rho-\rho')\,fy.d}{0.85\,fc'.} \qquad \rho=\frac{As}{bd} \qquad \rho'=\frac{As}{bd}$$

$Mn_1= (As-As')fy(d-\frac{a}{2})$

Part 2

$T_2=C_2$

$As_2fy=As'fy \longrightarrow As_2=As'$

$Mn_2=As'fy(d-d')$

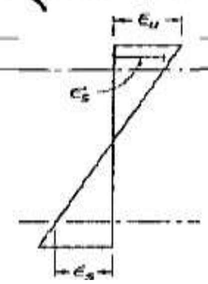
$Mn=Mn_1+Mn_2$

$$\rho'_b=\rho_b+\rho'$$

$$\rho'_{\max}=\rho_{\max}+\rho'$$

Compression steel below yield stress

To ensure yielding of the Compression steel at failure $\varepsilon_s' = \varepsilon_y$



$$\frac{c}{d'} = \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} \qquad or \qquad c = \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} d'$$

$$T=C \longrightarrow As\ fy = 0.85\ f'c\ .a\ .b + As'\ fy$$

The minimum tensile reinforcement ratio $\bar{\rho}_{cy}$

$$\rho^{-}_{cy} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{d'}{d} \frac{600}{600 - f_y} + \rho'$$

If $\rho \geq \rho^{-}_{cy} \Rightarrow f_s' = f_y$

$$\rho < \rho^{-}_{cy} \Rightarrow f_s' < f_y$$

$$\text{T=C} \Rightarrow \rho.b.d.f_y = \rho' b.d.f_s' + 0.85 f_c' \beta_1 \frac{600}{600 + f_y} b.d \Rightarrow \rho_b' = \rho'.\frac{f_s'}{f_y} + \rho_b$$

$$f_s' = E_s \varepsilon_s = E_s \left[\varepsilon_u - \frac{d'}{d} \left(\varepsilon_u + \varepsilon_y \right) \right] = 600 - \frac{d'}{d} \left(600 + f_y \right) \leq f_y$$

$$\rho_{max}\ (when\ \varepsilon_t = 0.004)$$

$$f_s' = E_s \left[\varepsilon_u - \frac{d'}{d} \left(\varepsilon_u + 0.004 \right) \right] \leq f_y$$

$$\rho'_{max} = \rho_{max} + \rho' \frac{f_s'}{f_y}$$

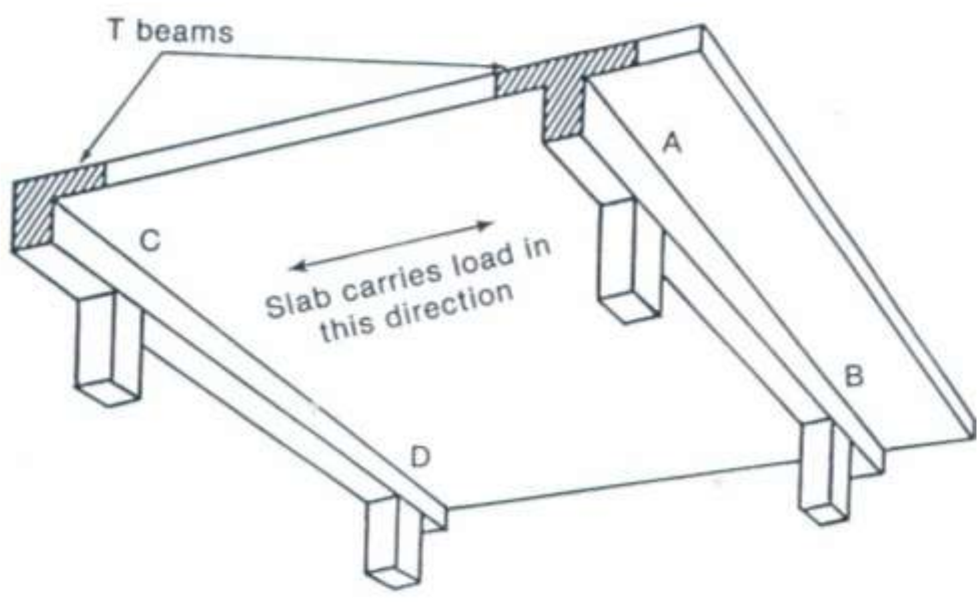
$$f_s' = 600 \left(1 - \frac{\beta_1 d'}{a} \right) \cdots\cdots\cdots\cdots\cdots (1)$$

$$A_s f_y = 0.85 f_c'.a.b + A_s' f_s' \cdots\cdots (2)$$

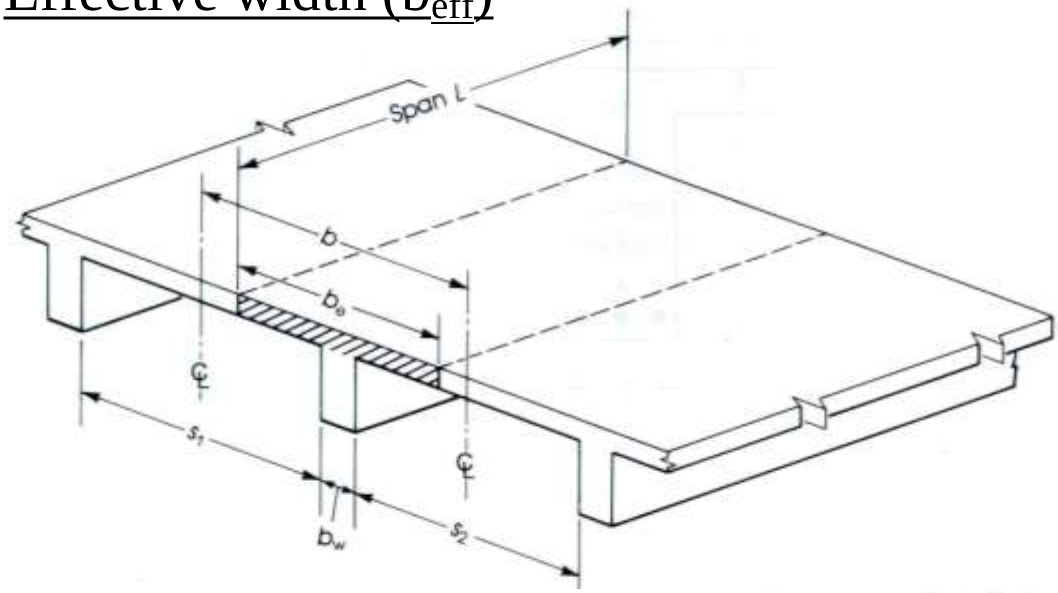
$$\text{Mn} = 0.85 \text{f}_c' a.b (d - \frac{a}{2}) + A_s' f_s' (d - d')$$

T-beams

Reinforced Concrete floor systems normally Consist of slabs and beams that are placed monolithically. As a result, the two parts act together to resist loads. In effect, the beams have extra width at their tops, Called Flanges, and the resulting T-shaped beams are Called T-beams. The part of a T-beam below the slab is referred to as the web or stem. The beam may be L-shaped if the stem is at the end of a slab.



Effective width (b_{eff})



- T-beam flange

$$b_{eff} \leq \frac{L}{4}$$

$$\leq 16\,h_f + b_w$$

$$\leq b_{actual}$$

- Inverted L shape flange

$$b_{eff} \leq \frac{L}{12} + b_w$$

$$\leq 6\,h_f + b_w$$

$$\leq b_{actual} = b_w + \frac{S}{2}$$ (S: clear distance to next web)

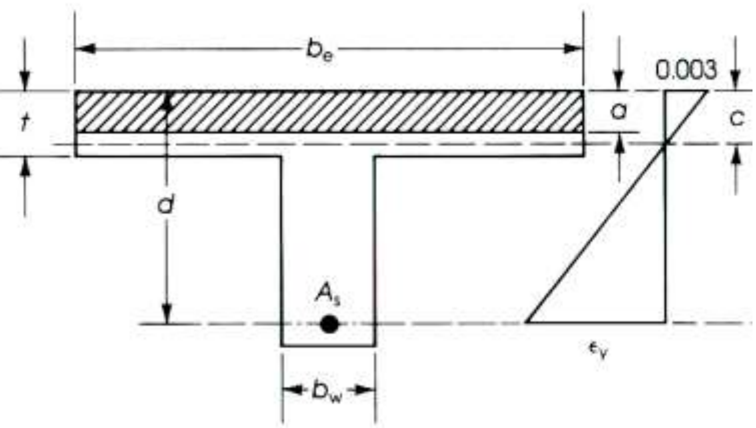
Isolated T-beams

$$h_f \geq \frac{b_w}{2}$$

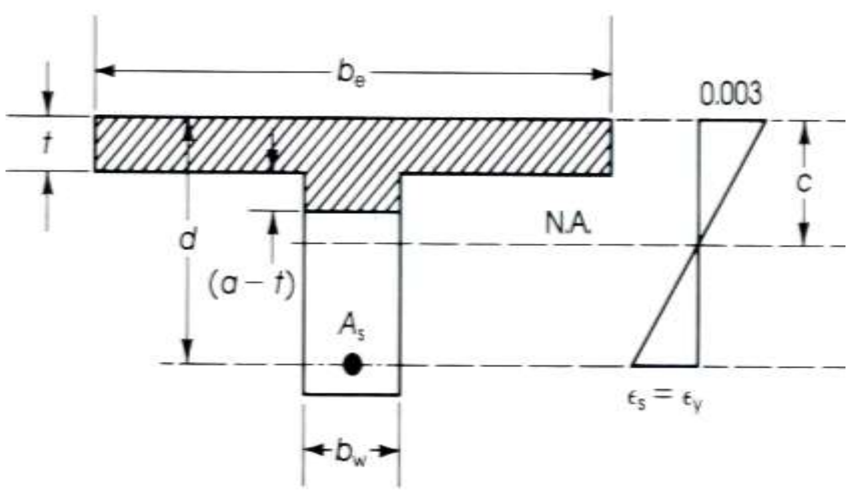
$$b_{eff} \leq 4\,b_w$$

If the neutral axis falls within the slab depth, analyze the beam as a rectangular beam, otherwise as a T-beam.

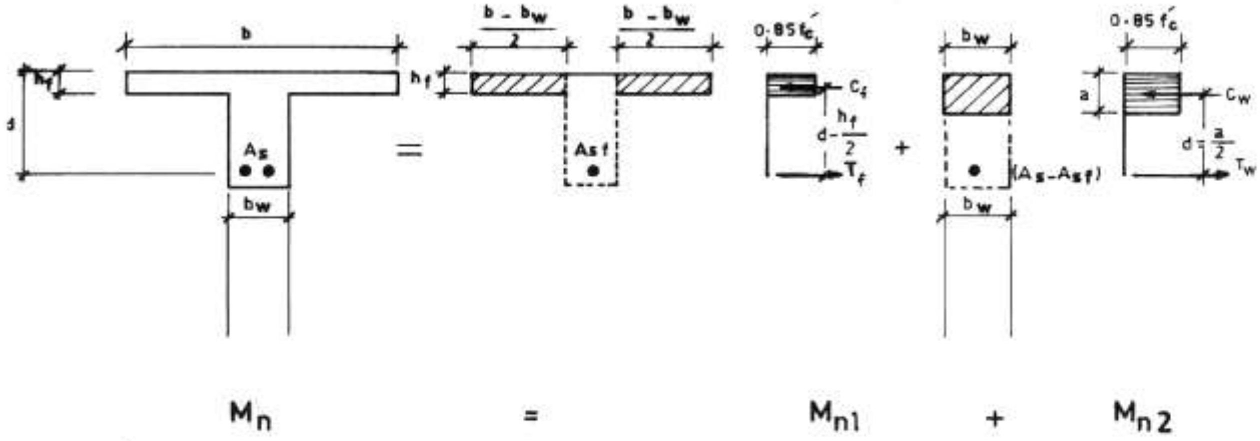
Case 1: $a \leq h_f$ Same as rectangular section



Case 2: $a > h_f$ T-beam



T- beams



assume $(f_s = f_y)$

Part 1

$T_1 = C_1 \Rightarrow A_{sf} \cdot f_y = 0.85 f'_c (b - b_w) h_f$

$$A_{sf} = \frac{0.85 f'_c (b - b_w) h_f}{f_y}$$

$$M_{n1} = A_{sf} \cdot f_y \left(d - \frac{h_f}{2}\right)$$

Part 2

$T_2 = C_2 \Rightarrow (A_s - A_{sf}) f_y = 0.85 f'_c \cdot a \cdot b_w$

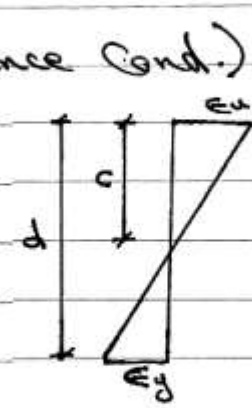
$$a = \frac{(A_s - A_{sf}) f_y}{0.85 f'_c b_w}$$

$$M_{n2} = (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right)$$

$$M_n = M_{n1} + M_{n2}$$

From strain diagram (balance Cond.)

$$\frac{c}{d} = \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$$



equilibrium eq. T = C

$$A_s f_y = 0.85 f'_c \beta_1 c\, b_w + A_{sf} f_y$$

$$\Rightarrow \frac{A_s}{b_w . d} = 0.85 \beta_1 \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + \epsilon_y} + \frac{A_{sf}}{b_w\, d}$$

$$\rho_w = \frac{A_s}{b_w . d} \quad , \quad \rho_f = \frac{A_{sf}}{b_w . d}$$

$$\rho_{wb} = 0.85 \beta_1 \frac{f'_c}{f_y} \cdot \frac{\epsilon_u}{\epsilon_u + f_y} + \rho_f$$

$$\rho_{wb} = \rho_b + \rho_f$$

ρ_w = web steel ratio

ρ_f = flange steel ratio

ρ_{wb} = balanced steel ratio for T-beam

ρ_b = balanced steel ratio for rectangular part

$$\rho_{w,max} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} + \rho_f$$

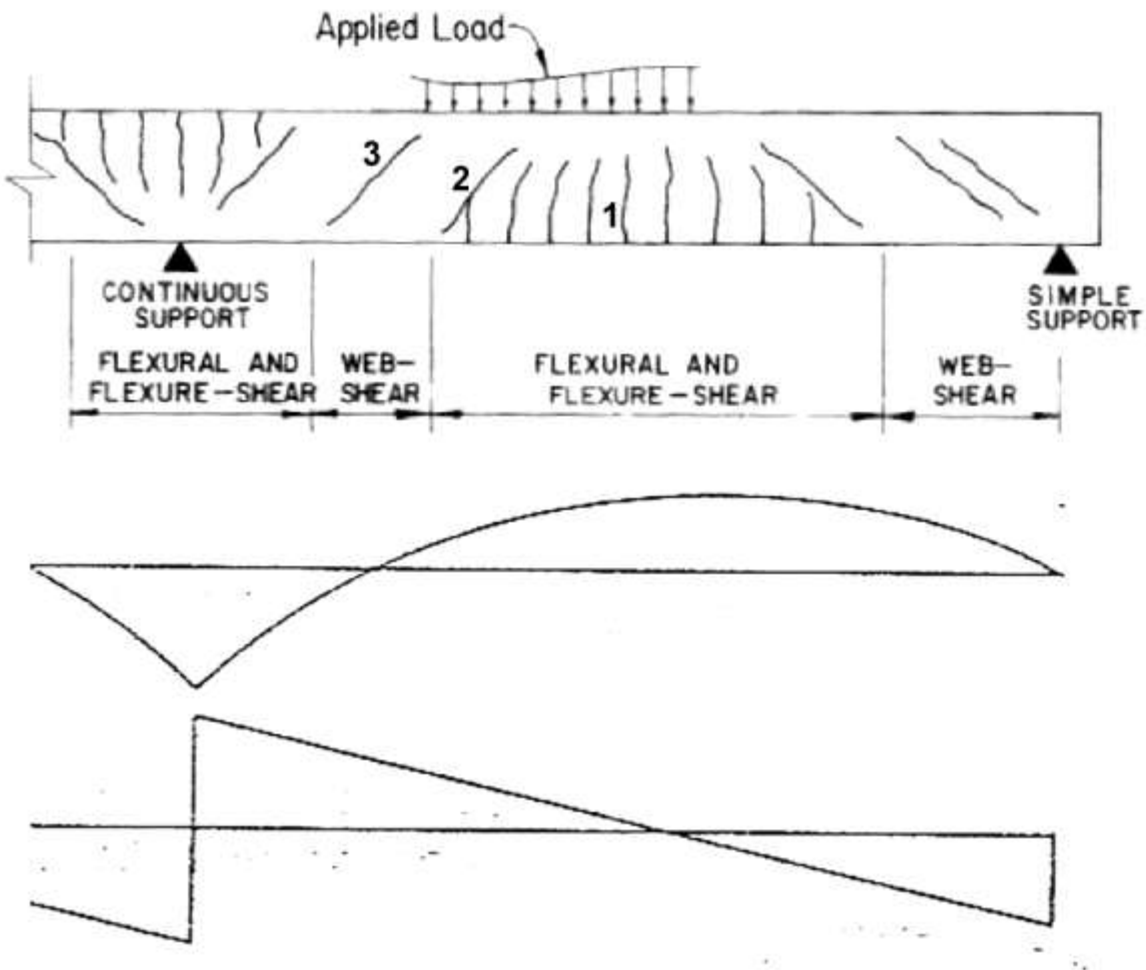
$$\rho_{w,max} = \rho_{max} + \rho_f$$

Shear and diagonal tension

According to moment (M)/shear (V) ratio, there are 3 types of cracks:—

1- Flextural cracks (large M, small V)
2- Flextural-shear cracks (large M, large V)
3- Web shear cracks (small M, large V)

Shear failures occur suddenly with no advance warning.



Shear reinforcement (stirrups)

$$\phi V_n \geq V_u$$

Capacity demand

V_u = factored shear force at section
V_n = Nominal shear strength
ϕ = 0.75 (shear) reduction factor

$$V_n = V_c + V_s$$

V_c = Nominal shear resistance provided by concrete
V_s = Nominal shear provided by the shear reinforcement

Shear strength provided by concrete

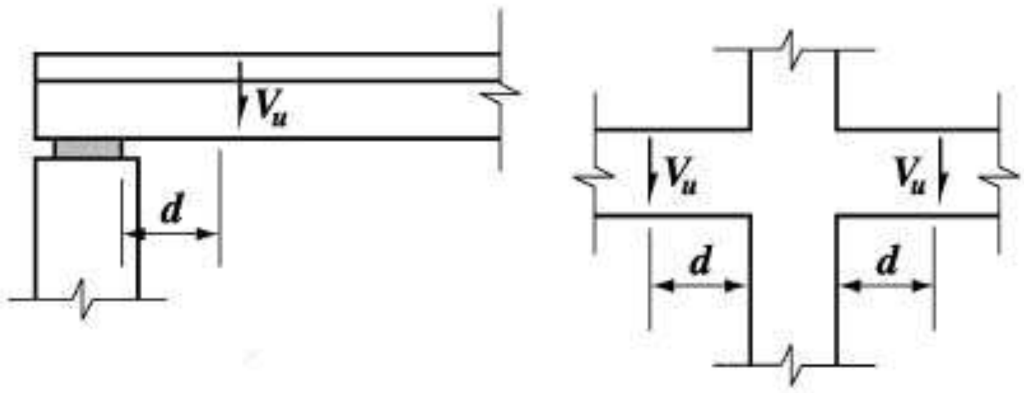
$$V_c = \frac{\sqrt{f'_c}}{6} b_w \cdot d$$

For more detailed:—

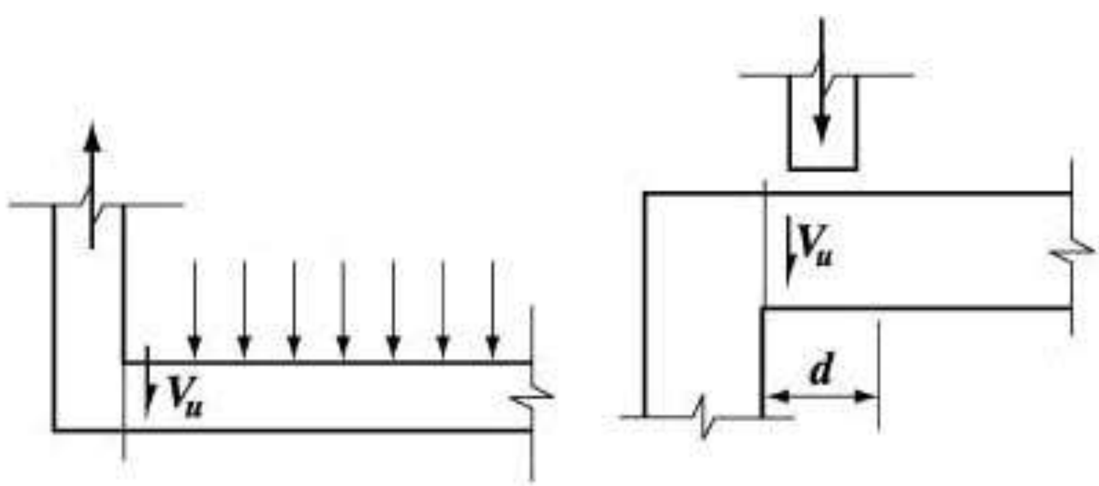
$$V_c = \left(\sqrt{f'_c} + 120 \rho_w \frac{V_u d}{M_u}\right) \frac{b_w d}{7} \leq 0.3\sqrt{f'_c}\, b_w d$$

where $\left(\frac{V_u d}{M_u}\right) \leq 1$

For section located less than a distanse d from face of support, ACI code permits to be designed for the same shear Vu as that computed at a distance d



but for members forming into a supporting member in tension and for section loaded between the support and a distance d the critical section is taken at the face of the support.



Shear reinforcement (stirrups)

- Shear reinforcement are not needed if $V_u < \frac{\phi V_c}{2}$

- Minimum shear reinforcement are needed if

$$\frac{\phi V_c}{2} < V_u < \phi V_c$$

$$A_{v\,min} = \text{larger of} \left[\frac{\sqrt{f'_c}}{16} \frac{b_w S}{f_y} , \frac{1}{3} \frac{b_w S}{f_y} \right]$$

S: spacing of stirrups

Spacing limit for shear reinforcement

- If $V_s \leq 2V_c = \frac{\sqrt{f'_c}}{3} b_w d$

$$S_{max} = \text{min of} \left[\frac{d}{2} , 600\,mm \right]$$

- If $V_s > 2V_c = \frac{\sqrt{f'_c}}{3} b_w d$

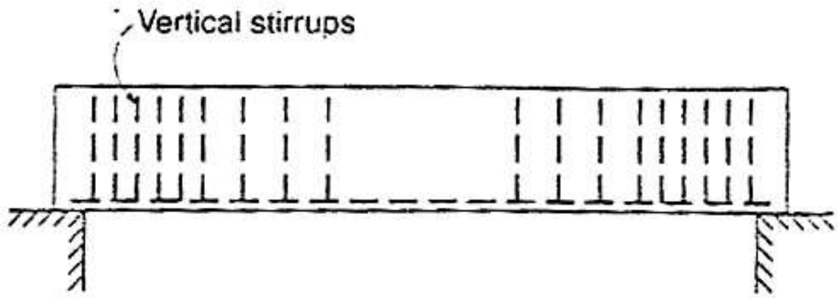
$$S_{max} = \text{min of} \left[\frac{d}{4} , 300\,mm \right]$$

* $S_{max} \leq \left[\frac{3 A_v f_y}{b_w} , \frac{16 A_v f_y}{\sqrt{f'_c}\, b_w} \right]$

Types of shear reinforcement

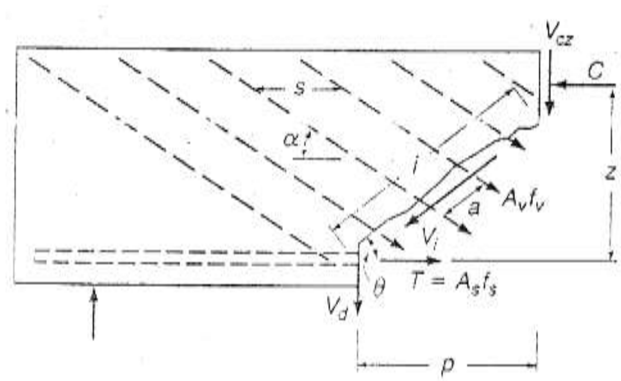
– Vertical stirrups

$$V_s = \frac{A_v f_y d}{S}$$



– Inclined stirrups

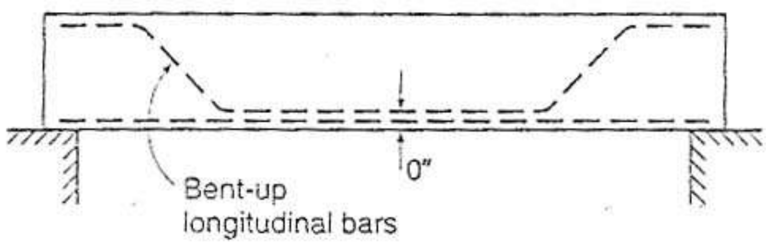
$$V_s = \frac{A_v f_y d}{S} (\sin\alpha + \cos\alpha)$$



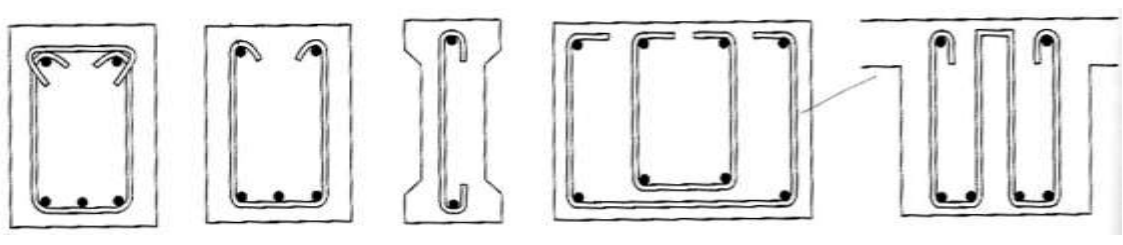
- Bent up longitudinal bars

$$V_s = n\, A_b\, f_y \sin\alpha \leq \frac{1}{4}\sqrt{f'_c}\, b_w\, d$$

n = No. of bent bars
A_b = Area of bar



$A_v = A_b * \text{legs}$

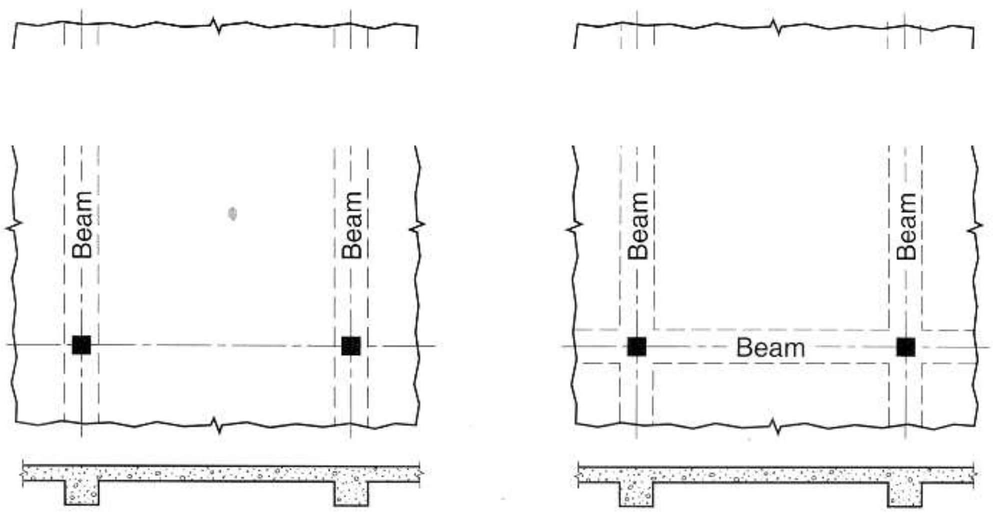


$A_v = 2A_b$ $A_v = A_b$ $A_v = 4A_b$

Slabs

Slabs can be Classified as:

- One-way slabs
- Two-way Slabs



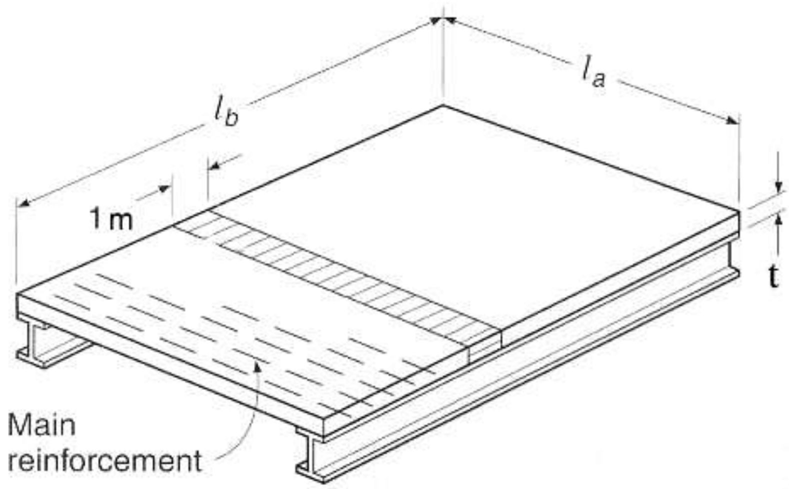
(*a*) One-way slab (*b*) Two-way slab

If $\frac{\text{Longer span } l_b}{\text{Shorter span } l_a} > 2$ One-way slab

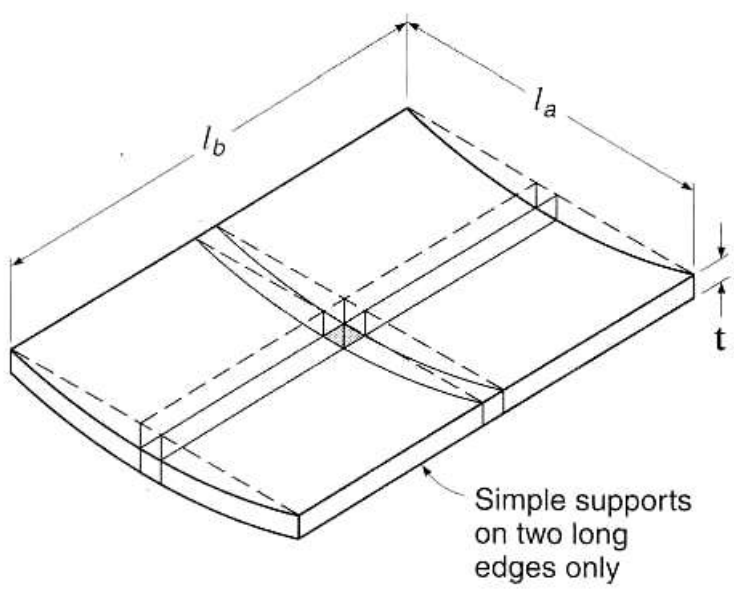
If $\frac{\text{Longer span } l_b}{\text{Shorter span } l_a} \leq 2$ Two-way slab

One Way Slab

A one-way slab is assumed to be a rectangular beam with a large ratio of width to depth. Normally a 1 m wide piece of such a slab is designed as a beam



The bending is in one direction only that is perpendicular to the supported edges.



For shrinkage and temperature changes, the minimum percentages of reinforcing are :-

$\rho_{min} = 0.002$ For $f_y = 300$ & 350 Mpa

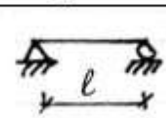
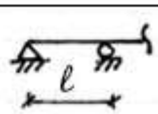
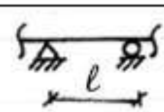
$\rho_{min} = 0.0018$ For $f_y = 420$ MPa

$\rho_{min} = 0.0018 * \frac{420}{f_y} \geq 0.0014$ For $f_y > 420$ MPa

$As_{min} = \rho_{min} * b * t$ (t = thickness of slab)

Minimum allowable slab thickness is determined using deflection requirments using ACI table (9.5a)

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, ***h***			
	Simply supported	One end continuous	Both ends continuous	Cantilever
				
Solid one-way slabs	ℓ/**20**	ℓ/**24**	ℓ/**28**	ℓ/**10**
Beams or ribbed one-way slabs	ℓ/**16**	ℓ/**18.5**	ℓ/**21**	ℓ/**8**

Notes:

Values given shall be used directly for members with normalweight concrete (density w_c = 2320 kg/m^3) and Grade 420 reinforcement. For other conditions, the values shall be modified as follows:

a) For structural lightweight concrete having unit density, w_c, in the range 1440-1920 kg/m^3, the values shall be multiplied by (**1.65 – 0.003**w_c) but not less than 1.09.

b) For f_y other than 420 MPa, the values shall be multiplied by (**0.4 +** f_y**/700**).

- Max Spacing (S_{max}) for flextural requirments

$$S_{max} = \text{min of } [3t \, , 500 \, mm]$$

- Max Spacing (S_{max}) for temp. & shrinkage

$$S_{max} = \text{min of } [5t \, , 500 \, mm]$$

- $$S_{req.} = \frac{A_b}{A_{s\,req}} * 1000$$

- Min. Concrete Cover = 20 mm

ACI Moment Coefficients

ACI Code 8.3 includes expressions that may be used for approximate calculation of maximum moments and shear in continous beams and one-way slabs.

They are applicable within the following limitations:-

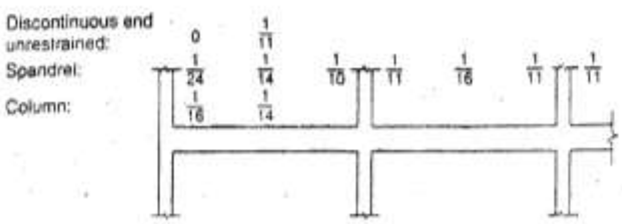
1- There are two or more spans.

2- Spans are approximately equal, with the longer of two adjacent spans not greater than the shorter more than 20%.

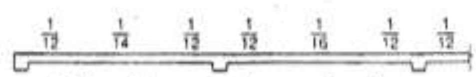
3- Loads are uniformly distributed

4- The unit live load does not exceed 3 times the unit dead load.

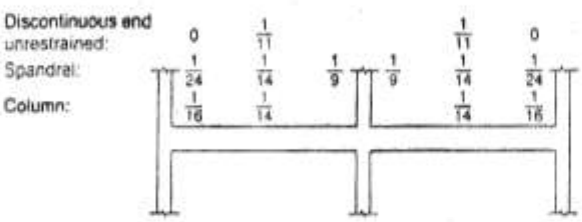
5- Members are prismatic.



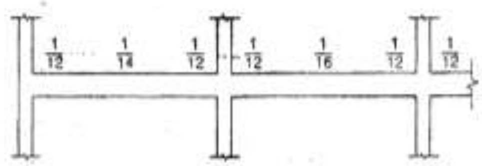
beams with more than two spans



slabs with spans not exceeding 3 m



beams with two spans only



beams in which the sum of column stiffnesses exceeds 8 times the sum of beam stiffnesses at each end of the span

$M_u = C \times w_u \, l_n^2$

where

C = moment Coefficient

l_n = clear span for positive moment, and average of two adjacent clear spans for negative moment

Shear in end members at first interior support	$1.15 \frac{w_u l_n}{2}$
Shear at all other supports	$\frac{w_u l_n}{2}$