Reinforced concrete

Introduction

Concrete is a mixture of sand, gravel, crushed rock, or other aggregates held together in a rocklike mass with a paste of cement and water, sometimes one or more admixtures are added to change certain characteristics of the concrete such as its workability, durability, and time of hardening.

As with most rocklike substances, concrete has a high compressive strength and a very low tensile strength. Reinforced concrete is a combination of concrete and steel wherein the steel reinforcement provides the tensile strength lacking in the concrete. Steel reinforcing is also capable of resisting compression forces.

Reinforced concrete may be the most important material available for construction. It is used in one form or another for almost all structures, great or small, buildings, bridges, pavements, dams, retaining walls, tunnels, drainage, and irrigation, facilities, tanks, and so on.
Types of cement

Type I: General purpose.
Type II: Lower heat of hydration than type I.
Type III: High early strength.
Type IV: Low heat of hydration.
Type V: Sulfate resisting.

Properties of Reinforced Concrete

1-Compressive Strength

The compressive strength of concrete ($f'_c$) is determined by testing to failure 28-day old (300 mm by 150 mm) concrete cylinders at a specified rate of loading. For the 28-day period, the cylinders are usually kept under water or in a room with constant temperature and 100% humidity. Normal used: 21, 24, 28 and 32 N/mm$^2$.

\[
(f'_c)_{ASTM} \equiv 0.85 (f'_c)_{BS}
\]
2- Tensile Strength

The tensile strength of concrete varies from about 8 to 15% of its compressive strength. A major reason for this small strength is the fact that concrete is filled with fine cracks.

The tensile strength of concrete in flexure is quite important when considering beam cracks and deflection. For these considerations, the tensile strength obtained with the modulus of rupture test have long been used. The modulus of rupture (which is defined as the flexural tensile strength of concrete) is usually measured by loading a $150 \text{mm} \times 150 \text{mm} \times 750 \text{mm}$ plain (i.e. unreinforced) rectangular beam with simple supports placed $600 \text{mm}$ on center to failure with equal concentrated load at its one third point.

$$f_r = 0.62\sqrt{f'_c} \quad \text{(MPa)}$$

$$f_r = \frac{Mc}{l} = \text{Modulus of rupture}$$
The tensile strength of concrete may also be measured with split-cylinder test.

\[ f_{ct} = \frac{2P}{\pi LD} \]

3-Modulus of Elasticity

\[ E_s = \text{modulus of elasticity} \]

This is actually a secant modulus with the line (whose slope equals the modulus) drawn from the origin to a point on the stress-strain curve.

\[ E_c = w_c^1.5(0.043)\sqrt{f'_c} \text{MPa} \quad \text{for } w_c = 1440 - 2480 \text{kg/m}^3 \]

\[ E_c = 4700\sqrt{f'_c} \text{MPa} \quad \text{for normal weight concrete} \]

Reinforcing Steel

The reinforcing used for concrete structures may be in the form of bar or welded wire fabric. Reinforcing bars are referred to as plain or deformed. (Deformed bars are used for almost all applications.)
Loading

Loads that act on structure can be divided into three broad categories: dead loads, live loads, and environmental loads.

Dead loads are loads of constant magnitude that remain in one position. They include the weight of the structure under consideration, as well as any fixtures that are permanently attached to it. For reinforced concrete building, some dead loads are the frames, walls, floors, ceilings, stairways, roofs & plumbing.

Live loads are loads that can change in magnitude and position. They include occupancy loads, warehouse material/construction loads, overhead service cranes, equipment operating loads and many others.

Environmental loads are loads caused by the environment in which the structure is located. They are caused by rain, snow, wind, temperature change and earthquakes.
Flexural Analysis and Design of Beams

The basic assumptions made in flexural design and analyses are:

1. Section perpendicular to the axis of bending that are plane before bending remain plane after bending.

2. A perfect bond exist between the reinforcement and the Concrete. Such that the strain in the reinforcement is equal to the strain in the concrete at the same level.

3. The strain in both the concrete and reinforcement are assumed to be directly proportional to the distance from the neutral axis.

4. Concrete is assumed to fail when the compressive strain reaches 0.003.

5. The tensile strength of Concrete is neglected.

6. The stresses in the concrete and reinforcement can be computed from the strain using stress-strain curves for concrete and steel, respectively.
Elastic stresses: uncracked section

At small loads when the tensile stresses are less than the modulus of rupture (the bending tensile stress at which the concrete begins to crack), the entire cross section of the beam resists bending with compression on one side and tension on the other.

Transformed area method is replacing the actual area of the reinforcement with an equivalent concrete area equal to \( n A_s \) located at the level of the steel.

\[
f = \frac{M.C}{I} \quad f_r = 0.62\sqrt{fc'}
\]

\[
E_s = 200\,000\, MPa\quad Ec = 4700\sqrt{fc'}\, MPa
\]

\[
n = \frac{E_s}{Ec}
\]

\[
\bar{y} = \frac{b.h(h/2) + (n - 1)A_s.d}{b.h + (n - 1)A_s}
\]

\[
M_{cr} = \frac{f_r.I}{y_b}
\]

\[
I = I_{Concrete} + A_s(n - 1)(d - \bar{y})^2
\]
**Elastic stresses: cracked section**

When the bending moment is sufficiently large to cause the tensile stress in the extreme fibers to be greater than the modulus of rupture, it is assumed that all of the concrete on the tensile side of the beam is cracked and must be neglected in the flexural calculations.

\[
C = \frac{f_c}{2} bkd
\]

\[
T = A_s f_s
\]

\[
T = C
\]

\[
\frac{f_c}{2} bkd = A_s f_s
\]

\[
kd.b(kd/2) = n.A_s(d-kd)
\]  \[\text{find} \ kd\]

Also, neutral axis location can be determined using:

\[
k = \sqrt{(\rho n)^2 + 2\rho n - \rho n}
\]

where \[\rho = \frac{A_s}{bd}\]

\[
j = 1 - \frac{k}{3}
\]
Ultimate Strength

The distribution of concrete compressive stress at or near ultimate load is modeled with equivalent stress blocks.

The equivalent rectangular concrete stress distribution has what is known as a $\beta_i$ coefficient, which is the proportion of average stress:

$$a = \beta_i c$$

$\beta_i = 0.85$ for $f_c' \leq 30\text{MPa}$

$$\beta_i = 0.85 - \frac{0.05}{7} (f_c' - 30)$$ for $f_c' > 30\text{MPa}$

$0.65 \leq \beta_i \leq 0.85$

Loading

The most general equation for the ultimate load is:

$$U = 1.2D + 1.6L$$
**Strength Reduction Factor**  \((\Phi)\)

- Tension-controlled Section  \(\phi = 0.9\)
- Compression-controlled Section
  - Members with spiral reinforcement  \(\phi = 0.7\)
  - Other reinforced members  \(\phi = 0.65\)
- Shear and Torsion  \(\phi = 0.75\)
- Bearing on Concrete  \(\phi = 0.65\)

Variation of strength reduction factor with net tensile strain

\[ \varepsilon_t = \varepsilon_u \frac{d_t - c}{c} \]

\(\varepsilon_t\) is the net tensile strain of the reinforcement farthest from the compression face of the concrete at depth \(d_t\).

- For beams with single layer of reinforcement \(d_t = d\)
- For beams with multiple layers of reinforcement \(d_t > d\)
Flexural behavior of singly rectangular reinforced concrete beam.

\[ C = 0.85 f'_c a b \]

\[ T = A_s f_s \]

\[ T = C \]

\[ A_s f_y = 0.85 f'_c a b \]

\[ a = \frac{A_s f_y}{0.85 f'_c b} \]

\[ a = \frac{\rho f_y d}{0.85 f'_c} \]

\[ M_n = A_s f_y (d - \frac{a}{2}) \]

\[ M_n = 0.85 f'_c a b (d - \frac{a}{2}) \]

\[ M_n = \rho f_y b d^2 (1 - 0.59 \rho \frac{f_y}{f_c}) \]

\[ M_u \leq \phi M_n \]
Types of failure

1- Tension Failure

The reinforced yields before the concrete crushes (under-reinforced beam)

2- Balanced Failure

The concrete crushed and the steel yields at the same time (balanced-reinforced beam)
3 - Compression Failure

The concrete will crush before the steel yields. This is sudden failure (over-reinforced beam).

The under-reinforced beam is the most desirable.

**Balanced reinforcement ratio** $\rho_b$

$$\varepsilon_c = \varepsilon_u = 0.003$$

$$\varepsilon_s = \varepsilon_y \Rightarrow f_s = f_y$$

$$c_b = \frac{\varepsilon_u}{d = \varepsilon_u + \varepsilon_y}$$

$$\rho_b = 0.85 \beta \frac{f_c'}{f_y \varepsilon_u + \varepsilon_y}$$

or

$$\rho_b = 0.85 \beta \frac{f_c'}{f_y 600 + \varepsilon_y} \quad \text{where} \quad \varepsilon_y = \frac{f_y}{200000}$$
Max. reinforcement ratio \( \rho_{\text{max}} \)

To ensure underreinforced behavior, ACI Code 10.3.5 establishes:

\[ \varepsilon_t \geq 0.004 \]

\[ \rho_b = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t} \]

when \( \varepsilon_t = 0.004 \)

\[ \rho_{\text{max}} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004} \]

when \( \varepsilon_t = 0.005 \) (\( \Phi = 0.9 \))

\[ \rho_{t=0.005} = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005} \]

Min. reinforcement ratio \( \rho_{\text{min}} \)

\[ \rho_{\text{min}} = \frac{\sqrt{f'_c}}{4 f_y} \geq \frac{1.4}{f_y} \]
Flexural behavior of Doubly rectangular reinforced concrete beam.

If a beam is limited in cross section because of architectural or other consideration, it may happen that the concrete cannot develop the compression force required to resist the given bending moment, in this case, reinforcement is added in compression zone, resulting in a so-called doubly reinforced beam, i.e. one with compression as well as tension reinforcement.

Tension and compression steel both at yield stress

A rectangular beam cross section is shown with compression steel $A_s'$ placed a distance $d'$ from the compression face and with tensile steel $A_s$ at effective depth $d$. It is assumed initially that both $A_s$ and $A_s'$ are stressed to $f_y$ at failure. ($F_{s'} = F_y, (F_s = F_y)$
Part 1

\( T_1 = C_1 \)

\((As-As')fy = 0.85 f'c \ a \ b\)

\[ a = \frac{(As - As') fy}{0.85 f'c b} \quad \text{or} \quad a = \frac{(\rho - \rho') fy d}{0.85 f'c}. \]

\[ \rho = \frac{As}{bd} \quad \rho' = \frac{As}{bd} \]

\[ Mn_1 = (As-As')fy(d - \frac{a}{2}) \]

Part 2

\( T_2 = C_2 \)

\[ As_2 fy = As'fy \rightarrow As_2 = As' \]

\[ Mn_2 = As'fy(d-d') \]

\[ Mn = Mn_1 + Mn_2 \]

\[ \rho'_b = \rho_b + \rho' \]

\[ \rho'_{\text{max}} = \rho_{\text{max}} + \rho' \]
Compression steel below yield stress

To ensure yielding of the compression steel at failure, \( \varepsilon_s = \varepsilon_y \)

\[
\frac{c}{d'} = \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} \quad \text{or} \quad c = \frac{\varepsilon_u}{\varepsilon_u - \varepsilon_y} d'
\]

\[T = C \rightarrow A_s f_y = 0.85 f'_c .a .b + A'_s f'_y \]

The minimum tensile reinforcement ratio \( \rho_{cy}^- \)

\[
\rho_{cy}^- = 0.85 \beta_1 \left( \frac{f'_c}{f_y} \right) d' \frac{600}{600 - f_y} + \rho'
\]

If \( \rho \geq \rho_{cy}^- \Rightarrow f'_s = f_y \)

\( \rho < \rho_{cy}^- \Rightarrow f'_s < f_y \)

\[T = C \Rightarrow \rho . b . d . f_y = \rho' . b . d . f'_s + 0.85 f'_c \beta_1 \frac{600}{600 + f_y} b . d \Rightarrow \rho' = \rho' \frac{f'_s}{f_y} + \rho_b \]

\[
f'_s = E_s \varepsilon_s = E_s \left[ \varepsilon_u - \frac{d'}{d} (\varepsilon_u + \varepsilon_y) \right] = 600 - \frac{d'}{d} (600 + f_y) \leq f_y
\]

\( \rho_{max} \) (when \( \varepsilon_s = 0.004 \))

\[
f'_s = E_s \left[ \varepsilon_u - \frac{d'}{d} (\varepsilon_u + 0.004) \right] \leq f_y
\]

\[
\rho'_{max} = \rho_{max} + \rho' \frac{f'_s}{f_y}
\]

\[
f'_s = 600 \left( 1 - \frac{\beta_d d'}{a} \right) \quad \text{.........(1)}
\]

\[
A_s f_y = 0.85 f'_c .a .b + A'_s f'_s \quad \text{.........(2)}
\]

\[
Mn = 0.85 f'_c .a .b (d - \frac{a}{2}) + A'_s f'_s (d - d')
\]
T-beams

Reinforced concrete floor systems normally consist of slabs and beams that are placed monolithically. As a result, the two parts act together to resist loads. In effect, the beams have extra width at their tops, called Flanges, and the resulting T-shaped beams are called T-beams. The part of a T-beam below the slab is referred to as the web or stem. The beam may be L-shaped if the stem is at the end of a slab.
Effective width ($b_{eff}$)

- **T-beam Flange**
  
  $b_{eff} \leq \frac{L}{4}$
  
  $\leq 16 h_f + b_w$
  
  $\leq b_{actual}$

- **Inverted L shape flange**
  
  $b_{eff} \leq \frac{L}{12} + b_w$
  
  $\leq 6 h_f + b_w$
  
  $\leq b_{actual} = b_w + \frac{s}{2}$ (s: clear distance to next web)

**Isolated T-beams**

$h_f \geq \frac{b_w}{2}$

$b_{eff} \leq 4 b_w$
If the neutral axis falls within the slab depth, analyze the beam as a rectangular beam, otherwise as a T-beam.

Case 1: \( a \leq h_f \)  Same as rectangular section

Case 2: \( a > h_f \)  T-beam
T-beams

\[ M_n = M_{n1} + M_{n2} \]

**Part 1**

\[ T_1 = C_1 \rightarrow Asf. \ f_y = 0.85 f'_c (b - bw) \frac{h_f}{f_y} \]

\[ Asf = 0.85 f'_c (b - bw) \frac{h_f}{f_y} \]

\[ M_{n1} = Asf. \ f_y (d - \frac{h_f}{2}) \]

**Part 2**

\[ T_2 = C_2 \rightarrow (As - Asf) \ f_y = 0.85 f'_c \ a \ \cdot bw \]

\[ a = \frac{(As - Asf) \ f_y}{0.85 f'_c \ \cdot bw} \]

\[ M_{n2} = (As - Asf) \ f_y (d - \frac{a}{2}) \]

\[ M_n = M_{n1} + M_{n2} \]
From strain diagram (balance cond.)

\[
\frac{c}{d} = \frac{E_u}{E_u + E_y}
\]

Equilibrium eq. \( T = C \)

\[
A_s f_y = 0.85 f'_c b, c b d_w + A_{sf} f_y
\]

\[
\Rightarrow \frac{A_s}{b w \cdot d} = 0.85 \frac{f'_c}{f_y} \cdot \frac{E_u}{E_u + E_y} + \frac{A_{sf}}{b w \cdot d}
\]

\[
P_w = \frac{A_s}{b w \cdot d}, \quad P_f = \frac{A_{sf}}{b w \cdot d}
\]

\[
P_{wb} = 0.85 b, f'_c \cdot \frac{E_u}{f_y E_u + f_y} + P_f
\]

\[
P_{wb} = P_b + P_f
\]

\( P_w = \) web steel ratio

\( P_f = \) flange steel ratio

\( P_{wb} = \) balanced steel ratio for T-beam

\( P_b = \) balanced steel ratio for rectangular part

\[
P_w, \text{ max } = 0.85 b, f'_c \frac{E_u}{f_y} \frac{E_u}{E_u + 0.004} + P_f
\]

\[
P_w, \text{ max } = P_{\text{max}} + P_f
\]
Shear and diagonal tension

According to moment (M)/Shear (V) ratio, there are 3 types of cracks:

1- Flexural cracks (large M, small V)
2- Flexural - Shear cracks (large M, large V)
3- Web shear cracks (small M, large V)

Shear Failures occur suddenly with no advance warning.
Shear reinforcement (stirrups)

\[ \phi V_n \geq V_u \]

\( \phi = 0.75 \) (shear) reduction factor

\[ V_u = \text{Factored Shear force at section} \]

\[ V_n = \text{Nominal Shear strength} \]

\[ V_n = V_c + V_s \]

\( V_c = \text{Nominal Shear resistance provided by concrete} \)

\( V_s = \text{Nominal Shear provided by the shear reinforcement} \)

Shear strength provided by concrete

\[ V_c = \frac{\sqrt{f_{ec}}}{6} b w \cdot d \]

For more detailed:

\[ V_c = (\sqrt{f_{ec}} + 120 \frac{P_w}{M_u} \frac{V_{ud}}{f_{cd}}) \frac{b w}{d} \leq 0.3 \sqrt{f_{ec}} b w d \]

where \( \left( \frac{V_{ud}}{M_u} \right) \leq 1 \)
For section located less than a distance \( d \) from face of support, ACI Code permits to be designed for the same shear \( V_u \) as that computed at a distance \( d \).

but for members forming into a supporting member in tension and for section loaded between the support and a distance \( d \) the critical section is taken at the face of the support.
Shear reinforcement (Stirrups)

- Shear reinforcement are not needed if \( V_u < \frac{\phi V_c}{2} \)

- Minimum shear reinforcement are needed if

\[
\frac{\phi V_c}{2} < V_u < \phi V_c
\]

\[ A_{\min} = \text{larger of } \left[ \frac{\sqrt{f_{ck}}}{16} b w S, \frac{1}{3} \frac{b w S}{f_y} \right] \]

- \( S \): Spacing of Stirrups

Spacing limit for shear reinforcement

- IF \( V_s \leq 2 V_c = \frac{\sqrt{f_{ck}}}{3} b w d \)

\[ S_{\max} = \min \left[ \frac{d}{2}, 600 \text{ mm} \right] \]

- IF \( V_S > 2 V_c = \frac{\sqrt{f_{ck}}}{3} b w d \)

\[ S_{\max} = \min \left[ \frac{d}{4}, 300 \text{ mm} \right] \]

* \( S_{\max} \leq \left[ \frac{3 A v f_y}{b w}, \frac{16 A v f_y}{\sqrt{f_{ck}} b w} \right] \)
Types of shear reinforcement

- Vertical stirrups

\[ V_S = \frac{A_v f_y d}{S} \]

- Inclined stirrups

\[ V_S = \frac{A_v f_y d}{S} (\sin \alpha + \cos \alpha) \]
Bent up longitudinal bars

\[ V_s = n A_b f_y \sin \alpha \leq \frac{1}{4} \sqrt{f_c' b w d} \]

- \( n \) = No. of bent bars
- \( A_b \) = Area of bar

Bent-up longitudinal bars

\[ A_v = A_b \times \text{legs} \]

- \( A_v = 2A_b \)
- \( A_v = A_b \)
- \( A_v = 4A_b \)
Slabs

Slabs can be classified as:
- one-way slabs
- two-way slabs

(a) One-way slab
(b) Two-way slab

\[
\frac{\text{Longer Span } l_b}{\text{Shorter Span } l_a} > 2 \quad \text{one-way slab}
\]

\[
\frac{\text{Longer Span } l_b}{\text{Shorter Span } l_a} \leq 2 \quad \text{two-way slab}
\]
One Way Slab

A one-way slab is assumed to be a rectangular beam with a large ratio of width to depth. Normally a 1 m wide piece of such a slab is designed as a beam.

The bending is in one direction only, that is perpendicular to the supported edges.
For shrinkage and temperature changes, the minimum percentages of reinforcing are:

\[ P_{\text{min}} = \begin{cases} 0.002 & \text{for } f_y = 300 \text{ and } 350 \text{ MPa} \\ 0.0018 & \text{for } f_y = 420 \text{ MPa} \\ \frac{0.0018 \times \frac{420}{f_y}}{0.0014} & \text{for } f_y > 420 \text{ MPa} \end{cases} \]

As \[ min = P_{\text{min}} \times b \times t \] (t = thickness of slab)

Minimum allowable slab thickness is determined using deflection requirements using ACI table (9.5a)

**TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED**

<table>
<thead>
<tr>
<th></th>
<th>Simply supported</th>
<th>One end continuous</th>
<th>Both ends continuous</th>
<th>Cantilever</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solid one-way slabs</strong></td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>( l/20 )</td>
<td>( l/24 )</td>
<td>( l/28 )</td>
<td>( l/10 )</td>
</tr>
<tr>
<td><strong>Beams or ribbed one-way slabs</strong></td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>( l/16 )</td>
<td>( l/18.5 )</td>
<td>( l/21 )</td>
<td>( l/8 )</td>
</tr>
</tbody>
</table>

Notes:
Values given shall be used directly for members with normalweight concrete (density \( w_c = 2320 \text{ kg/m}^3 \)) and Grade 420 reinforcement. For other conditions, the values shall be modified as follows:

a) For structural lightweight concrete having unit density, \( w_c \), in the range 1440-1920 kg/m\(^3\), the values shall be multiplied by \( (1.65 - 0.003w_c) \) but not less than 1.09.

b) For \( f_y \) other than 420 MPa, the values shall be multiplied by \( (0.4 + f_y/700) \).
- Max Spacing (Smax) for flexural requirements
  \[ S_{\text{max}} = \min \{ d, [3t, 500 \text{ mm}] \} \]

- Max Spacing (Smax) for temp. & shrinkage
  \[ S_{\text{max}} = \min \{ d, [5t, 500 \text{ mm}] \} \]

- \( S_{\text{req.}} = \frac{A_b}{A_{\text{as req}}} \times 1000 \)

- Min. Concrete Cover = 20 mm


**ACI Moment Coefficients**

ACI Code 8.3 includes expressions that may be used for approximate calculation of maximum moments and shear in continuous beams and one-way slabs. They are applicable within the following limitations:

1. There are two or more spans.
2. Spans are approximately equal, with the longer of two adjacent spans not greater than the shorter more than 20%.
3. Loads are uniformly distributed.
4. The unit live load does not exceed 3 times the unit dead load.
5. Members are prismatic.

\[
M_u = C \cdot w_u \cdot L^2
\]

where:
- \( C \) = moment coefficient
- \( L_n \) = clear span for positive moment, and average of two adjacent clear spans for negative moment

Shear in end members at first interior support:

\[
1.15 \frac{w_u L}{2}
\]

Shear at all other supports:

\[
\frac{w_u L}{2}
\]