Definition: The Hydrology

- It is the science that deals with water in the global, its appearance, circulation & distribution, its chemical & physical characteristics and its relation with the environment & living beings, and it's a branch of geology science so it deals with streams, rivers, and it is related with other sciences like chemistry, physical, and fluid.

Some branches of hydrology

1. Limnology (the science that studies the lakes)
2. Cryology (the science that studies the snow and ice)
3. Geohydrology (the science that deals with ground water)
4. Potamology (the science that studies the over ground or the rivers that run on the ground, or the surface water)
5. Hydrometeorology (the science that deals with hydrology & climate to gather)
6. Chemical hydrology is the study of the chemical characteristics of water

Some purposes for studying the hydrology

1. Design the water resources plants such as irrigation, water supply, water's energy, waste water plants, bridges.
2. Estimating the capacity of water reservoir and dams.
3. Quantity and capacity of floods to control them.
4-Minimum and maximum flow from resource.

5-Determination of probable maximum precipitation for channel and spillways, and also to design water and rain water pipes. Analyzing the impacts of antecedent moisture on sanitary sewer systems.

6-Determining the water balance of a region.

**Units you need**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Donname</td>
<td>2500 m²</td>
</tr>
<tr>
<td>1 Hectare (ha.)</td>
<td>10⁴ m²</td>
</tr>
<tr>
<td>1 Acre</td>
<td>0.4047 ha.</td>
</tr>
<tr>
<td>1 Acre</td>
<td>4047 ~4000 m²</td>
</tr>
<tr>
<td>Acre-ft</td>
<td>43560 ft²</td>
</tr>
<tr>
<td>1 ft</td>
<td>0.3048 m</td>
</tr>
<tr>
<td>1 ft</td>
<td>12 inches</td>
</tr>
<tr>
<td>1 m</td>
<td>3.28 ft</td>
</tr>
<tr>
<td>1 acre-ft</td>
<td>23*10⁻⁶ acre-ft</td>
</tr>
<tr>
<td>1 acre-ft</td>
<td>3048 m³</td>
</tr>
</tbody>
</table>

Note: Units of runoff (R) may be by: m or m³ or m³/s or acre-ft. The area from which the calculations be on it named Water shed, catchment area, basin area.
Water budget (balance equation)

In fact the water recirculation in the nature yields to the water budget equation and takes this formula:

\[ I - O = \Delta S \]

Where:
\[ I : \text{inflow}, \quad O: \text{out flow}, \quad \Delta S: \text{change in storage} \]

Each income to the watershed take the plus sign and each outcome take the minus sign

P : precipitation or rainfall (+)

R: direct runoff (+)
I: infiltration (−)
E: evaporation (−)
Qin (+)
Qout(−) and so on
So the equation may take another shape:
P+R+Qin−E−I−Qout=Δs
When there is no change in storage then Δs=0

Definitions
Over flow: the flow of water over surface
Interflow: the lateral flow of water in the surface of soil.
Direct flow: the summation of overland flow and inter flow.
Runoff: water leaving land surface to the stream.
Base flow: inter flow + ground flow.
Total flow: direct runoff + base flow.
Examples:

Ex.1-estimate the amount of depression storage in a 2.5 hectar parking for the following data, rainfall=0.88 in, runoff= 5 cfs for 1 hour ?by meters.

\[ P = \left( \frac{0.88}{12} \right) \times 0.3048 = 0.022 \text{ m} \]

\[ R = 5 \times (0.3048)^3 \times 1 \times 3600 = 509.7 \text{ m}^3 \]

\[ R = \frac{509.7}{2.5 \times 10^4} = 0.02 \text{ m} \]

\[ \Delta s = p + r \]

\[ = 0.022 + 0.02 \]

\[ = 0.042 \text{ m} \]

Ex.2-For a lake surface area=3000 acre and annual evaporation =50 inches ,what is the daily evaporation ?(m³/day)

\( \left( \frac{50 ''}{\text{yr}} \times 365 \right) = 0.137 ''/d \)

\( (0.137/12) \times 0.3048 \times 3000 \times 4000 = 4175 \text{ m}^3/d \)

Ex.3-What is the infiltration rate from 1000 ha. Lake area if the annual infiltration=40

Inches?(acre-ft)

\( 40''/12 = 3.34' \)

\( 1000 \text{ ha.} /0.4047 = 2470.9 \text{ acre} \)

\( 3.34 \times 2470.9 = 8253.02 \text{ acre-ft} \)
E4-Twelve cubic feet of water per second added to a vertical –walled reservoir

With surface area = 600 acres, how many hours will it take to raise the water level up to one ft?

\[ 600 \times 43560 \times 1 = 26.13 \times 10^6 \text{ ft}^3 \quad \text{(to convert area in acre to vol. in ft}^3) \]

\[ 12 \times 3600 = 43200 \text{ ft}^3/\text{hr}. \]

\[ \frac{26.13 \times 10^6}{43200} = 605 \text{ hrs}. \]

(\text{because } Q=\text{vol./t})
Precipitation

Precipitation is the general terms for all forms of moisture emanating from the Clouds and falling to the ground, from the time of its formation in the atmosphere until it reaches to the ground.

*Forms of precipitations:*

1. **Rain:** drops usually greater than 0.5 mm in dia. it may reach to 6 mm.

2. **Snow:** is a precipitation in the form of ice crystals resulting from sublimation (water vapor directly to ice) and its density = 0.1 gm/cm³.

3. **Drizzle:** water drops under 0.5 mm dia.

4. **Glaze:** ice coating formed when drizzle or rain freezes as it comes in contact with cold object at the ground.

5. **Sleet:** frozen rain drops cooled to the ice stage while falling through air at freezing temperature.

6. **Hail:** precipitation in the form of balls of ice over 8 mm.

7. **Storm:** heavy rains

*Measurement of precipitation*

A variety of instruments and techniques have been developed for gathering information on various phases of P.

*Disdrometer - precipitation characteristics*

- Radar - cloud properties, rain rate estimation, hail and snow detection
- Rain gauge - rain and snowfall
- Satellite - rainy area identification, rain rate estimation, land-cover/land-use, soil moisture
- Sling psychrometer – humidity

All form of P are measured on the base of the vertical depth of water that would accumulate on a level of surface if the P remained where it fell.

**Types of rain gauges**

Non recording rain gage

Recording gage
1-Non recording gauge

Any open receptacle with vertical sides it is a pan and a collector inside the pan with 12 cm dia. And 30 cm depth and there is a scale to read the water high, when there is a snow the collector is removed from the pan.

2-The recording gauge

- Tipping bucket gauge
- Weighing bucket gauge

The distribution of rain gauges depends on meteorological and topographical factors

**Estimating of missing data**

Many rain gage stations have a short breaks in their recorded because of absences of the observer because of instrumental failures. Here it is necessary to estimate the missing record.

If the normal annual $P$ at each of index station is within 10% of that with the station with the missing record, a simple arithmetic average of the $P$ at the index station provides the estimated amount.

If the average annual $P$ at any of the index station differs from that at station in question by more than 10% the normal ratio method is used.

$N_x$: the average annual $P$ @ missing sta.

$N_i$: the average annual $P$ @ others sta.

$P_x$: the missing data sta.

$P_i$: data for other stas.

For checking
(Ni-Nx)/Nx<=10%

\[ Px = \frac{1}{n} (p_1 + p_2 + p_3 + \ldots) \]

(Ni-Nx)/Nx>10%

\[ Px = \frac{N_x}{n} \left( \frac{p_1}{N_1} + \frac{p_2}{N_2} + \ldots \right) \]

**Examples**

Ex.1-the normal annual P of five stations (A,B,C,D & E) are respectively (125,102,76,118,137)cm during a storm the P recorded for stations (A,B,C,&D) are (13.2,9.2,6.8, & 10.2)cm, estimate the missing data at sta. E?

Check

\[ \frac{125-137}{137} = 0.08 < 0.1 \]

\[ \frac{102-137}{137} = 0.255 > 0.1 \]

Use normal ratio equ.

\[ P_x = \frac{N_x}{n} \left( \frac{p_1}{N_1} + \frac{p_2}{N_2} + \ldots \right) \]

\[ P_x = \frac{137}{4} \left( \frac{13.2}{125} + \frac{9.2}{102} + \frac{6.8}{76} + \frac{10.2}{113} \right) = 12.86 \text{ cm} \]
Average precipitation over an area:

1- Simple arithmetic mean

This method is used for a flat, wide and little number of gages

\[ \text{Pav.} = (p_1 + p_2 + p_3 + \ldots + p_n)/n \]

2- Thiessen method

This method is used at a flat (or nearly), uniform distribution and the area takes a geometrical shape

\[ \text{Pav.} = \frac{P_1 A_1}{A_t} + \frac{P_2 A_2}{A_t} + \ldots + \frac{P_n A_n}{A_t} \]

\[ \text{Pav.} = \sum_{i=1}^{n} (P_i * A_i) / A_t \]

3- Isohyetal method

When the arrangement of stations are non-uniform and the area is not flat (like mountain) with a lot of gages this method is used

\[ \text{Pav.} = \frac{(p_1 + p_2)}{2} \times \frac{A_1}{A_t} + \frac{(p_2 + p_3)}{2} \times \frac{A_2}{A_t} + \ldots + \]
Examples

Ex.1 - A square area of 100 km² is gauged by three rainfall gauges @ 2.5 km from sides (fig.) estimate the average precipitation?

<table>
<thead>
<tr>
<th>Sta.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(mm)</td>
<td>106</td>
<td>152</td>
<td>127</td>
</tr>
</tbody>
</table>

A1 = square + triangular

\[ A1 = 5 \times 5 + 0.5 \times 5 \times 5 \]

\[ = 37.5 \text{ km}^2 = A3 \]

A2 = 5 \times 5

\[ = 25 \text{ km}^2 \]

At = 10 \times 10

\[ = 100 \text{ km}^2 \]

\[ P_{av.} = p_1 \left( \frac{A_1}{A_t} \right) + p_2 \left( \frac{A_2}{A_t} \right) + p_3 \left( \frac{A_3}{A_t} \right) \]

\[ = \frac{(106 \times 37.5 + 25 \times 152 + 127 \times 37.5)}{100} \]

\[ = 125.4 \text{ mm} \]

Ex.2 - Isohyets drawn for a storm gave the following data

<table>
<thead>
<tr>
<th>ppcm</th>
<th>15-12</th>
<th>12-9</th>
<th>9-6</th>
<th>6-3</th>
<th>3-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>area km²</td>
<td>92</td>
<td>128</td>
<td>120</td>
<td>175</td>
<td>85</td>
</tr>
</tbody>
</table>

Estimate the average precipitation over the catchment?

\[ P_{av.} = \left( \frac{5+12}{2} \right) \times 92 + \left( \frac{12+9}{2} \right) \times 128 + \left( \frac{9+6}{2} \right) \times 120 + \left( \frac{6+3}{2} \right) \times 175 + \left( \frac{3+1}{2} \right) \times 85 \] / At

\[ = \text{ cm} \]
Ex3-

A circle shaped area of 50 km radius gauges fixed @ the points 1, 2, 3, 4 & 5 with data below, compute the value of average P by Thiessen’s method?

<table>
<thead>
<tr>
<th>Sta.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pcm</td>
<td>3.2</td>
<td>4.8</td>
<td>5.4</td>
<td>6</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Total area = \(50^2 \pi\)

= 7580 km²

\(A_5 = 50 \times 50 = 2500 \text{ km}^2\)

\(A_1 = A_2 = A_3 = A_4 = (7850 - 2500)/4 = 1337.5 \text{ km}^2\)

\(P_{av.} = (A_1P_1 + A_2P_2 + A_3P_3 + A_4P_4 + A_5P_5)/\text{AT}\)

= \(2500 \times 4.5 + (1337.5(3.2 + 4.8 + 5.4 + 6))/7850\)

= 3.4 cm
Rainfall information

Some definitions related to the rainfall information:

Intensity (i):

It is a measuring of the quantity of rainfall during a given time

\[ i = \frac{dept \ h}{time} = \frac{p}{t} \text{(mm/hr.)} \]

Duration (t):

It is a period of time during which rain falls. (Hr, second…)

Frequency (N):

This refers to the expectation that a given depth of rainfall will fall in a given time such an amount may be equal or exceeded in a given number of days or years.

i.e. how many times during 10 years the rain fall more than the normal.

Return period (T):

The average period within which rain of a given depth will equaled or exceeded once.

This mean during a long period (40 yrs.) How many times the up normal amount of rainfall does frequent?
Relations between rainfall information

Depth –Area –Duration (D-A-D)

The relation between the depth of rainfall and the area of catchment is diversely by the time (this mean within the time the depth increase but when the area increase the depth decrease)

![Graph showing the relationship between depth and area](image)

Area (km²)

The depth of rainfall in the center of the catchment h maximum and it decrease with the area increasing (at area =0, P =maximum)
The previous curve is drawn by depending on the following formula:

\[
\frac{P'}{P} = 1 - \frac{0.3 - \sqrt{A}}{t^*}
\]

Where :P average depth

P point depth at the center (mm)

\* t* invers gamma function

Example:

What is the average rainfall intensity over an area =5 km², during one hour storm, if t* =5.6 & i=23 mm/hr.?

\[
\frac{P'}{P} = 1 - \frac{0.3 - \sqrt{A}}{t^*}
\]

P=i * t

=23*1=23 mm

\[
\frac{P'}{23} = 1 - \frac{0.3 - \sqrt{5}}{5.6}
\]

p' =20 mm , i=20 /1 =20 mm/hr.
Intensity -duration relation

The relation between the intensity & the duration take the formula

\[ I = \frac{a}{t+b} \text{ when } t \leq 2 \text{ hr.} \]

\[ I = \frac{c}{t^n} \text{ when } t > 2 \text{ hr.} \]

Where amebic and \( n \) are constants.

When two variables situations be \( x \) & \( y \) values are measured and a relation between these two is determined. The relation can be linear. Assume a linear or nonlinear does exist and given by

\[ y' = a + bx + \epsilon \]

\( y', x \) variables

\( a, b \) constants and \( \epsilon \) error

Constants may be obtained from many methods like least square method and matrix.

Using the least square method to find amebic & \( n \)

1- Write the equation by the formula of straight line eq.

\[ Y = Ax + B \]

2- \[ \sum y = A \sum x + NB \]

\[ \sum X = A \sum x^2 + B \sum x \]

3 After solving the last two equations simultaneously
\[ A = \frac{\sum xy - Ny'y'}{\sum x^2 - Nx'^2} \]

\[ B = y' - Ax' \]

\[ x' = \frac{\sum x}{N} \]

\[ y' = \frac{\sum y}{N} \]

Example

Given the relation between the intensity and the duration \( i = \frac{a}{t + b} \) calculate the variables \( a \) and \( b \) depending on the following data:

\( i \) (mm/hr.)  | 30 | 20 | 15
--- | --- | --- | ---
\( t \) (min) | 20 | 40 | 60

\[ y = Ax + B \]

\[ \frac{1}{i} = \frac{1}{a} t + \frac{b}{a} \]

\[ \therefore y = \frac{1}{i} \quad x = t \quad A = \frac{1}{a} \quad B = \frac{b}{a} \]

<table>
<thead>
<tr>
<th>( t = x )</th>
<th>( y = 1/i )</th>
<th>( \text{my} )</th>
<th>( x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.033</td>
<td>0.676</td>
<td>400</td>
</tr>
<tr>
<td>40</td>
<td>0.05</td>
<td>2</td>
<td>1600</td>
</tr>
<tr>
<td>60</td>
<td>0.67</td>
<td>4</td>
<td>3600</td>
</tr>
<tr>
<td>( \sum 120 )</td>
<td>( \sum 0.15 )</td>
<td>( \sum 6.67 )</td>
<td>( \sum 5600 )</td>
</tr>
</tbody>
</table>

\[ x' = \sum \frac{x}{N} = 40 \]

\[ y' = \sum \frac{y}{N} = 0.05 \]

\[ A = \frac{0.67 - 3 \times 40 \times 0.05}{5600 - 3 \times 40^2} = 0.00083 \]
\[
B = 0.05 - 0.00083 \times 40 = 0.0167 \\
a = \frac{1}{A} = 1200 \\
b = a \times B = 20 \\
\therefore \ i = \frac{1200}{t + 20}
\]

**Intensity-duration-frequency (IDF)**

Structures designed to control stormwater volumes and flows need quantitative criteria to determine their size. Two important stormwater parameters, intensity and duration, can be statistically related to a frequency of occurrence. The graphical representation of this relationship is the intensity-duration-frequency (IDF). The (IDF) curve is a plot of average rainfall intensity versus rainfall duration for various frequency of occurrence is shown in fig. below.

![Rainfall intensity (mm/hr.)](image-url)
Within the time the intensity become decrease for any frequency, this curve can be expressed as the following formula:

\[ I = \frac{aT^m}{(b + t)^n} \]

Where:

- \( I \): intensity (mm/hr.)
- \( T \): frequency (yr.)
- \( T \): duration (hrs.)
- \( a, b, m, \) & \( n \): coefficient varying from one region to another

The common form of the last equation used for hydrologic analysis is one that fixes the frequency of occurrence, thus we eliminate \( t \) and \( m \) from the equation and assume the exponent \( n \) to equal unity resulting in:

\[ I = \frac{a}{b + t} \]

Also within the time the depth increases as shown in fig.:
Example: Drive the IDF curve from data below and find intensity for duration 6 sec. and frequency 5 yrs.?

<table>
<thead>
<tr>
<th>T(yr.)</th>
<th>(sec)</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.4</td>
<td>0.82</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.56</td>
<td>0.9</td>
<td>1.21</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0.67</td>
<td>0.98</td>
<td>1.35</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T(yr.)/ t(sec)</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.08</td>
<td>0.082</td>
<td>0.068</td>
</tr>
<tr>
<td>10</td>
<td>0.11</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td>25</td>
<td>0.13</td>
<td>0.098</td>
<td>0.09</td>
</tr>
</tbody>
</table>
There are another formulas that connect the depth, duration, frequency & return period one of them is Bilham formula as follows

\[ N = \frac{10}{T} = 1.214 \times 10^5 t (p + 2.54)^{-3.55} \]

N: no. of occurrence in 10 yrs.
T: return period
P: depth
t: duration

Example:

Determine the rainfall intensity (cm/hr.) for 40 yrs. return period storm occurred during 30 min. on 10 hectare watershed, what is the volume of water applied on this area?

\[ \frac{10}{T} = 1.214 \times 10^5 t (p + 2.54)^{-3.55} \]

\[ P = \left( \frac{1.214 \times 10^5}{10} \right)^{1/3.55} - 2.54 \]

\[ = \left( \frac{1.214 \times 10^5}{10} \times 40 \times \frac{30}{60} \right)^{1/3.55} - 2.54 \]

\[ = 30.3 \text{ mm} \]

I = \frac{p}{t} = \frac{30.3}{30/60} = 60.6 \text{ mm/hr.} \]

Vol. = 30.3 \times 10^{-3} \times 10 \times 10^{-4} = 3034.3 \text{ m}^3
Abstractions from precipitation

*Evaporation

Evaporation is understood to be a cooling process because heat is removed from the surface where evaporation has taken place. Energy must be available for the evaporation process and are chiefly solar vapor pressure and advective (wind).

There are three general methods commonly in use for measuring evaporation which mainly indirect methods

1-Measurement from evaporation pan.

2-Water budget.

3-Correlation with climatic data.

Evaporation pans: the class A pan is the most widely used (fig.) it is a cylinder made of galvanized iron with 122 cm dia. & 2 cm depth, the pan restson a leveled wooden.

It is usually filled to a depth of 20 cm and re filled when the depth has fallen to ≤18 cm. the water surface is measured daily with a hook gage.

Also there are another types of pans like Indians standard pan (the same as class A) and Colorado sunken pan (is square ,1 m on a side and 0.5 m deep it buried in the ground to within 5 cm so it look like a lake or water surfaces.

Pan evaporation is used to calculate lake evaporation (El) by

Using a pan coef. (Pc)

El=PcEp
2-Water budget:
Another estimate depends on an accurate water budget in which evaporation is the only unknown variable = \( P + R - O + \Delta S \) .................1

3-Correlation to climatic data
Empirical formulas has been developed to rate either pan or actual evaporation to atmospheric measures. The form of the equation are similar and in general are related to vapor pressure and wind speed.

\[ E = f(\Delta e, U) \] (mass transfer eq.) ...............2

where
\[ \Delta e = \text{change in vapor pressure from the water to the air.} \]
\[ U = \text{wind speed.} \]

Atypical equation developed in connection with 1&2 (Hefner)

\[ E_t = 0.0024(e_o - e_a)U^8 \] .................3

\[ E = \text{evaporation (in/d)} \]
\[ e_o = \text{saturation vapor (in)} \]
\[ e_a = \text{vapor pressure (inches of Ag)} \]
\[ U^8 = \text{wind speed (miles/d)} \]

Also the correlation was further as follow (by Kohler et al.)

\[ E_p = (e_o - e_a)^n (m + Bu) \] ...............4 where

\[ E_p = \text{daily evaporation (in/d)} \]
\[ e_o = \text{saturation vapor (in of Hg)} \]
\[ e_a = \text{atm. Vapor pressure (in of Hg)} \]
U = wind movement (mpd) - 6 in. above pan rim.

n, m & b constants.

Other important climate variables are mean daily air and water temperature, wind movement, and solar radiation, not all the advective energy is used for evaporation.

To estimate lake evaporation, a general formula can be used with different in temperature, mean daily air speed, and elevation above sea level.

Transpiration from vegetation can be estimated by the formula:

\[ T = ET - E \]

- **T** = transpiration rate (mm/time)
- **ET** = evapotranspiration (mm/time)
- **E** = evapo. (mm/time)
**Infiltration (f)**

It is a passage of water through the soil surface into the soil under gravity and capillary force.

**Infiltration capacity I.C.**

It is the maximum rate at which water can enter the soil at a particular point at any time (cm/hr).

Soil and fluid characteristics affect the I.C.

The rate and quantity of water which infiltrate is a function of soil type, soil moisture, soil permeability, ground cover, drainage conditions, depth of water table, and intensity and volume of precipitation.

**Infiltration measurement:**

It measures by infiltrometer, a device used to measure the rate of water infiltration into soil or other porous media. Commonly used infiltrometers are single ring or double ring infiltrometer.

![Single ring infiltrometer](image)

The single ring involves driving a ring into the soil and supplying water in the ring either at constant head or falling head condition. Constant head refers to a condition where the amount of water in the ring is always held constant. Because infiltration capacity is the maximum infiltration rate, and if infiltration rate exceeds the infiltration capacity, runoff will be the consequence, therefore maintaining constant head means the rate of water supplied corresponds to the infiltration capacity. The supplying of water is done with a Mariotte's bottle. Falling head refers to a condition where water is supplied in the ring, and the water is allowed to drop with time. The operator records how much water goes into the soil for a given time period. The rate of which water goes into the soil is related to the soil's hydraulic conductivity.
Infiltration calculations:

1- Green – Ampt method

Is based on darcy’s law

\[ Kt = F(t) - \eta \Psi \ln \left( \frac{\eta \Psi + F(t)}{\eta \Psi} \right) \]

\( K = \) hydraulic conductivity of the soil (L/T)

\( \Psi = \) capillary suction of the soil at the wetting front (L)

\( \eta = \) effective soil porosity

\( T = \) time

\( F(t) = \) cumulative infiltration volume at time \( t \) (L)

2- Horton equations

Infiltration can be written by Horton’s equation, this method gives an expression for varying infiltration:

\[ F(t) = f_c + (f_0 - f_c) e^{-kt} \]

Where \( f(t) = \) infiltration rate as a function of time cm/hr.

\( f_c = \) final or ultimate infiltration rate

\( f_0 = \) initial infiltration rate \( F(t) \)

\( K = \) constant (hr\(^{-1}\))

\( T = \) time.
3-Water budget

If infiltration is the only unknown in the water budget (and the other variables can be measured then the water budget would produce accurate results.

3-Infiltration indices

-Ø index.
-\( w \) index.

Ø index: It is the average rainfall above which the volume of rainfall equal the volume of runoff

It depends on the soil, vegetation cover, moisture, duration, intensity,
p=the whole area
I=lower part
R=upper part

w-index: it is the average infiltration index  \( w = \frac{P-R}{T} \)

examples:
ex1: for total rainfall of 75 mm and runoff 33 mm, find \( \bar{\phi} \) index for the data below

<table>
<thead>
<tr>
<th>I(mm/hr.)</th>
<th>7</th>
<th>18</th>
<th>25</th>
<th>12</th>
<th>10</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(hr.)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

\[ 7(1)+6(2)+2(3)+x(4)=33 \]

X=2

\( \bar{\phi} = 10 - 2 = 8 \text{ mm/hr.} \)

\( W = \frac{P-R}{T} = 75-33/6 = 7 \text{ mm/hr.} \)

Ex2: the table below shows the data of a number of storms are observed on a river; compute the w-index for all storms and its average?

<table>
<thead>
<tr>
<th>No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Av.P</td>
<td>2.82</td>
<td>2.98</td>
<td>4.55</td>
<td>14.2</td>
<td>2.87</td>
<td>3.91</td>
<td>8.1</td>
</tr>
<tr>
<td>Av.R</td>
<td>1.32</td>
<td>1.02</td>
<td>2.46</td>
<td>7.42</td>
<td>0.43</td>
<td>0.48</td>
<td>1.93</td>
</tr>
<tr>
<td>T</td>
<td>12</td>
<td>48</td>
<td>24</td>
<td>72</td>
<td>18</td>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>P-R/t</td>
<td>0.125</td>
<td>0.041</td>
<td>0.087</td>
<td>0.094</td>
<td>0.136</td>
<td>0.143</td>
<td>0.172</td>
</tr>
</tbody>
</table>
\[
\frac{P-R}{t} = w \quad \text{w av.} = 0.798/7 = 0.114 \text{ cm/hr.}
\]

Ex 3:

If you have the following data \( f_c = 0.53 \text{ in/hr.} \), \( f_o = 3 \text{ in/hr.} \), \( k = 4.18 \text{ hr}^{-1} \), find the infiltration rate \( f \) after 0, 0.5, 1.0, 1.5 and 2 hr. with curve?

\[
F = f_c + (f_o - f_c) e^{-kt}
\]

\[
F = 0.53 + (3 - 0.53) e^{-4.18t}
\]

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>3</td>
<td>0.83</td>
<td>0.567</td>
<td>0.534</td>
<td>0.5305</td>
</tr>
</tbody>
</table>

\[
F(\text{in/hr.})
\]

\[
T(\text{hr.})
\]
Stream flow

Measurement of stream flow (discharge)

<table>
<thead>
<tr>
<th>Direct meas. (non uni.)</th>
<th>Indirect meas. (uni.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Area velocity</td>
<td>1-Structures</td>
</tr>
<tr>
<td>2-Dilution technique</td>
<td>-weir</td>
</tr>
<tr>
<td>3-Electromagnetic</td>
<td>-flum</td>
</tr>
<tr>
<td>4-Ultra sonic</td>
<td>-gates</td>
</tr>
<tr>
<td></td>
<td>2-formulas</td>
</tr>
</tbody>
</table>

Water stage: is the elevation of the water surface at a specified station above some arbitrary zero datum sometime taken as mean sea level.

The simplest way to measure river stage is by means of staff gage. A scale set so that a portion of it is immersed in the water at all time. The gage may be attached to a bridge pier or other structure.
Another type of manual gage is the **suspended-weight gage** in which is lowered from a bridge or other overhead structure until it reaches the water surface. By subtracting the length of line paid out from the elevation of a fixed reference point on the bridge, the water level can be determined.

Also, there are other types of gages like recording gage, crest gage, float gage, and bubble gage.

1. **Area velocity method:**

   a- **Mean section**

   \[ Q = \sum \frac{V_i + V_{i+1} \cdot d_i + d_{i+1} \cdot 1}{2} (b_i + 1 - b_i) \]

   b- **Mid section**

   \[ Q = \sum v_i \cdot d_i \cdot \left( \frac{b_i + 1 - b_{i-1}}{2} \right) \]

   \( V_i \): is a velocity at each section in the river measured at one point at each distance of (0.6 D) from the surface or two points at (0.2 & 0.8 D) from the surface.
\[ V_{av.} = \frac{V_{0.2} + V_{0.8}}{2} \]

2-Dilution technique

By spray a tracer powder (no dissolved) with stream flow till it dilute then the discharge calculate.

3-Electromagneti

4-Ultrasonic method

By using the ultrasonic to calculate the velocity and then the discharge.

1-Structurs

Flume: by venture flume
A1 = b1 * y1
A2 = b2 * y2
A1 V1 = A2 V2
V1 = \frac{A2 + V1}{A1} 

\frac{p1}{\gamma} + \frac{v1}{2g} + z1 = \frac{p2}{\gamma} + \frac{v2}{2g} + z2 
\text{Bernoulli}

Qth = \frac{A1A2}{\sqrt{A1 - A2}} \sqrt{2g(y2 - y1)}

Qact = cd * Qth.

Cd = 0.95 - 0.99
-weirs

It is a notch in a wall built across a stream, it may be rectangular, trapezoidal or triangular.

It may be a board crest built parallel to the stream flow @ the floor of channel.

Or may be a sharp crested weir.

2-Formulas

<table>
<thead>
<tr>
<th></th>
<th>Chezy’s</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Manning’s</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q = \frac{n}{n} R^{2/3} S^{1/2} A$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydraulic radius</td>
<td>$Q = C A \sqrt{RS}$</td>
<td></td>
</tr>
<tr>
<td>$P$ wetted parameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S$ slope of bed channel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N$ manning coef.</td>
<td>$C$ Chezy’s coef.</td>
<td></td>
</tr>
<tr>
<td>$A = B*Y$</td>
<td>$A = B<em>Y + Z</em>Y^2$ (trapezoidal)</td>
<td></td>
</tr>
<tr>
<td>$P = B + Z*Y$</td>
<td>$P = B + 2*Y\sqrt{1 + Z^2}$</td>
<td></td>
</tr>
</tbody>
</table>

Al-Mustansiriyah University | College of Engineering | Transportation Dep.
2015-2016 | Engineering Hydrology | Lecture no.
Examples:

Ex1:

Compute the stream flow discharge for the measurements data below

<table>
<thead>
<tr>
<th>Distance(m)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>76</th>
<th>86</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth(m)</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Velocity(m/s)</td>
<td>0</td>
<td>0.5</td>
<td>0.8</td>
<td>1</td>
<td>0.9</td>
<td>0.6</td>
<td>0</td>
</tr>
</tbody>
</table>

By mean section

\[
Q = \sum \frac{v_i + v_{i+1} + d_i + d_{i+1}}{2} \times (b_{i+1} - b_i)
\]

\[
= \frac{0 + 0.5}{2} \times 0.5 + \frac{0.5 + 0.8}{2} \times 2 + \frac{0.8 + 1}{2} \times 3 + \frac{1 + 0.9}{2} \times 4 + \frac{0.9 + 0.6}{2} \times 5 + \frac{0.6 + 0}{2} \times 6 + \frac{0 + 0}{2} \times 7
\]

\[
= 152.5 m^3/s
\]

By mid-section

\[
Q = \sum v_i \times d_i \times \frac{b_{i+1} - b_i}{2}
\]

\[
Q = 0.5 \times 0.5 \times 0.5 + 0.8 \times 0.5 \times 2 + 1 \times 0.5 \times 3 + 0.9 \times 0.5 \times 4 + 0.6 \times 0.5 \times 5 + 0.6 \times 0.5 \times 6 + 0.6 \times 0.5 \times 7
\]

\[
= 201.25 m^3/s
\]

Ex2

A trapezoidal channel lined with concrete, c=130, B=10 m, side slope=1:1, depth=5m, channel slope =0.0004, find discharge?

\[
Q = C B \sqrt{R S}
\]

\[
A = B + Y Z^2, R = A/P, P = B + 2Y \sqrt{1 + Z^2} = 24.1 m \rightarrow R = 75/24.1 = 3.11 m
\]

\[
Q = 130 \times 75 \times \sqrt{3.11} \times 0.0004 = 343 m^3/s
\]
Runoff

It is the flow or discharge of precipitation on the catchment or through a surface channel during a time till it reach to the surface water. The flow over land occurs when soil is infiltrated to full capacity and excess water from rain.

During a precipitation a mass of total volume of rainfall onto and flow on soil. Initial abstraction is water intercepted by vegetation IA, also there are evaporation Transpiration T, infiltration F and initial abstraction, then the storage change is written as

\[ R(\text{rainfall excess}) = P - E - T - F - I_a \]

Over land flow: flow of water over the surface of land.

Inter flow: lateral flow of water in the surface of soil.

Flow open channel: flow of water in the through many channels to the stream.

Direct runoff: the sum. Of over land flow and inter flow.

Base flow: inter flow and ground water.
Volume of runoff

More complex methods for the determination of runoff are available. They require a more detailed mathematical description of the watershed characteristics. The volume of runoff may estimate as an annual or monthly or daily or even for hours.

1-relations between P & R
2-Emprirical equations
3-catchment area
4-infiltration indices
5-rational equation
6-hydrograph

1 & 2 through many relations between R&P like:

\[ R = CP \text{ or } R = aP + b \]
\[ R = 0.85P - 30.5 \]

3-it is the budget eq. \[ O - I = \Delta S \]
4-is was explained before
5-\[ Q = C*I*A \]
6- (next section)

Rational equation (CIA)

The rational methods are one of the oldest and were originally used to estimate the peak discharge. The simplest model of watershed runoff is the rational equation (Q=CIA).

Q: peak discharge, C: runoff coefficient, I: rainfall intensity, A: watershed area. In this equation, the watershed is modeled with two watershed characteristics, the rational coefficient (c) and the
watershed area the infiltration and depression storage are incorporated into the value of (c). so the volume of runoff will depend on the watershed area, it is the total surface area of the drainage basin. This area can subdivide into two areas: pervious area and the impervious area. The pervious area allows for soil infiltration where the impervious does not. If the area were 100% impervious then the infiltration term of the mass balance would be zero.

**Time of concentration:**
The time of concentration is the longest travel time it takes a particle of water to reach a discharge point in a watershed.

1- **Izzard’s formula**

\[ T_c = \frac{4.1KL}{i} \]

\( T_c: \text{time (min)} \), \( L: \text{overflow distances (ft)} \), \( i: \text{rainfall intensity (in/hr.)} \)

\[ K = 0.0007 \frac{i + c^2}{s} \]

\( S: \text{slope} \), \( c: \text{retardance coef. (kind of surface)} \)

2- **Kerby’s equation**

\[ T_c = c(Lns) \]

\( T_c: \text{time} \), \( L: \text{length of flow (ft)} \), \( s: \text{slope} \), \( c: \text{0.83 (when using ft) 1.44 (when using m)} \)

\( N: \text{retardance roughness coeff.} \)

3- **Kirpich’s equation**

\[ t_c = 0.0078(L^{0.77}/S^{0.385}) \]

\( t_c: \text{time (min)} \)

\( L: \text{length of travel (ft, m)} \)

\( S: \text{slope (m/m)} \)
Examples

Ex1

Design a pipe of storm sewer system that receive a drainage water from area 10000m². the length = 189 m, slope channel = 0.004 m/rainfall depth = 28.7 mm, runoff coef. = 0.6, relevant slope = 0.005 m/m, ?

\[ Tc = 0.02 \times (189)^{0.77} \times (0.004)^{-0.365} \]

= 9.487 min

\[ I = \frac{p}{t} = \frac{28.7}{9.487/60} = 181.5 \text{ mm/hr} \]

\[ Q = cia \]

\[ = 0.6 \times \frac{181.5}{1000} \times \frac{1}{3600} \times 10000 = 0.302 \text{ m}^3/\text{s} \]

\[ Q = \frac{1}{n} R^{2/3} S^{1/2} A \]

\[ = \frac{1}{n} (A/P)^{2/3} S^{1/2} A \]

\[ R = A/P = D/4 \]

\[ Q = \frac{1}{n} \left( \frac{D}{4} \right)^{2/3} S^{1/2} D^{2} \cdot L \]

D = 0.52 m

Ex2

Compute the diameter of the outfall sewer required to drain storm water from the watershed (fig.) which given the length of lines, area & times. c=0.3, 5 year, velocity = 0.75 m/s?

<table>
<thead>
<tr>
<th>Area (km²)</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 = 0.016</td>
<td>T = 5 min.</td>
</tr>
<tr>
<td>A2 = 0.023</td>
<td>T = 5 min.</td>
</tr>
<tr>
<td>A3 = 0.024</td>
<td>T = 8 min.</td>
</tr>
</tbody>
</table>
1 to 2

\[ T = \frac{l}{v} = \frac{120}{0.75 \times 60} = 2.7 \text{ min} \]

2 to 3

\[ t = \frac{180}{0.75 \times 60} = 4 \text{ min} \]

For A1, \( t = 5 + 2.7 + 4 = 11.7 \text{ min} \)

for A2, \( t = 5 + 4 = 9 \text{ min} \)

for A3, \( t = 8 \text{ min} \)

from fig., for 11.8 min & 5 yr

\( i = 115 \text{ mm/hr.} \)

\( Q = cia \)

\[ = 0.27 \times 0.3 \times 115 (0.016 + 0.032 + 0.024) = 0.67 \text{ m}^3/\text{s} \]

From monograph, \( Q = 670 \text{l/s, v = 0.75 m/s, D ~ 1050 mm, slope = 0.00055 m/m} \)
Ex3

For 10 year storm on a given area the data as follow

<table>
<thead>
<tr>
<th>T(min)</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(mm)</td>
<td>8</td>
<td>12</td>
<td>18</td>
<td>23</td>
<td>26</td>
</tr>
</tbody>
</table>

Find 10-yr design flow at the outlet from this composite area parking (60*300 m²), (c=0.9)& playground(240*30 m²), (c=0.3) the lateral flow time = 5min(parking) & 40 min (playground), channel flow velocity=0.9m/s?

Channel flow time = \( \frac{300}{0.9*60} \) = 5.56 min

tc1 = 5.56 + 5 = 10.56 min

tc2 = 5.56 + 40 = 45.54 min

Q = cia

\[
\text{I} = \frac{p}{t} \left( \frac{26 - p}{60 - 5.56} \right) = \frac{p - 23}{45.56 - 30}
\]
p = 24.5 mm

\[
\text{imax} = \frac{24.56 * 10}{45.56} = 0.537 * 10^{-3} \text{ m/min}
\]

Q = cia1 + cia2

= 0.9 * 0.537 * 10^{-3} (60*300) + 0.3 * 0.537 * 10^{-3} * 240 * 300

= 0.34 m³/s
The Hydrograph

1) The hydrograph is a graph of flow rate versus time. It is also referenced as a listing of flow rate data versus time. It is one of the more useful concepts of hydrology and is used frequently in stormwater management.

Atypical surface runoff is shown in the figure, the hydrograph consists of three general parts, (1) rising limb, (2) crest segment, (3) falling limb, the runoff hydrograph will have the following properties:

1-Time of peak ($t_p$).
2-Recession time ($t_r$).
3-Time of base ($t_b$).

The shape of the hydrograph depends on many factors, watershed shape, area, slope, depth, the earth impervious, the land use, the rainfall intensity, evaporation... etc.

DRO: direct runoff
B.F: base flow
Hydrograph separation:

Several techniques exist to separate DRO from B.F based on the analysis of ground water recession curves or type and amount of measured data available. The direct runoff hydrograph is the difference between the total runoff and the base flow function.

1-Straight line (constant slope) method. N=0.83 A^{0.2} (N days and A area km^2)

2-Fixed based (concave baseflow) method.

3-Variable slope (constant discharge) method.
Unit Hydrograph

U.H defined as; basin outflow resulting from one centimeter or one inch of direct runoff generated uniformly over the drainage area at a uniform rainfall rate during a specified period.

For a specific watershed, the U.H for a given quantity of rainfall excess can be used to generate another hy. If the storm duration is the same.

The following general rules should be observed in developing U.H:

1-Storms should be selected with a simple structure with relatively uniform spatial and temporal distribution.

2-Watershed size should generally fall 1000 ac. -1000 mi².

3-Direct runoff should range from 0.5-2 inch.

4-Duration of rainfall excess should be 25-30% of $t_p$.

5-Anumber of storms of similar duration should be analyzed to obtain an average unit hydrograph for that duration.

The following are essential steps for developing a U.H from a single storm hydrograph.

1-Analyze the hydrograph and separate the base flow.

2-Measure the total volume of direct runoff (DRO) under the hydrograph and convert this to in., cm, over the watershed.

3-Convert the total rainfall to rainfall excess and evaluate duration for the DRO and U.H.

4-Divide the ordinate of the DRO hy. By the volume and plot these results as the U.H for the basin. The time is assumed constant for storms of equal duration and thus it will not change.

$t_1 = t_2$

$i_1 \neq i_2$

$\frac{Q_1}{Q_2} = \frac{d_1}{d_2}$

$d = \frac{\sum DRO \cdot \Delta t}{A}$
Examples

Ex1: given the ordinate of a flood hy. For 1700 km² during 12 hrs. drive 12 hrs. U.H?

\[ \frac{d1}{d2} = \frac{Q1}{Q2} \]

\[ Q_1 = Q_2 \cdot \frac{d1}{d2} \]

\[ d = \frac{\sum \text{DRO} \cdot \Delta t}{A} = \frac{4000 \cdot 12 \cdot 3600}{1700 \cdot 10^6} = 0.1 \, \text{m} \]

\[ Q_1 = Q_2 \cdot \frac{0.01}{0.1} = 0.1 \times Q_2 \]

<table>
<thead>
<tr>
<th>T(hr)</th>
<th>Q1(YH.)</th>
<th>Q2(U.H.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>36</td>
<td>365</td>
<td>36.5</td>
</tr>
<tr>
<td>48</td>
<td>645</td>
<td>64.5</td>
</tr>
<tr>
<td>60</td>
<td>700</td>
<td>70</td>
</tr>
<tr>
<td>72</td>
<td>585</td>
<td>58.5</td>
</tr>
<tr>
<td>84</td>
<td>475</td>
<td>47.5</td>
</tr>
<tr>
<td>96</td>
<td>360</td>
<td>36</td>
</tr>
<tr>
<td>108</td>
<td>275</td>
<td>27.5</td>
</tr>
<tr>
<td>120</td>
<td>180</td>
<td>18</td>
</tr>
<tr>
<td>132</td>
<td>125</td>
<td>12.5</td>
</tr>
<tr>
<td>144</td>
<td>85</td>
<td>8.55</td>
</tr>
<tr>
<td>156</td>
<td>50</td>
<td>5</td>
</tr>
<tr>
<td>168</td>
<td>25</td>
<td>2.5</td>
</tr>
<tr>
<td>180</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Ex2:

Convert the DRO hy. Into a 2 hrs. U.H. The rainfall hyetograph is given in fig. and the Φ index for the storm was 0.5 in/hr. The base flow in the channel was 100 cfs. What are the t_p and t_b?

1.5 in/hr * 2 hr -0.5 in /hr * 2 = 2 in

\[ T = 2 \, \text{hrs.} \]
How to Convert the duration of unit hydrograph

The linear property of U.H. can be used to generate U.H. of a larger or smaller duration. There are two methods to convert U.H. duration:

1-Superposition method:

It is applied to convert duration from short to long time, and \( t_2/t_1 = \text{integer number} \).

To generate the U.H. of \( t_2 \), the U.H. of \( t_1 \) is legged till it reaches to \( t_2 \), then by taking the summation of the lagged unit hydrographs.

Example 1:

Given the ordinate of 2 hr. U.H., drive the ordinate of 6-hr U.H.?

- Lagging 2-hr.
- Lagging 2-hr.
- Sum. of three storms.
- Multiply by 2/6.
2-S-curve method:

Allows construction of a U.H. of any duration. Assume that a U.H. of duration $t$ is known and that we wish to generate a U.H. of $t'$, the first step by adding a series of unit hydrographs of duration $t$, each lagged by $t$, then by shifting the S-curve by $t'$, then subtract the accumulated U.H. Finally, we must multiply all hydrograph by $t/t'$.

- Shift by $t_1$ with accumulation.
- Add the last two columns.
- Shift by $t_2$ without accumulation.
- Subtract last two columns.
- Multiply by $t_1/t_2$. 

<table>
<thead>
<tr>
<th>T(hr.)</th>
<th>2-hr.U.H</th>
<th>LAG.2- HR</th>
<th>LAG.2HR.</th>
<th>SUM</th>
<th>6HR.U.H.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>0</td>
<td>-</td>
<td>300</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>720</td>
<td>300</td>
<td>0</td>
<td>1020</td>
<td>340</td>
</tr>
<tr>
<td>6</td>
<td>800</td>
<td>720</td>
<td>300</td>
<td>1820</td>
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<td>100</td>
<td>160</td>
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<td>0</td>
<td>10</td>
<td>50</td>
<td>60</td>
<td>20</td>
</tr>
</tbody>
</table>
Example:

Convert the following tabulated 2-hr. U.H. To 3-hr U.H. using the S-curve method?

<table>
<thead>
<tr>
<th>T</th>
<th>2-hr U.H</th>
<th>Lag 2hr.</th>
<th>Sum.</th>
<th>Lag 3hr</th>
<th>Sub.</th>
<th>*2/3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>75</td>
<td>-</td>
<td>75</td>
<td>-</td>
<td>75</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>0</td>
<td>250</td>
<td>-</td>
<td>250</td>
<td>166.6</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>75</td>
<td>375</td>
<td>0</td>
<td>375</td>
<td>250</td>
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<tr>
<td>4</td>
<td>275</td>
<td>250</td>
<td>525</td>
<td>75</td>
<td>450</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>375</td>
<td>575</td>
<td>250</td>
<td>325</td>
<td>216.6</td>
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<tr>
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<td>100</td>
<td>525</td>
<td>625</td>
<td>375</td>
<td>250</td>
<td>166.6</td>
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<tr>
<td>7</td>
<td>75</td>
<td>575</td>
<td>650</td>
<td>525</td>
<td>125</td>
<td>83.3</td>
</tr>
<tr>
<td>8</td>
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<td>575</td>
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<td>9</td>
<td>25</td>
<td>650</td>
<td>675</td>
<td>625</td>
<td>50</td>
<td>33.3</td>
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</tr>
<tr>
<td>11</td>
<td>675</td>
<td></td>
<td>675</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Culverts are common hydraulic structure. As such highway design manuals are devoted to the wide range of possible culverts. A culvert is a pipe or box that is located under a roadway, embankment, or service area to allow the passage of storm runoff. While culverts are often circular pipes made of either concrete or corrugated metal, other shapes including elliptical and arch pipes and rectangular box culvert are widely used. A culvert consists of an inlet structure, the pipe or box, and the outlet structure. In addition to the sheep, culverts are also classified according to the flow condition in the inlet and outlet:

1- Un-submerged inlet & outlet.

2- Submerged inlet, um-submerged outlet.

3- Submerged inlet & outlet.
Culverts are designed for the peak flow rate of the design storm. The peak flow is obtained from a U.H. at the culvert site.

Other factors affect the culvert design:

1-Head water depth: it is the invert water depth at the culvert inlet.

2-Tail water: it is the depth of water above the culvert outlet invert as the water flow out of the culvert.

*Culvert hydraulic design:*
1-Inlet control:

There are many factors that affect the performance of a culvert under inlet control:

- Inlet shape.
- Inlet configuration.
- Inlet area.
- Headwater depth.

The relation between H.W. and flow is:

- For un-submerged condition:

  \[ Q = k - 0.5S \]

- For submerged condition:

  \[ Q = c(\frac{D}{2})^2 + Y - 0.5S \]

HWi: headwater depth.

D: height of culvert.

Hc: specific head @ critical depth (dc + vc^2 / 2g)

Dc: critical depth.

Q: discharge.
A: barrel cross sec. area.

S: culvert slope.

K, M, C, Y constants depend on shape, material, cross sec.)

V: velocity.

Vc: critical velocity.

Computing invert inlet:

Example:

Determine the required inlet invert for 5*5 ft² box culvert under inlet control if the peak discharge 250 ft³/s, design HW ale. ELh=230.00 ft, stream bed ele. Inlet ELs=224.00 ft?

From fig. 3 select points A & B

A: high of box and B = 50 ft³/s/ft.

For points A&B draw a straight line, then extend it find HW/D

HW/D = 1.14

The required headwater = 1.41 * 5 = 7.1 ft

When velocity is not neglected HWi = HW – V²/2g

When it is neglected vHWi = 7.1 ft.

The required depression HWd = ELh - ELs

230 - 224 = 6.5 ft

HWi - HWd = 7.1 - 6.5 = 0.6 ft. (+, 0, -) when (0 or _ use 0)

Invert ele. = 224 - 0.6 = 223.4 ft.

2-Outlet control:

When the barrel is capable of transporting as much flow as the inlet opening, then the design will be under outlet control.

The factors affect the outlet control will be:
- Inlet shape.
- Inlet configuration.
- Inlet area.
- Headwater Depth. In addition:
- Tailwater depth.

Culvert characteristics (roughness, slope & length)

For the outlet there is a hydraulic analysis of flow based on the energy balance

\[ H_L + H_e + H_f + H_b + H_j + H_g \]

\( L \): total loss.
\( E \): energy loss at entrance.
\( F \): friction loss.
\( E \): energy loss at exit.
\( B \): bend loss.
\( J \): loss at junction.
\( G \): loss at grates.

If we neglect some losses

\[ H_L = (1 + K_e + \frac{v^2}{2g}) \]

\( K_e \) factor based on various configurations (from table).

\( R \): hydraulic radius.

\( L \): length.
\( V \): velocity.

Example for outlet control:
Determine the HW ele. For a last example, if the culvert is full under outlet control, the tail water depth above the invert = 6.5 ft, the length of the culvert = 200 ft and the natural stream slope @ 2%, ?

From chart 5 find the critical depth

\[
\frac{Q}{B} \quad dc = 4.3 \text{ ft or } \quad Dc = 0.315
\]

\(H_{TW}\) is the depth from the outlet to the hydraulic grade line =

Or tail water depth TW whichever is greater.

\(TW = 6.5 \text{ ft.}\)

\(H_{TW} = 4.7 \text{ ft}\)

Locate the size, length & \(k_e\) of the culvert at A and B, then draw a straight line from A to B and locate the intersection C.

Locate D on the discharge scale and draw a straight line C to D, extend this line to head loss scale at E.

The required outlet head water ELh is

\[
ELh = EL_{inv} + H + H_{TW}
\]

EL inv.: invert ele. At outlet.

\(EL_{inv.} = 223.4 - 0.02 * 200 = 219.4 \text{ ft.}\)

\(ELh = 219.4 + 3.3 + 6.5 + 229.2 \text{ ft.}\)

229.4 < 230

Outlet HW < design HW ele.
Circular Culvert Design:

1-For un-submerged inlet and outlet:

Manning's equation is used to estimate the diameter of the circular pipe culvert.

\[ Q = A \frac{R^{2/3}}{S^{1/2}} \]

\[ D = 1.33(Qn)^{3/8}/S^{3/16} \]

Example:

The discharge between inlet and outlet is 5.7 cfs. Assume the pipe have roughness coef. 0.014, slope 0.36% and the flow is full, calculate the diameter?

\[ D = (\frac{1}{n})^{0.375} = (0.375)^0.375 \]

= 1.48 ft.

\[ V = \frac{Q}{A} = 3.2 \text{ ft/s}. \]
2-Submerged inlet, un-submerged outlet:

In this case the system can be treated as an orifice.

\[ Q = C_d A \] \hspace{1cm} 1

\[ H^+ = H \] \hspace{1cm} 2

Substituting 2 in 1 yields

\[ Q = C_d \]

\[ (\ldots h) \ldots \]

Solving for \( D \)

\[ D^5 - 2hiD^4 + = 0 \]

3-Submerged inlet and outlet:

This case is treated as a special case where the depth of flow at the outlet equals the pipe diameter, thus the energy equation between points on the head water and tail water surface is

\[ +z1 -hL = +z2 \]

For large area \( v1 \& v2 \) are neglected

\[ Hi = \pi / \]

\[ Zi = S \cdot L \]

\[ h1+0 + SL - hL = h2 + 0 + 0 \]

\[ hL = h1 - h2 + SL \]

After using Manning's equation and continuity equation the equation become:

\[ h1 - h2 + SL = (K_{in} + K_{ex}) (\ldots) + \]

\[ K_{in}=0.5 \quad , \quad K_{ex}=1 \]

Example
A new culvert is being discharged, ponding cannot exceed 8 ft above the pipe invert inlet, at the outlet, the maximum pond of 5 ft is permitted. Mannings \( n = 0.013 \), length = 110 ft, \( S = 0.02 \text{ ft/ft} \), \( Q = 82 \text{ cfs} \), calculate the diameter?

\[
8 - 5 + 0.02 (110) = (0.5 + 1) ( + , D=)
\]
Flood forecasting

Flood forecasting is the use of real-time precipitation and streamflow data in rainfall-runoff and streamflow routing models to forecast flow rates and water levels for periods ranging from a few hours to days ahead, depending on the size of the watershed or river basin. Flood forecasting can also make use of forecasts of precipitation in an attempt to extend the lead-time available.

Annual flood: maximum discharge of a river in one year

Flood estimating

1-Flood formula.

2-Frequency analysis.

Flood formula:

\[ Q = C A^n \]

From which

Q=flood (m³/s)

A=catchment area (km²,mile²)

N=index.(0.5 – 1.25)

C=locality coefficient.

But each formula indicates an area for example in India Deckne's formula can be applied:

\[ Q(\text{ft}^3/\text{s}) = 825 A^{0.75}(\text{sq.mile}) \]

In Scotland Wales other formula applied.
\[ Q(\text{ft}^3/\text{s}) = 3000A^{0.5} (\text{sq.mile}) \]

**Frequency analysis:**

If the probability that flood will equal or larger than \( X \) for any year = \( P \)

Then \( T \) is the return period.

\[ P = \frac{1}{T} \] (probability to happening)

\[ .q = (1 - p) = (1 - \frac{1}{T}) \] (not happening)

\[ 0 \leq p (x_i) \leq 1 \]

\[ \sum_{i=1}^{n} p(x_i) = 1 \]

\[ P_{r,n} = \frac{n!}{(n-r)!r!} p^r q^{n-r} \]

N no. of years , r no. of happening

\[ P_{0,n} = q^n = (1 - p)^n \] احتمالية الحدوث صفر

\[ P_{1,n} = 1 - q^n = 1 - (1 - p)^n \] احتمالية الحدوث مرة واحدة على الاقل

When \( p \leq x_o = (1 - \frac{1}{T})^n \)

\[ p \geq x_o = 1 - (1 - \frac{1}{T})^n \]

Example:

Analysis of data on maximum one-day rainfall depth indicated that a depth of 280 mm had a return period of 50 yr. Determine the probability of a one–day rainfall depth equal to or greater than 280 mm , (1) once in 20 successive years (2) two times in 15 successive years (3) at least once in 20 yrs.?

\[ P = \frac{1}{50} = 0.02 \]

(1) \( N = 20 \) yrs. \( r = 1 \) so \[ P_{1,20} = \frac{20!}{19!1!} \times 0.02 \times (0.98)^{19} = 0.272 \]
(2) N=15 \hspace{1cm} r=2 \hspace{1cm} \text{so} \hspace{0.5cm} p_{2,15} = \frac{15!}{13!2!} \times 0.02^2 \times (0.98)^{13} = 0.323

(3) \hspace{1cm} P=1-(1-0.02)^{20}=0.332

**determination of N year flood:**

1-Graphical methods.

2-Mathematical methods.

Graphical methods:

a-compute T for all observed floods from

\[ T = \frac{n+1}{m} \] (Weibull formula)

From which:

N: number of observed flood.

M: rank of flood with data arranged in descending order and m=1 for the largest flood.

Or \[ T = \frac{n}{m} \] (California formula)

Then plot

1-Observed flood (X) against T
2 - X against T on a semi-log paper

3 - y = \ln(X) against P(X \geq x) = 1/T on a normal probability paper

Example

Determine 100-year flood from flood observation show

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
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<tr>
<td>Flood</td>
<td>3120</td>
<td>2780</td>
<td>1710</td>
<td>2960</td>
<td>7500</td>
<td>4540</td>
<td>3450</td>
<td>6790</td>
<td>5040</td>
<td>5240</td>
</tr>
<tr>
<td>Rank</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>8</td>
<td>1</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>T=n+1/m</td>
<td>1.57</td>
<td>1.22</td>
<td>1.1</td>
<td>1.37</td>
<td>11</td>
<td>2.2</td>
<td>1.83</td>
<td>5.5</td>
<td>2.75</td>
<td>3.6</td>
</tr>
<tr>
<td>Log T</td>
<td>0.195</td>
<td>0.086</td>
<td>0.041</td>
<td>0.136</td>
<td>1.04</td>
<td>0.34</td>
<td>0.26</td>
<td>0.74</td>
<td>0.44</td>
<td>0.56</td>
</tr>
</tbody>
</table>

After plot at log (100) then X = 12100m³/s.
Mathematical method:
a-fitting log-normal distribution.

1-compute \( y = \ln x \).
2-compute \( y = \frac{\sum y}{n} \) (mean).
3-compute \( y = \sqrt{\frac{\sum (y_i - y)^2}{n-1}} \) (standard deviation of \( y \))
4-compute \( f(y_o) = P(y \leq y_o) = 1 - \frac{1}{T} \)
5-let \( f(y_o) = f(z_o) \).
6-\( f(y_o) = \int_{-\infty}^{y_o} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-y)^2}{2}} dy \)
7-from statistical table find \( z_o \)
8-\( z_o = \frac{y_o - y}{\sigma} \) then \( y_o = \ln X_o \quad X_o = e^{y_o} \)

Example

Find 100-yrs. Flood by fitting log distribution

<table>
<thead>
<tr>
<th></th>
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<tr>
<td>flood</td>
<td>3120</td>
<td>2780</td>
<td>1710</td>
<td>2960</td>
<td>7500</td>
<td>4540</td>
<td>3450</td>
<td>6790</td>
<td>5040</td>
<td>5240</td>
</tr>
<tr>
<td>( Y = \ln x )</td>
<td>8.05</td>
<td>7.93</td>
<td>7.44</td>
<td>7.99</td>
<td>8.92</td>
<td>8.42</td>
<td>8.15</td>
<td>8.82</td>
<td>8.53</td>
<td>8.5</td>
</tr>
</tbody>
</table>

\( \frac{\sum y}{n} = \frac{82.18}{10} = 8.281 \)

\( \sqrt{\frac{\sum (y_i - y)^2}{n-1}} = 0.453 \)

\( F(y_o) = 1 - \frac{1}{T} = 1 - \frac{1}{100} = 0.99 \)

\( F(z_o) = 0.99 - 0.5 = 0.49 \)

From table \( z_o = 2.325 \)
\[ z_0 = y_0 - y' / y \]

\[ 2.325 = \frac{y_0 - 8.281}{0.453} \]

\[ z_0 = 9.33 \]

\[ 2.325 = \frac{y_0 - 8.281}{0.453} \]

\[ Y_0 = 9.33 \]

\[ X_0 = e^{y_0} = 11318.85 \text{ m}^3/\text{s} \]

B-Fitting the Gumble distribution

\[ 1 - P(X \geq X_0) = \frac{1}{T} = 1 - e^e \]

Or \( b = \ln \left( \frac{1}{\ln \left( \frac{1}{1 - T} \right)} \right) \)

\[ 2 - b = \frac{1}{0.78} (x_0 - x' + 0.45) \]

3-find \( x' = \frac{\Sigma x}{n} \)

4-find \( S = \sqrt{\frac{\Sigma (x_i - x')^2}{n - 1}} \)

Example

The same previous example:

\[ x' = \frac{\Sigma x}{n} = 4313 \]

\[ = 1852 \quad 1/100 = 0.01 \]

\[ B = \ln \left( \frac{1}{\ln \left( \frac{1}{1 - 0.01} \right)} \right) = 4.6 \]

\[ 4.6 = \frac{1}{0.78 \times 1852} (x_0 - 4313 + 0.45 \times 1852) \]

\[ x_0 = 10124 \text{ m}^3/\text{s} \]
Flood routing

The flood hydrograph is in fact a wave. The stage and discharge hydrographs represent the passage of waves of stream depth and discharge respectively. As this wave moves down, the shape of the waves gets modified due to channel storage, resistance, lateral addition or withdrawal of flows. Flood routing is the technique to determining the flood hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream section. Flood routing is used in (1) flood forecasting, (2) flood protection, (3) reservoir design, and (4) design of spillway and outlet structures.

Flood routing types:

1. Reservoir routing
2. Channel routing

A variety of routing methods are available and can grouped into:

a. Hydrologic
b. Hydraulic

Hydrologic methods employ essentially the continuity equation, and hydraulic method use continuity equation along with the equation of motion of unsteady flow.
Basic equation for hydrologic routing:

The passage of a flood hydrograph through a reservoir or a channel is gradually varied unsteady flow. If we consider some hydrologic system with input $I(t)$, output $O(t)$, and storage $S(t)$, then the equation of continuity in hydrologic routing method is the following:

$$I - O = \frac{dS}{dt}$$

$I \Delta t - O \Delta t = \Delta$

$$\frac{I_1 + I_2}{2} \Delta t - \frac{O_1 + O_2}{2} \Delta t = S_2 - S_1$$

$$\frac{I_1 + I_2}{2} \Delta t + S_1 - \frac{O_1 \Delta t}{2} = S_2 + \frac{O_2 \Delta t}{2}$$

$$I_1 + I_2 + \left( \frac{2S_1 \Delta t}{\Delta t} - O_1 \right) = \left( \frac{2S_2 \Delta t}{\Delta t} + O_2 \right)$$

Steps for routing:

1- drive the relation between $O$ & $\frac{2S_1 \Delta t}{\Delta t}$

2- select $O_1$ on the curve 1, then find a value

The calculate $I_1 + I_2 + \left( \frac{2S_1 \Delta t}{\Delta t} - O_1 \right) = \frac{2S_2 \Delta t}{\Delta t} + O_2$

(1) (2)

3- the value(2) we obtained from step 2 we select it on the curve to obtain $O$

4- the value of $O$ select it to find(1) and calculate $I_1 + I_2 + (1) = (2)$

5- the value of (2) select it on the curve to obtain $O$ and so on.
Example

Find the outflow hydrograph for a given 3 hours inflow hydrograph at a reservoir of initial outflow = 1 m³/s.

<table>
<thead>
<tr>
<th>t</th>
<th>I(m³/s)</th>
<th>2S/Δt-O</th>
<th>2S/Δt+O</th>
<th>O</th>
<th>O(m³/s)</th>
<th>S(m³*10⁶)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>49</td>
<td>207</td>
<td>1</td>
<td>0.3</td>
<td>1.21</td>
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<td>610</td>
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<td>1692</td>
<td>88</td>
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<td>22.36</td>
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<td>1726</td>
<td>93</td>
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<td>27.32</td>
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<td>1537</td>
<td>1721</td>
<td>95</td>
<td>3.25</td>
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<tr>
<td>27</td>
<td>85</td>
<td>1532</td>
<td>1691</td>
<td>94</td>
<td>4.5</td>
<td>44.1</td>
</tr>
</tbody>
</table>

2-Channel routing

In a very long channels the entire flood wave also travels a considerable distance resulting in a time redistribution and time of translation as well. Thus, in a river, the redistribution due to storage effects modifies the shape while the translation changes its position in time. In the
reservoir the storage was unique function of the outflow discharge \( S=f(O) \), however in channel the storage is a function of both outflow and inflow discharge. The water surface in a channel reach is not only parallel to the channel bottom but also varies with time. The total volume in storage considered as a prism + wedge. The prism storage is constant while the wedge storage is changes from a positive at an advancing flood to a negative during a receding flood. Assuming the volume of prism is \( KO \), where \( K \) is a proportionality coefficient (the time of travel of flood through the channel) and the volume of wedge is \( KX(I-O) \), \( X \) is \( 0 < X < 0.5 \) the total storage is the sum of:

\[
S = K(XI+(1-X)O) \quad \text{(Muskingum)}
\]

Then the value of storage at time \( j \) and \( j+1 \) is:

\[
S_j = K(XI_j+(1-X)O_j) \quad \text{and} \quad S_{j+1} = K(XI_{j+1}+(1-X)O_{j+1})
\]

The change in storage over time \( \Delta t \) is:

\[
S_{j+1} - S_j = K(X(I_{j+1} - I_j) + (1-X)(O_{j+1} - O_j))
\]

From continuity eq.:

\[
\left( \frac{I_j + I_{j+1}}{2} \right) \Delta t - \left( \frac{O_j + O_{j+1}}{2} \right) \Delta t = S_{j+1} - S_j
\]

Equating two eq.

\[
\left( \frac{I_j + I_{j+1}}{2} \right) \Delta t - \left( \frac{O_j + O_{j+1}}{2} \right) \Delta t = K(X(I_{j+1} - I_j) + (1-X)(O_{j+1} - O_j))
\]

The after simplifying

\[
O_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 O_j
\]

\[
(O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1)
\]

From which:

\[
C = \frac{0.5 \Delta t - KX}{K(1-X)+0.5\Delta t}
\]

\[
C_1 = \frac{0.5 \Delta t + KX}{K(1-X)+0.5\Delta t}
\]
\[ C_2 = \frac{K(1-X)-0.5 \Delta t}{K(1-X)+0.5 \Delta t} \]

\[ C_0 + C_1 + C_2 = 1 \]

**EXAMPLE**

Rout the flood for a river when \( K=12 \) hr, \( X=0.2 \), for initial outflow=10 m\(^3\)/s

<table>
<thead>
<tr>
<th>T(hr)</th>
<th>0</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
<th>36</th>
<th>42</th>
<th>48</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qin(m(^3)/s)</td>
<td>10</td>
<td>20</td>
<td>50</td>
<td>60</td>
<td>55</td>
<td>45</td>
<td>35</td>
<td>27</td>
<td>20</td>
<td>15</td>
</tr>
</tbody>
</table>

\[ c_0 = \frac{-12 \times 0.2 + 0.5 \times 6}{12 - 12 \times 0.2 + 0.5 \times 6} = 0.048 \]

\[ c_1 = \frac{-12 \times 0.2 + 0.5 \times 6}{12.6} = 0.429 \]

\[ c_2 = \frac{12 - 12 \times 0.2 + 0.5 \times 6}{12.6} = 0.523 \]

\( I_1 = 10 \quad C_1 I_1 = 4.29 \)

\( I_2 = 20 \quad C_0 I_2 = 0.96 \)

\( O_1 = 10 \quad C_2 O_1 = 5.23 \)

\( O = 10.48 \) M\(^3\)/S

\( (O_2 = C_3 I_2 + C_1 I_1 + C_2 O_1) \)

<table>
<thead>
<tr>
<th>T(HR)</th>
<th>I</th>
<th>( C_0 I_2 )</th>
<th>( C_1 I_1 )</th>
<th>( C_2 O_1 )</th>
<th>O</th>
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<tbody>
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<td>4.29</td>
<td>5.23</td>
<td>10</td>
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