Introduction & Rectilinear Kinematics:

Mechanics: The study of how bodies react to forces acting on them.

Statics: The study of bodies in equilibrium.

Dynamics:

1. ***Kinematics*** – concerned with the geometric aspects of motion without any reference to the cause of motion.

2. ***Kinetics*** - concerned with the forces causing the motion

Vectors: **are quantities which are fully described by both a magnitude and a direction.**

eg: **displacement , velocity , force ,acceleration etc.,**

Scalars: **are quantities which are fully described by a magnitude alone.**

eg : **distance , mass ,time ,volume etc**

Introduction & Rectilinear Kinematics:

Linear motion: when a body moves either in a straight line or along a curved path, then we say that it is executing linear motion.

1. when a body moves in a straight line then the linear motion is called ***rectilinear motion.***

eg ., an athlete running a 100 meter race along a straight track is said to be a *linear motion or rectilinear motion*.

2. when a body moves along a curved in two or three dimensions path then the linear motion is called ***curvilinear motion.***

eg., the earth revolving around the sun.

Rotatory motion: A body is said to be in rotatory motion when it stays at one place and turns round and round about an axis.

example: a rotating fan, a rotating pulley about its axis.

Oscillatory motion: a body is said to be in oscillatory motion when it swings to and fro about a mean position.

example: the pendulum of a clock, the swing etc.,

What is motion?

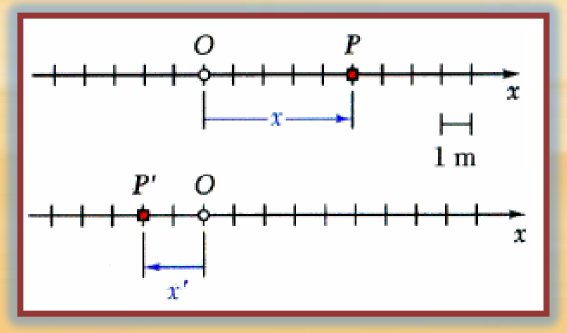
➢ **when a body is continuously changing its position with respect to the surroundings, then we say that the body is in motion.**

Motion in Relative:

➢ **When we discuss the motion of something, we describe motion *relative* to something else.**

➢ **When sitting on a chair, your speed is zero relative to the Earth but 30 km/s relative to the sun**

❖ **when we discuss the speeds of things in our environment we mean *relative* to the surface of the Earth.**



Rectilinear Kinematics: Continuous Motion

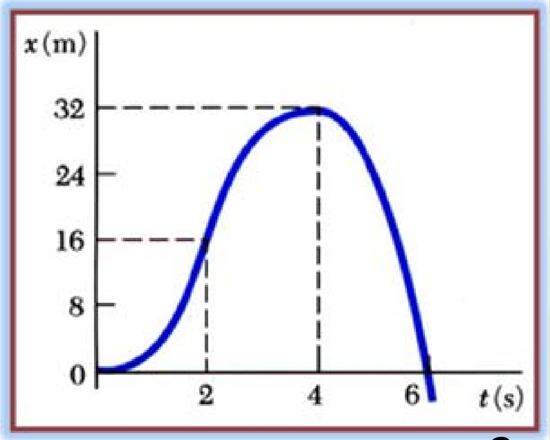
❖ A particle travels along a straight-line path defined by the **coordinate axis (x)**.

❖The *motion of a particle is known if the position* (*location*) coordinate for particle is known for every value of time (t).

❖ *Position coordinate of a particle is defined by* **positive or negative** distance of particle from a *fixed origin on the line*.

❖ *Motion of the particle may be expressed* in the form of a function, e.g.,

$$\boxed{x = 6t^2 - t^3}$$

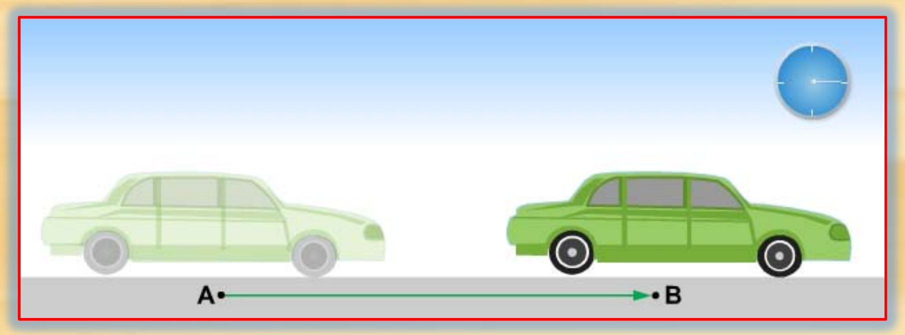


❖ Or in the form of a graph (x *vs.* t).

Rectilinear Kinematics: Continuous Motion

Distance and Displacement

- In the picture, the car moves from (**point A to point B**). So it changes its position during a time interval (**t**).



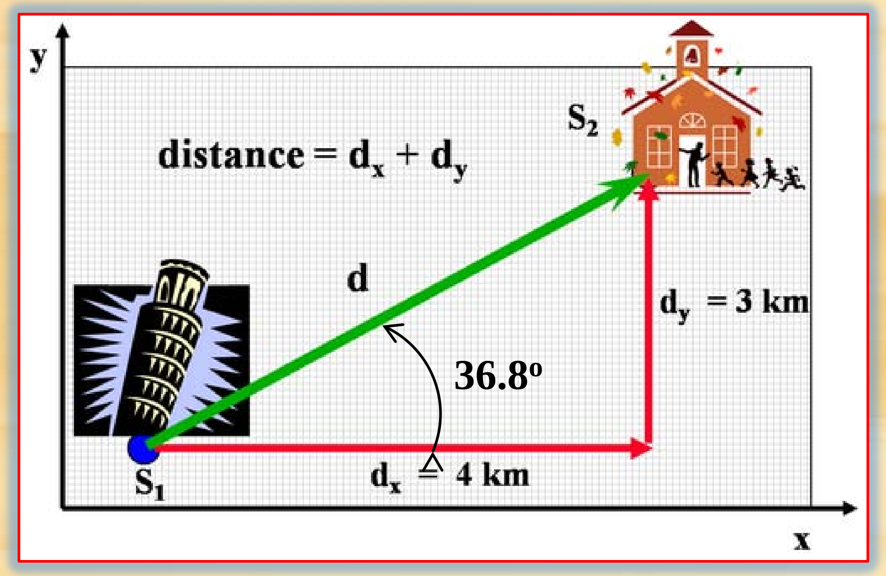
Distance: refers to how far an object travels. This is a *scalar quantity*. For example: the car traveled (*35 km*).

❖ We are only concerned with the length of travel we don't distinguish between directions.

Distance = d_x + d_y = 4 + 3 = 7 km

Displacement: refers to how far the object travels, but also adds direction. This is a *vector quantity*. For example: the car traveled (*35km to the east*).

$$\bar{d} = \sqrt{3^2 + 4^2} = 5\ km\ @\ 36.8^o$$



Speed:

- Speed is a measure of *how fast* something moves.
- Speed is a *scalar* quantity, specified only by its magnitude.
- Speed is defined as the ***distance*** covered per ***unit time***:

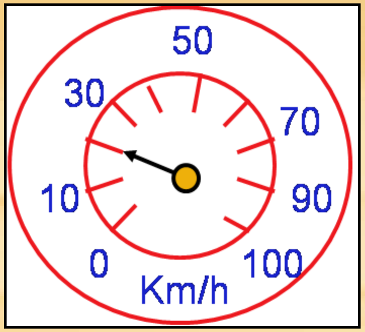
speed = distance / time = x / t (m/s, km/h, foot/min, … etc)

Average Speed:

- Average speed is the *whole distance* covered divided by *the total time* of travel. General definition:

Average speed = total distance covered / time interval

- Distinguish between *instantaneous* speed and *average speed*:
  - On most trips, we experience a variety of speeds, so the average speed and instantaneous speed are often quite different.



Instantaneous Speed:

- The speed at any instant is the ***instantaneous speed***.
- The speed registered by an automobile speedometer is the ***instantaneous speed***.

Example 1: If we travel 320 km in 4 hours, what is our average speed? If we drive at this average speed for 5 hours, how far will we go?

Answer: v_{avg} = 320 km/4 h = 80 km/h.

$d = v_{avg}$ x time = 80 km/h x 5 h = 400 km.

Example 2: A plane flies 600 km away from its base at 200 km/h, then flies back to its base at 300 km/h. What is its average speed?

Answer: total distance traveled, d = 2 x 600 km = 1200 km;

total time spent (for the round trip):

t = (600 km / 200 km/h) + (600 km / 300 km/h) = 3 h + 2 h = 5 h.

Average speed, $v_{avg} = d / t$ = 1200 km/5 h = 240 km/h.

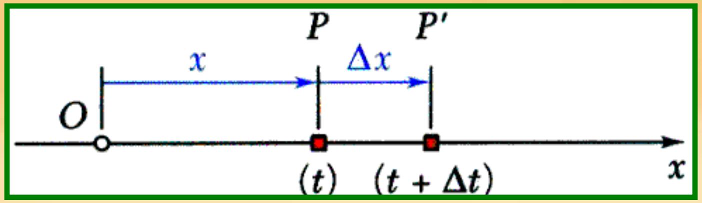
Velocity:

- Velocity is speed in a given direction; when we describe *speed and direction* of motion, we are describing ***velocity***.

Velocity = **speed** and **direction**; velocity is a *vector*.

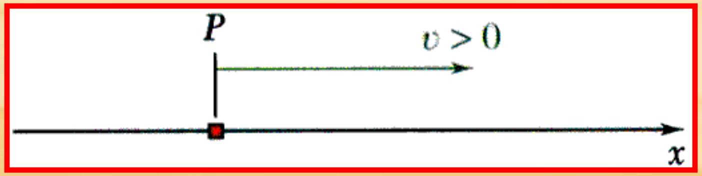
Constant velocity = *constant speed* and ***no change in direction***

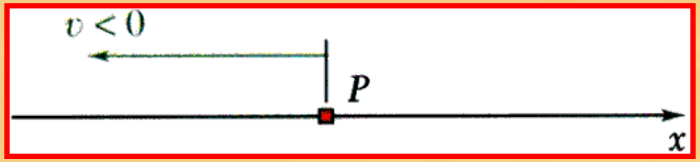
- Consider particle which occupies position (***P***) at time (***t***) and (***P'***) at (***t + Δt***).



$$\textit{Average velocity} = \frac{\Delta x}{\Delta t}$$

$$\textit{Instantaneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$





- *Instantaneous velocity* may be positive or negative. *Magnitude of velocity* is referred to as ***particle speed***.

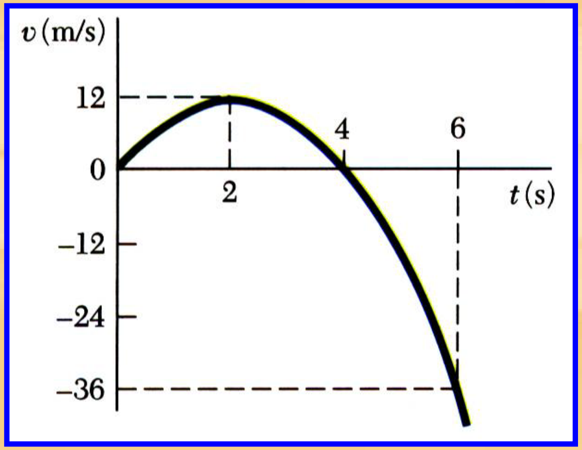
- From the definition of a derivative,

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

e.g.,

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

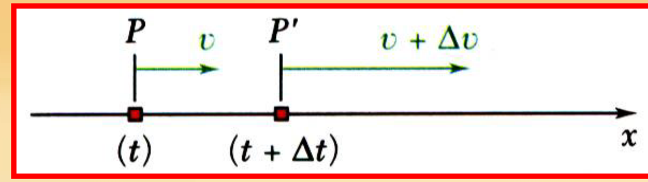


Acceleration:

Acceleration is the rate of change in the velocity of a particle. It is a vector quantity. Typical units are m/s^2 or ft/s^2.

- The *instantaneous acceleration* is the time derivative of velocity.
- Consider particle with velocity (v) at time (t) and (v') at ($t + \Delta t$):

Instantaneous acceleration $= a = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t}$



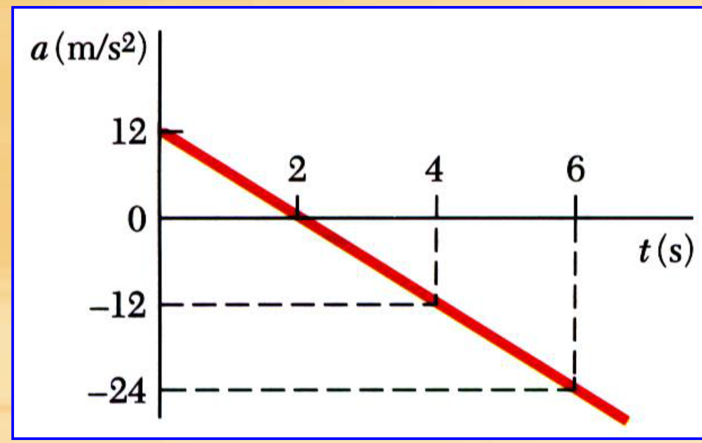
- Acceleration can be positive (**speed increasing**) or negative (**speed decreasing**).

- From the definition of a derivative:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

e.g. $v = 12t - 3t^2$

$$a = \frac{dv}{dt} = 12 - 6t$$ *Variable acceleration*



- The derivative equations for velocity and acceleration can be manipulated to get:

$\boxed{v = \dfrac{dx}{dt}}$ $\boxed{a = \dfrac{dv}{dt}}$ $\boxed{a = \dfrac{d^2 x}{dt^2}}$ $\boxed{a = \dfrac{dv}{dt} = \dfrac{dv}{dx}\dfrac{dx}{dt} = v\dfrac{dv}{dx}}$ $\Rightarrow$ $\boxed{a.\, dx = v\, dv}$

Constant Acceleration:

The three kinematic equations can be integrated for the special case when acceleration is constant (**$a = a_c$**) to obtain ***very useful equations***. A common example of ***constant acceleration is gravity***; i.e., a *body freely falling toward earth*. In this case, ($a_c = g = 9.81\ m/s^2 = 32.2\ ft/s^2$) downward. These equations are:

$$\frac{dv}{dt} = a = \text{constant} \qquad \int_{v_0}^{v} dv = a\int_{0}^{t} dt \qquad v - v_0 = at$$

$$v = v_0 + at$$

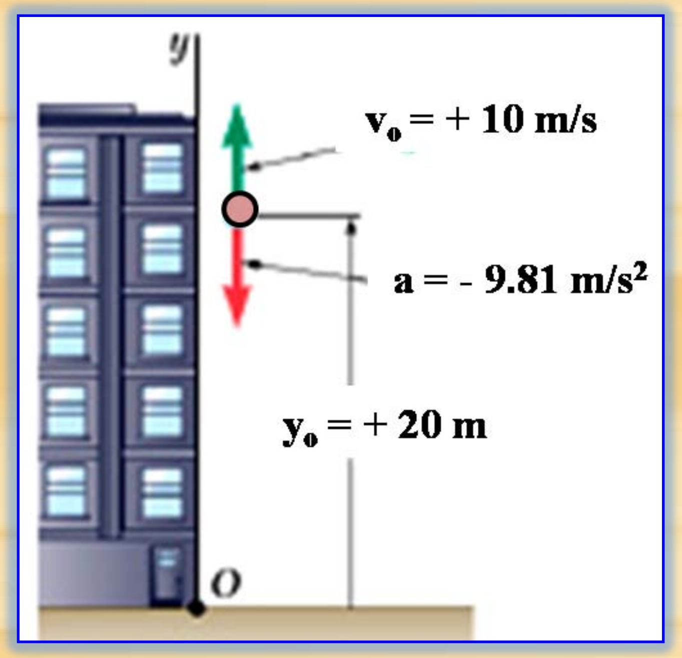
$$\frac{dx}{dt} = v_0 + at \qquad \int_{x_0}^{x} dx = \int_{0}^{t} (v_0 + at)\,dt \qquad x - x_0 = v_0 t + \tfrac{1}{2} a t^2$$

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v\frac{dv}{dx} = a = \text{constant} \qquad \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \qquad \tfrac{1}{2}\left(v^2 - v_0^2\right) = a\left(x - x_0\right)$$

$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$

Example 1:



Ball tossed with (10 m/s) vertical velocity from window (20 m) above ground.

Determine:

- velocity and elevation above ground at time *t*,
- highest elevation reached by ball and corresponding time, and
- time when ball will hit the ground and corresponding velocity.

Solution:

- The ball experiences a constant downward acceleration.

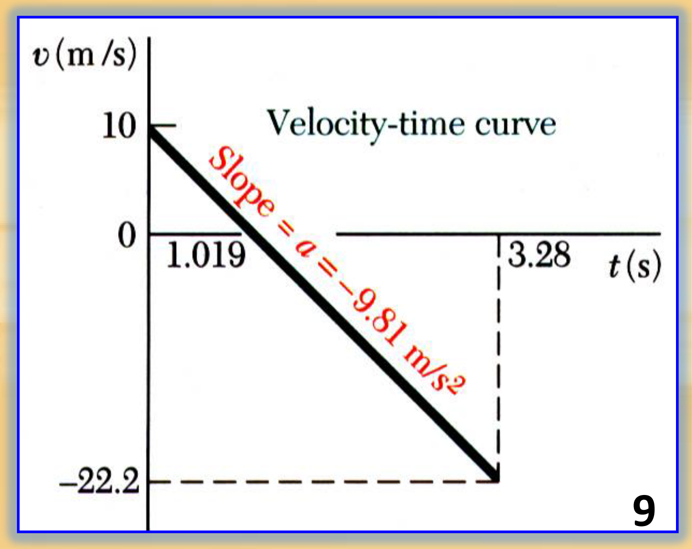
$\boxed{v = v_o + a_c \,.\, t}$ ⇒ $v = 10 + (-9.81)\,.\,t$ ⇒ $v = 10 - 9.81\,t$

$v = (dy/dt) = 10 - 9.81\,t$ ⇒ $dy = (10 - 9.81\,t)\,.\,dt$

$$\int_{y_o}^{y} dy = \int_{0}^{t} (10 - 9.81\,t)\,.\,dt$$

$$y = y_o + 10\,t - 9.81\,t^2/2$$

$$y = 20 + 10\,t - 4.905\,t^2$$



- Solve for (t) at which *velocity equals zero* and evaluate corresponding altitude.

$v = 10 - 9.81\,t = 0$ ⇒ $t = 1.019$ sec.

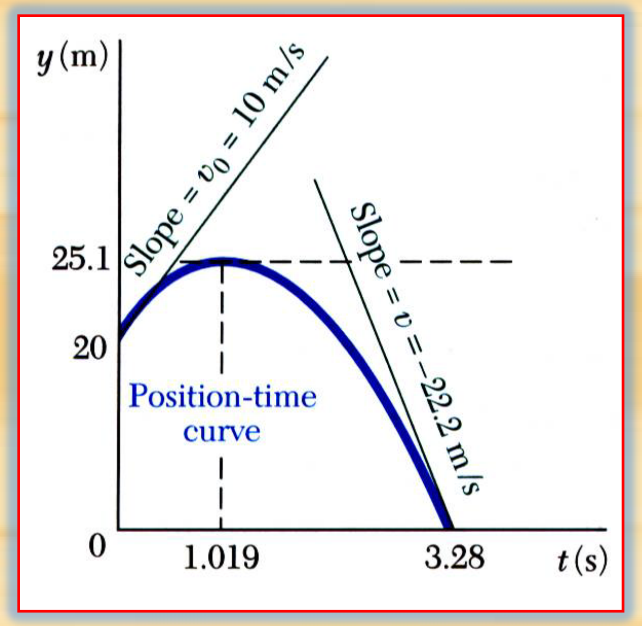
Example 1: Continue

- At time ($t = 1.019$ *sec.*) the corresponding altitude (y), where the velocity equal to zero.

$$y = 20 + 10\,t - 4.905\,t^2$$

$$y = 20 + 10\,.\,(1.019) - 4.905\,.\,(1.019)^2$$

$$y = 25.1\ m$$



- Solve for (t) at which *altitude equals zero* and evaluate corresponding velocity (**v**).

$$y = 20 + 10\,t - 4.905\,t^2 = 0$$

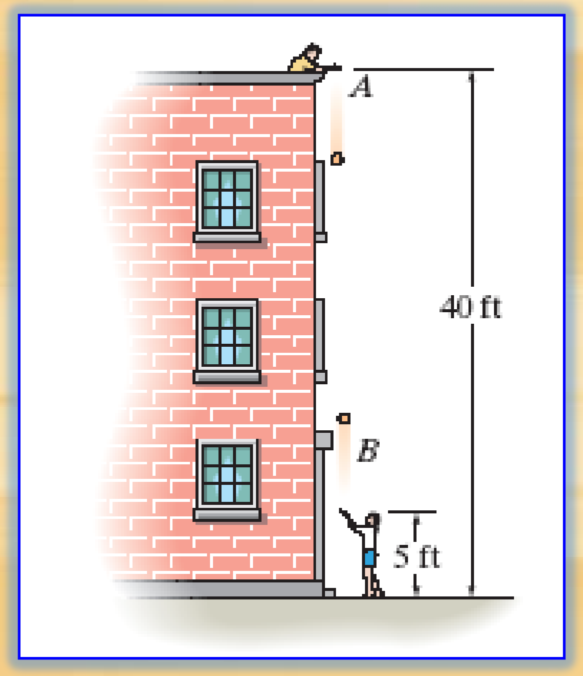
$$\boxed{\begin{array}{c} t = -1.243 \text{ sec. (meaningless)} \\ t = 3.28 \text{ sec.} \end{array}}$$

$$v = 10 - 9.81\,t = 10 - 9.81 * 3.28 = -22.2 \text{ m/s}$$

Example 2:

Ball (**A**) is released from rest at a height of (**40 ft**) at the same time a ball (**B**) is thrown upward, (**5 ft**) from the ground. The balls pass one another at a height of (**20 ft**).

Find: The speed at which ball (**B**) was thrown upward.



Solution: Both balls experience a constant downward acceleration of ($a_c = 32.2$ ft/s^2).

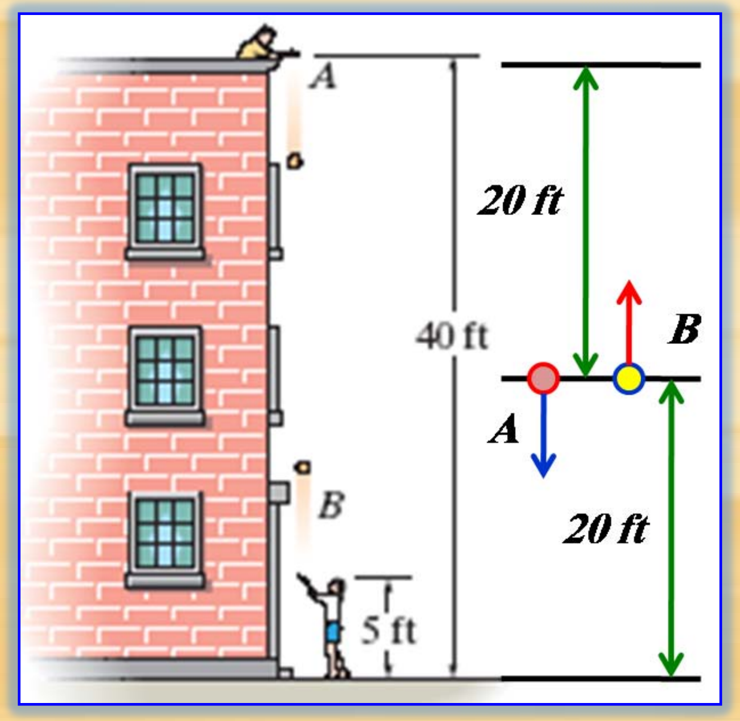
Example 2: Continue

1) *First consider ball A.* With the origin defined at the ground, ball (A) is released from rest [$(v_A)_o = 0$] at a height of (40 ft), [$(s_A)_o = 40$ ft]. Calculate *the time* required for ball (A) to drop to (20 ft), [$s_A = 20$ ft] using a position equation.

$$s_A = (s_A)_o + (v_a)_o \cdot t + (1/2)\, a_c \cdot t^2$$

$$20 \text{ ft} = 40 \text{ ft} + (0) \cdot (t) + (1/2) \cdot (-32.2) \cdot (t^2)$$

$$t = 1.115 \text{ s}$$



2) **Now consider ball B.** It is throw upward from a height of (5 ft) [$(s_B)_o = 5$ ft]. It must reach a height of (20 ft) [$s_B = 20$ ft] at the same time ball (A) reaches this height ($t = 1.115$ s). Apply the position equation again to ball (B) using ($t = 1.115$ sec).

$$s_B = (s_B)_o + (v_B)_o \cdot t + (1/2) \cdot a_c \cdot t^2$$

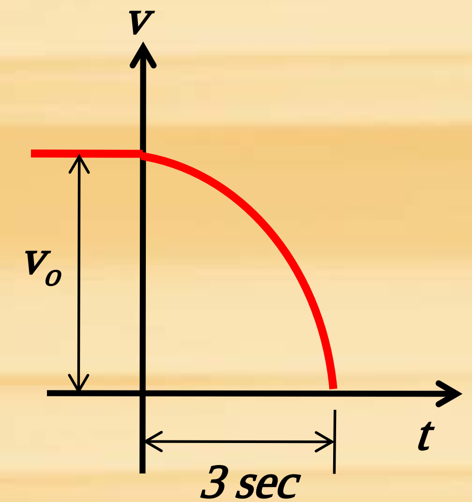
$$20 \text{ ft} = 5 + (v_B)_o \cdot (1.115) + (1/2) \cdot (-32.2) \cdot (1.115)^2$$

$$(v_B)_o = 31.4 \text{ ft/s}$$

Example 3: Variable acceleration

A motorcyclist travels along a straight road at a speed of (**27 m/s**). When the brakes are applied, the motorcycle decelerates at a rate of **(- 6 t m/s²**).

Find: The distance the motorcycle travels before it stops.



Solution:

Establish the positive coordinate (S) in the direction the motorcycle is traveling. Since the acceleration is given as a function of time, integrate it once to calculate the velocity and again to calculate the position.

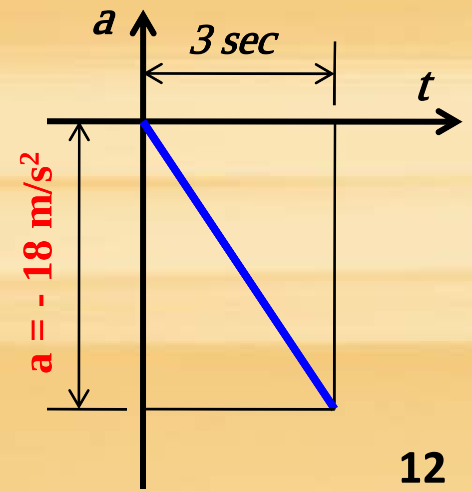
1) Integrate acceleration to determine the velocity:

a = dv / dt => dv = a dt

$$\Rightarrow \int_{v_o}^{v} dv = \int_{o}^{t} (-6\,t)\,.\,dt \quad \Rightarrow \quad v - v_o = -3\,.\,t^2 \quad \Rightarrow \quad v = -3\,.\,t^2 + v_o$$

2) We can now determine the amount of time required for the motorcycle to stop (**v = 0**). Use [v_o **= 27 m / s**].

$$0 = -3\,.\,t^2 + 27 \quad \Rightarrow \quad t = 3 \text{ sec.}$$



3) Now calculate the distance traveled in (**3 sec**) by integrating the velocity using [$S_o = 0$]:

$$v = dS / dt \quad \Rightarrow \quad dS = v\,dt \quad \Rightarrow \quad \int_{s_o}^{s} ds = \int_{o}^{t} (-3\,.\,t^2 + v_o)\,dt$$

$$\Rightarrow \quad S - S_o = -t^3 + v_o t \quad \Rightarrow \quad S - 0 = (3)^3 + (27)\,.\,(3) \quad \Rightarrow \quad S = 54 \text{ m}$$

Example 4: Variable acceleration

Given that the velocity of a particle changes with time such that ($v = 16 - t^2$), find:

A. Draw a velocity-time graph for the first (4 seconds) of motion.
B. Obtain the distance covered in the first (4 seconds).
C. The acceleration when (i) t = 1 seconds, (ii) t = 3 seconds.

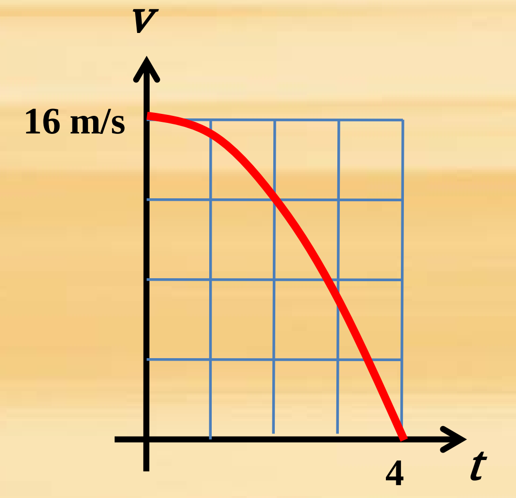
Solution:

B) Distance travelled in the first 4 seconds, (Integration to find the distance).

$$v = dS/dt \Rightarrow dS = v \cdot dt$$

Distance covered (S) $= \int_0^4 (16 - t^2)\,dt = \left[16t - t^3/3\right]_0^4$

$$S = 42.67 \text{ m}$$



B) The accelaration can be find by differentiating the velocity.

$$a = dv/dt = \frac{d}{dt}(16 - t^2) = -2t$$

i.) $a = -2t$

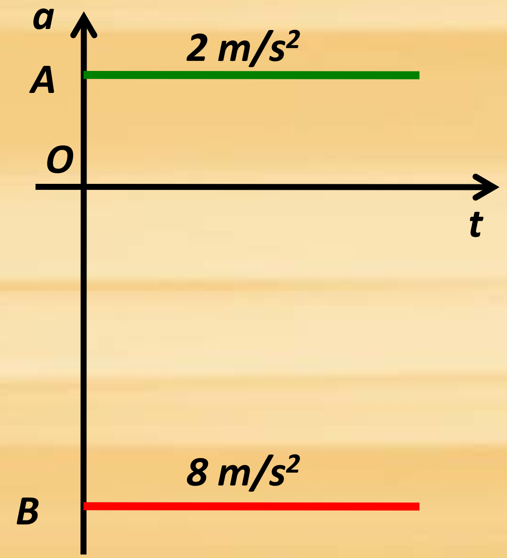
At $t = 1$ **sec.** $\Rightarrow$ $a = -2 \text{ m/s}^2$

ii.) $a = -2t$

At $t = 3$ **sec.** $\Rightarrow$ $a = -6 \text{ m/s}^2$

Example 5: Constant acceleration

Two cars (**A** & **B**) start simultaneously from point (**O**) and move in a straight line. One with a velocity of (**66 m/s**), and an acceleration of (**2 m/s²**), the other car with a velocity of (132 m/s) and with a retardation of (**8 m/s²**). Find the ***time*** which the velocities of the cars are **same** and the **distance** from point (**O**) where they **meet** again.



Solution:

For Car (A):

$a = dv/dt \Rightarrow dv = a\,.\,dt \Rightarrow dv = 2\,dt$

$\boxed{v = v_o + a_c\,.\,t} \Rightarrow v_A = 66 + 2\,.\,t$

For Car (B):

$\boxed{v = v_o + a_c\,.\,t} \Rightarrow v_B = 132 - 8\,.\,t$

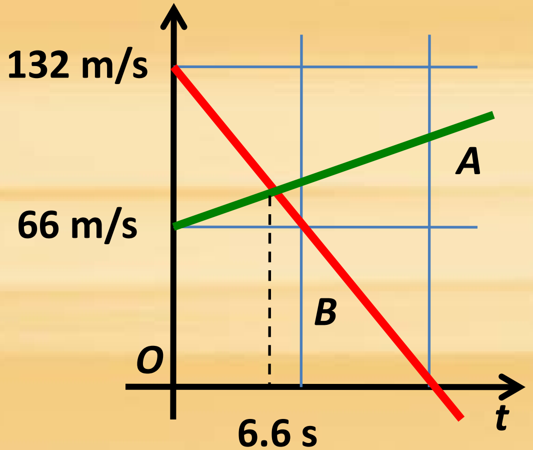
❖ When car (**A**) and car (**B**) have the same velocity:

$$v_A = v_B$$

$$66 + 2\,.\,t = 132 - 8\,.\,t$$

$$66 = 10\,.\,t$$

$$t = 6.6 \text{ sec}$$



❖ So, after (**6.6 sec.**), the velocities of car (**A**) and car (**B**) have the same velocity.

Example 5: Continue

➢ To find at what distance these two cars will meet and when, let find first the equations of their distances:

For Car (A):

$$v_A = 66 + 2 \cdot t = dS_A / dt \Rightarrow dS_A = (66 + 2 \cdot t)\, dt$$

$$\boxed{s = s_o + v_o t + (1/2) \cdot a_c \cdot t^2} \Rightarrow S_A = 66\,t + (1/2) \cdot 2 \cdot t^2 = 66\,t + t^2$$

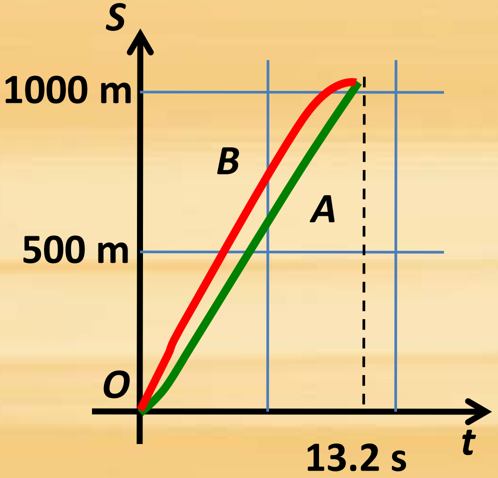
For Car (B):

$$v_B = 132 - 8 \cdot t = dS_B / dt \Rightarrow dS_B = (132 - 8 \cdot t)\, dt$$

$$\boxed{s = s_o + v_o t + (1/2) \cdot a_c \cdot t^2} \Rightarrow S_B = 132\,t - (1/2) \cdot 8 \cdot t^2 = 132\,t - 4\,t^2$$

Where: $S_{Ao} = S_{Bo} = 0$

➢ To find when these two cars will meet, it means they will move the same distances:



$$S_A = S_B$$

$$66\,t + t^2 = 132\,t - 4\,t^2$$

$$66\,t = 5\,t^2$$

$$t = 13.2 \text{ sec}$$

$$S_B = S_A = 66\,t + t^2 = 66 * 13.2 + (13.2)^2 = 1045.44 \text{ m}$$

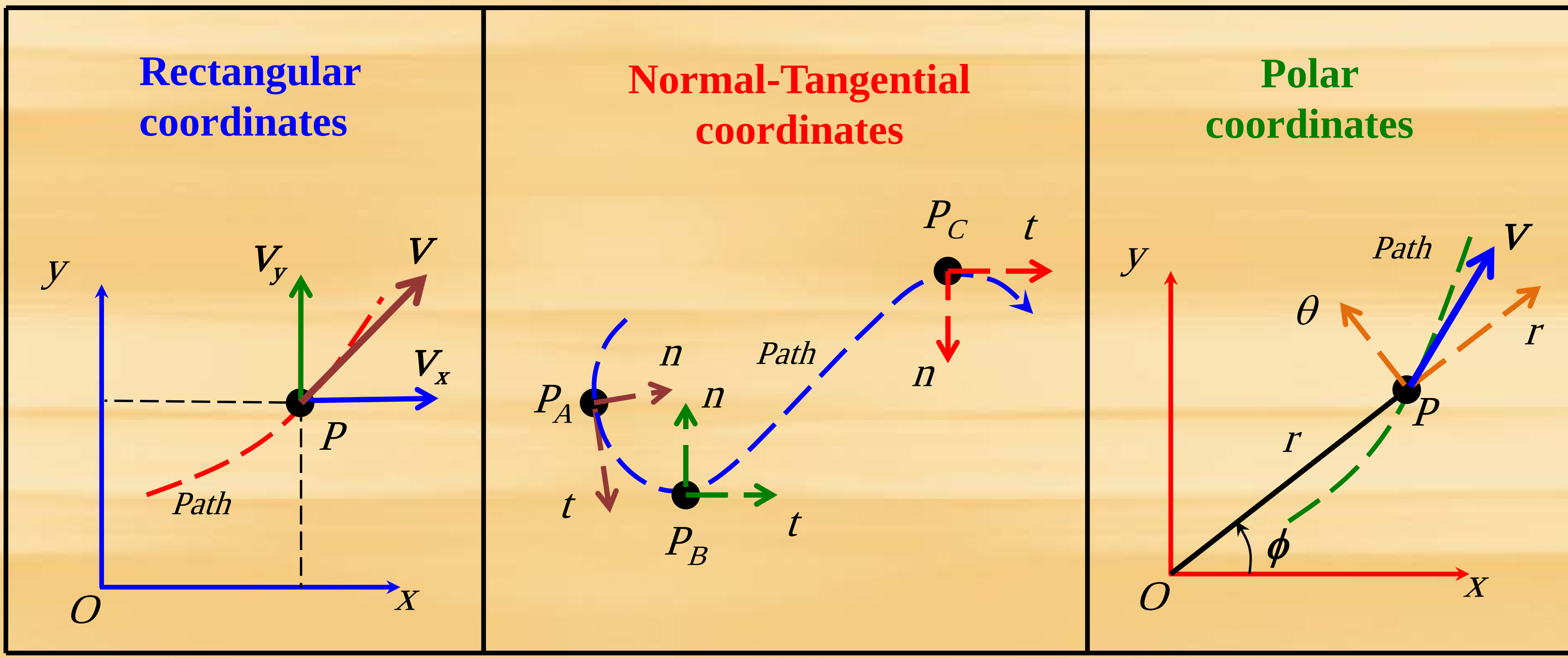
Curvilinear Motion:

➢ When a ***Particle*** moving along **a curve** other than ***a straight line***, we say that the ***particle*** is in ***curvilinear motion***.

Coordinates Used for Describing the Plane Curvilinear Motion:

The motion of a particle can be described by using three coordinates, these are:

1. Rectangular coordinates.
2. Normal – Tangential coordinates.
3. Polar coordinates.



Plane Curvilinear Motion – without Specifying any Coordinates:

Position:

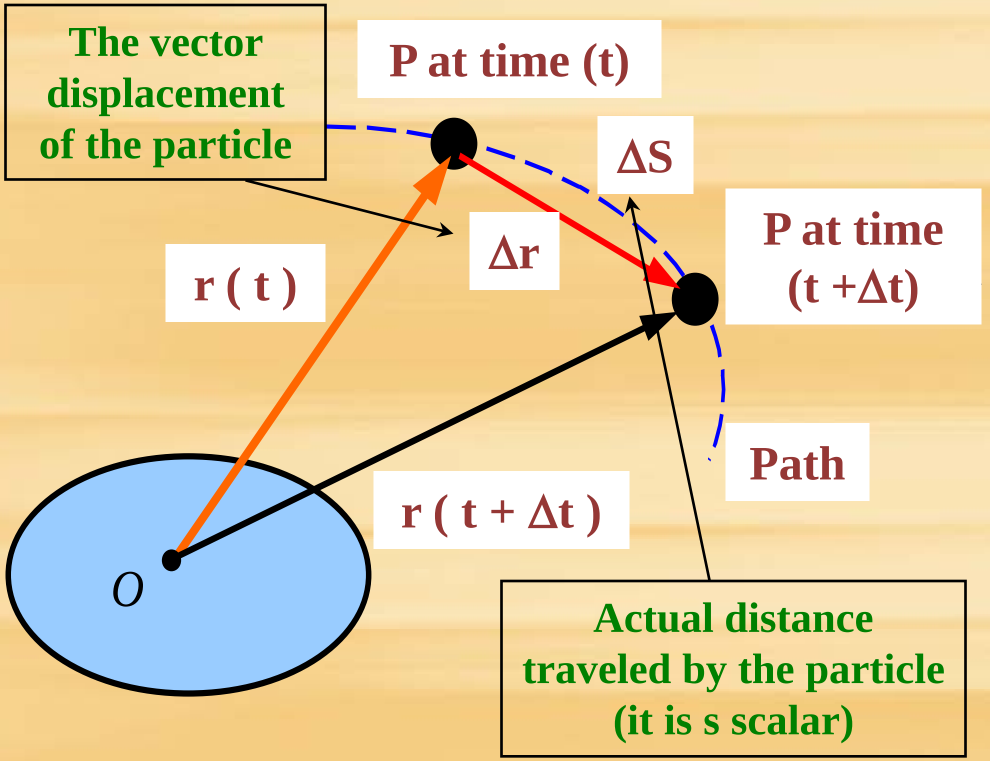
➢ The position of the particle measured from a fixed point (O) will be designated by the position vector [$r = r (t)$].

➢ This vector is a function of time (t), since in general both its magnitude and direction change as the particle moves along the curve path.

(Displacement)

❖ Suppose that particle which occupies position (P) defined by (r) at time (t) moving along a curve a distance (ΔS) to new position (P') defined by ($r' = r + \Delta r$) at a time ($t + \Delta t$).

❖ ($\Delta r = r' - r$ = *displacement*).



Note: Since, here, the particle motion is described by two coordinates components, both the ***magnitude and the direction*** of the position, the velocity, and the acceleration have to be specified.

Curvilinear Motion:

Average Velocity:

➢ The average velocity (V_{av}) is a ***vector***, it has the direction of (Δr) and its magnitude equal to the magnitude of (Δr) divided by (Δt).

$$\bar{V}_{av} = \frac{\Delta r}{\Delta t}$$

Average Speed:

➢ The average speed of the particle is the ***scalar*** ($\Delta S/\Delta t$). The magnitude of the speed and (V_{av}) approach one another as (Δt) approaches zero.

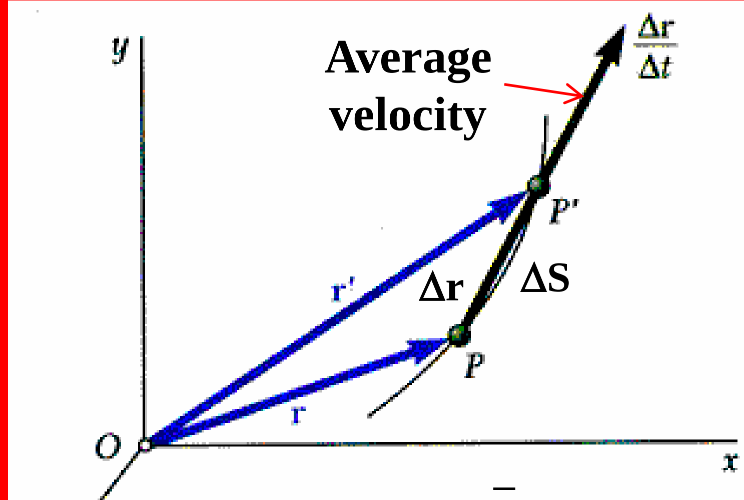
$$V_{av} = \frac{\Delta S}{\Delta t}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$ As (Δt) approach zero

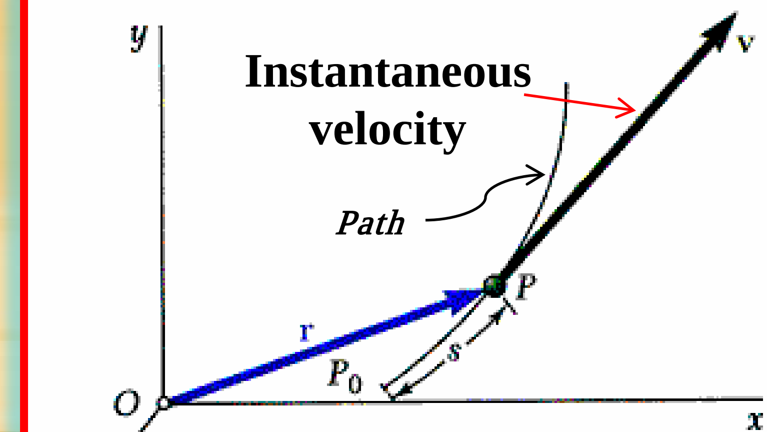
$$= \text{instantaneous velocity (vector)}$$

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$ As (Δt) approach zero

$$= \text{instantaneous speed (scalar)}$$



the magnitude of ($\bar{V}$) is called the speed, i.e. $V = |V| = dS/dt$



the velocity vector $\bar{V}$ is always tangent to the path.

Curvilinear Motion:

Average Acceleration ($\bar{a}_{av}$):

$$\bar{a}_{av} = \frac{\Delta V}{\Delta t}$$

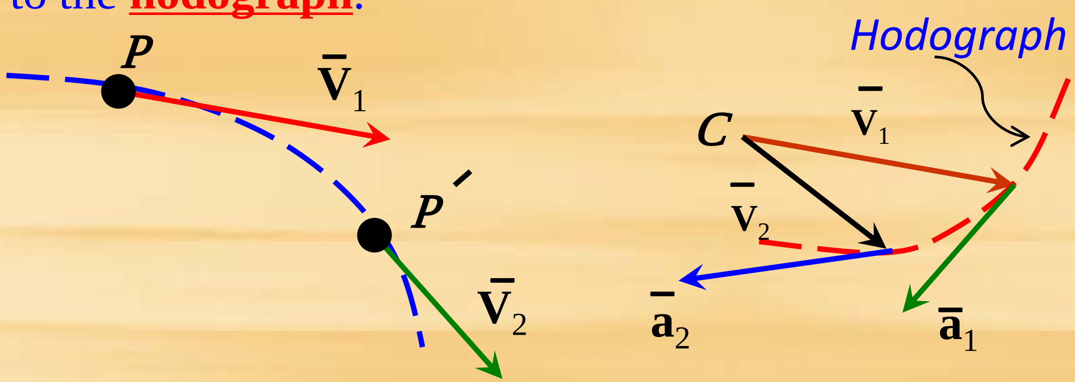
Note: $\bar{a}_{av}$ has the direction of ($\Delta \mathbf{V}$) and its magnitude is the magnitude of ($\Delta \mathbf{V}$) divided by (Δt).

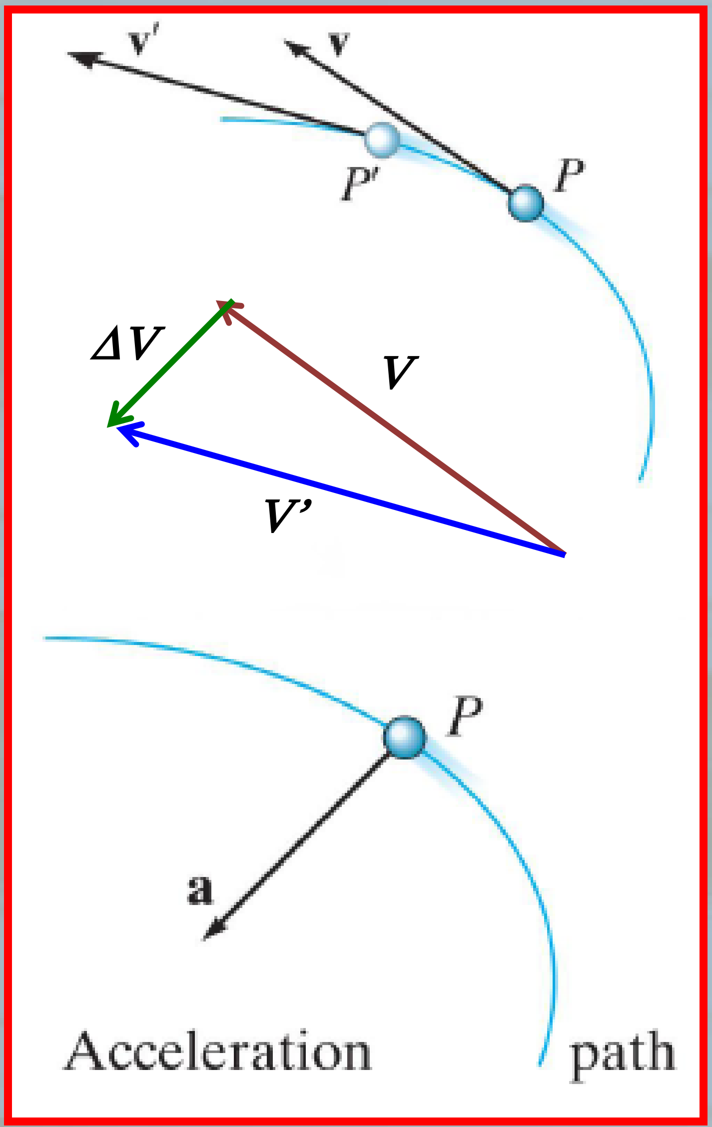
Instantaneous Acceleration ($\bar{a}$):

As (Δt) approaches zero:

$$\bar{a} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt} = \dot{V}$$

Note: In general, the acceleration vector ($\bar{a}$) is neither tangent nor normal to the path. However, ($\bar{a}$) is tangent to the **hodograph**.

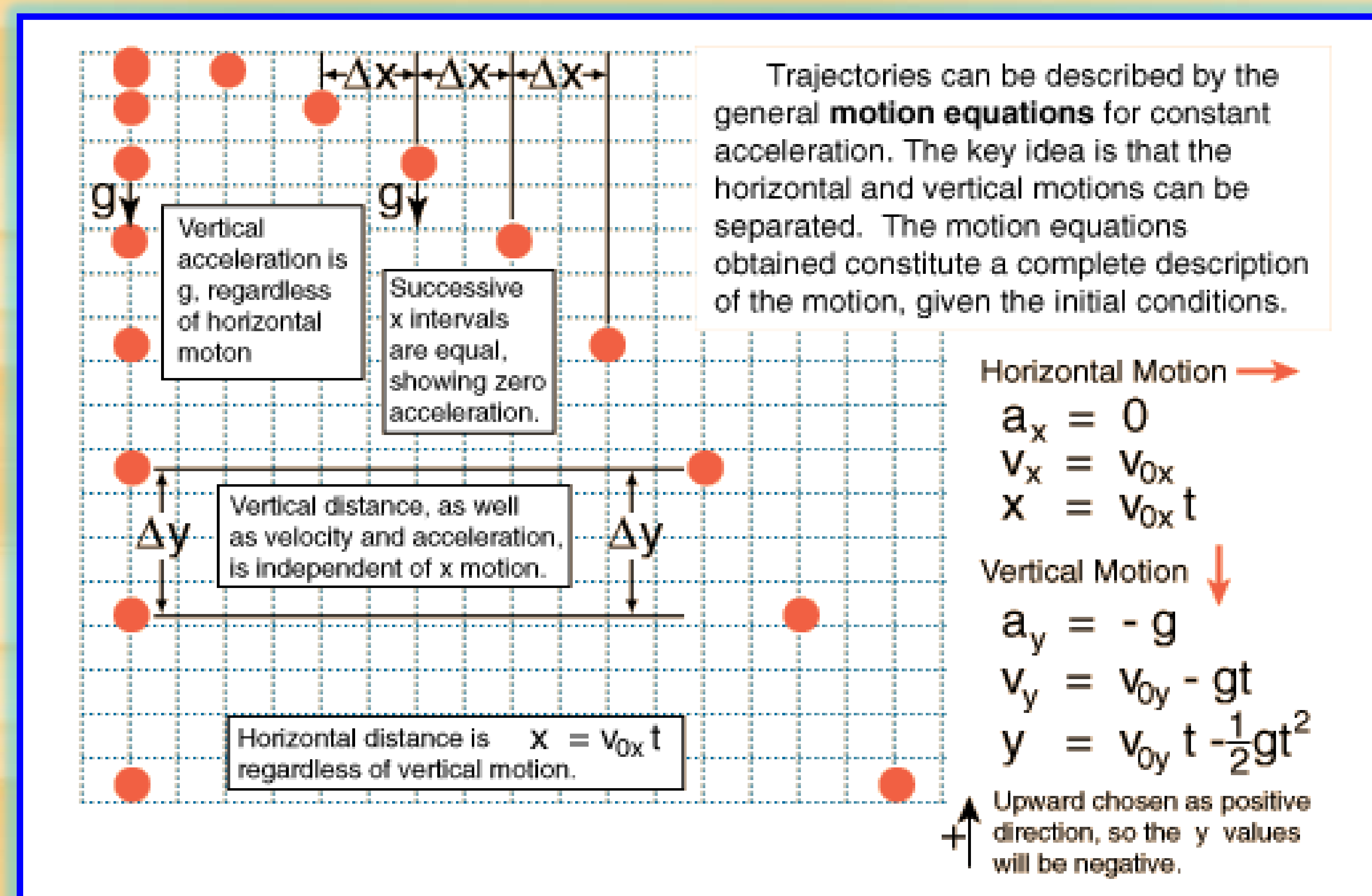




A plot of the locus of points defined by the arrowhead of the velocity vector is called a **hodograph**. The acceleration vector is tangent to the hodograph, but not, in general, tangent to the path function.

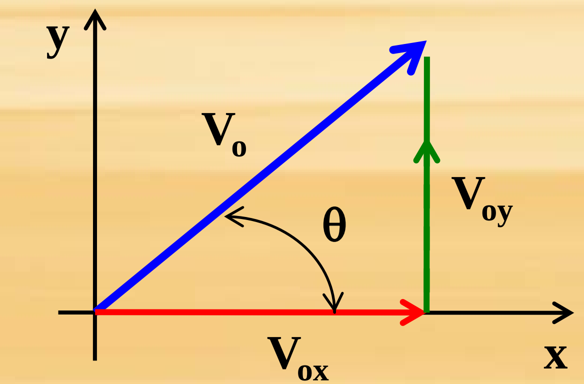
Motion of a Projectile:

Projectile motion can be treated as **two rectilinear motions**, **one in the horizontal** direction (**constant velocity**) experiencing ***zero acceleration*** and the **other in the vertical direction** experiencing ***constant acceleration*** (i.e., *gravity*).

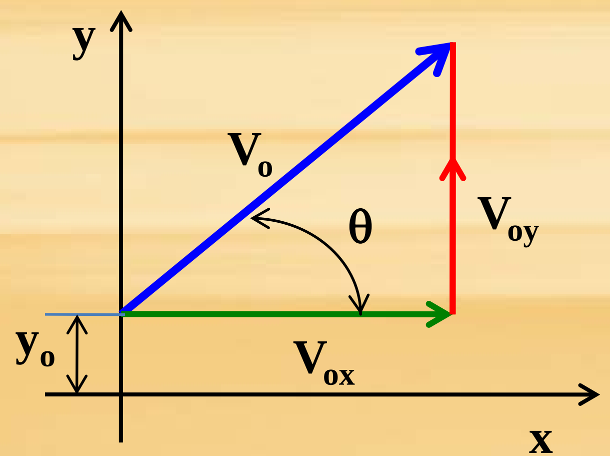


Motion of a Projectile: Path of a Projectile:

- ➢ The velocity vector ($\mathbf{v_o}$) changes with ***time*** in both *magnitude and direction*. This change is the result of *acceleration in the negative* ($\mathbf{y}$) direction (*due to gravity*).
- ➢ The horizontal component (**x component**) of the velocity ($\mathbf{v_o}$) remains constant ($\mathbf{v_{oX}}$) over time because there is ***no acceleration*** along the horizontal direction
- ➢ The ***vertical component*** ($\mathbf{v_y}$) of the velocity ($\mathbf{v_o}$) is ***zero*** at the peak of the trajectory. However, there is a horizontal component of velocity, ($\mathbf{v_x}$), at the peak of the trajectory.



- ❖ v_o = initial velocity or resultant velocity
- ❖ v_x = horizontal velocity
- ❖ v_{yi} = initial vertical velocity
- ❖ v_{yf} = final vertical velocity
- ❖ X_{max} (or R) = maximum horizontal distance (range)
- ❖ x = horizontal distance
- ❖ Δy = change in vertical position
- ❖ y_i = initial vertical position
- ❖ y_f = final vertical position
- ❖ θ = angle of projection (launch angle)
- ❖ H = maximum height
- ❖ g = gravity = 9.8 m/s^2



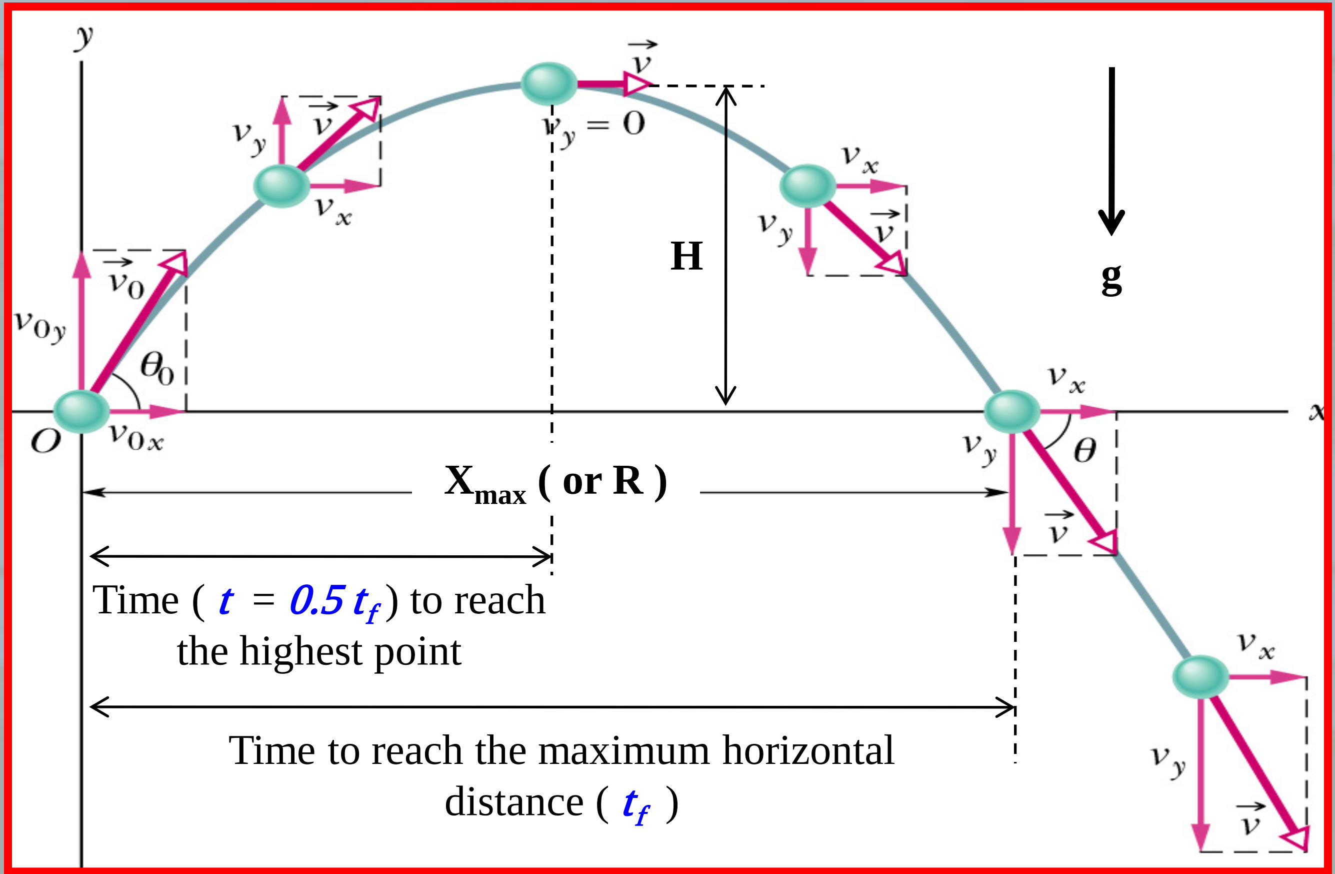
Motion of a Projectile: HORIZONTAL MOTION :

- Since: $a_x = 0$
- The velocity in the horizontal direction remains constant .

$$v_x = v_{ox} = v_o \,.\, \cos\theta$$

- The position in the x direction can be determined by:

$$x = x_o + V_{ox} \,.\, t \Rightarrow x = V_{ox} \,.\, t \;\ldots\; (\text{if } x_o = 0)$$



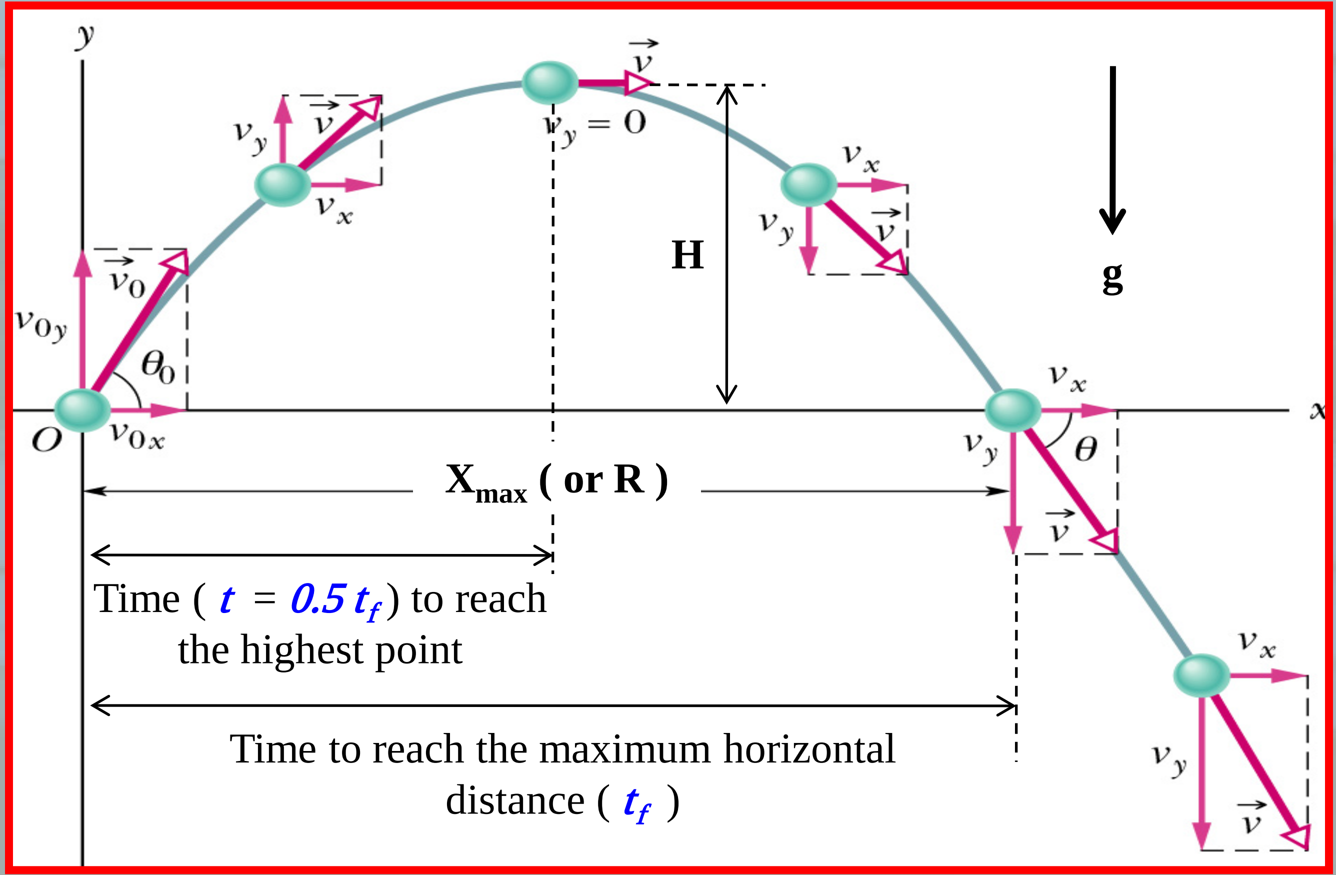
Motion of a Projectile: VERTICAL MOTION :

➢ Since the *positive y-axis* is directed *upward*, ($a_y = -g$). Application of the constant acceleration equations yields:

$$v_y = v_{oy} - g \,.\, (t)$$

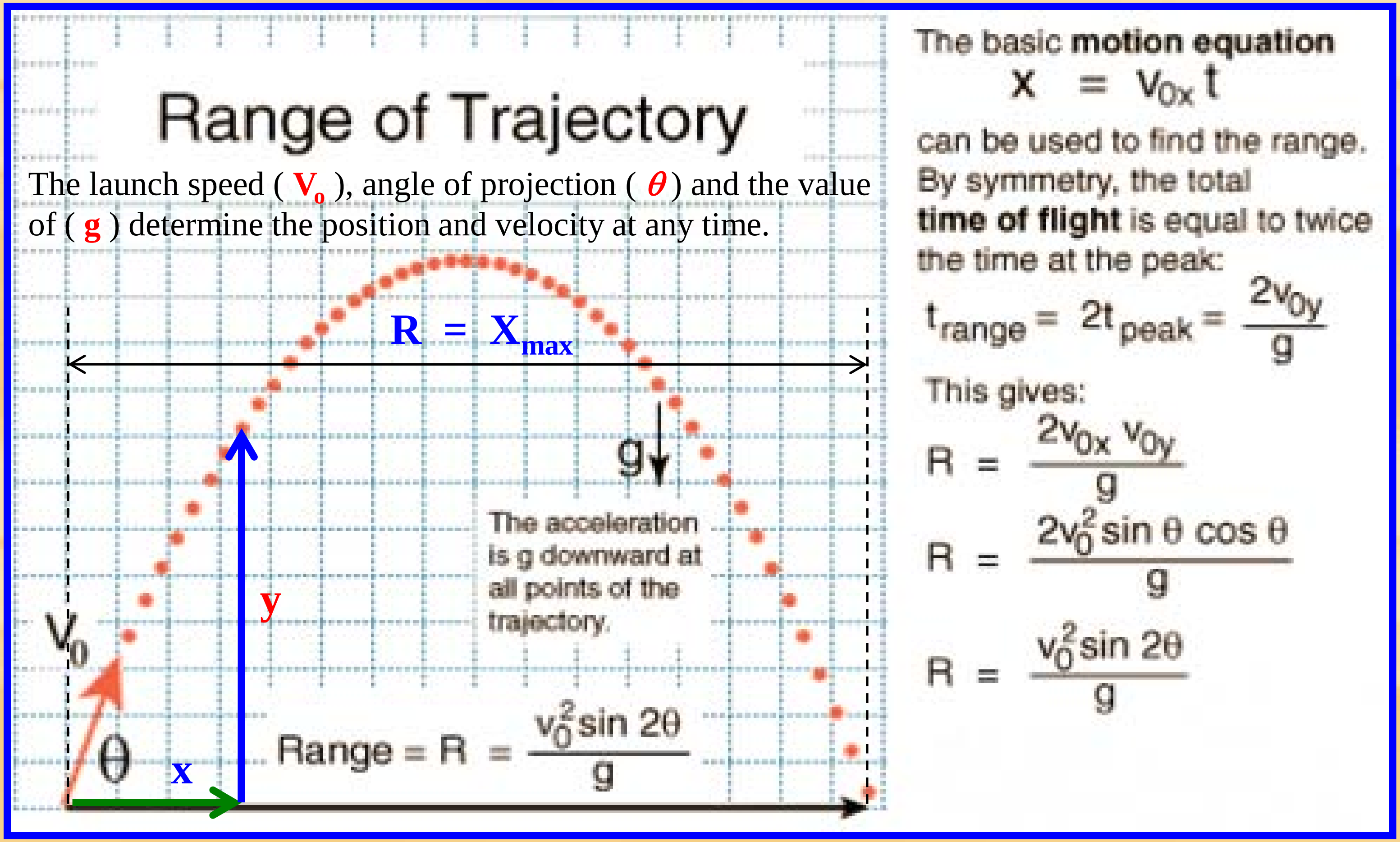
$$y = y_o + (v_{oy}) \,.\, (t) - \tfrac{1}{2}g(t)^2$$

$$v_y^{\,2} = v_{oy}^{\,2} - 2g \,.\, (y - y_o)$$



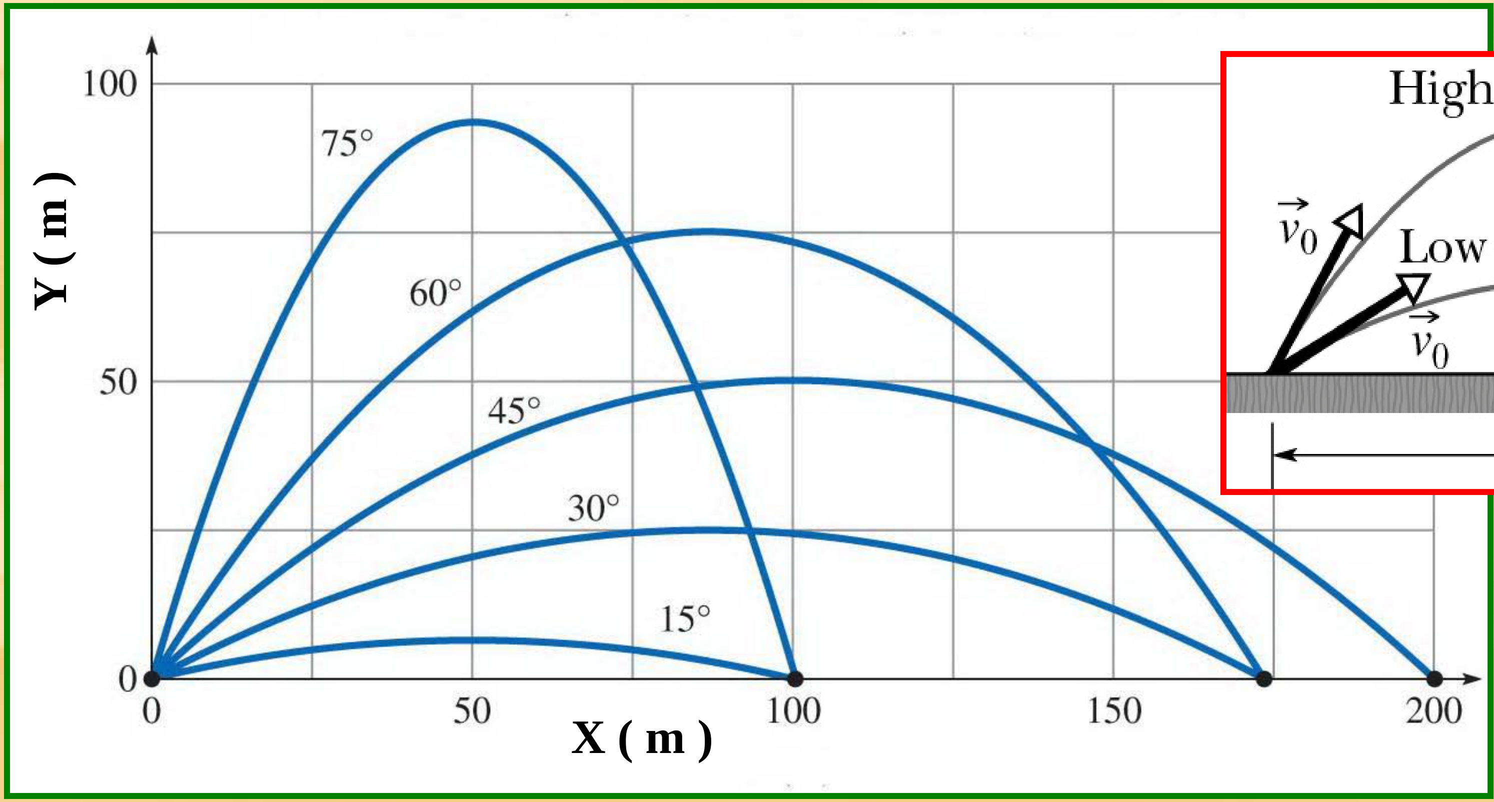
Motion of a Projectile: GENERAL NOTES:

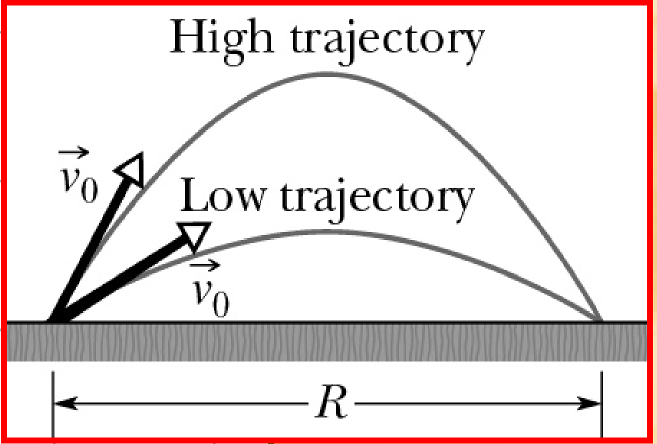
- The motion of a projectile is determined only by the object's initial velocity and gravity.
- The vertical motion of a projected object is independent of its horizontal motion.
- The vertical motion of a projectile is nothing more than free fall.
- The one common variable between the horizontal and vertical motions is ***time***.



Range and Angle of Projection:

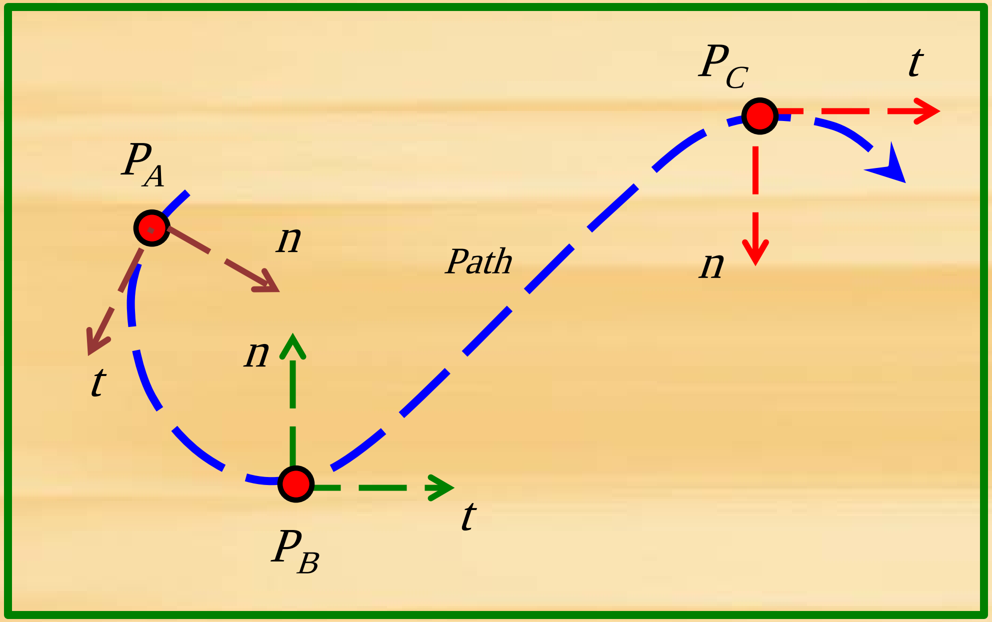
- The range is a maximum at 45° because sin (2·45) = 1.
- For any angle θ other than 45°, a point having coordinates (x,0) can be reached by using either one of two complimentary angles for θ, such as 15 ° and 75 ° or 30 ° and 60 °.
- The maximum height and time of flight differ for the two trajectories having the same coordinates (x, 0).
- A launch angle of 90° (straight up) will result in the maximum height any projectile can reach.





Normal & Tangential Components:

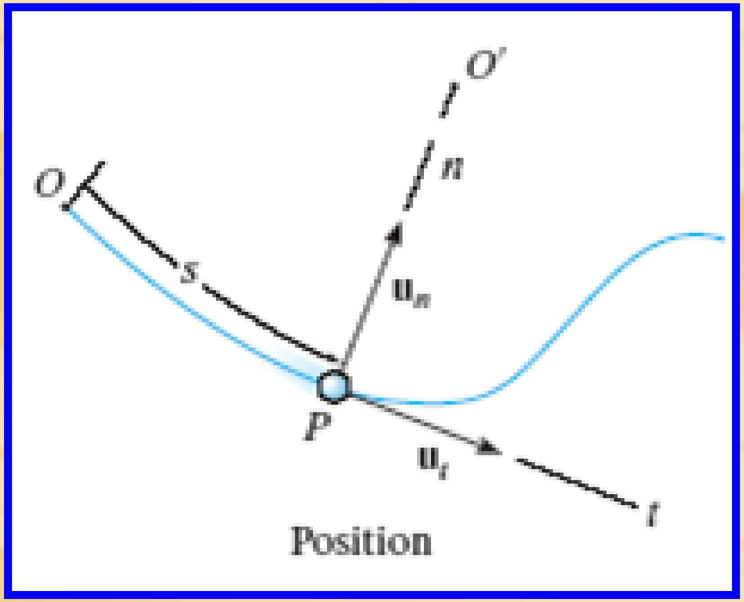
- As mentioned previously that the velocity of a particle is a vector tangent to the path of the particle's movement.
- The acceleration is not tangent not normal to the path.
- It is convenient to resolve the acceleration into components directed respectively along the tangent and normal to the path of the particle's movement.

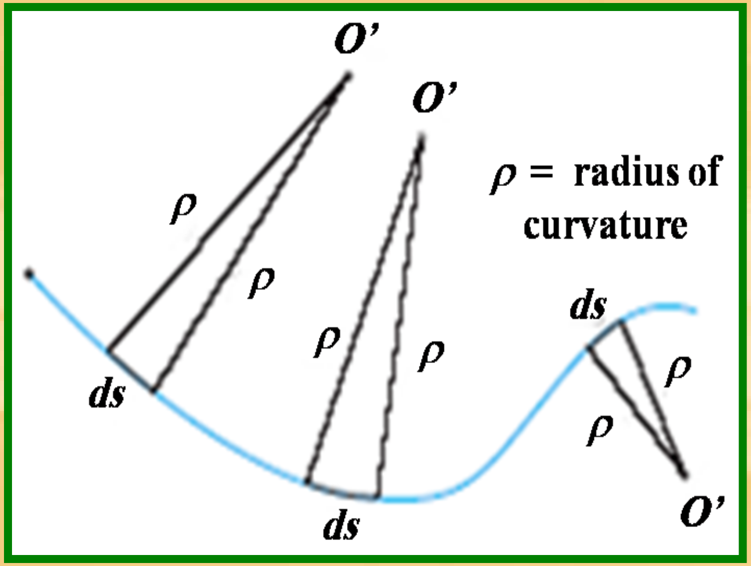


- Cars traveling along a clover-leaf interchange experience an acceleration due to a change in speed as well as due to a change in direction of the velocity.
- If the car's speed is increasing at a known rate as it travels along a curve, how can we determine the magnitude and direction of its total acceleration?

Normal & Tangential Components:

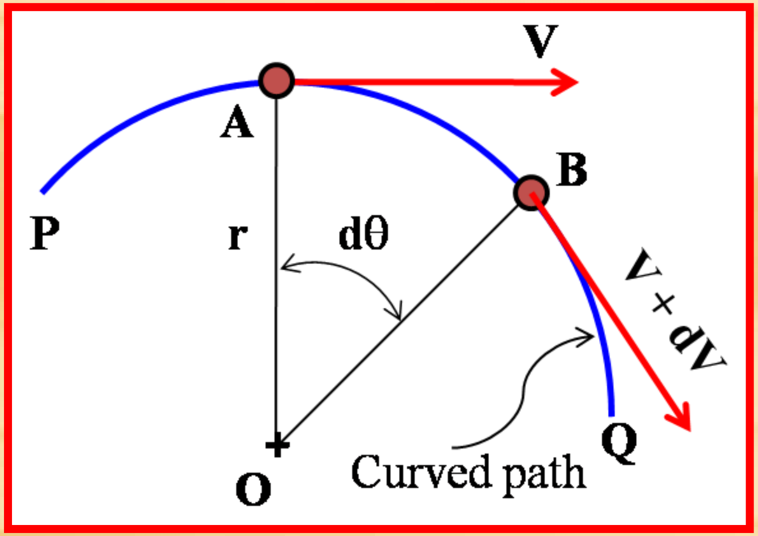
- When a particle moves along a curved path, it is sometimes convenient to describe its motion using coordinates other than Cartesian.
- When the path of motion is known, normal (**n**) and tangential (**t**) coordinates are often used.
- In the (**n – t**) coordinate system, the origin is located on the particle (the origin moves with the particle).
- The (**t-axis**) is tangent to the path (**curve**) at the instant considered, positive in the direction of the particle's motion.
- The (**n-axis**) is perpendicular to the (**t-axis**) with the positive direction toward the center of curvature of the curve.
- The center of curvature, (**O'**), always lies on the concave side of the curve.
- The radius of curvature, (ρ), is defined as the perpendicular distance from the curve to the center of curvature at that point.
- The position of the particle at any instant is defined by the distance, (**s**), along the curve from a fixed reference point.





Normal & Tangential Components:

- Consider (A & B), the two positions of a particle moving on a curved path displaced through an angle ($d\theta$) in time (dt) as shown in the figure.



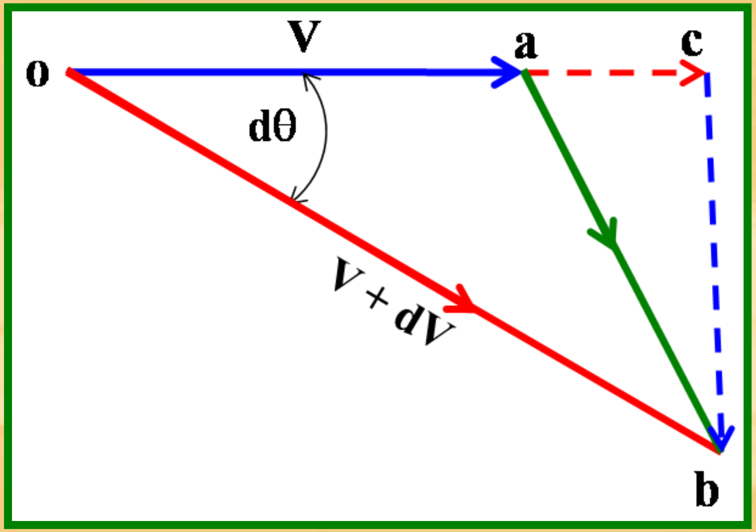
Lte:

r = radius of curvature of the curved path.

V = velocity of the particle at (A).

V + dV = velocity of particle at (B).

- The change of velocity as the particle moves from (**A**) to (**B**) may be obtained by drawing the vector triangle (**oab**) as shown in figure. In triangle (**oa**) represents the velocity (**V**) and (**ob**) represents the velocity (**V + dV**). The change of velocity in time (**dt**) is represented by (**ab**).



- Now resolving (**ab**) into two components (**parallel & perpendicular**) to (**oa**). Let (**ac & cb**) be the components parallel and perpendicular to (**oa**) respectively:

$$\therefore \quad \mathbf{ac = oc - oa = ob \cos d\theta - oa}$$

$$\mathbf{= (V + dV) \cos d\theta - V}$$

And $\quad cb = ob \sin d\theta = (V + dV) \sin d\theta$

Normal & Tangential Components:

Since the change of velocity of a particle (**represented by a vector ab**) has two mutually perpendicular components, therefore the acceleration of a particle moving along a circular path has the following components of acceleration:

1. Tangential component of the acceleration:

The acceleration of a particle at any instant moving along a circular path in a direction tangential at that instant, is known as tangential component of acceleration:

$$a_t = \frac{ac}{dt} = \frac{(V + dV)\,.\cos d\theta - V}{dt}$$

As (dθ) is too small, then: $\cos d\theta = 1$

$$a_t = \frac{dV}{dt}$$

2. Normal component of the acceleration:

The acceleration of a particle at any instant moving along a circular path in a direction normal to the tangent to that instant, is known as normal component of acceleration and directed towards the center of the circular path:

$$a_n = \frac{cb}{dt} = \frac{(V + dV)\,.\sin d\theta}{dt}$$

Since for very small angel, then: $\sin d\theta = d\theta \quad and \quad dV\,.\,d\theta \cong 0$

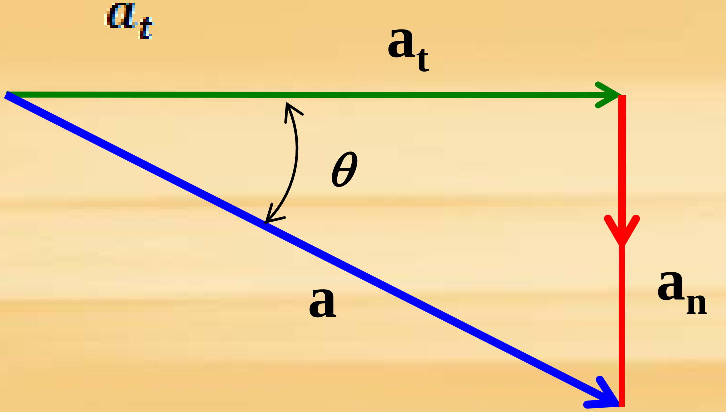
$$a_n = V\,.\frac{d\theta}{dt} = V\,.\,\omega = V\,.\frac{V}{r} = \frac{V^2}{r} = \omega^2\,.\,r$$

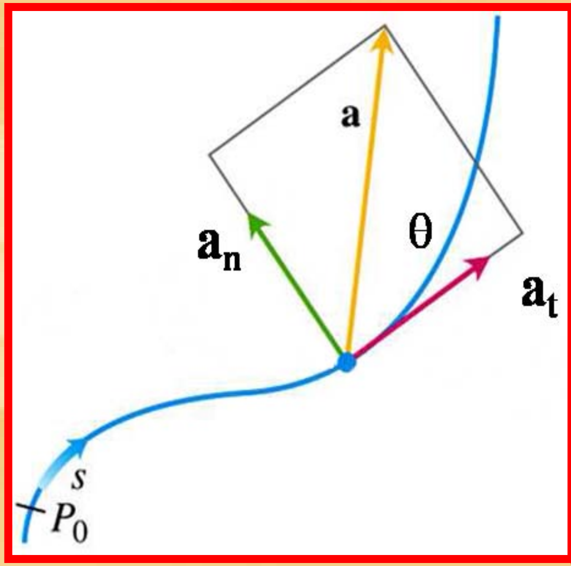
Normal & Tangential Components:

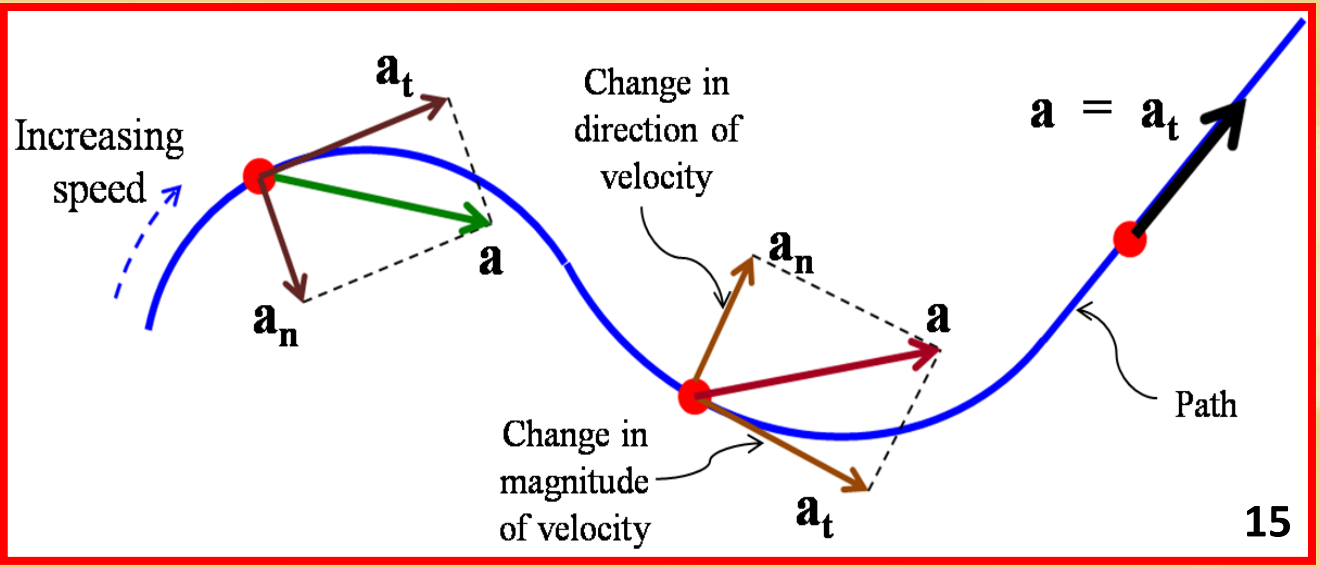
Since the tangential acceleration (a_t) and the normal acceleration (a_n) of the particle (**A**) at any instant are perpendicular to each other as shown, therefore total acceleration of the particle is:

$$a = \sqrt{a_t^2 + a_n^2} \quad and \quad \tan\theta = \frac{a_n}{a_t}$$

- Tangential component of acceleration reflects change of speed and normal component reflects change of direction.
- Tangential component may be positive or negative. Normal component always points toward center of path curvature.







Normal & Tangential Components:

Radius of curvature:

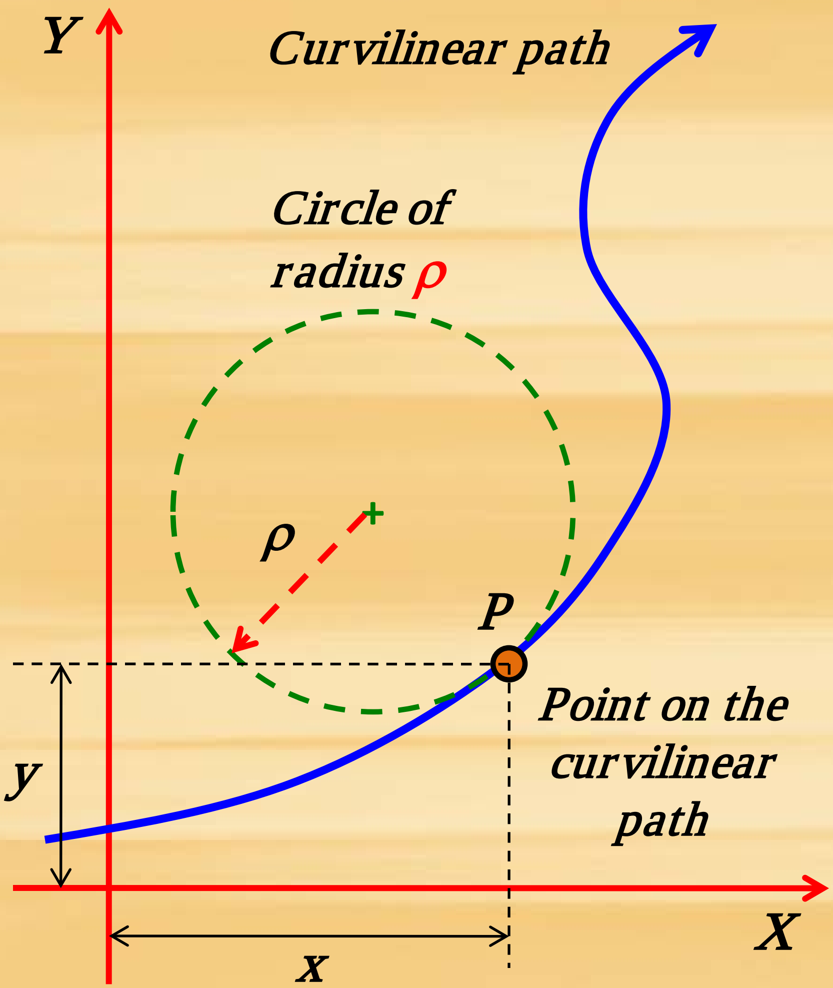
Any point on a curvilinear path can be approximated by a circle of radius (ρ).

The particle moves along a path expressed as:

$$y = f(x)$$

The radius of curvature, (ρ), at any point on the path can be calculated from

$$\rho = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}}$$



Where:

$$\frac{dy}{dx} = \textit{the first derivative of } y \textit{ to } x$$

$$\frac{d^2y}{dx^2} = \textit{the second derivative of } y \textit{ to } x$$

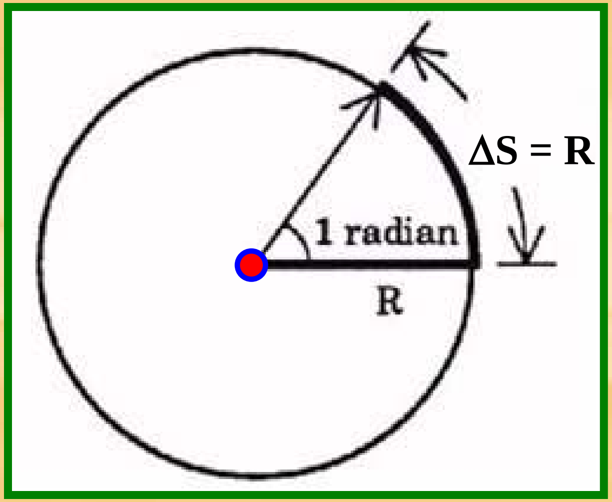
Rotary Motion:

The Radian:

Rotational - motion involving a rotation or revolution around a fixed chosen axis (*an axis which does not move*).

Rotational quantities are usually defined with units involving a ***radian*** measure.

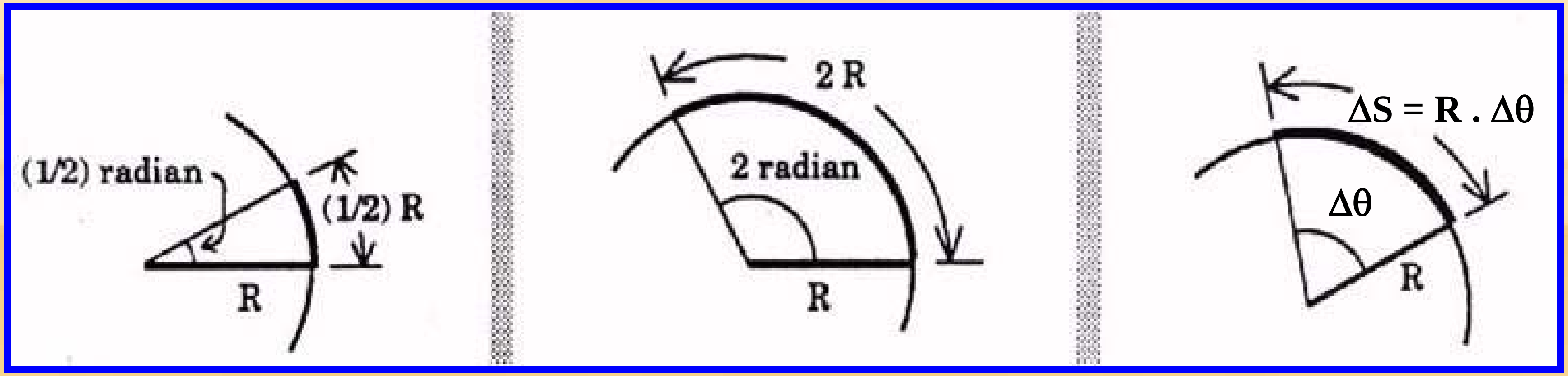
- If we take the radius of a circle and **LAY IT DOWN** on the circumference, it will create an angle whose arc length is equal to(**R**).
- In other words, one radian angle is subtends an arc length(**ΔS**)equal to the radius of the circle (***R***)
- A general Radian Angle (**Δθ**) subtends an arc length [**ΔS**] equal to (**R . Δθ**). The **theta (θ)** in this case represents **ANGULAR DISPLACEMENT** (in radian).



$$\boxed{\Delta S = R \, . \, \Delta\theta}$$

$$1 \; radian = \frac{R}{R} = 57.3^{o}$$

$$1 \, revolution = 2\pi \; radians = 360^{\circ}$$



Rotary Motion:

Angular Velocity:

Since velocity is defined as the rate of change of displacement. **ANGULAR VELOCITY** is defined as the rate of change of **ANGULAR DISPLACEMENT**.

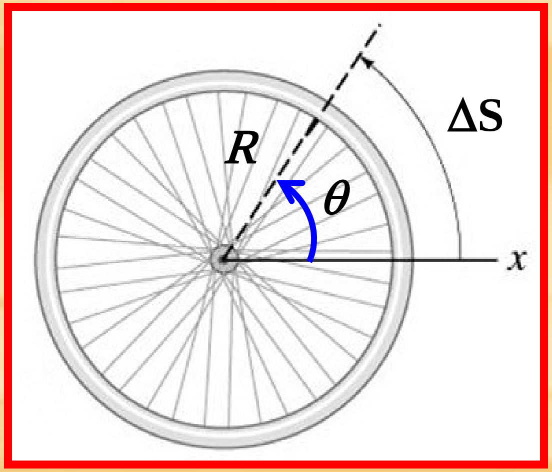
$$\bar{v} = \frac{\Delta x}{\Delta t} \rightarrow translational\ velocity$$

$$\bar{\omega} = \frac{\Delta \theta}{\Delta t} \rightarrow rotational\ velocity$$

$$v = \frac{dx}{dt}, \omega = \frac{d\theta}{dt}$$

$$\theta = \frac{\Delta S}{R} \quad (rad)$$

$$\omega = \lim_{\Delta t \to 0} \underbrace{\frac{\Delta \theta}{\Delta t}}_{\omega_{ave}} = \frac{d\theta}{dt} \quad (rad/s)$$

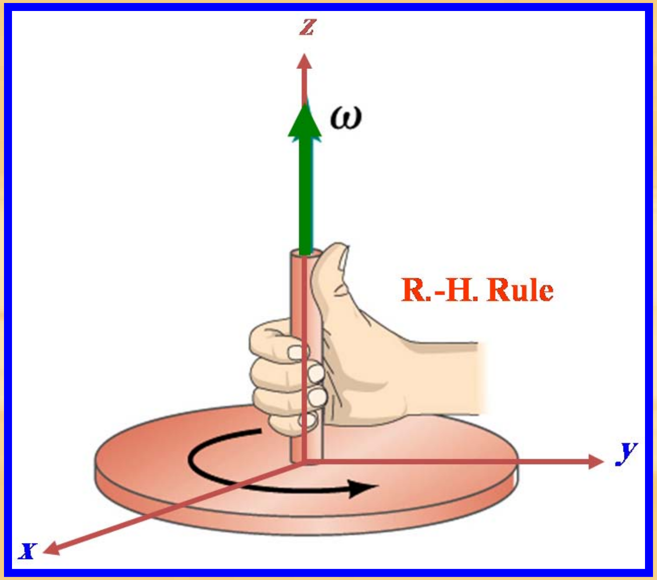


Rotational motion tells THREE THINGS:

- ➢ Magnitude of the motion and the units.
- ➢ The PLANE in which the object rotates in the directional sense (counterclockwise or clockwise).

❑ **Counterclockwise rotations are defined as having a direction of POSITVE motion on the "z" axis.**

❑ **Clockwise rotations are defined as having a direction of NEGAITVE motion on the "z" axis.**



Rotary Motion:

Angular Acceleration:

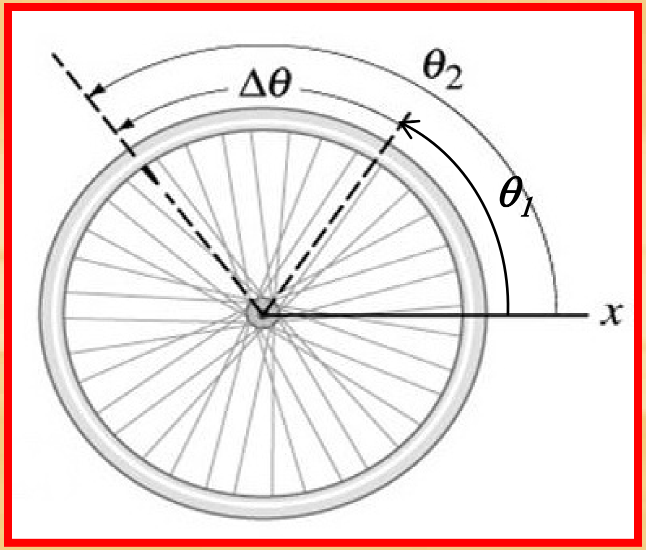
Once again, following the same lines of logic. Since acceleration is defined as the rate of change of velocity. We can say the **ANGULAR ACCELERATION is defined as the rate of change of the angular velocity.**

$$\overline{a} = \frac{\Delta v}{\Delta t} \rightarrow translational\ acceleration$$

$$\overline{\alpha} = \frac{\Delta \omega}{\Delta t} \rightarrow rotational\ acceleration$$

$$a = \frac{dv}{dt}, \alpha = \frac{d\omega}{dt}$$

$$\boxed{\alpha = \lim_{\Delta t \to 0} \underbrace{\frac{\Delta \omega}{\Delta t}}_{\alpha_{ave}} = \frac{d\omega}{dt} \quad (rad/s^2)}$$



❖ Also, we can say that the **ANGULAR ACCELERATION is the TIME DERIVATIVE OF THE ANGULAR VELOCITY**.

❖ **All the rules for integration apply as well.**

$$\boxed{\begin{aligned} x &= \int v\,dt, & \theta &= \int \omega\,dt \\ v &= \int a\,dt, & \omega &= \int \alpha\,dt \end{aligned}}$$

Rotary Motion:

Combining motions –Tangential velocity:

- **First we take our equation for the radian measure and divide BOTH sides by a change in time (Δt).**
- The left side is simply the equation for **LINEAR** velocity. **BUT** in this case the velocity is **TANGENT** to the circle. Therefore we call it **TANGENTIAL VELOCITY**.
- Inspecting the right side we discover the formula for **ANGULAR VELOCITY.**

$$\Delta s = R\Delta\theta$$

$$\frac{\Delta s}{\Delta t} = R\frac{\Delta\theta}{\Delta t}$$

$$v_T = \frac{\Delta s}{\Delta t}$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$v_T = r\omega$$

Tangential acceleration:

- **Using the same kind of mathematical reasoning we can also define Linear tangential acceleration.**
- *Inspecting each equation we discover that there is a DIRECT relationship between the Translational quantities and the Rotational quantities.*

$$V_t = r\,.\,\omega = r\,.\,\frac{d\theta}{dt}$$

$$\frac{V_t}{dt} = r\,.\,\frac{d\omega}{dt} = r.\,\frac{d^2\theta}{dt^2} \Rightarrow a_t = r\,.\,\alpha$$

Rotary Motion:

Linear motion – Rotation (Angular) motion:

$$\textit{Conversion}: x \rightarrow \theta,\ v \rightarrow \omega,\ a \rightarrow \alpha$$

$$v = v_o + at$$

$$x = x_o + v_o t + \tfrac{1}{2} a t^2$$

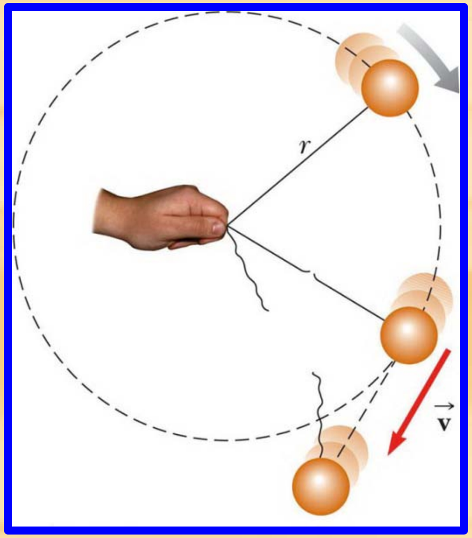
$$v^2 = v_o^2 + 2a\Delta x$$

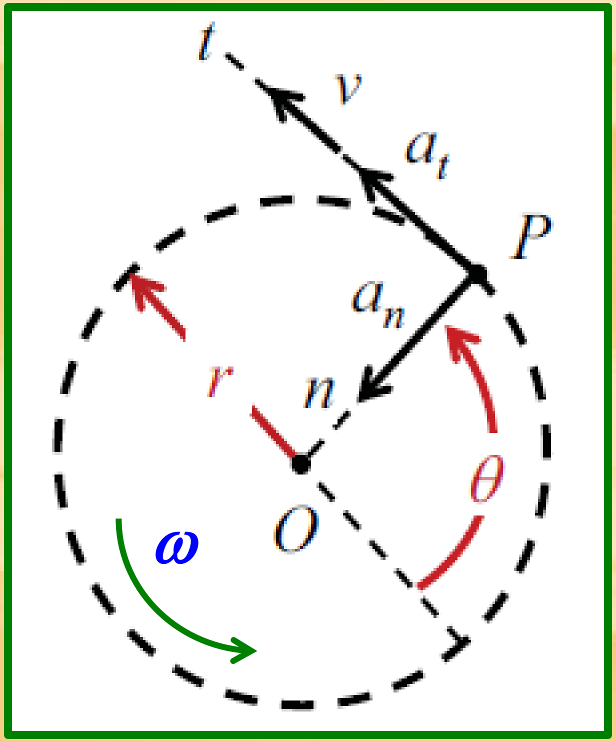
$$(1)\ \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$(2)\ \omega = \omega_0 + \alpha t$$

$$(3)\ \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\textit{Note}: \alpha = \textit{constant}$$



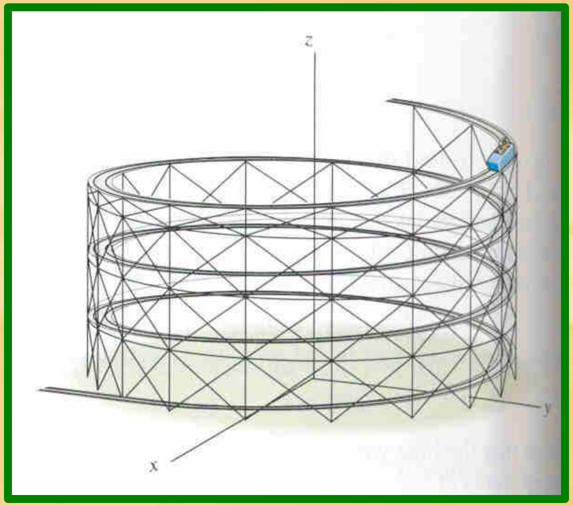


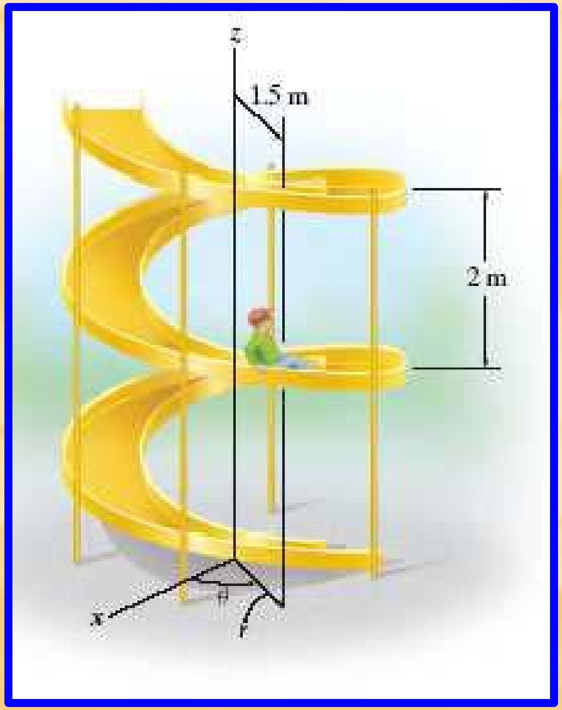
Other Types of Motions:

Cylindrical motion:

The cylindrical coordinate system is used in cases where the particle moves along a 3-D curve.

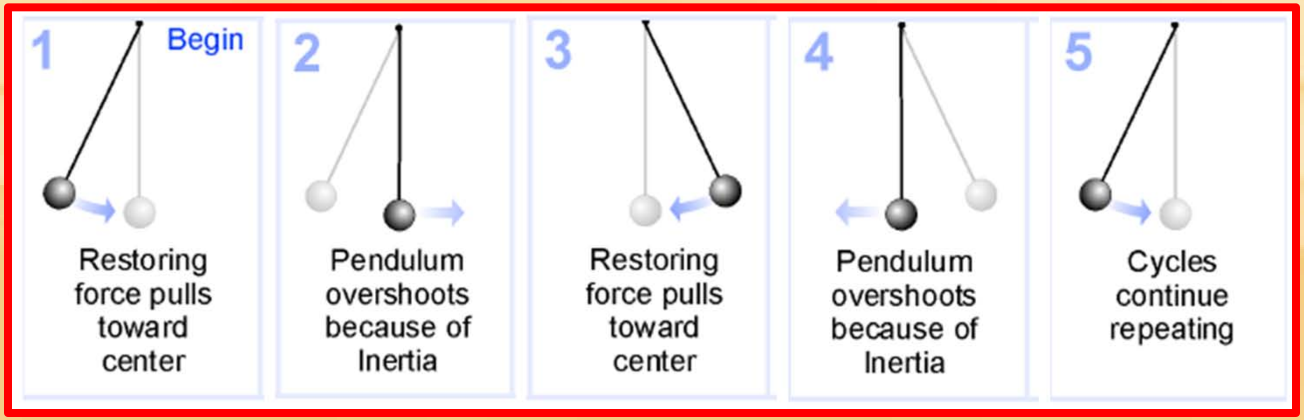
Example: A roller coaster car travels down a fixed, helical path at a constant speed.





Oscilatory motion:

Systems that have harmonic motion move back and forth around a central or equilibrium position. It describes the back and forth motion of a **pendulum**.



Newton's Laws of Motion:

➢ The motion of a particle is governed by Newton's three laws of motion.

❖ **First Law:** A particle originally at rest, or moving in a straight line at constant velocity, will remain in this state if the resultant force acting on the particle is zero.

❖ **Second Law:** If the resultant force on the particle is not zero, the particle experiences an acceleration in the same direction as the resultant force. This acceleration has a magnitude proportional to the resultant force.

❖ **Third Law:** Mutual forces of action and reaction between two particles are equal, opposite, and collinear.

❑ The **first and third laws** were used in developing the **concepts of statics**. **Newton's second law** forms the basis of **the study of dynamics**.

❑ **Mathematically, Newton's second law of motion can be written:**

$$\boldsymbol{F} = \mathbf{m} \cdot \mathbf{a}$$

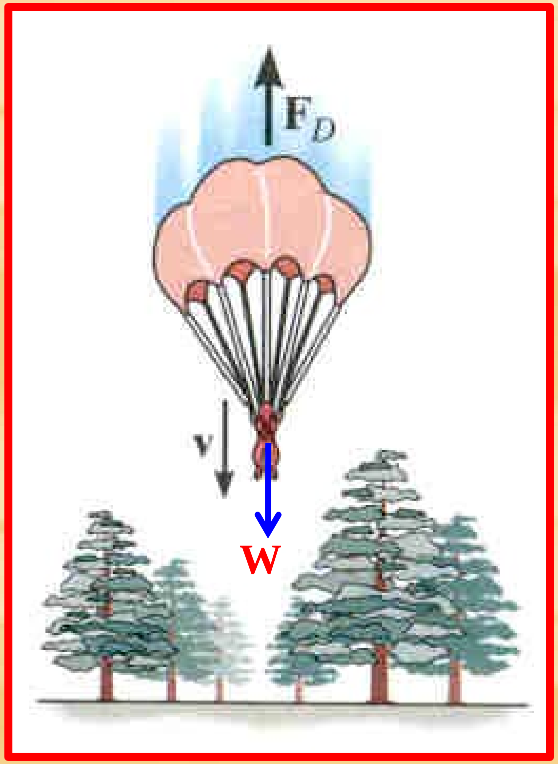
Where: (F) is the resultant unbalanced force acting on the particle, and (a) is the acceleration of the particle. The positive scalar (**m**) is called the mass of the particle.

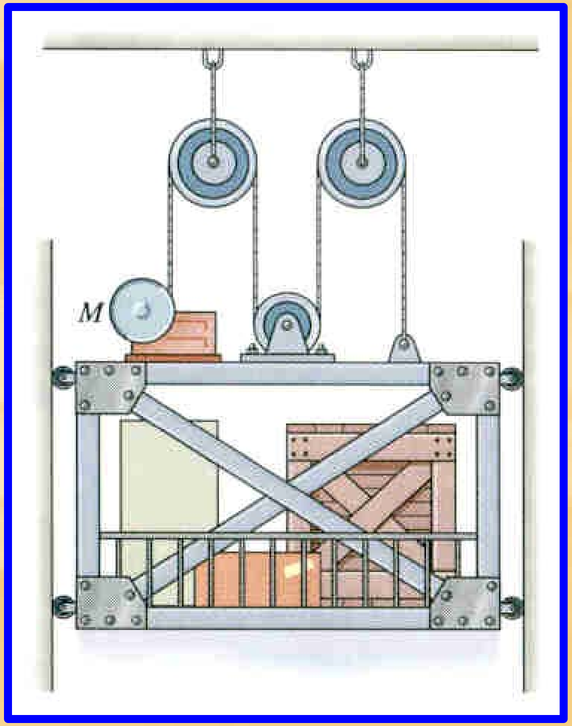
❑ **Newton's second law** cannot be used when the **particle's speed** approaches the **speed of light**, or if **the size of the particle is extremely small** (~ size of an atom).

Applications:

The motion of an object depends on the forces acting on it.

- ❑ A parachutist relies on the atmospheric drag resistance force to limit his velocity.
- ❑ Knowing the drag force, how can we determine the acceleration or velocity of the parachutist at any point in time?





- ❖ A freight elevator is lifted using a motor attached to a cable and pulley system as shown.
- ❖ How can we determine the tension force in the cable required to lift the elevator at a given acceleration?
- ❖ Is the tension force in the cable greater than the weight of the elevator and its load?

Newton's Second Law of Motion:

Newton's Second Law: If the *resultant force* acting on a particle is not zero, the particle will have an acceleration *proportional to the magnitude of resultant* and in the *direction of the resultant.*

- Consider a particle subjected to constant forces:

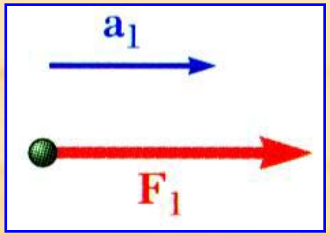
$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \cdots = \text{constant} = \text{mass}, m$$

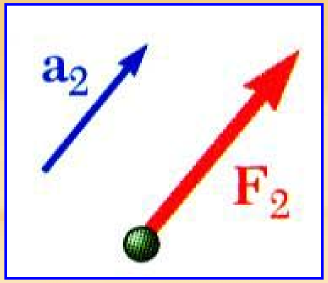
- When a particle of mass (***m***)is acted upon by a force $\vec{F}$, the acceleration of the particle must satisfy:

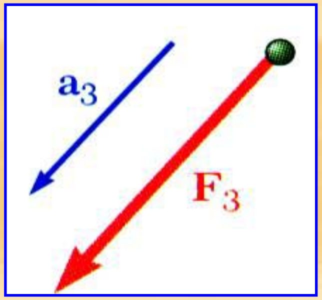
$$\boxed{\vec{F} = m\vec{a}}$$

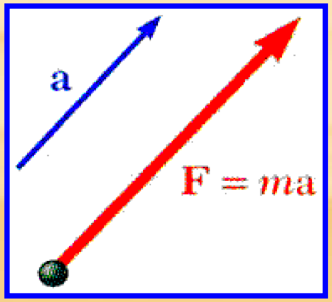
- Acceleration must be evaluated with respect to a *Newtonian frame of reference*, i.e., for problems concerned with motions at or near the earth's surface, we typically assume our " inertial frame " to be fixed to the earth. We neglect any acceleration effects from the earth's rotation.

- If force acting on particle is zero, particle will not accelerate, i.e., it will remain stationary or continue on a straight line at constant velocity.









Equation of Motion:

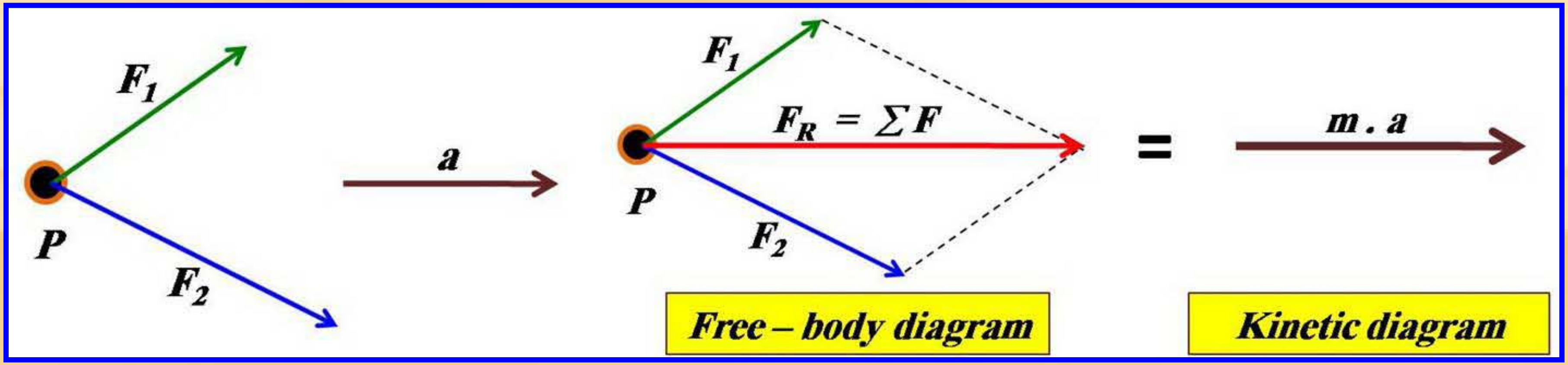
The motion of a particle is governed by **Newton's second law**, relating the **unbalanced forces** on a particle to its acceleration. **If more than one force** acts on the particle, **the equation of motion can be written:**

$$\sum F = F_R = m \cdot a$$

Where: (F_R) is the resultant force, which is a vector summation of all the forces.

To illustrate the equation, consider a particle acted on by two forces as shown in the figure below:

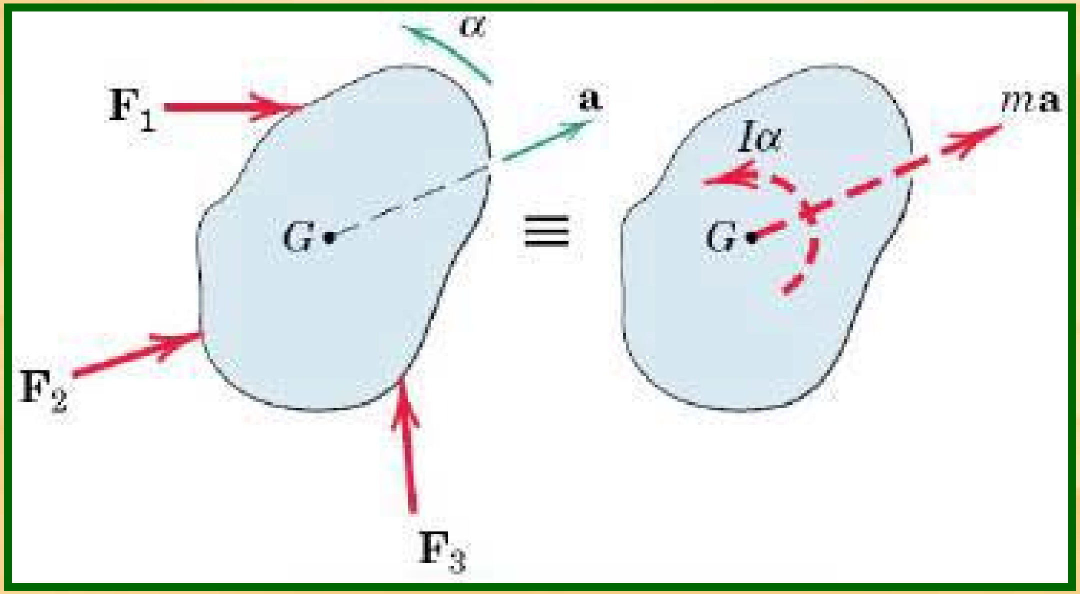
- First, draw the particle's free-body diagram, showing all forces acting on the particle.
- Next, draw the kinetic diagram, showing the inertial force (ma) acting in the same direction as the resultant force (F_R).



Plane Motion of a Rigid Body:

❖ Motion of a rigid body in plane motion is completely defined by the ***resultant and moment resultant*** about (G) of **the external forces**.

Remember, unbalanced forces cause acceleration!



$$\sum F_x = m\bar{a}_x \qquad \sum F_y = m\bar{a}_y \qquad \sum M_G = \bar{I}\alpha$$

<u>Where:</u>

m = the mass of the rigid body (kg)

$\mathbf{a_x}$ **&** $\mathbf{a_y}$ = acceleration in (x & y direction) respectively.

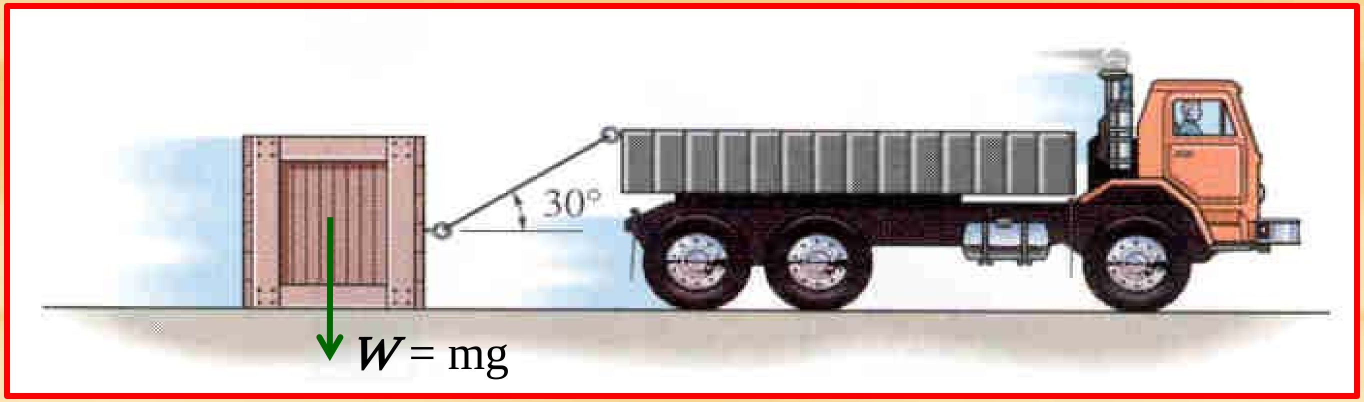
$\mathbf{M_G}$ = the moment (Torque) of the resultant force about the center of gravity (G).

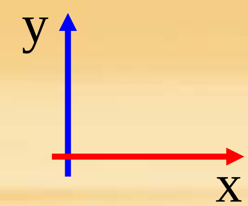
I = the moment of inertia about the center of gravity (G).

α = the angular acceleration of the rigid body about the center of gravity (G).

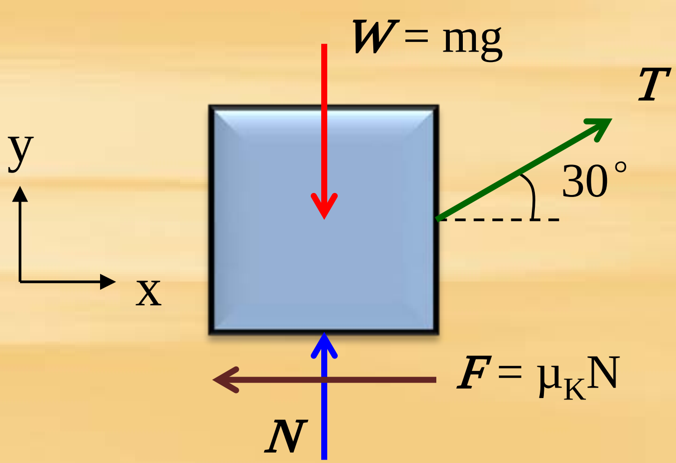
Procedure For The Application Of The Equation Of Motion:

1) Select a convenient inertial coordinate system. Rectangular, normal/tangential, or cylindrical coordinates may be used.





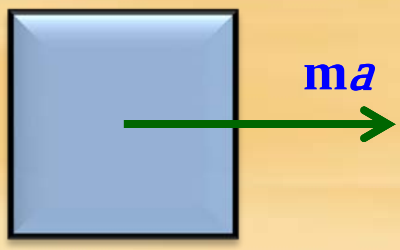
2) Draw a free-body diagram showing all external forces applied to the particle. Resolve forces into their appropriate components.



The weight force (***W***) acts through the crate's center of mass. (***T***) is the tension force in the cable. The normal force (***N***) is perpendicular to the surface. The friction force (***F*** $= \mu_K N$) acts in a direction opposite to the motion of the crate.

Procedure For The Application Of The Equation Of Motion:

3) Draw the kinetic diagram, showing the particle's inertial force, (**m*a***). Resolve this vector into its appropriate components.



The crate will be pulled to the ***right***. The ***acceleration*** vector can be directed to ***the* right** if the truck is ***speeding up***, or to ***the left*** if it is ***slowing down***.

4) Apply the **equations of motion** in their scalar component form and solve these equations for the unknowns.

$$\sum F_x = F_R$$

$$T\,.\ \cos 30 - F_k = m\,.\,a \quad \Longrightarrow \quad T\,.\ \cos 30 - \mu_k\,.\ N = m\,.\,a$$

5) It may be necessary to apply the proper kinematic relations to generate additional equations.

➢ The second law only provides solutions for ***forces and accelerations***. If ***velocity* or *position*** have to be found, ***kinematics equations*** are used once the acceleration is found from the equation of motion.

$$x = x_o + V_x\,.\,t + \frac{1}{2}\,.\,a\,.\,(t)^2$$

$$V_x = V_{ox} + a\,.\,t$$

$$V_x{}^2 = V_{ox}{}^2 + 2\ a\,.\,(x - x_o)$$

WORK AND ENERGY:

Another equation for working kinetics problems involving particles can be derived by integrating the equation of motion ($\boldsymbol{F}$ = m . **a**) with respect to displacement.

By substituting:

$$\mathbf{a = v\,.\,(dv / ds)} \qquad \textbf{into} \qquad \mathbf{F = m\,.\,a}$$

The result is integrated to yield an equation known as the principle of work and energy:

$$F = m\,.\,V\,.\left(\frac{dV}{dS}\right)$$

$$\boxed{F\,.\,dS = m\,.\,V\,.\,dV} \Rightarrow \text{Same units as work}$$

The integration of the left term yield to equation of **Work**, and the integration of the right term yield to equation of **Energy**. That means we need **Energy** to perform **Work**.

$$F\,.\,(S - S_o) = \frac{1}{2}\,m\,.\,V^2$$

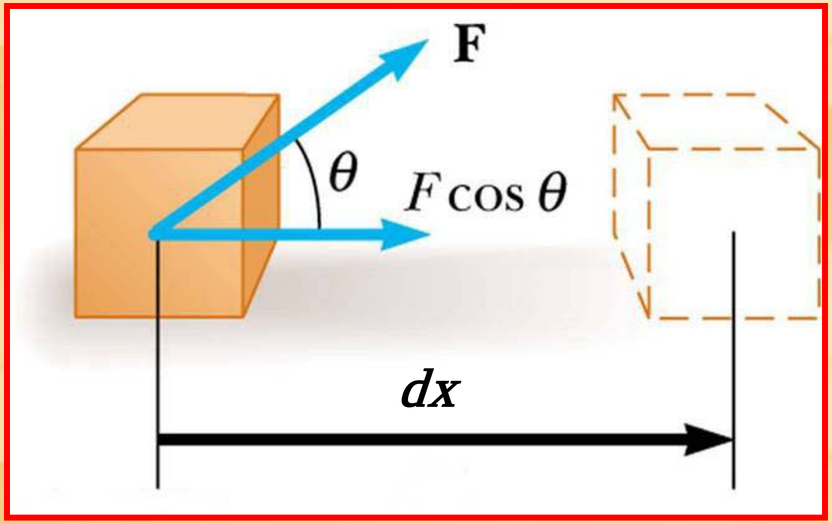
WORK OF A FORCE:

- A force does **work** on a particle when the particle undergoes a **displacement** along the **line of action of the force**.
- Work is defined as the **product** of **force** and **displacement components** acting in the **same direction**.

WORK OF A FORCE:

So, if the angle between the force and displacement vector is (θ), the increment of work (**dU**) done by the force is:

$$\boxed{dU = F \cdot \cos\theta \cdot dx}$$



By integrating, **the total work** can be written as:

$$U_{1-2} = \int_{x_1}^{x_2} F \cdot \cos\theta \cdot dx$$

$$U_{1-2} = F \cdot \cos\theta \cdot (x_2 - x_1)$$

By definition the work is equal to the product of force and distance:

$$U = F \cdot x$$

The work's unit becomes:

SI unit = Joule

1 J = 1 N · m = 1 kg · m^2 / s^2

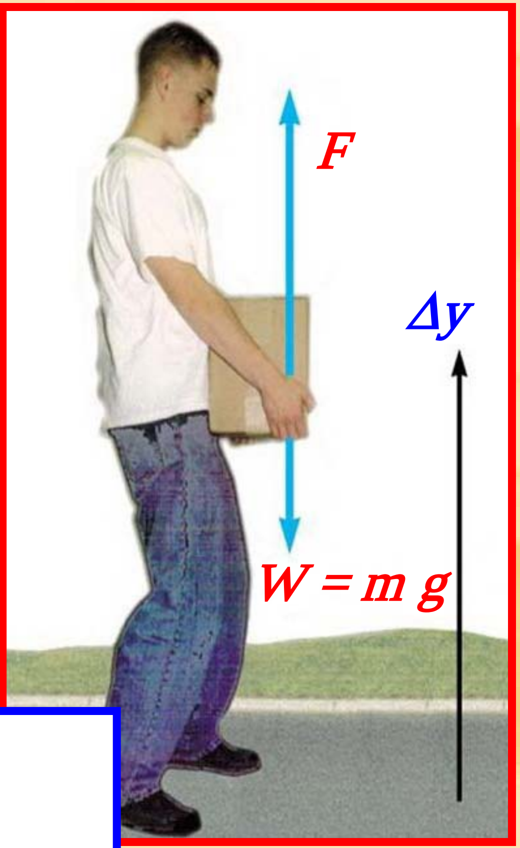
Work is positive if the force and the movement are in the same direction. If they are opposing, then the work is negative. If the force and the displacement directions are perpendicular, the work is zero.

WORK OF A WEIGHT

The work done by the gravitational force acting on a particle (or *weight of an object*) can be calculated by using :

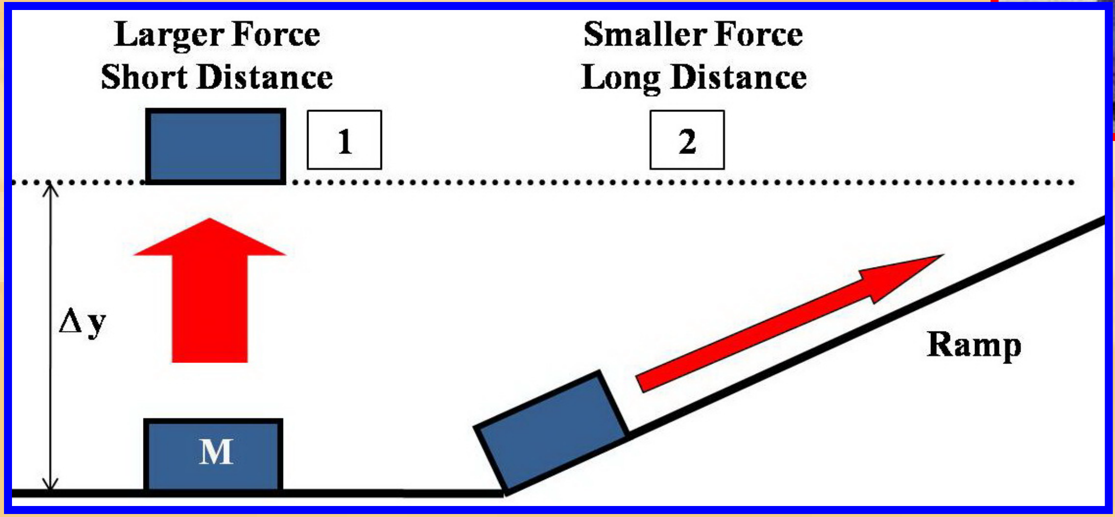
$$U_{1-2} = \int_{y_1}^{y_2} -W\,.dy$$

$$= -W\,.(y_2 - y_2) = -W\,.\Delta y$$



➢ The work of a weight is the product of the magnitude of the particle's weight and its vertical displacement. If (**Δy**) is **upward**, the work is **negative** since the weight force always acts downward.

➢ Exert a **smaller force** over a **larger distance** to achieve the same change in gravitational potential energy (**height raised**).



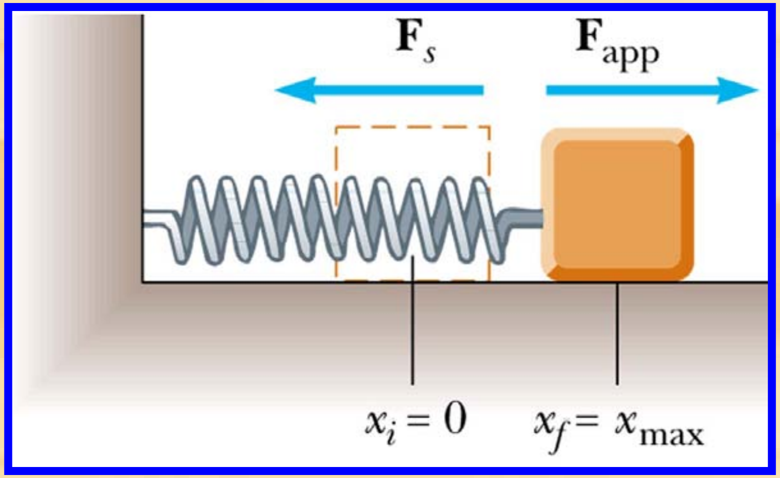
WORK OF A SPRING FORCE:

When a spring is stretched, a linear elastic spring develops a force of magnitude:

$$F_s = k \,.s$$

Where:

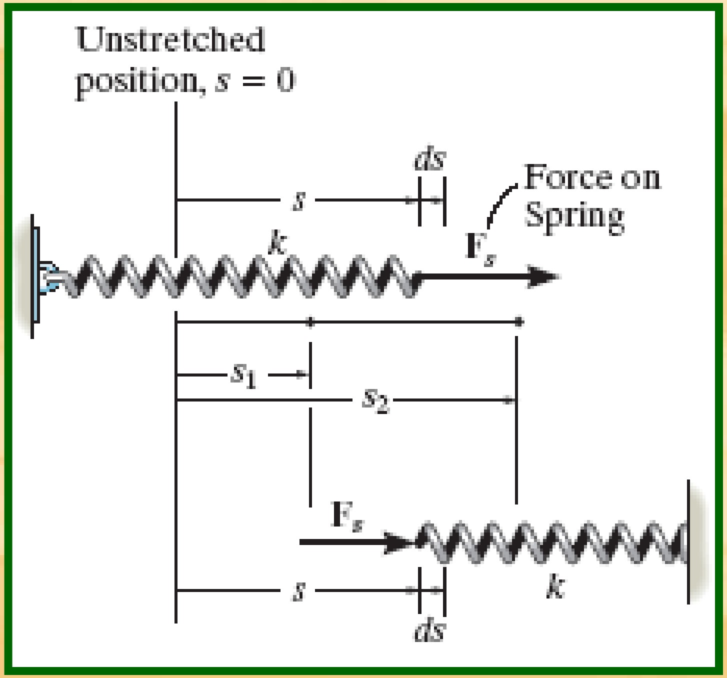
- (**k**) is the spring stiffness and,
- (**s**) is the displacement from the un-stretched position.



➢ The work of the spring force moving from position (s_1) to position (s_2) is:

$$U_{1-2} = \int_{s_1}^{s_2} F_s \,.ds = \int_{s_1}^{s_2} k \,.s \,.ds$$

$$= 0.5 \,.k \,.(s_2)^2 - 0.5 \,.k \,.(s_1)^2$$



➢ If a particle is attached to the spring, the force (F_s) exerted on the particle is opposite to that exerted on the spring. Thus, the work done on the particle by the spring force will be negative :

$$U_{1-2} = -\left[0.5 \,.k \,.(s_2)^2 - 0.5 \,.k \,.(s_1)^2\right]$$

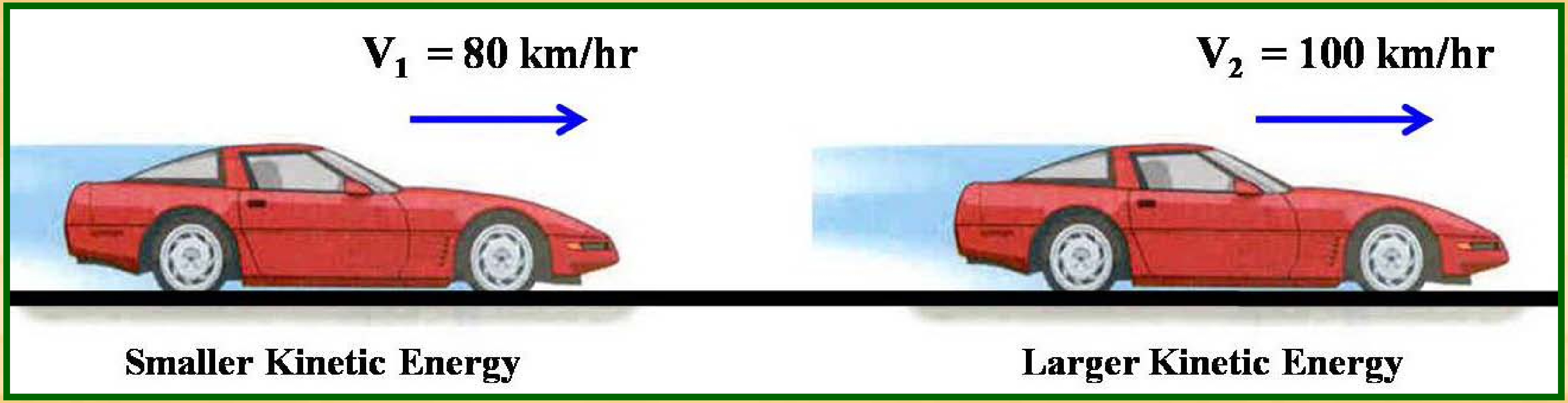
Kinetic Energy:

❖ Kinetic energy exists whenever an object which has mass is in motion with some velocity.

❖ Everything you see moving about has kinetic energy. The kinetic energy of an object in this case is given by the relation:

$$K.E_{1-2} = \int_{V_1}^{V_2} m\,.\,V\,.\,dV$$

$$= \frac{1}{2} m\,.\left(V_2^{\,2} - V_1^{\,2}\right)$$



❖ Kinetic energy is proportional to (V^{2}). The greater the mass or velocity of a moving object is the greater kinetic energy it has.

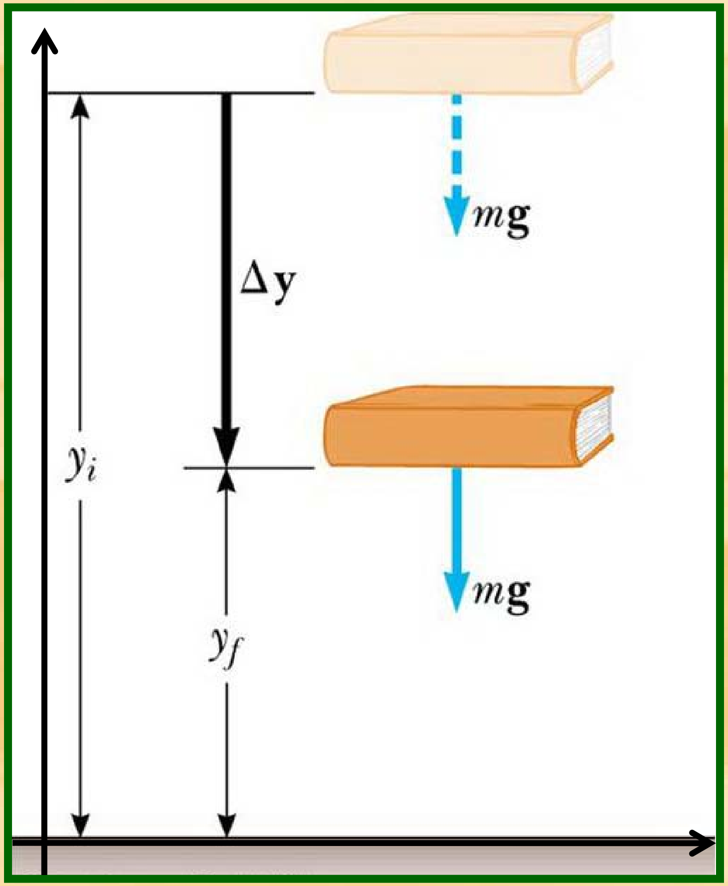
Potential Energy:

➢ Potential energy exists whenever an object which has mass has a position within a force field.

➢ The most everyday example of this is the position of objects in the earth's gravitational field. The potential energy of an object in this case is given by the relation:

$$\Delta PE = -F \,.\, \Delta y$$

$$\Delta PE = m . g \,.\, \Delta y$$



Where:

PE = Potential Energy (in Joules)

m = mass (in kilograms)

g = gravitational acceleration of the earth (9.8 m/sec2)

Δy = height above earth's surface (in meters)

$= (\, y_f - y_i \,)$

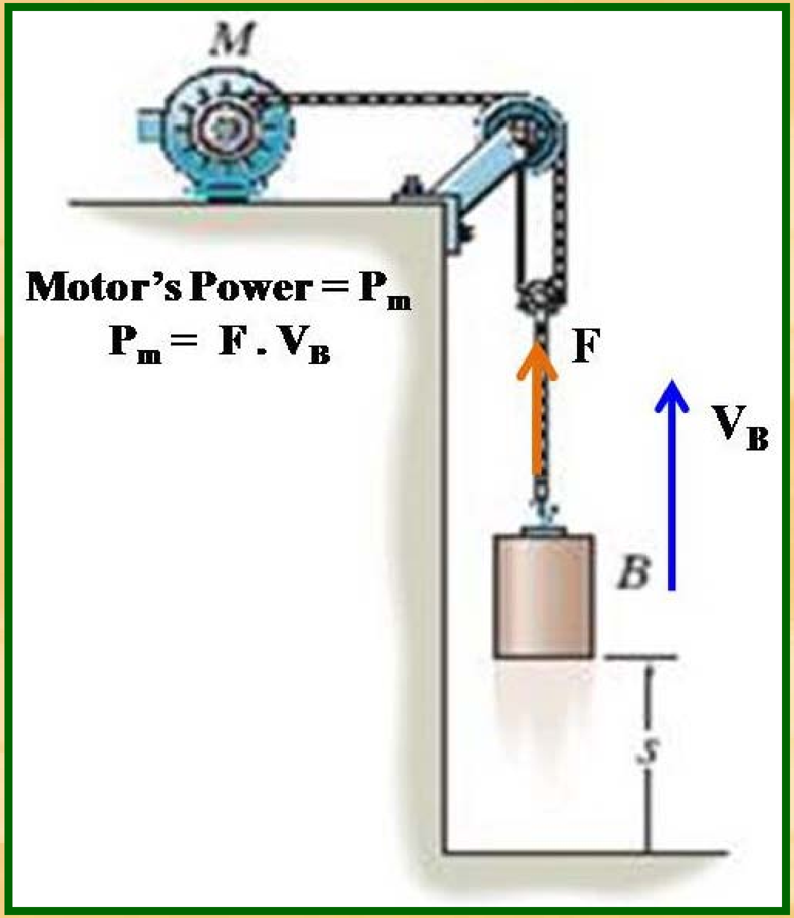
POWER AND EFFICIENCY:

Power is defined as the amount of work performed per unit of time.

❖ If a machine or engine performs a certain amount of work, dU, within a given time interval, dt, the power generated can be calculated as

$$P = \frac{dU}{dt}$$

$$P = \frac{F \,.\, dx}{dt} = F \,.\, V$$



❖ SI units of the Power is Watts (W):

$$1 \ W = 1 \ J/s = 1 \ kg \frac{m^2}{s^3}$$

❖ Thus, power is a scalar defined as the product of the force and velocity components acting in the same direction.

❖ Using scalar notation, power can be written

$$P = F \,.\, V = F \,.\, V \,.\cos\theta$$

❖ where (θ) is the angle between the force and velocity vectors.

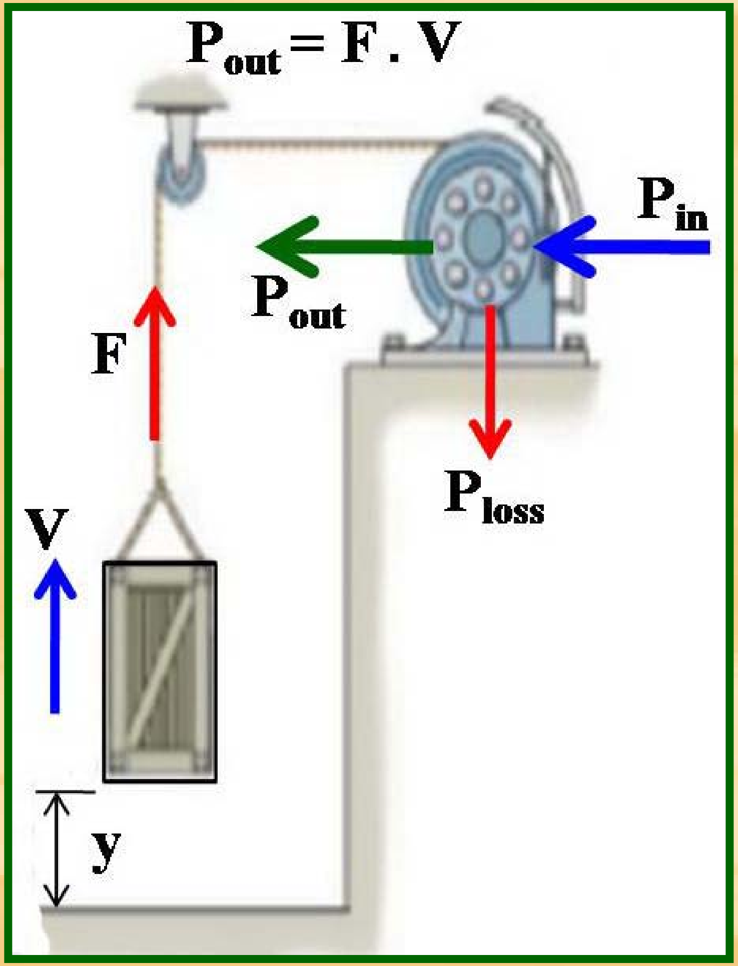
POWER AND EFFICIENCY:

EFFICIENCY:

The mechanical efficiency of a machine is the ratio of the useful power produced (output power) to the power supplied to the machine (input power) or:

η = (power output) / (power input)

$$\eta = \frac{P_{in} - P_{loss}}{P_{in}} = 1 - \frac{P_{loss}}{P_{in}}$$



- If energy input and removal occur at the same time, efficiency may also be expressed in terms of the ratio of output energy to input energy or

η = (energy output) / (energy input)

- Machines will always have frictional forces. Since frictional forces dissipate energy, additional power will be required to overcome these forces.

- Consequently, the efficiency of a machine is always less than 1.

- The higher the efficiency, more of the work input is converted to work output; less of the work output is lost to overcoming friction.

POWER AND EFFICIENCY:

PROCEDURE FOR ANALYSIS:

1. Find the resultant external force acting on the body causing its motion. It may be necessary to draw a free-body diagram.

2. Determine the velocity of the point on the body at which the force is applied. The equation of motion and appropriate kinematic relations, may be necessary.

3. Multiply the force magnitude by the component of velocity acting in the direction of (F) to determine the power supplied to the body:

$$P = F \cdot V \cdot \cos\theta$$

4. In some cases, power may be found by calculating the work done per unit of time:

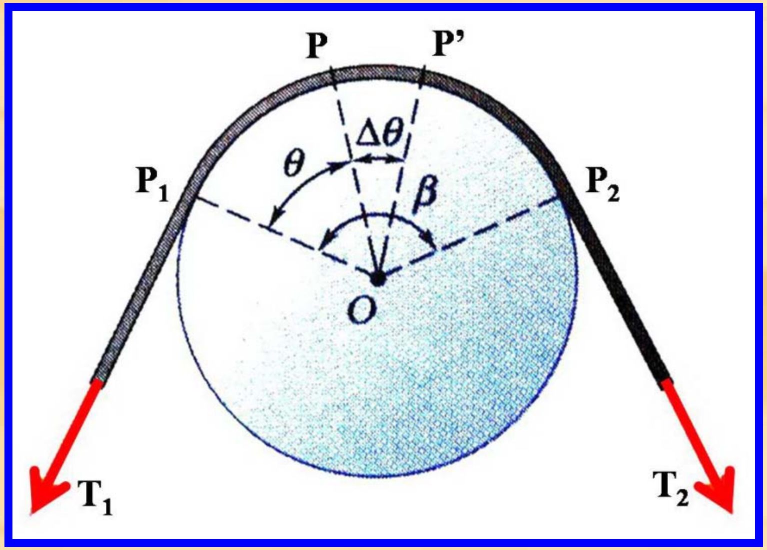
$$P = \frac{dU}{dt}$$

5. If the mechanical efficiency of a machine is known, either the power input or output can be determined.

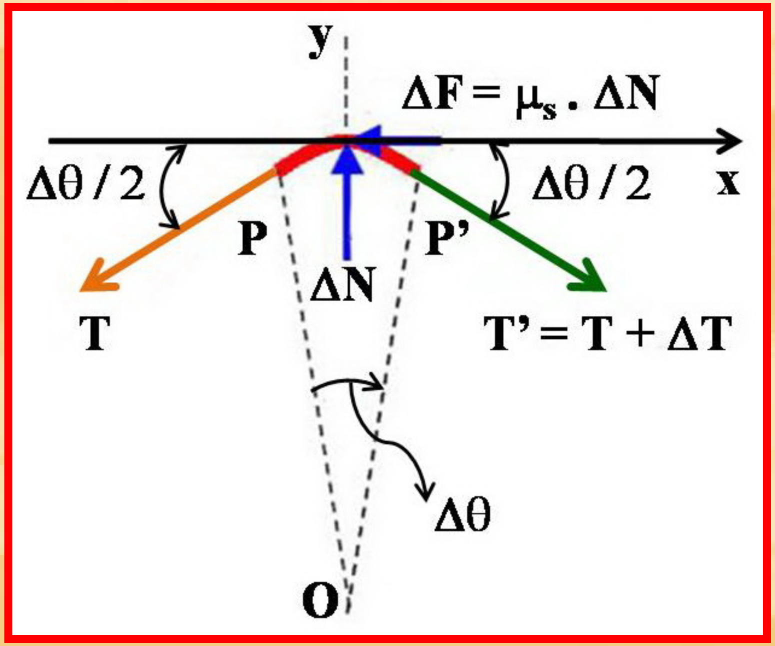
Belts & Pulleys:

Belts:

Consider a flat belt passing over a fixed cylindrical drum. We propose to determine the relation existing between the values (T_1 and T_2) of the tension in the two parts of the belt when the belt is just about to slide toward the right.



- Let us detach from the belt a small element (PP') subtending an angle ($\Delta\theta$). We draw the FBD of the element of the belt:



Where:

T = the tension at **P**
T + ΔT = the tension at **P'**
ΔN = Normal reaction
ΔF = friction force (opposite to the direction of motion). The motion is assumed to be impending:

$$\Delta F = \mu_s \Delta N$$

Equations of equilibrium:

$$\sum F_x = 0: \quad (T + \Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \mu_s \Delta N = 0$$

Belts & Pulleys:

Belts:

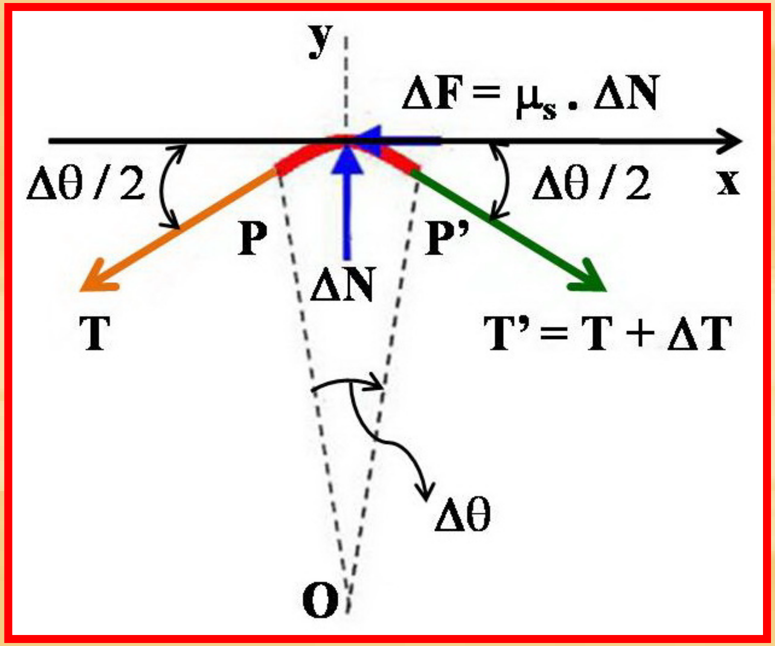
$$\sum F_y = 0: \quad \Delta N - (T + \Delta T)\sin\frac{\Delta\theta}{2} - T\sin\frac{\Delta\theta}{2} = 0$$

- Combine to eliminate (ΔN), divide through by ($\Delta\theta$):

$$\frac{\Delta T}{\Delta\theta}\cos\frac{\Delta\theta}{2} - \mu_s\left(T + \frac{\Delta T}{2}\right)\frac{\sin(\Delta\theta/2)}{\Delta\theta/2} = 0$$

- In the limit as ($\Delta\theta$) goes to zero, (ΔT) goes to zero too:

$$\frac{dT}{d\theta} - \mu_s T = 0 \quad \Longrightarrow \quad \frac{dT}{T} = \mu_s \, . \, d\theta$$



- Separate variables and integrate from:

$$\theta = 0 \text{ to } \theta = \beta$$

$$\ln\frac{T_2}{T_1} = \mu_s\beta \quad \text{or} \quad \frac{T_2}{T_1} = e^{\mu_s\beta}$$