Chapter 3

Non-Newtonian fluid

3-1. Introduction:

The study of the deformation of flowing fluids is called rheology; the rheological behavior of various fluids is sketched in Figure 3-1. Newtonian fluids, defined as *fluids for which the shear stress is linearly proportional to the shear strain rate.* Newtonian fluids (stress proportional to strain rate). Many common fluids, such as air and other gases, water, kerosene, gasoline, and other oil-based liquids, are Newtonian fluids. Fluids for which the shear stress is *not* linearly related to the shear strain rate are called non-Newtonian fluids. Examples include slurries and colloidal suspensions, polymer solutions, blood, paste, and cake batter. Some non-Newtonian fluids exhibit a "memory"-the shear stress depends not only on local strain rate, but also on its *history*. A fluid that returns (either fully or partially) to its original shape after the applied stress is released is called viscoelastic.

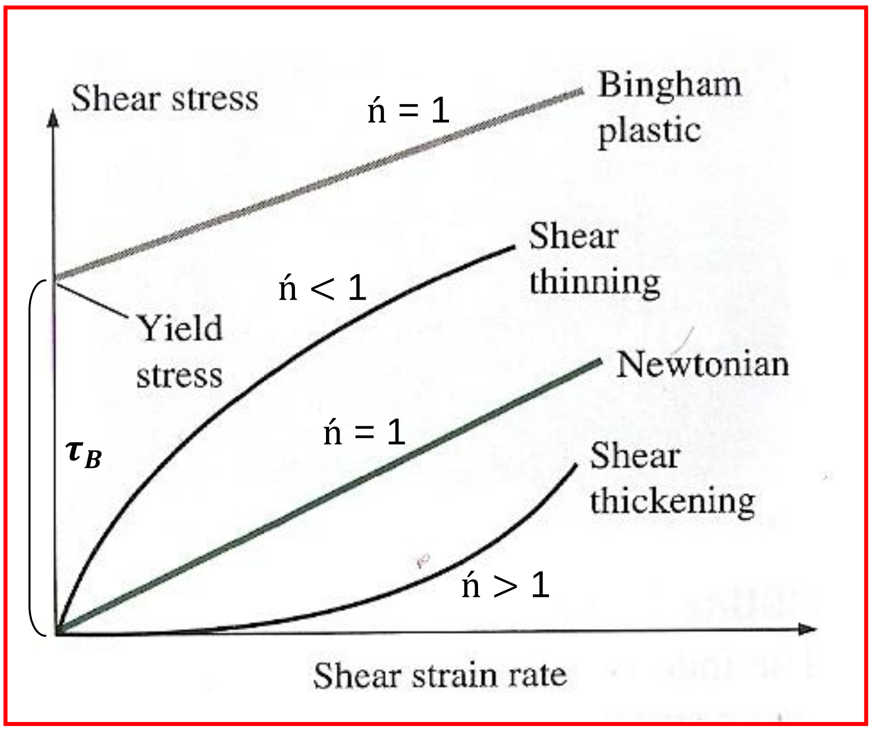

Figure 3-1. Rheological behavior of fluids –shear Stress as a function of shear strain rate.

Some non-Newtonian fluids are called shear thinning fluids or pseudoplastic fluids, because the more the fluid is sheared, the less viscous it becomes. A good example is paint. Paint is very viscous when poured from the can or when picked up by a paintbrush, since the shear rate is small. However, as we apply the paint to the wall, the thin layer of paint between the paintbrush and the wall is subjected to a large shear rate , and it becomes much less viscous. Plastic fluids are those in which the shear thinning effect is extreme. In some fluids a finite stress called the yield stress is required before the fluid begins to flow at all; such fluids are called Bingham plastic fluids. Certain pastes such as acne cream and toothpaste are examples of Bingham plastic fluids. If you hold the tube upside down, the paste does not flow, even though there is a nonzero stress due to gravity. However, if you squeeze the tube (greatly increasing the stress), the paste flows like a very viscous fluid. Other fluids show the opposite effect and are called shear thickening fluids or dilatant fluids; the more the fluid is sheared, the more viscous it becomes. The best example is quicksand, a thick mixture of sand and water. As we all know from Hollywood movies, it is easy to move *slowly* through quicksand, since the viscosity is low; but if you panic and try to move quickly, the viscous resistance increases considerably and you get "stuck" (Figure 3-2). You can create your own quicksand by mixing two parts cornstarch with one part water-try it! Shear thickening fluids are used in some exercise equipment-the faster you pull, the more resistance you encounter.

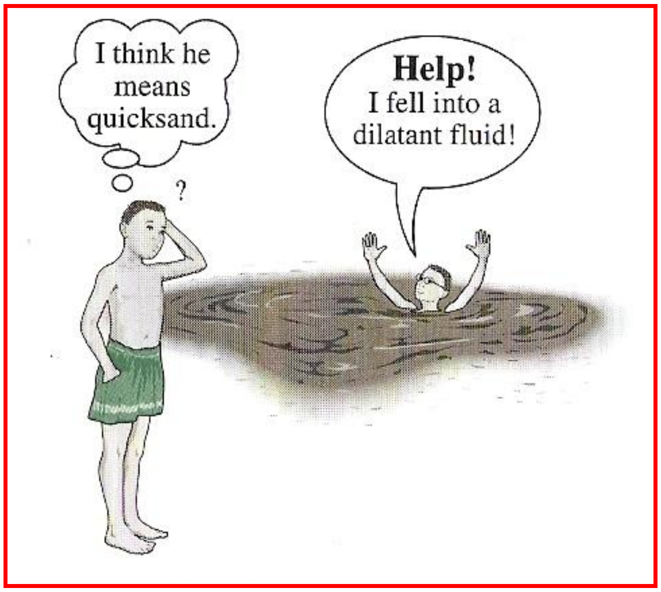

Figure 3-2.

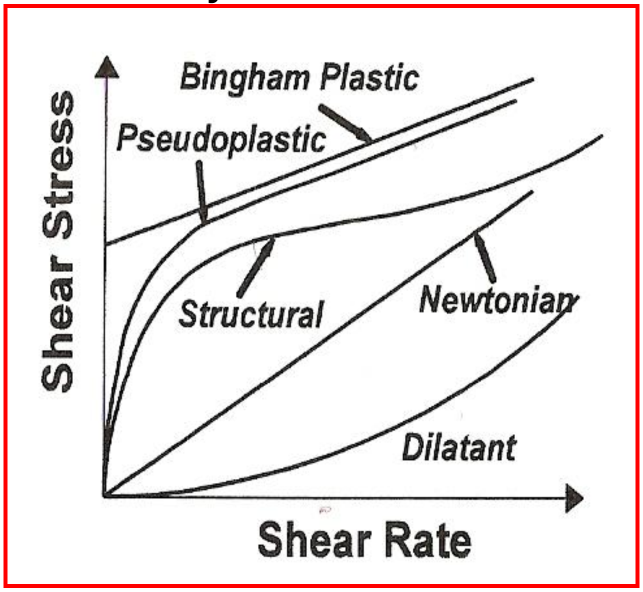

Figure 3-3.

The viscosity takes various forms, depending on the nature of the fluid. Some of these are illustrated in Figure 3-3 and in Figure 3-4. The classes of behavior illustrated are the Bingham plastic, pseudoplastic, dilatant, and structural. Both the Bingham plastic and the pseudoplastic are shear thinning, since the viscosity decreases with increasing shear rate, whereas the dilatant fluid is shear thickening. The structural fluid exhibits newtonian behavior at very low and very high values of shear rate and is shear thinning at intermediate shear rates.

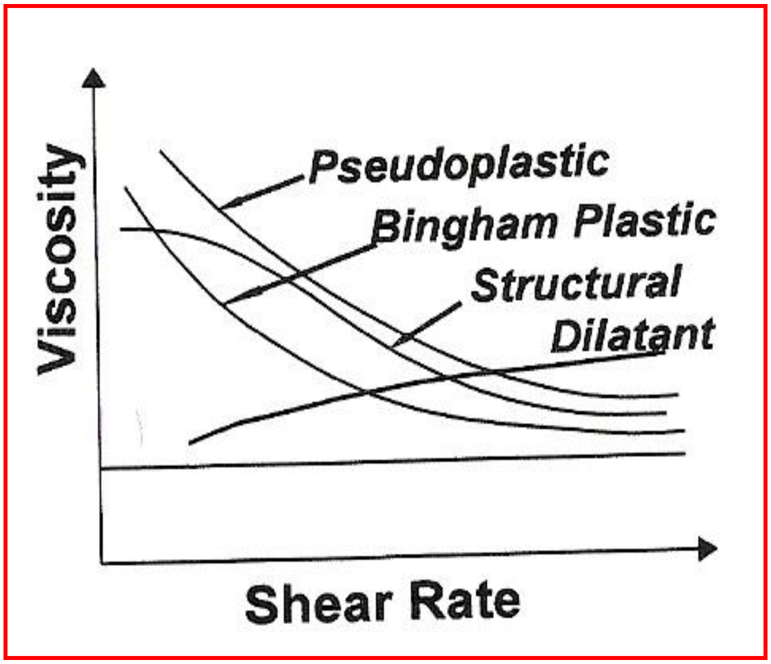

Figure 3-4.

In general the definitions of Newtonian and non-Newtonian fluids as follows:

Newtonian fluids: fluids which obey Newton's law of viscosity [$\tau_w = \mu(-\frac{du}{dy})$].

Non-Newtonian fluids: fluids which do not obey Newton's law of viscosity [for Bingham plastic fluids[$\tau_w = \tau_B + \mu_a(-\frac{du}{dy})$].

3-2. Generalized Reynolds number for flow in pipes:

For Newtonian flow in a pipe, the Reynolds number is defined by:

$$Re = \frac{\rho u d_i}{\mu} \quad (3.1)$$

In case of Newtonian flow, it is necessary to use an appropriate apparent viscosity (μ_a). In flow in a pipe, where the shear stress varies with radial location, the value of μ_a varies. The value of μ_a evaluated at wall is given by:

$$\mu_a = -\frac{\text{shear stress at wall}}{\text{shear rate at wall}} = \frac{\tau_w}{(-du/dy)_w} \quad (3.2)$$

Another definition is based, not on the true shear rate at the wall, but on the flow characteristic. This quantity, which may be called the apparent viscosity for pipe flow, is given by:

$$\mu_{ap} = \frac{\text{shear stress at wall}}{\text{flow characteristic}} = \frac{\tau_w}{8u/d_i} \quad (3.3)$$

For laminar flow, μ_{ap} has the property that it is the viscosity of a Newtonian fluid having the same flow characteristic as the non-Newtonian fluid when subjected to the same value volumetric flow rate for the same pressure gradient in the same pipe. This suggests that μ_{ap} might be a useful quantity for correlating flow rate-pressure gradient data for non-Newtonian flow in pipes. This is found to be the case and it is on μ_{ap} that a generalized Reynolds number Re' is based:

$$Re' = \frac{\rho u d_i}{\mu_{ap}} \quad (3.4)$$

Representing the fluid's laminar flow behavior in terms of K' and n':

$$\tau_w = K' \left(\frac{8u}{d_i}\right)^{n'} \quad (3.5)$$

The pipe flow apparent viscosity, defined by equation 3.3, is given by:

$$\mu_{ap} = \frac{\tau_w}{8u/d_i} = K' \left(\frac{8u}{d_i}\right)^{n'-1} \qquad (3.6)$$

When this equation for μ_{ap} is substituted into equation 3.4, the generalized Reynolds number takes the from:

$$Re' = \frac{\rho u^{2-n'} d_i^{n'}}{8^{n'-1} K'} \qquad (3.7)$$

Use of this generalized Reynolds number was suggested by Metzner and Reed (1955). For Newtonian behavior, K' = μ and n' = 1 so that the generalized Reynolds number reduces to the normal Reynolds number.

3-3. Turbulent flow of inelastic non-Newtonian fluids in pipes:

Turbulent flow of Newtonian fluids is described in terms of the Fanning friction factor, which is correlated against the Reynolds number with the relative roughness of the pipe wall as a parameter. The same approach is adopted for non-Newtonian flow but the generalized Reynolds number is used,

The fanning friction factor is defined by:

$$f = \frac{\tau_w}{\frac{1}{2}\rho u^2} \qquad (3.8)$$

For laminar flow of a non-Newtonian fluid, the wall shear stress can be expressed in terms of K' and n' as:

$$\tau_w = K' \left(\frac{8u}{d_i}\right)^{n'} \qquad (3.9)$$

On substituting for τ_w equation 3.8, the Fanning friction factor for laminar non-Newtonian flow becomes:

$$f = \frac{16}{Re'} \tag{3.10}$$

This is of the same form as equation 2.13 for Newtonian flow and is one reason for using this form of generalized Reynolds number. Equation 3.10 provides another way calculating the pressure gradient for a given flow rate for laminar non-Newtonian flow.

3-3-1. Laminar-turbulent transition:

A stability analysis made by Ryan and Johnson (1959) suggests that the transition from laminar to turbulent flow for inelastic non-Newtonian fluids occurs at a critical value of the generalized Reynolds number that depends on the value n'. The results of this analysis are shown in Figure 3-5. This relationship has been for shear thinning and for Bingham plastic fluids and

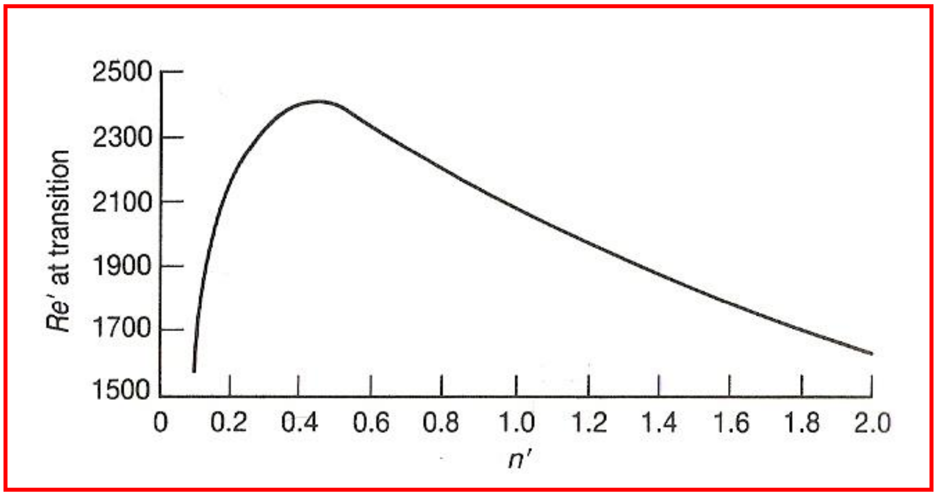

Figure 3-5 Variation of critical value of the Reynolds number with n'.

has been found to be accurate. Over the range of shear thinning behavior encountered in practice, $0.2 \leq n' < 1.0$, the critical value of Re' is in the range $2100 \leq Re' \leq 2400$.

3-3-2. Friction factors for turbulent flow in smooth pipes :

Experimental results for the Fanning friction factor for turbulent flow of shear thinning fluids in smooth pipes have correlated by Dodge and Metzner (1959) as a generalized form of the von Kármán equation:

$$\frac{1}{f^{1/2}} = \frac{4.0}{(n')^{0.75}} \log\left[f^{(1-n'/2)}\, Re'\right] - \frac{4.0}{(n')^{1.2}} \qquad (3.11)$$

This correlation is shown in Figure 3-6. The broken lines represent extrapolation of equation 3.11 for values of n' and Re' beyond those of the measurements made by Dodge and Metzner. More recent studies tend to confirm the findings of Dodge and Metzner but do not significantly extend the range of applicability.

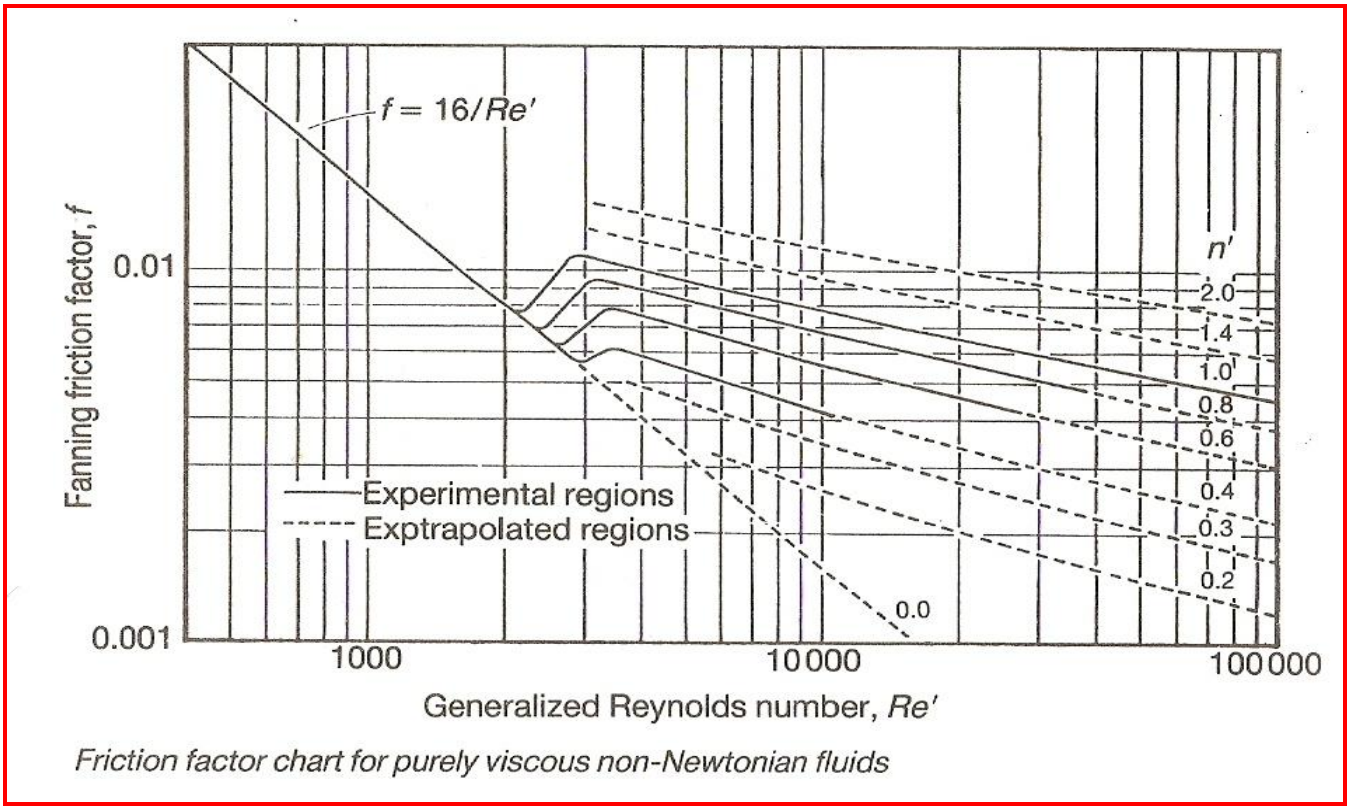

Figure 3-6 Friction factor chart for purely viscous non-Newtonian fluids.

Having determined the value of the friction factor f for specified flow rate and hence Re', the pressure gradient can be calculated in the normal way using equation 2.12.

$$\Delta P_f = 4f\left(\frac{L}{d_i}\right)\frac{\rho u^2}{2} = \frac{2fL\rho u^2}{d_i} \qquad (3.12)$$

3-4. Power law fluids:

The methods presented in mentioned Sections are general and do not require the assumption of any particular flow model. While the flow of power law fluids and Bingham plastics can be treated by those methods, some results specific to these materials will be considered in this section.

It was mentioned earlier that for laminar flow of a non-Newtonian fluid, the wall shear stress can be expressed in terms of K' and n' as:

$$\tau_w = K' \left(\frac{8u}{d_i}\right)^{n'} \quad (3.13)$$

The relationship between n' and n, and K' and K will now be demonstrated.

For the conditions at the pipe wall, denoted by the subscript w, the equation of power law fluid can be written as:

$$\tau_w = K\,(-\dot{\gamma}_w)^n = K\left(-\frac{\mathrm{d}u}{\mathrm{d}y}\right)_w^n \quad (3.14)$$

The minus sign has been placed inside the parentheses recognizing the fact that the shear rate $\dot{\gamma}$ (equal to $\mathrm{d}u/\mathrm{d}y$) is negative. $\dot{\gamma}_w$ is the true shear rate at wall and is related to the flow characteristic ($8u/d_i$) by the Rabinowitsch- Mooney equation:

$$-\dot{\gamma}_w = \frac{8u}{d_i}\left(\frac{3n'+1}{4n'}\right) \quad (3.15)$$

Therefore the behavior of a power law fluid, evaluated at the wall conditions, is given by:

$$\tau_w = K\left(\frac{8u}{d_i}\right)^n \left(\frac{3n'+1}{4n'}\right)^n \quad (3.16)$$

Equation 3.15 shows that a plot of ln τ_w against ln ($8u/d_i$) has a constant gradient of n. Consequently, for a power law fluid:

$$n' = n \quad (3.17)$$

Comparing equation 3.16 with equation 3.13 (both with $n' = n$), shows that:

$$K' = K\left(\frac{3n+1}{4n}\right) \quad (3.18)$$

Nomenclature

di	inside diameter of the pipe, m
f	fanning friction factor, dimensionless
K	consistency coefficient, Pa s^n
K'	consistency coefficient for pipe flow, Pa s^n
L	length of pipe, m
n	power law index, dimensionless
n'	flow characteristic, dimensionless
Re	Reynolds number, dimensionless
Re'	generalized Reynolds number, dimensionless
u	volumetric average velocity, m/s
$\dot{\gamma}$	shear rate = $\mathrm{d}u/\mathrm{d}y$, s^{-1}
μ	dynamic viscosity, Pa s
μ_a	apparent viscosity at wall, Pa s
μ_{ap}	apparent viscosity for pipe flow, Pa s
ρ	fluid density, kg /m^3
τ	shear stress, Pa
τ_w	shear stress at wall, Pa
τ_B	yield stress, Pa

Examples-Chapter 3

Example 3-1

A general time-independent non-Newtonian liquid of density 961 kg/m^3 flows steadily with an average velocity of 2.0 m/s through a tube 3.048 m long with an inside diameter of 0.0762 m. for these conditions, the pipe flow consistency coefficient K' has a value of 1.48 Pa s$^{0.3}$ and n' a value of 0.3. Calculate the values of the apparent viscosity for pipe flow μ_{ap}, the generalized Reynolds number Re' and the pressure drop across the tube, neglecting end effects.

Solution:

Apparent viscosity for pipe flow is given by:

$$\mu_{ap} = K' \left(\frac{8u}{d_i}\right)^{n'-1}$$

and the generalized Reynolds number by:

$$Re' = \frac{\rho u d_i}{\mu_{ap}}$$

The flow characteristic is given by:

$$\frac{8u}{d_i} = \frac{8\ (2.0\ \text{m/s})}{0.0762\ \text{m}} = 210\ \text{s}^{-1}$$

and

$$\left(\frac{8u}{d_i}\right)^{n'-1} = (210\ \text{s}^{-1})^{(0.3-1.0)} = 0.0237\ \text{s}^{0.7}$$

Hence

$$\mu_{ap} = (1.48\ \text{Pa s}^{0.3})(0.0237\ \text{s}^{0.7}) = 0.0351\ \text{Pa s}$$

and

$$Re' = \frac{(961\ \mathrm{kg/m^3})(2.0\ \mathrm{m.s})(0.0762\ \mathrm{m})}{(0.0351\ \mathrm{Pa\ s})} = 4178$$

From Figure 3-6, the Fanning friction factor f has a value 0.0047.

Therefore the pressure drop is given by:

$$\Delta P_f = 4f\left(\frac{L}{d_i}\right)\frac{\rho u^2}{2} = \frac{2fL\rho u^2}{d_i}$$

$$\Delta P_f = \frac{2\ (0.0047)(3.048\ \mathrm{m})(961\mathrm{kg/m^3})(2.0\ \mathrm{m/s})^2}{(0.0762\mathrm{m})} = 1445\ \mathrm{Pa}$$

Example 3-2

Calculate the frictional pressure gradient $\Delta P_f/L$ for a time independent non-Newtonian fluid in steady state flow in a cylindrical tube if:

the liquid density	ρ = 1000 kg/m^3
the inside diameter of the tube	d_i = 0.08 m
the mean velocity	u = 1 m/s
the point pipe consistency coefficient	K' = 2 Pa s$^{0.5}$
and the flow behavior index	n' = 0.5

Solution:

Apparent viscosity for pipe flow is given by:

$$\mu_{ap} = K'\left(\frac{8u}{d_i}\right)^{n'-1}$$

and the generalized Reynolds number by:

$$Re' = \frac{\rho u d_i}{\mu_{ap}}$$

The flow characteristic is given by:

$$\frac{8u}{d_i} = \frac{8\ (1.0\ \mathrm{m/s})}{0.08\ \mathrm{m}} = 100\ \mathrm{s}^{-1}$$

and

$$\left(\frac{8u}{d_i}\right)^{n'-1} = (100\ \mathrm{s}^{-1})^{(0.5-1.0)} = 0.1\ \mathrm{s}^{0.5}$$

Hence

$$\mu_{ap} = (2\ \mathrm{Pa\ s}^{0.5})(0.1\ \mathrm{s}^{0.5}) = 0.2\ \mathrm{Pa\ s}$$

and

$$Re' = \frac{(1000\ \mathrm{kg/m^3})(1.0\ \mathrm{m.s})(0.08\ \mathrm{m})}{(0.2\ \mathrm{Pa\ s})} = 400$$

From Figure 3-6, at Re' = 400, the Fanning friction factor $f = \frac{16}{Re'}$:

then $$f = \frac{16}{400} = 0.04.$$

Therefore the frictional pressure gradient $\Delta P_f/L$ is given by:

$$\Delta P_f/L = \frac{2 f \rho u^2}{d_i}$$

$$\Delta P_f/L = \frac{2\ (0.04)(1000\ \mathrm{kg/m^3})(1.0\ \mathrm{m/s})^2}{(0.08\ \mathrm{m})} = 1000\ \mathrm{Pa/m}$$

Example 3-3

The rheological properties of a particular suspension can be approximate reasonably well by either a "power law" or a "Bingham plastic" model and the shear rate range of 10 to 50 s^{-1}. If the consistency coefficient, K is 10 N s^n/m^2 and the flow behavior index, n is 0.2 in the power law model. What will the approximate values of the yield stress and of the plastic viscosity in the Bingham plastic model?

$$\tau_w = K\left(-\frac{du}{dy}\right)^n$$

$$\tau_w = 10\left(-\frac{du}{dy}\right)^{0.2}$$

When

$$\frac{du}{dy} = 10\ s^{-1} \quad , \quad \tau_w = 10 * (10)^{0.2} = 15.85\ N/m^2$$

$$\frac{du}{dy} = 50\ s^{-1} \quad , \quad \tau_w = 10 * (50)^{0.2} = 21.87\ N/m^2$$

Using Bingham plastic model:

$$\tau_w = \tau_B + \mu_a\left(-\frac{du}{dy}\right)$$

$$15.85 = \tau_B + \mu_a (10),$$

$$21.8\ 7 = \tau_B + \mu_a (50),$$

Thus: μ_a **= 0.15 N.s/m^2**

and : τ_B **=14.35 N/m^2**

Thus, Bingham plastic equation is: $\tau_w = 14.35 + 0.15\left(-\frac{du}{dy}\right)$

Example 3-4

A Newtonian liquid of viscosity, μ is 0.1 N.s/m^2 is flowing through a pipe. As a result of a process change a small quantity of polymer is added to the liquid and this causes the liquid to exhibit non-Newtonian characteristics; its rheology is described adequately by the "power law" model and the flow index, n is 0.33. The apparent viscosity (μ_a) of the modified fluid is equal to the viscosity (μ) of the original liquid at a shear rate, du/dy is 1000 s^{-1}. Determine the rheological equation?

Solution:

$$\tau_w = K\,(-\frac{du}{dy})^n$$

$$\tau_w = K\left(-\frac{du}{dy}\right)^{n-1} * \left(-\frac{du}{dy}\right) \qquad \text{.......... (1)}$$

Since $$\mu_a = \frac{\tau_w}{\left(-\frac{du}{dy}\right)} \qquad \text{.......... (2)}$$

Substitute equation (1) in equation (2), get equation (3):

$$\mu_a = K\left(-\frac{du}{dy}\right)^{n-1} \qquad \text{.......... (3)}$$

$$\mu_a = K\,(1000)^{0.33-1}$$

$$0.1 = K\,(1000)^{-0.67}$$

K = 10 Pa s^n

Rheological equation is: $\tau_w = 10\,(-\frac{du}{dy})^{0.33}$