Chapter 3

The First Law of Thermodynamics-Closed Systems

3-1. Introduction to the first law of thermodynamics

The first law of thermodynamics can simply be stated as follows: During an interaction between a system and its surroundings, the amount of energy gained by the system must be exactly equal to the amount of energy lost by surroundings. Energy can cross the boundary of closed system in two distinct forms: heat and work (Fig. 3-1). It is important to distinguish between these two forms of energy.

![Energy can cross the boundaries of a closed system in the form of heat and work.](image)

3-2. Heat

Heat is defined as the form of energy that is transferred between two systems (or a system and its surroundings) by virtue of a temperature difference. That is, an energy interaction is heat only if it takes place because of a temperature difference (Fig. 3-2).
In thermodynamics, however, heat and internal energy are two different things. Energy is a property, but heat is not. A body contains energy, but not heat. Energy is associated with a state; heat is associated with a process.

Heat is energy in transition. It is recognized only as it crosses the boundaries of a system. Consider the hot baked potato one more time. The potato contains energy, but this called heat only as it passes through the skin of potato (the system boundary) to reach the air, as shown in Fig. 3-3. Once in the surroundings, the heat becomes part of the internal energy of the surroundings. Thus in thermodynamics the term heat simply means heat transfer.
A process during which there is no heat transfer is called an adiabatic process (Fig. 3-4).

As a form of energy, heat has energy units, kJ (or Btu) being the most common one. The amount of heat transferred during the process between two states (states 1 and 2) is denoted \( Q_{12} \), or just \( Q \). Heat transfer *per unit mass* of a system is denoted \( q \) and is determined from:

\[
q = \frac{Q}{m} \quad \text{(kJ/kg)}
\]  

(3-1)

Sometimes it is desirable to know the rate of heat transfer (the amount of heat transferred per unit time) instead of the total heat transferred over some time interval (Fig. 3-5). The heat transfer rate is denoted \( \dot{Q} \), and has the unit kJ/s, which is equivalent to kW.

**Figure 3-4.** During an adiabatic process, a system exchanges no heat with its surroundings.

**Figure 3-5.** The relationships among \( q, Q, \) and \( \dot{Q} \).
The universally accepted sign convention for heat is as follows: *Heat transfer to a system is positive, and heat transfer from a system is negative* (Fig. 3-6).

**3-2. Work**

*Work*, like heat, is an energy interaction between a system and its surroundings. As mentioned earlier, *energy* can cross the *boundary* of a closed system in the form of *heat* or *work*. Therefore, *if the energy crossing the boundary is not heat, it must be work* (Fig. 3-7).
Work is also a form of energy like heat and, therefore, has energy units such as kJ. The work done during a process between state 1 and 2 is denoted $W_{12}$, or simply $W$. The work done per unit mass of the system is denoted $w$ and is defined as:

$$w = \frac{W}{m} \text{ (kJ/kg)}$$  \hspace{1cm} (3-2)

The work done per unit time is called power and denoted $\dot{W}$ (Fig.3-8). The unit of power is kJ/s, or kW.

![Figure 3-8. The relationships among $w$, $W$, and $\dot{W}$](image)

The production of work by a system is viewed as a desirable, positive effect and the consumption of work as an undesirable, negative effect. The sign convention for work adapted in this text reflects this philosophy: Work done by a system is positive, and work done on a system is negative (Fig.3-9).
Heat and work are *interactions* between a system and its surroundings, and there are many similarities between the two:

1. Both are recognized at the boundaries of the system as they cross them. That is, both heat and work are *boundary phenomena*.

2. Systems possess energy, but not heat or work. That is, heat and work are *transient phenomena*.

3. Both are associated with a *process*, not a state. Unlike properties, heat or work has no meaning at a state.

4. Both are *path functions* (i.e., their magnitudes depend on the path followed during a process as well as the end states).
**Example 3-3. (3-4)**

A well-insulated electric oven is being heated through its heating element. If the entire oven, including the heating element, is taken to be the system, determine whether this is a heat or work interaction.

**Solution**

For this problem the interior surfaces of the oven form the system boundary, as shown in Fig. 3-13. The energy content of the oven obviously increases during this process, as evidence by a rise in temperature. This energy transfer to the oven is not caused by a temperature difference between the oven and the surrounding air. It is caused by negatively charged particles called *electrons* crossing the system boundary and thus doing work. Therefore, this is a work interaction.

![Figure 3-13. Schematic for Example 3-3.](image)
Example 3-4. (3-5)

Answer the equation in Example 3-3 if the system is taken as only the air in the oven without the heating element.

Solution

This time the system boundary will include the outer surface of the heating element and will not cut through it, as shown in Fig. 3-14. Therefore, **no electrons will be crossing the system boundary** at any point. Instead, the energy generated in the interior of the heating element will be transferred to the air around it as a result of the **temperature difference** between the heating element and the air in the oven. Therefore, this is a heat transfer process.

For both cases, the **amount of energy transfer** to the air is **the same**. These two examples show that the same interaction can be heat or work depending on how the system is selected.
Electrical Work.

In an electric field, electrons in a wire move under the effect of electromotive forces, doing work. When \( N \) electrons move through a potential difference \( V \), the electrical work done is:

\[
W_e = VN \text{ (kJ)}
\]

This can also be expressed in the rate form as:

\[
\dot{W}_e = VI \text{ (kW)} \quad (3-3)
\]

Where \( \dot{W}_e \) is the electrical power and \( I \) is the number of electrons flowing per unit time, i.e., the current (Fig. 3-15). In general, both \( V \) and \( I \) vary with time, and the electrical work done during a time interval \( \Delta t \) is expressed as:

\[
W_e = \int_1^2 VI \, dt \text{ (kJ)} \quad (3-4)
\]

If both \( V \) and \( I \) remain constant during the time interval \( \Delta t \), this equation will reduce to:

\[
W_e = VI \Delta t \text{ (kJ)} \quad (3-5)
\]

3-4. Mechanical forms of work

In elementary mechanics, the work \( W \) done by a constant force \( F \) on a body which is displaced distance \( s \) in the direction of the force (Fig. 3-17) is given by:

\[
W = Fs \text{ (kJ)} \quad (3-6)
\]
If the force $F$ is not constant, the work done is obtained by adding (i.e., integrating) the differential amount of work (force times the differential displacement $ds$):

$$W = \int_{1}^{2} F \, ds \quad \text{(kJ)} \quad (3-7)$$

In many thermodynamic problems, mechanical work is the only form of work involved. It is associated with the movement of the boundary of a system or with the movement of the entire system as a whole. Some common forms of mechanical work are discussed below.

1. **Moving boundary work**

One form of mechanical work frequently encountered in practice is associated with the expansion or compression of a gas in a piston-cylinder device. The expansion and compression work is often called moving boundary work, or simply boundary work (Fig. 3-19).
Consider the gas enclosed in the piston-cylinder arrangement shown in Fig. 3-20. The initial pressure of the gas is $P$, the total volume is $V$ and the cross-sectional area of the piston is $A$. If the piston is allowed to move a distance $ds$ in a quasi-equilibrium manner, the differential work done during this process is:

$$\delta W_b = F ds = PA ds = P dV \quad (3-8)$$

![Figure 3-20](image.png)

The total boundary work done during the entire process as the piston moves is obtained by adding all the differential works from the initial state to the final state:

$$W_b = \int_1^2 P \, dV \quad \text{(kJ)} \quad (3-9)$$

That is, $P = f(V)$ showed be a available. Note that $P = f(V)$ is simply the equation of the process path on a $P-V$ diagram.

The quasi-equilibrium expansion process described above is shown on a $P-V$ diagram in Fig.3-21. On this diagram, the differential area $dA$ is equal to $P dV$, which is the differential
work. The total area $A$ under the process curve 1-2 is obtained by adding these differential areas:

$$\text{Area} = A = \int_{1}^{2} dA = \int_{1}^{2} P \, dV$$

![Figure 3-21. The area under the process curve on a $P$-$V$ diagram represents the boundary work.](image)

A comparison of this equation with Eq. 3-9 reveals that the area under the process curve on a $P$-$V$ diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system. (On the $P$-$v$ diagram, it represents the boundary work done per unit mass).

A gas can follow several different paths as it expands from state 1 to state 2. In general, each path will have a different area underneath it, and since this area represents the magnitude of the work, the work done will be different for each process (Fig.3-22). This is expected since work is path function (i.e., it depends on the path followed as well as the end states).
The cycle shown in Fig. 3-23 produces a net work output because the work done by the system during expansion process (area under path $A$) is greater than the work done on the system during the compression part of the cycle (area under path $B$), and the difference between these two is the net work done during the cycle (the color area).

The use of the boundary work relation (Eq. 3-9) is not limited to the quasi-equilibrium processes of gas only. It can also be used for solids and liquids.

*Figure 3-22. The boundary work done during a process depends on the path followed as well as the end states.*

*Figure 3-23. The net work done during a cycle is difference between the work done by the system and the work done on the system.*
Example 3-6. (3-7)
A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.

Solution
A sketch of the system and the $P$-$V$ diagram of the process are shown in Fig. 3-24. Assuming the process to be quasi-equilibrium, the boundary work can be determined from Eq. 3-9:

$$W_b = \int_1^2 P \, dV = 0$$

This is expected since a rigid tank has a constant volume and $dV = 0$ in the above equation. Therefore, there is no boundary work done during this process. That is, the boundary work done during a constant-volume process is always zero. This is also evident from the $P$-$V$ diagram of the process (the area under the process curve is zero).

Example 3-7. (3-8)
A frictionless piston-cylinder device contains 10 lbm of water vapor at 60 psia and 320°F. Heat is now added to the steam until the temperature reaches 400°F. If the piston is not attached to a
shaft and its mass is constant, determine the work done by the steam during this process.

Solution

A sketch of the system and the \( P-V \) diagram of the process are shown in Fig. 3-25. Even though it is not explicitly stated, the pressure of the steam within the cylinder remains constant during this process since both the atmospheric pressure and the weight of the piston remain constant. Therefore, this is a constant-pressure process, and from Eq. 3-9:

\[
W_b = \int_1^2 P \, dV = P_0 \int_1^2 dV = P_0 (V_2 - V_1)
\]

or

\[
W_b = mP_0(v_2 - v_1) \quad (3-10)
\]

since \( V = mv \). From the superheated vapor table (Table A-6E), the specific volumes are determined to be \( v_1 = 7.485 \text{ ft}^3/\text{lbm} \) at state 1 (60 psia, 320°F) and \( v_2 = 8.353 \text{ ft}^3/\text{lbm} \) at state 2 (60 psia, 400°F). Substituting these values yields:
\[ W_b = (10 \text{ lbm})(60 \text{ psia}) \left( (8.353 - 7.485) \text{ ft}^3/\text{lbm} \right) \left( \frac{1 \text{ Btu}}{5.404 \text{ psia ft}^3} \right) \]

\[ = 96.4 \text{ Btu} \]

The positive sign indicates that the work is done by the system. That is, the steam used 96.4 Btu of its energy to do this work. The magnitude of this work could also be determined by calculating the area under the process curve on the \( P-V \) diagram, which is \( P_0 \Delta V \) for this case.

**Example 3-8.** (3-9)

A piston-cylinder device initially contains 0.4 m\(^3\) of air at 100 kPa and 80\(^\circ\)C. The air is now compressed to 0.1 m\(^3\) in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

**Solution**

A sketch of the system and the \( P-V \) diagram of the process are shown in Fig. 3-26. At the specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values \( T_{cr} = -147^\circ \text{C}, \ P_{cr} = 3390 \text{ kPa} \) for nitrogen, the main constituent of air). For an ideal gas at constant temperature \( T_0 \),

![Figure 3-26. Schematic and \( P-V \) diagram for Example 3-8.](image-url)
\[ PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V} \]

where \( C \) is a constant. Substituting this into Eq. 3-9, we have:

\[ W_b = \int_1^2 P \, dV = \int_1^2 \frac{C}{V} \, dV = C \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{V_2}{V_1} \quad (3-11) \]

In the above equation, \( P_1 V_1 \) can be replace by \( P_2 V_2 \) or \( mRT_0 \). Also, \( V_2 / V_1 \) can be replaced by \( P_2 / P_1 \) for this case since \( P_1 V_1 = P_2 V_2 \). Substituting the numerical values into the above equation yields:

\[ W_b = (100 \text{ kPa})(0.4 \text{ m}^3) \left( \ln \frac{0.1}{0.4} \right) \left( \frac{1 \text{ kJ}}{1 \text{ kPa.m}^3} \right) = -55.45 \text{ kJ} \]

The negative sign indicates that this work is done on the system, which is always the case for compression process.

**Example 3-9.**

During expansion and compression processes of real gases, pressure and volume are often related by \( PV^n = C \), where \( n \) and \( C \) are constants. A process of this kind is called a polytropic process. Develop a general expression for the work done during a polytropic process.

**Solution**

A sketch of the system and the \( P-V \) diagram of the process are shown in Fig. 3-27. The pressure for a polytropic process can be expressed as:

\[ PV^n = C \]

Substitution this relation into Eq. 3-9, we obtain:
\[ W_b = \int_1^2 P \, dV = \int_1^2 CV^{-n} \, dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2 V_2 - P_1 V_1}{1-n} \]  

(3-12)

Since \( C = P_1 V_1^n = P_2 V_2^n \). For an ideal gas \((PV = mRT)\), this equation can also be written as:

\[ W_b = \frac{mR(T_2 - T_1)}{1-n} \text{ (kJ)} \]  

(3-13)

The special case of \( n = 1 \) is equivalent to the isothermal process discussed in the previous example.

2 Gravitational work

The gravitational work can be defined as the work done by or against a gravitational force field. In a gravitational field, the force acting on a body is:

\[ F = mg \]
Where \( m \) is mass of the body and \( g \) is the acceleration of gravity which is assumed to be constant. Then the work required to raise this body from level \( z_1 \) to level \( z_2 \).

\[
W_g = \int_{z_1}^{z_2} F \, dz = mg \int_{z_1}^{z_2} dz = mg (z_2 - z_1) \quad \text{(kJ)} \tag{3-14}
\]

where \( z_2 - z_1 \) is the vertical distance traveled (Fig. 3-28). This expression is easily recognized as the change in potential energy. We conclude from Eq. 3-14 that the gravitational work depends on the end states only and is independent of the path followed.

**Example 3-10.**

Determine the work done by a person to lift a 50-lbm suitcase shown in Fig. 3-29 by 1 ft.

**Solution**

By assuming a standard gravitation and using Eq. 3-14, the work done is:

\[
W_g = mg (z_2 - z_1)
\]

\[
W_g = (50 \text{ lbm})(32.174 \text{ ft/s}^2)(1 \text{ ft}) \left( \frac{1 \text{ Btu}}{25.037 \text{ ft}^2/\text{s}^2} \right) = 0.064 \text{ Btu}
\]
That is, \(0.064\) Btu of work is needed to perform this task. The potential energy of the system (the suitcase) increases by \(0.064\) Btu during this process.

3 Accelerational work

The work associated with a change in velocity of a system is defined as the **accelerational work**. The accelerational work required to accelerate a body of mass \(m\) from an initial velocity of \(V_1\) to final velocity \(V_2\) (Fig. 3-30) is determined from the definition of acceleration and Newton’s second law:

\[
F = ma = \frac{dV}{dt}
\]

\[
F = m \frac{dV}{dt}
\]

The differential displacement \(ds\) is related to velocity \(V\) by:

\[
V = \frac{ds}{dt} \quad \rightarrow \quad ds = V \, dt
\]

Substituting the \(F\) and \(ds\) relations into the expression (Eq. 3-7), we obtain:
\[ W_a = \int_{1}^{2} F ds = \int_{1}^{2} \left( m \frac{dV}{dt} \right) (V dt) \]

\[ = m \int_{1}^{2} V \, dV = \frac{1}{2} m (V_2^2 - V_1^2) \quad \text{(kJ)} \quad (3-15) \]

The work done to accelerate a body is independent of path and is equivalent to the change in the kinetic energy of the body.

The sign of accelerational work is determined by inspection: positive if done by the system and negative if done on the system.

**Example 3-11.**

Determine the power required to accelerate a 900kg car shown in Fig. 3-31 from rest to a velocity of 80 km/h in 20 s on a level road.

**Figure 3-30.** Vehicles require more power (accelerational work per unit time) as they accelerate.

**Figure 3-31.** Schematic for Example 3-11.
Solution

The accelerational work is determined from Eq. 3-15 to be:

\[ W_a = \frac{1}{2} m (v_f^2 - v_i^2) \]

\[ W_a = \frac{1}{2} (900 \text{ kg}) \left[ \left( \frac{80000 \text{ m}}{3600 \text{ s}} \right)^2 - 0^2 \right] \left( \frac{1 \text{ kJ}}{1000 \text{ kg.m}^2/\text{s}^2} \right) \]

\[ = 222.2 \text{ kJ} \]

The average power is determined from:

\[ \dot{W}_a = \frac{W_a}{\Delta t} = \frac{222.2 \text{ kJ}}{20 \text{ s}} = 11.1 \text{ kW} \quad \text{(or 14.9 hp)} \]

Where, kW = 1.341 hp

4 Shaft work

Energy transmission with a rotating shaft is very common in engineering practice (Fig. 3-32). Often the torque \( \tau \) applied to the shaft is constant, which means that the force \( F \) applied is also constant. For a specified constant torque, the work done during \( n \) revolutions is determined as follows: A force \( F \) acting through a moment arm \( r \) generates a torque \( \tau \) (Fig. 3-33) which is determined from:

\[ \tau = Fr \quad \rightarrow \quad F = \frac{\tau}{r} \]

This force acts through a distance \( s \) which is related to the radius \( r \) by:

\[ s = (2\pi r) n \]

Then the shaft work is determined from Eq. 3-6:
\[ W_{sh} = FS = \left( \frac{\tau}{r} \right)(2\pi n) = 2\pi n \tau \quad (kJ) \quad (3-16) \]

The power transmitted through the shat is the work done per unit time, which can be expressed as:

\[ \dot{W}_{sh} = 2\pi \dot{n} \tau \quad (kW) \quad (3-17) \]

Where \( \dot{n} \) is the number of revolutions per unit time.

The sign of the shaft work is also determined by inspection: \textit{positive} if done by the system and \textit{negative} if done on the system.

**Example 3-12.**

Determine the power transmitted through the shaft of a car when the torque applied is 200 N.m and the shaft rotates at a rate of 4000 revolutions per minute (rpm).
Solution

A sketch of the car is given in Fig. 3-34. The shaft power is determined from Eq. 3-17:

\[ W_{\text{sh}} = 2\pi n\tau = (2\pi)(4000 \frac{1}{\text{min}})(200\text{N.m})\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{1\text{ kJ}}{1000\text{ N.m}}\right) \]

\[ = 83.7\text{ kW} \quad \text{(or 112.2 hp)} \]

5 Spring work

It is common knowledge that when a force is applied on a spring, the length of the spring changes (Fig. 3-35). When the length of the spring changes by a differential amount \(dx\) under the influence of force \(F\), the work done is:

\[ \delta W_{\text{spring}} = F \quad (3-18) \]

For linear elastic springs, the displacement \(x\) is proportional to the force \(F\) applied (Fig. 3-36). That is,

\[ F = kx \quad \text{(kN)} \quad (3-19) \]

Where,

\[ k = \text{spring constant (kN/m)}, \quad x = \text{displacement of the spring (m)} \]

\[ x = 0 \text{ when } F = 0 \]
Substituting Eq. 3-18 and integrating yield:

\[ W_{\text{spring}} = \frac{1}{2} k (x_2^2 - x_1^2) \]  \hspace{1cm} (3-20)

Where,

\( x_1 \) = initial displacement of the spring

\( x_2 \) = final displacement of the spring

Example 3-13.

A piston-cylinder device contains 0.05 m³ of a gas initially at 200kPa. At this state a linear spring which has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m²,
determine (a) the final pressure inside the cylinder, (b) the total work done by the gas, and (c) the fraction of this work done against the spring to compress it.

Solution

(a) A sketch of the system and the $P-V$ diagram of the process are shown in Fig. 3-37. The enclosed volume at the final state is:

$$V_2 = 2V_1 = (2)(0.05 \text{ m}^3)$$

$$= 0.1 \text{ m}^3$$

Then the displacement of the piston (and the spring) becomes:

$$x = \frac{\Delta V}{A} = \frac{(0.1 - 0.05)\text{m}^3}{0.25 \text{ m}^2} = 0.2 \text{ m}$$

The force applied by the linear spring at the final state is determined from Eq. 3-19 to be:

$$F = kx = (150 \text{ kN/m})(0.2 \text{ m}) = 30 \text{ kN}$$

The additional pressure applied by the spring on the gas at this state is:

$$P = \frac{F}{A} = \frac{30 \text{ kN}}{0.25 \text{ m}^2} = 120 \text{ kPa}$$

Under the effect of the spring, the pressure rises linearly from 200 kPa to:

Final pressure = (200 + 120) kPa = 320 kPa
(b) From Fig. 3-37, the area under the process curve is determined to be:

\[ |W| = \text{area} = \frac{(200 + 320)\text{kPa}}{2} \times [(0.1 - 0.05)\text{m}^3] \left( \frac{1\text{kJ}}{1\text{kPa.m}^3} \right) = 13\text{ kJ} \]

The sign of the work is positive, meaning that work is done by the system.

(c) The work represented by the rectangular area (region I) is done against the piston and the atmosphere, and the work represented by the triangular area (region II) is done against the spring. Thus:

\[ W_{\text{spring}} = \frac{1}{2} \times [(320 - 200)\text{kPa}] \times (0.05\text{m}^3) \left( \frac{1\text{kJ}}{1\text{kPa.m}^3} \right) = 3\text{ kJ} \]

or this result could also be obtained from Eq. 3-20:

\[ W_{\text{spring}} = \frac{1}{2} k (x_2^2 - x_1^2) \]

\[ = \frac{1}{2} (150\text{kN/m})[(0.2\text{m})^2 - 0^2] \left( \frac{1\text{kJ}}{1\text{kN.m}} \right) = 3\text{ kJ} \]

3-5. The first law of thermodynamics

The first law of thermodynamics, also known as the conservation of energy principle. Based on experimental observations, the first law of thermodynamics states that energy can be neither created nor destroyed; it can only change forms.

We all know that a rock at some elevation possesses some potential energy, and part of this potential energy is converted to kinetic energy as the rock falls (Fig. 3-39).

Consider first some processes that involve heat transfer but no work interactions. As a result of heat transfer to the potato as shown in (Fig. 3-40), the energy of the potato will increase. If we
disregard any mass transfer (moisture loss from the potato), the increase in the total energy of the potato becomes equal to the amount of heat transfer. Therefore, the conservation of energy principle for this case can be expressed as:

\[ Q = \Delta E. \]

Consider the heating of water in a pan on top of a range (Fig. 3-41). If 15kJ of heat is transferred to the water from the heating element and 3 kJ of it is lost from the water to the surrounding air, the increase in energy of the water will be equal to the net heat transfer to water, which is 12 kJ. That is:

\[ Q = Q_{\text{net}} = \Delta E \]
The above conclusions can be summarized as follows: *In the absence of any work interactions between a system and its surroundings, the amount of net heat transfer is equal to the change in energy of a closed system*. That is:

\[ Q = \Delta E \quad \text{when} \quad W = 0 \quad (3-21) \]

Now consider a well-insulated (i.e., adiabatic) room heated by an electric heater as our system (Fig. 3-42). The conservation of energy principle dictates that the electrical work done on the system must equal the increase in energy of the system. That is:

\[ W_e = \Delta E \quad \text{when} \quad Q = 0 \]

The negative sign is due to the Sign convention that work done on the system is *negative*.

Now let us replace the electric heater with a paddle wheel (Fig. 3-43). Again since there is no heat interaction between the system and its surroundings \((Q = 0)\), the paddle-wheel work done on the system must show up as an increase in the energy of the system. That is:

\[ -W_{pw} = \Delta E \]

*Figure 3-42. The work (electrical) done on an adiabatic system is equal to the increase in the energy of the system.*

*Figure 3-43. The work (shaft) done on an adiabatic system is equal to the*
Many of you have probably noticed that the temperature of air rises when it is compressed (Fig. 3-44). This is because energy is added to the air in the form of boundary work. In the absence of any heat transfer ($Q = 0$), the entire boundary work will be stored in the air as part of its total energy. The conservation of energy principle again requires that:

$$-W_b = \Delta E.$$  

It is clear from the foregoing examples that for adiabatic processes, the amount of work done is equal to the change in the energy of a closed system. That is:

$$-W = \Delta E \quad \text{when} \quad Q = 0 \quad (3-22)$$

To generalize our conclusions, the first law of thermodynamics, or the conservation of energy principle for a closed system or a fixed mass, may be expressed as follows:

$$\text{Net energy transfer to (or from) the system as heat and work} = \text{Net increase (or decrease) in the total energy of the system}$$

or

$$Q - W = \Delta E \quad (3-23)$$

where

$$Q = \text{net heat transfer across system boundaries} = \sum Q_{\text{in}} - \sum Q_{\text{out}}$$
\( W = \text{net work done in all forms} \ (= \sum W_{\text{out}} - \sum W_{\text{in}}) \)

\( \Delta E = \text{net change in total energy of system, } E_2 - E_1 \)

The total energy \( E \) of a system is considered to consist of three parts: internal energy \( U \), kinetic energy \( KE \), and potential energy \( PE \).

Then the \textbf{change in total energy} of a system during a process can be expressed:

\[ \Delta E = \Delta U + \Delta KE + \Delta PE \quad \text{(kJ)} \quad (3-24) \]

Substituting this relation into Eq. 3-23, we obtain:

\[ Q - W = \Delta U + \Delta KE + \Delta PE \quad \text{(kJ)} \quad (3-25) \]

Where \( \Delta U = m(u_2 - u_1) \)

\[ \Delta KE = \frac{1}{2} m(v_2^2 - v_1^2) \]

\[ \Delta PE = mg(z_2 - z_1) \]

Most \textbf{closed systems} encountered in practice are \textbf{stationary}, i.e., they do not involve any changes in their \textbf{velocity} or the \textbf{elevation} of their center of gravity during a process (Fig. 3-46). Thus for stationary closed systems, the change in kinetic and potential energies are negligible (that \( \Delta KE = \Delta PE = 0 \)), and the \textbf{first-law} relation reduces to:

\[ Q - W = \Delta U \quad \text{(kJ)} \quad (3-26) \]

\[ z_1 = z_2 \Rightarrow \Delta PE = 0 \]

\[ V_1 = V_2 \Rightarrow \Delta KE = 0 \]

\[ \Delta E = \Delta U \]

\textbf{Figure 3-46.} For stationary systems, \( \Delta KE = \Delta PE = 0 \); thus \( \Delta E = \Delta U \).
Sometimes it is convenient to consider the work term in two parts: \( Q - W_{other} \), where \( W_{other} \) represents all forms of work except the boundary work. The first law takes the following form:

\[
Q - W_{other} - W_b = \Delta E \quad \text{(kJ)} \tag{3-27}
\]

**Other forms of the first-law relation**

The first-law relation for closed systems can be written in various forms (Fig. 3-47). Dividing Eq. 3-23 by the mass of the system gives:

\[
q - w = \Delta e \quad \text{(kJ/kg)} \tag{3-28}
\]

The rate form of the first-law is obtained by dividing Eq. 3-23 by the time interval \( \Delta t \) and taking the limit as \( \Delta t \to 0 \). It yields:

\[
\dot{Q} - \dot{W} = \frac{dE}{dt} \quad \text{(kW)} \tag{3-29}
\]

Where, \( \dot{Q} \) is the rate of net heat transfer, \( \dot{W} \) is the power, and \( dE/dt \) is the rate of change of total energy.

Equation 3-23 can be expressed in the differential form as:

\[
\delta Q - \delta W = dE \quad \text{(kJ)} \tag{3-30}
\]

or

\[
\delta q - \delta w = de \quad \text{(kJ/kg)} \tag{3-31}
\]
For a cyclic process, the initial and final states are identical, and therefore \( \Delta E = E_2 - E_1 = 0 \). Then the first-law relation for a cycle simplifies to:

\[
Q - W = 0 \quad \text{(kJ)} \quad (3-32)
\]

That is, the net heat transfer and the net work done during a cycle must be equal (Fig. 3-48).

**Example 3-14.**

A rigid tank contains a hot fluid which is cooled while being stirred by a paddle wheel. Initially, the internal energy of the fluid is 800 kJ. During the cooling process, the fluid loses 500 kJ of heat, and the paddle wheel does 100 kJ of work on the fluid. Determine the final internal energy of the fluid. Neglect the energy stored in the paddle wheel.

**Solution**

We choose the fluid in the tank as our system. The system boundaries are indicated in Fig. 3-49. By applying the conservation of energy principle as given by Eq. 3-25 to this process, \( U_2 \) is determined to be:
\[ Q - W = \Delta U + \Delta KE + \Delta PE \]
\[ = \Delta U = U_2 - U_1 \]
\[-500 \text{ kJ} - (-100 \text{ kJ}) = U_2 - 800 \text{ kJ}\]
\[ U_2 = 400 \text{ kJ} \]

3-6. A systematic approach to problem solving

Thermodynamic problems, particularly the complicated ones, also require a systematic approach. By using a step-by-step approach, an engineer can solve a series of simple problems instead of one large, formidable problem (Fig. 3-50).

The proper approach to solving thermodynamic problems is illustrated below with the help of a sample problem.

Sample problem

A 0.1 m³ rigid tank contains steam initially at 500 kPa and 200°C. The steam is now allowed to cool until its temperature drops to 50°C. Determine the amount of heat transfer during this process and the final pressure in the tank.
**Step 1:** Draw a sketch and identify the system (Fig. 3-51):

![Figure 3-51](image)

**Step 2:** List the given information on the sketch (Fig. 3.52):

![Figure 3-52](image)

**Step 3:** Check for special processes (Fig. 3-53):

In our case, both the temperature and the pressure vary but the specific volume remains constant since rigid tanks have a fixed volume ($V = \text{constant}$), and the mass is fixed ($m = \text{constant}$).

![Figure 3-53](image)
Step 4: State any assumptions (Fig. 3-54):

The system in our case can be assumed to be stationary since there is no indication to the contrary. Thus the changes in kinetic and potential energies can be neglected.

Step 5: Apply the conservation equations (Fig. 3-55):

Step 6: Draw a process diagram (Fig. 3-56):

Process diagrams, such as the $P$-$v$ or $T$-$v$ diagrams, are extremely helpful in visualizing the initial and final states of a system and the path of the process.
Step 7: Determine the required properties and unknowns (Fig. 3-57):

Steam exists as a superheated vapor at the state 1 since the temperature is greater than the saturation temperature at the given pressure [that is, $200^\circ C > T_{\text{sat} @ 500 \text{ kPa}} = 151.9^\circ C$ (Table A-5)].

Example 3-15.

A piston-cylinder device contains 25 g of saturated water vapor which is maintained at a constant pressure of 300 kPa. A resistance heater within the cylinder is turned on and passes a current of 0.2 A for 5 min from a 120 V source. At the same time, a heat loss of 307 kJ occurs. (a) Show that for a closed system the boundary work $W_b$ and the change in internal energy $\Delta U$ in the first-law relation can be combined into one term, $\Delta H$, for a
constant-pressure process. \( (b) \) determine the final temperature of the steam.

**Solution**

\( (a) \) Neglecting the changes in kinetic and potential energies and expressing the work as the sum of boundary and other forms of work, Eq.3-25 simplifies to:

\[
0 \quad 0
\]

\[
Q - W = \Delta U + \Delta KE + \Delta PE \quad \text{when} \quad W = W_{\text{other}} + W_b \quad \text{then:}
\]

\[
Q - W_{\text{other}} - W_b = \Delta U = U_2 - U_1
\]

For a constant-pressure process, the boundary work is given by Eq. 3-10 as:

\[
W_b = P_0(V_2 - V_1). \quad \text{Substituting this into the above relation gives:}
\]

\[
Q - W_{\text{other}} - P_0(V_2 - V_1) = U_2 - U_1
\]

But

\[
P_0 = P_2 = P_1 \quad \rightarrow \quad Q - W_{\text{other}} = (U_2 + P_2 V_2) - (U_1 + P_1 V_1)
\]
Also $H = U + PV$, and thus:

$$Q - W_{\text{other}} = H_2 - H_1 \quad \text{(kJ)}$$

which is the desired relation (Fig. 3-59):

\[ Q - W_{\text{other}} = \Delta U \]

\[ Q - W_{\text{other}} = \Delta H \]

\(\text{Fig. 3-59. For a closed system undergoing a quasi-equilibrium } P = \text{constant process, } \Delta U + W_b = \Delta H\)

\(b\) For our case, the only other form of work is the electrical work which, which can be determined from Eq. 3-5:

$$W_e = VI\Delta t = (120 \text{ V})(0.2 \text{ A})(300 \text{ s}) \left( \frac{1 \text{ kJ}}{1000 \text{ VA.s}} \right) = 7.2 \text{ kJ}$$

\(\text{State 1: } P_1 = 300 \text{ kPa} \text{ sat. vapor} \}

\[ h_1 = h_g \text{ @ 300KPa} = 2725.3 \text{ kJ/kg} \quad \text{(Table A-5)} \]

The enthalpy at the final state can be determined from the conservation of energy relation for closed systems undergoing a constant-pressure process (Eq. 3-33):

$$Q - W_e = m(h_2 - h_1)$$

$$- 3.7 \text{ kJ} - (-7.2 \text{ kJ}) = (0.025 \text{ kg})( h_2 - 2725.3 \text{ kJ/kg})$$

$$h_2 = 2865.3 \text{ kJ/kg}$$
sine \( h_2 > h_1 \), the state 2 is superheated water

**State 2:**

\[
\begin{align*}
P_2 &= 300 \text{ kPa} \\
h_2 &= 2865.3 \text{ kJ/kg} \\
T_2 &= 200^\circ\text{C}
\end{align*}
\]  
(Table A-6)

**Example 3-16.**

A rigid tank with a volume of 3 ft\(^3\) is initially filled with refrigerant-12 at 120 psia and 140\(^\circ\)F. The refrigerant is now cooled to 20\(^\circ\)F. Determine (a) the mass of the refrigerant, (b) the final pressure in the tank, and (c) the heat transferred from the refrigerant.

**Solution.**

A sketch of the system and the \( T-v \) diagram of the process are given in Fig. 3-60.

(a) To find the mass, we need to know the specific volume of the refrigerant at the initial state, which is determined from Table A-13E:

**State 1:**

\[
\begin{align*}
P_1 &= 120 \text{ psia} \\
T_1 &= 140^\circ\text{F} \\
v_1 &= 0.389 \text{ ft}^3/\text{lbm} \\
u_1 &= 86.098 \text{ Btu/lbm}
\end{align*}
\]

Thus,

\[
m = \frac{V}{v_1} = \frac{3 \text{ ft}^3}{0.389 \text{ ft}^3/\text{lbm}} = 7.71 \text{ lbm}
\]

(b) This is a constant-volume process, and therefore \( v_2 = v_1 = 0.389 \text{ ft}^3/\text{lbm} \).

At 20\(^\circ\)F:

\[
\begin{align*}
v_f &= 0.01130 \text{ ft}^3/\text{lbm} \\
u_f &= 12.79 \text{ Btu/lbm} \\
v_g &= 1.0988 \text{ ft}^3/\text{lbm} \\
u_g &= 72.12 \text{ Btu/lbm}
\end{align*}
\]

(Table A-11E)
The refrigerant is a saturated liquid-vapor mixture at the final state since \( v_f < v_2 < v_g \). Therefore, the pressure must be the saturation pressure at \( 20^\circ\text{F} \):

\[
P_2 = P_{\text{sat}}@20^\circ\text{F} = 35.736 \text{ psia} \quad \text{(Table A-11E)}
\]

(c) Assuming \( \Delta KE = \Delta PE = 0 \) and realizing that there are no boundary or other forms of work interactions, we see that the first-law relation (Eq. 3-25) simplifies to:

\[
Q - W = \Delta U + \Delta KE + \Delta PE
\]

\[
Q = \Delta U = m(u_2 - u_1)
\]

To determine \( u_2 \), we first need to know the quality \( x_2 \) at the final state. It is determined from:

\[
x_2 = \frac{v_2 - v_f}{v_f g}
\]

\[
= \frac{[(0.389-0.0113)\text{m}^3/\text{kg}]}{[(1.0988-0.0113)\text{m}^3/\text{kg}]} = 0.348
\]

---

Figure 3-60. Schematic and \( T-v \) diagram for Example 3-16.
That is, 34.8 percent of the refrigerant is in the vapor form at the final state. Then,

\[ u_2 = u_f + x_2u_{fg} \]

\[ = 12.79 \text{ Btu/lbm} + (0.348)[(72.12 - 12.79) \text{ Btu/lbm}] \]

\[ = 33.44 \text{ Btu/lbm} \]

Finally, substituting these values into the first-law relation will give us the heat transfer:

\[ Q = (7.71 \text{ lbm})[(33.44 - 86.098) \text{ Btu/lbm}] \]

\[ = -406.0 \text{ Btu} \]

3-7. Specific heats

We know from experience that it takes different amounts of energy to raise the temperature of identical masses of different substances by one degree (Fig. 3-62). Therefore, it is desirable to have a property that will enable us to compare the energy storage capability of various substances. This property is the specific heat.

The specific heat is defined as the energy required to raise the temperature of a unit mass of a substance by one degree (Fig. 3-63). In thermodynamics, we are interested in two kinds of specific heats: specific heat at constant volume \( C_v \) and specific heat at constant pressure \( C_p \).
The specific heat at constant pressure $C_p$ is always greater than $C_v$ because at constant pressure the system is allowed to expand and the energy for this expansion work must also be supplied to the system as shown in (Fig. 3-64).

From the definition of $C_v$, this energy must be equal to $C_v \, dT$, where $dT$ is the differential change in temperature. Thus,

$$C_v \, dT = du$$

at constant volume

$$C_v = \left( \frac{\partial u}{\partial T} \right)_v \quad (3-34)$$

Similarly, an expression for the specific heat at constant pressure $C_p$ can be obtained by considering a constant-pressure process ($w_b + \Delta u = \Delta h$).

It yields

$$C_p = \left( \frac{\partial h}{\partial T} \right)_p \quad (3-35)$$

A common unit for specific heats is:

kJ/(kg.°C) or kJ/(kg.K),

these two units are identical since $\Delta T \text{ (°C)} = \Delta T \text{ (K)}$
Equations 3-34 and 3-35 are the defining equations for $C_v$ and $C_p$, and their interpretation is given in Fig. 3-65.

The energy required to raise the temperatures of substance by one degree will be different at different temperatures and pressures (Fig. 3-66). But the difference usually not large.

### Figure 3-66. The specific heat of a substance changes with temperature.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AIR</td>
<td>AIR</td>
<td></td>
</tr>
<tr>
<td>$m = 1$ kg</td>
<td>$m = 1$ kg</td>
<td></td>
</tr>
<tr>
<td>$300 \rightarrow 301$ K</td>
<td>$1000 \rightarrow 1001$ K</td>
<td></td>
</tr>
<tr>
<td>0.718 kJ</td>
<td>0.855 kJ</td>
<td></td>
</tr>
</tbody>
</table>

### Figure 3-65. Mathematical definitions of $C_p$ and $C_v$.

- $C_v = \frac{\partial u}{\partial T}$
  - the change in internal energy with temperature at constant volume

- $C_p = \frac{\partial h}{\partial T}$
  - the change in enthalpy with temperature at constant pressure

The specific heats are sometimes given on a molar basis. They are denoted by $\bar{C}_v$ and $\bar{C}_p$ and have the units kJ/(kmol.°C) or kJ/(kmol.K).

### 3-8. Internal energy, enthalpy and specific heats of ideal gases

We defines an ideal gas as a substance whose temperature, pressure, and specific volume are related by:
\[ Pv = RT \]

For ideal gas the internal energy is a function of the temperature only. That is,

\[ u = u(T) \]  \hspace{1cm} (3-36)

In the Joule's experiment, Joule submerged two tanks connected with a pipe and a valve in a water bath, as shown in Fig. 3-67. Since there was also no work done, he concluded that the internal energy of the air did not change even though the volume and the pressure changed. Therefore, he reasoned, the internal energy is a function of temperature only and not a function of pressure or specific volume.

Using the definition of enthalpy and the equation of state of an ideal gas, we have:

\[
\begin{align*}
    h &= u + Pv \\
    Pv &= RT \\
    h &= u + RT
\end{align*}
\]

Since \( R \) is constant and \( u = u(T) \), it follows that the enthalpy of an ideal gas is also a function of temperature only:

\[ h = h(T) \]  \hspace{1cm} (3-37)

Since \( u \) and \( h \) depend only on temperature for an ideal gas, the specific heats \( C_v \) and \( C_p \) also depend, at most, on temperature only. Therefore, at a given temperature \( u, h, C_v, \) and \( C_p \) of an ideal gas will have fixed values regardless of the specific volume or pressure (Fig. 3-68).
Then the differential changes in the internal energy and enthalpy of an ideal gas can be expressed as:

\[
du = C_v(T) \, dT \quad (3-38)
\]

and \( dh = C_p(T) \, dT \quad (3-39) \)

The change in internal energy or enthalpy for an ideal gas during a Process from state 1 to state 2 is

Determined by integration these equations:

\[
\Delta u = u_2 - u_1 = \int_{T_1}^{T_2} C_v(T) \, dT \quad \text{(kJ/kg)} \quad (3-40)
\]

and \( \Delta h = h_2 - h_1 = \int_{T_1}^{T_2} C_p(T) \, dT \quad \text{(kJ/kg)} \quad (3-41) \)

At low pressures all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only. The specific heats of real gases at low pressures are called ideal-gas specific heats or zero pressure specific heats, and are often denoted \( C_{p0} \) and \( C_{v0} \) (Table A-2 \( a, b, c \)). A plot of \( C_{p0}(T) \) data for some gases is given in Fig. 3-69.

![Figure 3-68. For ideal gases, \( u, h, C_v, \) and \( C_p \) vary with temperature only.](image)

![Figure 3-69. Ideal-gas constant – pressure specific heats for some gases (see Table A-2c for \( \bar{C}_{p0} \) equations).](image)
Some observations can be made from Fig. 3-69. First, the specific heats of gases with complex molecules (molecules with two or more atoms) are higher and increase with temperature. Also, the variation of specific heats with temperature is smooth and may be approximated as linear over small temperature intervals (a few hundred or less). Then the specific heat functions in Eqs. 3-40 and 3-41 can be replaced by the constant average specific heat values. Now the integration in these equations can be performed, yielding:

\[ u_2 - u_1 = C_{v,av}(T_2 - T_1) \quad \text{(kJ/kg)} \quad (3-42) \]
and
\[ h_2 - h_1 = C_{p,av}(T_2 - T_1) \quad \text{(kJ/kg)} \quad (3-43) \]

The specific heat values for some common gases are listed as a function of temperature in Table A-2b. The average specific heats \( C_{p,av} \) and \( C_{v,av} \) are evaluated from this table at the average temperature \( (T_2 + T_1)/2 \), as shown in Fig. 3-71.

Another way of determining the average specific heats is to evaluate them at \( T_1 \) and \( T_2 \) and then take their average. Usually both methods give reasonably good results, and one is not necessarily better than the other.

Another observation that can be made from Fig. 3-69 is that the ideal-gas specific heats of monatomic gases such as argon, neon, and helium remain constant over the entire temperature range. Thus \( \Delta u \) and \( \Delta h \) of monatomic gases can easily be evaluated from Eqs. 3-42 and 3-43.
Specific-heat relations of ideal gases

A special relationship between \( C_v \) and \( C_p \) for ideal gases can be obtained by differentiating the relation \( h = u + RT \), which yields:

\[
dh = du + RdT
\]

Replacing \( dh \) by \( C_p \, dT \) and \( du \) by \( C_v \, dT \) and dividing the resultant expression by \( dT \), we obtain:

\[
C_p = C_v + R \quad [\text{kJ/(kg.K)}] \quad (3-44)
\]

This is an important relationship for ideal gases since it enables us to determine \( C_v \) from a knowledge of \( C_p \) and the gas constant \( R \).

When the specific heats are given on a molar basis, \( R \) in the above equation should be replaced by the universal gas constant \( R_u \) (Fig. 3-74). That is,

\[
\bar{C}_p = \bar{C}_v + R_u \quad [\text{kJ/(kmol.K)}] \quad (3-45)
\]

At this point, we introduce another ideal-gas property called the specific heat ratio \( k \), defined as:

\[
k = \frac{C_p}{C_v} \quad (3-46)
\]

Figure 3-74. The \( C_p \) of an ideal gas can be determine from a knowledge of \( C_v \) and \( R \).
The specific heat ratio \((k)\) also varies with temperature, but this variation is very small. For monatomic gases, its value is essentially constant at \(k = 1.667\). Many diatomic gases, including air, have specific heat ratio of about \(k = 1.4\) at room temperature.

**Example 3-18.**

Air at 300 K and 200 kPa is heated at constant pressure to 600 K. Determine the change in internal energy of air per unit mass\((\Delta u)\), using (a) data from the air table (Table A-17), (b) the functional form of the specific heat (Table A-2c), and (c) the average specific heat value (Table A-2b).

**Solution.**

Air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values \((T_{cr} = -147^\circ C, P_{cr} = 3390 \text{ kPa} \text{ for nitrogen, the main constituent of air}).\) \(\Delta u\) of ideal gases depends on the initial and final temperatures only, and not on the type of process.

(a)

\[
\begin{align*}
\bar{u}_1 &= \bar{u} @ 300 \text{ K} = 214.07 \text{ kJ/kg} \\
\bar{u}_2 &= \bar{u} @ 600 \text{ K} = 434.78 \text{ kJ/kg} \\
\end{align*}
\]

Thus, \(\Delta u = \bar{u}_2 - \bar{u}_1 = (434.78 - 214.07) \text{ kJ/kg} = 220.71 \text{ kJ/kg}\)

(b)

The \(\bar{C}_p(T)\) of air is given in Table A-2c in the form of a third-degree polynomial expressed as:

\[
\bar{C}_p(T) = a + bT + cT^2 + dT^3
\]
Where \( a = 28.11, \ b = 0.1967 \times 10^{-2}, \ c = 0.4802 \times 10^{-5}, \ d = -1.966 \times 10^{-9}. \) From Eq. 3-45,
\[
\overline{C}_v(T) = \overline{C}_p - R_u = (a - R_u) + bT + cT^2 + dT^3
\]
From Eq. 3-40,
\[
\Delta \bar{u} = \overline{C}_v(T) \int \Delta T
\]
\[
= \int_{T_1}^{T_2} [(a - R_u) + bT + cT^2 + dT^3] dT
\]
Performing the integration and substitution the values, we obtain:
\[
\Delta \bar{u} = 6447.15 \text{ kJ/kmol}
\]
The change in the internal energy on a unit-mass basis (\( \Delta u \)) is determined by dividing this value by the molar mass of air (Table A-1):
\[
\Delta u = \frac{\Delta \bar{u}}{M} = \frac{6447.15 \text{ kJ/kmol}}{28.97 \text{ kg/kmol}} = 222.55 \text{ kJ/kg}
\]
This differs from the exact result by 0.8 percent:
\[
\% \text{ error} = \frac{(222.55 - 220.71) \text{ kJ/kg}}{(220.71) \text{ kJ/kg}} \times 100 \cong 0.8 \%
\]
(c) The average value of the constant-volume specific heat \( C_{v,av} \) is determined from Table A-2b at the average temperature \( (T_2 + T_1)/2 = (300 + 600) \text{ K}/2 = 450 \text{ K} \) to be:
\[
C_{v,av} = C_v @ 450 \text{ K} = 0.733 \text{ kJ/(kg.K)} \quad \text{(Table A-2b)}
\]
Thus
\[
\Delta u = C_{v,av}(T_2 - T_1) = [0.733 \text{ kJ/(kg.K)}][600 - 300] \text{ K}
\]
\[
= 219.9 \text{ kJ/kg}
\]
This answer differs the exact result (220.71 kJ/kg) by only 0.4 percent:
\[
\% \text{ error} = \frac{(220.71 - 219.9) \text{ kJ/kg}}{(220.71) \text{ kJ/kg}} \times 100 \approx 0.4 \% 
\]

**Example 3-19.**

An insulated rigid tank initially contains 1.5 lbm of helium at 80°F and 50 psia. A paddle wheel with a power rating of 0.02 hp is operated within the tank for 30 min. Determine (a) the final temperature and (b) the final pressure of the helium gas.

**Solution.**

A sketch of the system and the \( P-v \) diagram of the process are shown in Fig. 3-75. The helium gas at the specified conditions can be considered to be an ideal gas since it is at a very high temperature relative to its critical-point temperature (\( T_{cr} = -451°F \) for helium Table A-1E).

(a) The amount of paddle-wheel work done on the system is:

\[
W_{pw} = \dot{W}_{pw} \Delta t = (-0.02 \text{ hp})(0.5 \text{ h}) \left( \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right) = -25.45 \text{ Btu}
\]

Since, \( W_b = 0 \) (rigid tank have not moving boundaries)

\( Q = 0 \) (insulated tank)

\( \Delta KE = \Delta PE = 0 \) (The system is assumed to be stationary)

Then the conservation of energy equation for this closed system reduces to:

\[
\begin{align*}
0 & \quad 0 \quad 0 \quad 0 \quad 0 \\
\dot{Q} - W_{pw} - \dot{W}_b & = \Delta U + \Delta KE + \Delta PE \\
- W_{pw} & = \Delta U = m(u_2 - u_1) \cong mC_{v,av}(T_2 - T_1)
\end{align*}
\]
As we pointed out earlier, the ideal-gas specific heats of monatomic gases (helium being one of them) are constant. The \( C_v \) value of helium is determined from Table A-2Ea to be \( C_v = 0.753 \) Btu/(lbm.\(^\circ\)F ). Substituting this and other known quantities into above energy equation, we obtain:

\[
-(-25.45 \text{ Btu}) = (1.5 \text{ lbm})(0.753 \text{ Btu/(lbm.\(^\circ\)F)})(T_2 - 80^\circ\text{F})
\]

\[
T_2 = 102.5^\circ\text{F}
\]

(b) The final pressure is determined from the ideal-gas relation:

\[
\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}
\]

Where \( V_1 \) and \( V_2 \) are identical and cancel. Then the final pressure becomes:

\[
\frac{50 \text{ psia}}{(80+460) R} = \frac{P_2}{(102.5+460)R}
\]

\[
P_2 = 53.1 \text{ psia}
\]

Figure 3-75. Schematic and \( P-v \) diagram for Example 3-19.
Example 3-20.

A piston-cylinder device initially contains 0.5 m³ of nitrogen gas at 400 kPa and 27°C. An electric heater within the device is turned on and is allowed to pass a current of 2 A for 5 min from a 120 V source. Nitrogen expands at constant pressure, and a heat loss of 2800 J occurs during the process. Determine the final temperature of the nitrogen, using data from the nitrogen table (Table A-18).

Solution.

A sketch of the system and the P-V diagram of the process are given in Fig. 3-76.

![Figure 3-76. Schematic and P-V diagram for Example 3-20.](image)

At the specified conditions, the nitrogen gas can be considered to be an ideal gas since it is at a high temperature and low...
pressure relative to its critical-point values \( (T_{cr} = -147^\circ C, P_{cr} = 3390 \text{ kPa}, \text{Table A-1}) \).

First, let us determine the electrical work done on the nitrogen:

\[
W_e = V I \Delta t = (120 \text{ V})(2 \text{ A})(5 \times 60 \text{ s}) \left( \frac{1 \text{ kJ}}{1000 \text{ VA.s}} \right) = -72 \text{ kJ}
\]

The negative sign is added because the work is done on the system.

The number of moles of nitrogen is determined from the ideal-gas relation:

\[
N = \frac{P_1 V_1}{R_u T_1} = \frac{(400 \text{ kPa})(0.5 \text{ m}^3)}{[8.314 \text{ kJ/}(\text{kmol.K})](300 \text{ K})} = 0.080 \text{ kmol}
\]

When gases other than air are involved, it is more convenient to work with mole numbers instead of masses since all the \( u \) and \( h \) data are given on a mole basis (Fig. 3-77). Assuming no change in kinetic and potential energies \( (\Delta KE = \Delta PE = 0) \), the conservation of energy equation for this closed system can be written as:

\[
Q - W_e - W_b = \Delta U
\]

For the constant-pressure process of a closed system, \( \Delta U + W_b \) is equivalent to \( \Delta H \). Thus,

\[
Q - W_e = \Delta H = m(h_2 - h_1) = N(h_2 - h_1)
\]

From the nitrogen table, \( h_1 = \bar{h}_{300} \text{ k} = 8723 \text{ kJ/kmol} \). The only unknown quantity in the above equation is \( \bar{h}_2 \), and it is found to be:

\[
-2.8 \text{ kJ} - (-72 \text{ kJ}) = (0.08 \text{ kmol})(\bar{h}_2 - 8723 \text{ kJ/kmol})
\]

\[
\bar{h}_2 = 9588 \text{ kJ/kmol}
\]
The temperature corresponding to this enthalpy value is:

\[ T_2 = 329.7 \text{ K} = 56.7^\circ \text{C} \quad (\text{Table A-18}) \]

3-9. **Internal energy, enthalpy, and specific heats of solids and liquids**

A substance whose specific volume (or density) is constant is called an *incompressible substance*. The specific volumes of solids and liquids essentially remain constant during a process (Fig. 3-79). Therefore, liquids and solids can be approximated as incompressible substances without sacrificing much in accuracy.

The constant-volume and constant-pressure specific heats are **identical** for incompressible substances (Fig. 3-80).
Therefore, for solids and liquids the subscripts on \( C_p \) and \( C_v \) can be dropped, and both specific heats can be represented by a single symbol \( C \). That is,

\[
C_p = C_v = C
\]  

(3-47)

Specific heat values for several common liquids and solids are given in Table A-3.

Like those of ideal gases, the specific heats of incompressible substances depend on temperature only. Thus the partial differentials in the defining equation of \( C_v \) (Eq. 3-34) can be replaced by ordinary differentials, which yields:

\[
du = C_v \, dT = C(T) \, dT
\]  

(3-48)

The change in internal energy between state 1 and 2 is then obtained by integration:

\[
\Delta u = u_2 - u_1 = \int_1^2 C(T) \, dT \quad \text{(kJ/kg)} \]  

(3-49)

For small temperature intervals, a \( C \) values at the average temperature can be used and treated as a constant, yielding:

\[
\Delta u \approx C_{av}(T_2 - T_1) \quad \text{(kJ/kg)} \]  

(3-50)

For enthalpy change of solids or liquids during process 1-2 can be determined from the definition of enthalpy \( h = u + Pv \) to be:

\[
h_2 - h_1 = (u_2 - u_1) + v(P_2 - P_1)
\]  

(3-51)

since \( v_1 = v_2 = v \). It can also be expressed in a compact form as:

\[
\Delta h = \Delta u + v \Delta P \quad \text{(kJ/kg)}
\]  

(3-52)
The second term \( (\nu \Delta P) \) in Eq.3-52 is often small compared with the first term \((\Delta u)\) and can be neglected without significant loss in accuracy.

For a constant-temperature process \((\Delta T = 0)\), the internal energy change of an incompressible substance is \((\Delta u = 0)\). The from Eq. 3-51 the change in enthalpy change will be:

\[
h_2 - h_1 = \nu(P_2 - P_1)
\]

By taking state 2 as the compressed liquid state and state 1 as the saturated liquid state at the same temperature, the enthalpy of the compressed liquid at a given \(P\) and \(T\) can be determined from:

\[
h_{@P,T} \cong h_{f@T} + \nu f_{@T}(P - P_{sat})
\]

(3-53)

where \(P_{sat}\) is the saturation pressure at the given temperature.

This is an improvement over the assumption that the enthalpy of the compressed liquid could be taken as \(h_f\) at the given temperature (that is, \(h_{@P,T} \cong h_{f@T}\)). However, the contribution of the last term is often very small, so it is usually neglected.

Example 3-22.

A 50 kg iron block at 80°C is dropped into an insulated tank which contains 0.5 m³ of liquid water at 25°C. Determine the temperature when thermal equilibrium is reached.

Solution.

We take the iron block and water as our system. The inner surfaces of the tank walls form the system boundary, as shown in Fig. 3-81.

Since the tank is insulated, no heat will cross these boundaries \((Q = 0)\). Also since there is no movement of the boundary \((W_b = \)
and no indication of other forms of work \( W_{\text{other}} = 0 \), the work for this process is zero \( W = 0 \). Then the conservation of energy equation for this process will reduce to:

\[
\varphi - W_{\text{other}} - W_{b} = \Delta U + \Delta K_{E} + \Delta P_{E}
\]

or \( \Delta U = 0 \)

The total energy \( U \) is an extensive property, and therefore it can be expressed as the sum of the internal energies of the parts of the system. Then the total internal energy change of the system is:

\[
\Delta U_{\text{system}} = \Delta U_{\text{iron}} + \Delta U_{\text{water}} = 0
\]

\[
[mC(T_2 - T_1)]_{\text{iron}} + [mC(T_2 - T_1)]_{\text{water}} = 0
\]

The specific volume of liquid water at or about room temperature can be taken to be \( v_{\text{water}} = 0.001 \text{ m}^3/\text{kg} \) (Table A-4).

Then mass of the water is:

\[
m_{\text{water}} = \frac{V}{v} = \frac{0.5 \text{ m}^3}{0.001 \text{ m}^3/\text{kg}} = 500 \text{ kg}
\]

The specific heats of iron and liquid water are determined:

\[
C_{\text{iron}} = 0.45 \text{ kJ/(kg.} \cdot \text{C)} \quad \text{(Table A-3 a) at } 25\text{C}
\]

\[
C_{\text{water}} = 4.184 \text{ kJ/(kg.} \cdot \text{C)}
\]

Subsisting these values into the energy equation, we obtain:

\[
(50 \text{ kg})[0.45 \text{ kJ/(kg.} \cdot \text{C})](T_2 - 80^\circ \text{C}) + (500 \text{ kg})[4.184 \text{ kJ/(kg.} \cdot \text{C})](T_2 - 25^\circ \text{C}) = 0, \quad T_2 = 25.6^\circ \text{C}
\]
Example 3-23.

Determine the enthalpy of liquid water at 100°C and 15 MPa (a) from the compressed liquid tables, (b) by approximating it as saturated liquid, and (c) by using the correction given by Eq. 3-53.

Solution.

At 100°C, the saturation pressure of water $P_{\text{sat}} = 101.35$ kPa from (Table A-4), and since $P > P_{\text{sat}}$, the water exists as a compressed liquid at the specified state.

(a) From the compressed liquid tables, we read:

\[
P = 15 \text{ MPa}
\]
\[
T = 100°C
\]
\[
h = 430.28 \text{ kJ/kg} \quad \text{(Table A-7)}
\]

This is the exact value.

(b) Approximating the compressed liquid as a saturated liquid at 100°C, as is commonly done, we obtain:

\[
h \approx h_f @ 100°C = 419.04 \text{ kJ/kg} \quad \text{(Table A-4)}
\]

This value is in error by about 2.6 percent.

(c) From Eq. 3-53:

\[
h_{\overline{P,T}} \equiv h_f @ T + v_f @ T(P - P_{\text{sat}})
\]
\[
\approx (419.04 \text{ kJ/kg}) + (0.001 \text{ m}^3/\text{kg})[(15000 - 101.35)\text{kPa}]^* \\
\]
\[
\quad \times \left( \frac{1 \text{ kJ}}{1 \text{ kPa.m}^3} \right)
\]
\[
\approx 434.60 \text{ kJ/kg}
\]

The correction term reduced error from 2.6 to about 1 percent. But this improvement in accuracy is often not worth the extra effort involved.