Chapter 3

The First Law of Thermodynamics-Closed Systems

3-1. Introduction to the first law of thermodynamics

The first law of thermodynamics can simply be stated as follows: During an **interaction** between a **system** and its **surroundings**, the amount of **energy gained** by the system must be exactly equal to the amount of **energy lost** by surroundings. Energy can cross the boundary of closed system in two distinct forms: *heat* and *work* (**Fig. 3-1**). It is important to distinguish between these two forms of energy.
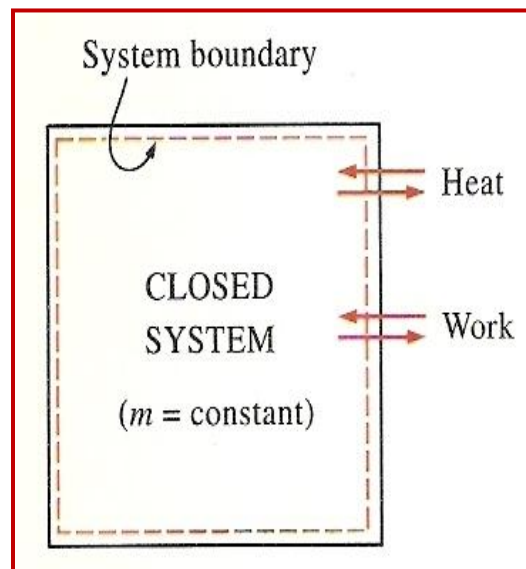

Figure 3-1. Energy can cross the boundaries of a closed system in the form of **heat** and **work**.

3-2. Heat

Heat is defined as *the form of energy that is transferred between two systems* (*or a system and its surroundings*) *by virtue of a temperature difference.* That is, an **energy interaction** is **heat** only if it takes place because of a temperature difference (**Fig. 3-2**).
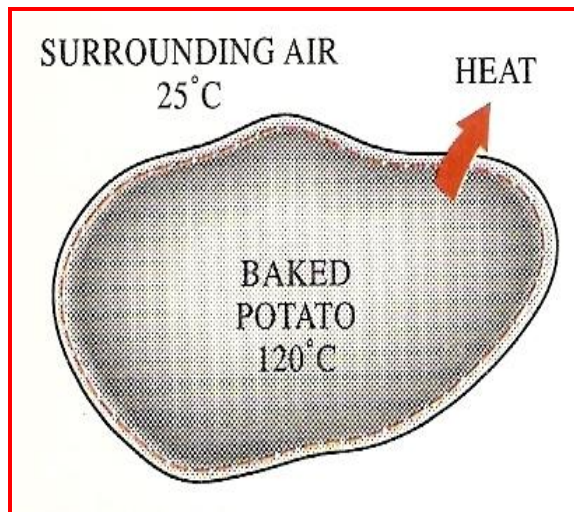
Figure 3-2. Heat is transferred from hot bodies to colder ones by virtue of a temperature difference.

In **thermodynamics**, however, heat and internal energy are two different things. **Energy** is a **property**, but **heat** is **not**. A body contains energy, but not heat. **Energy** is associated with a **state**; **heat** is associated with a **process**.

Heat is energy in transition. It is recognized only as it crosses the boundaries of a system. Consider the hot baked potato one more time. The potato contains energy, but this called heat only as it passes through the skin of potato (the system boundary) to reach the air, as shown in **Fig. 3-3**. Once in the surroundings, the heat becomes part of the internal energy of the surroundings. Thus in **thermodynamics** the term *heat* simply means *heat transfer*.
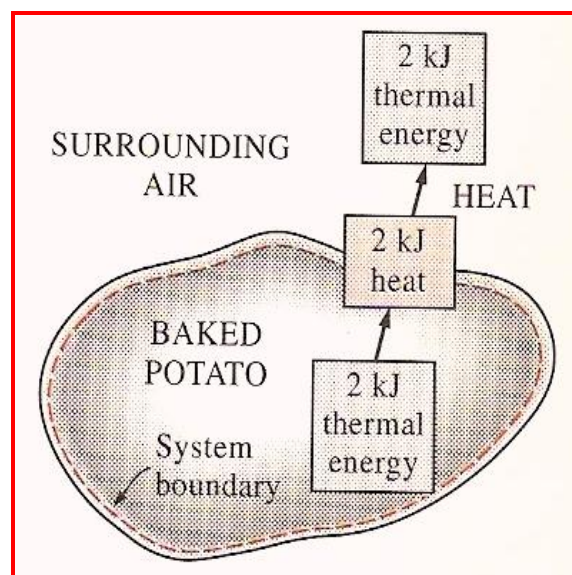

Figure 3-3. Energy is recognized as heat only as it crosses the system boundary.

A process during which there is **no heat transfer** is called an **adiabatic** process (**Fig. 3-4**).
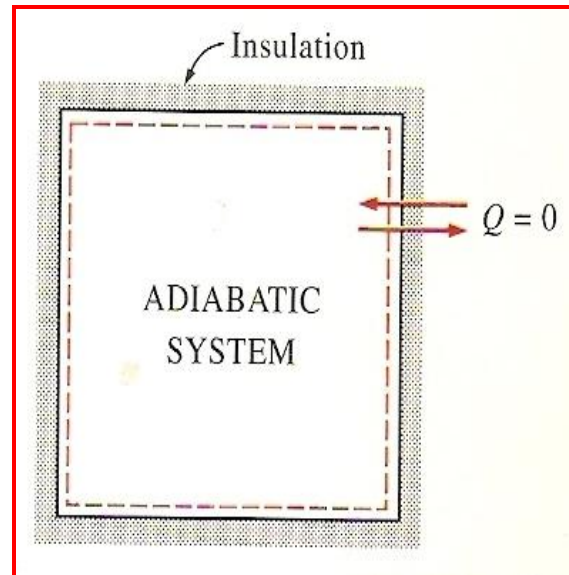

Figure 3-4. During an adiabatic process, a system exchanges no heat with its surroundings.

As a form of energy, **heat** has energy units, **kJ** (or **Btu**) being the most common one. The amount of heat transferred during the process between two states (states **1** and **2**) is denoted Q_{12}, or just Q. Heat transfer *per unit mass* of a system is denoted q and is determined from:

$$q = \frac{Q}{m} \qquad \text{(kJ/kg)} \qquad \qquad (3\text{-}1)$$

Sometimes it is desirable to know the **rate of heat transfer** (the amount of heat transferred per unit time) instead of the total heat transferred over some time interval (**Fig. 3-5**).The heat transfer rate is denoted $\dot{Q}$, and has the unit **kJ/s**, which is equivalent to **kW**.

$Q = 30$ kJ
$m = 2$ kg
$\Delta t = 5$ s

30 kJ heat
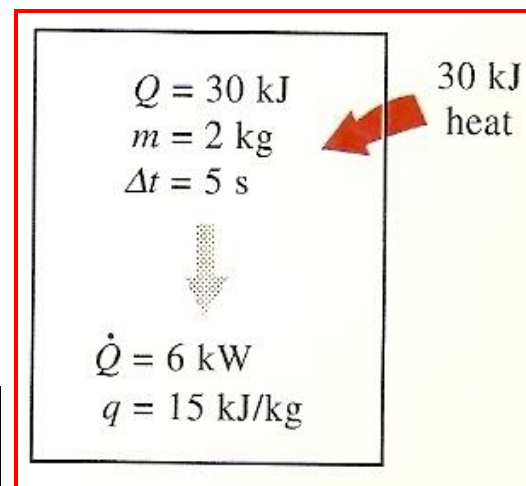
$\dot{Q} = 6$ kW
$q = 15$ kJ/kg

Figure 3-5. The relationships among q, Q, and $\dot{Q}$.

The universally accepted sign convention for heat is as follows: *Heat transfer to a system is positive, and heat transfer from a system is negative (Fig. 3-6).*
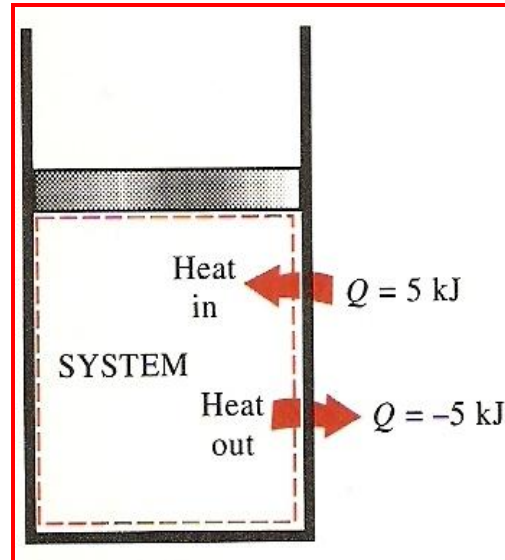

Figure 3-6. Sign convention for heat: positive if to the system, negative if from the system.

3-2. Work

Work, like heat, is an energy interaction between a system and its surroundings. As mentioned earlier, energy can cross the boundary of a closed system in the form of heat or work. Therefore, *if the energy crossing the boundary is not heat, it must be work* (Fig. 3-7).
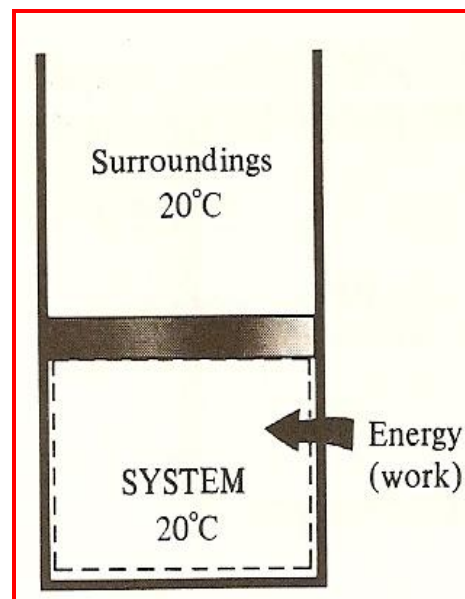

Figure 3-7. An energy interaction which is not heat is work.

Work is also a form of energy **like heat** and, therefore, has energy units such as **kJ**.The work done during a process between state **1** and **2** is denoted W_{12}, or simply W. The work done *per unit mass* of the system is denoted w and is defined as:

$$w = \frac{W}{m} \quad \text{(kJ/kg)} \tag{3-2}$$

The **work** done *per unit time* is called **power** and denoted $\dot{W}$ (**Fig.3-8**). The unit of power is **kJ/s**, or **kW**.

$$W = 30 \text{ kJ} \quad 30 \text{ kJ}$$
$$m = 2 \text{ kg} \quad \text{work}$$
$$\Delta t = 5 \text{ s}$$

$$\dot{W} = 6 \text{ k}\dot{W}$$
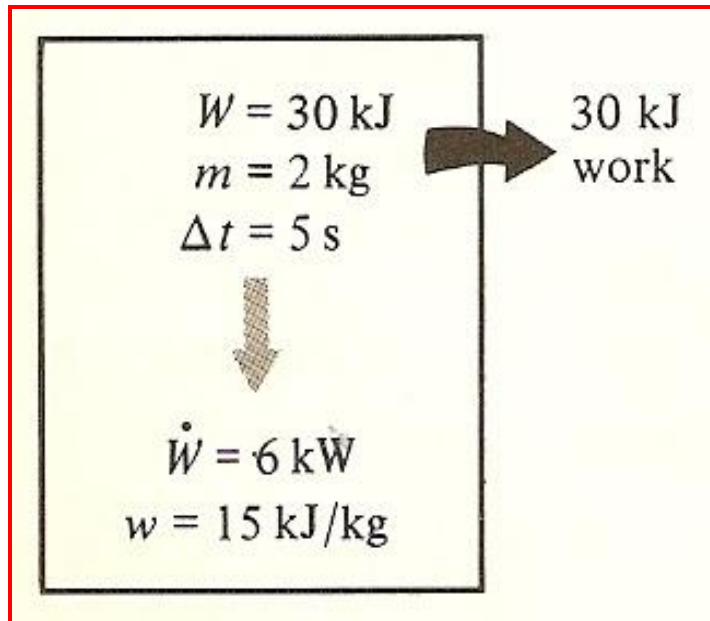$$w = 15 \text{ kJ/kg}$$

Figure 3-8. The relationships among w, W, and $\dot{W}$.

   The production of work by a system is viewed as a desirable, **positive** effect and the consumption of work as an undesirable, **negative** effect. The sign convention for work adapted in this text reflects this philosophy: *Work done by a system is positive, and work done on a system is negative* (**Fig.3-9**).
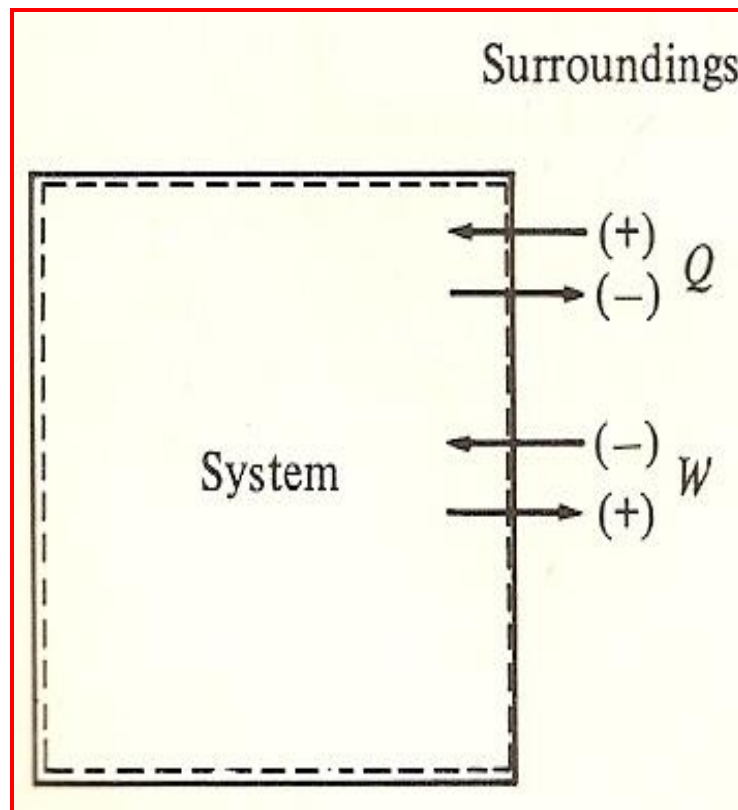
Figure 3-9. Sign convention for **heat** and **work**.

Heat and **work** are *interactions* between a system and its surroundings, and there are many similarities between the two:

1. Both are recognized at the boundaries of the system as they cross them. That is, both heat and work are *boundary* phenomena.

2. Systems possess energy, but not heat or work. That is, heat and work are *transient* phenomena.

3. Both are associated with a *process*, not a state. Unlike properties, heat or work has no meaning at a state.

4. Both are *path functions* (i.e., their magnitudes depend on the path followed during a process as well as the end states).

<u>**Example 3-3**</u>. **(3-4)**

A well-insulated electric oven is being heated through its heating element. If the entire oven, including the heating element, is taken to be the system, determine whether this is a heat or work interaction.

<u>**Solution**</u>

For this problem the interior surfaces of the oven form the system boundary, as shown in Fig. 3-13. The energy content of the oven obviously increases during this process, as evidence by a rise in temperature. This energy transfer to the oven is not caused by a temperature difference between the oven and the surrounding air. It is caused by negatively charged particles called *electrons* crossing the system boundary and thus doing work. Therefore, this is a work interaction.

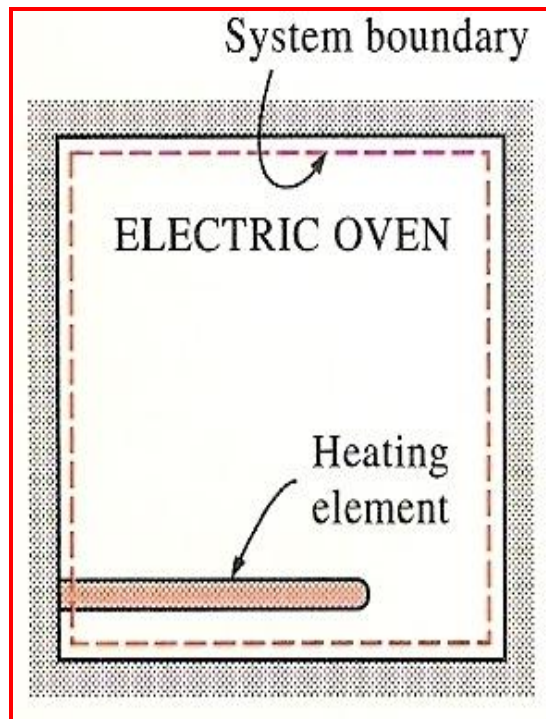**Figure 3-13. Schematic for Example 3-3.**

Example 3-4. (3-5)

Answer the equation in Example 3-3 if the system is taken as only the air in the oven without the heating element.

Solution

This time the system boundary will include the outer surface of the heating element and will not cut through it, as shown in Fig. 3-14. Therefore, no electrons will be crossing the system boundary at any point. Instead, the energy generated in the interior of the heating element will be transferred to the air around it as a result of the temperature difference between the heating element and the air in the oven. Therefore, this is a heat transfer process.

   For both cases, the amount of energy transfer to the air is the same. These two examples show that the same interaction can be heat or work depending on how the system is selected.

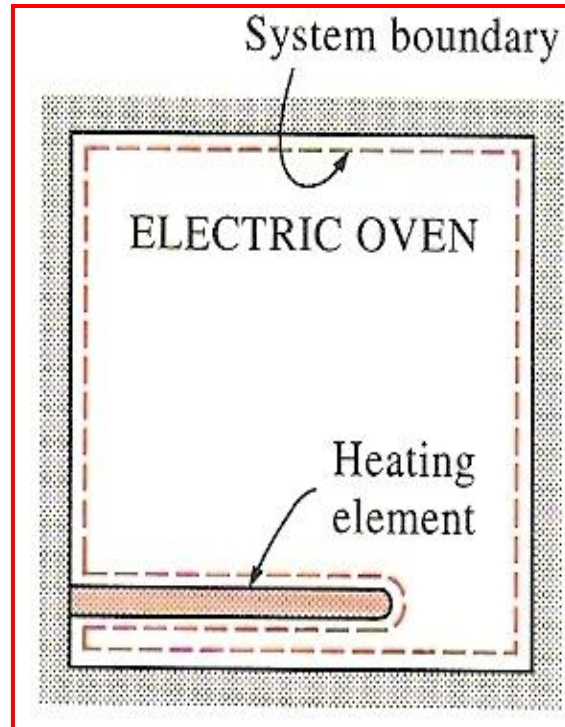**Figure 3-14**. Schematic for Example 3-4.

Electrical Work.

In an electric field, electrons in a wire move under the effect of **electromotive forces**, doing work. When N electrons move through a potential difference V, the electrical work done is:

$W_e = VN$ **(kJ)**

This can also be expressed in the rate form as:

$\dot{W}_e = VI$ **(kW)** (3-3)

Where $\dot{W}_e$ is the **electrical power** and I is the number of electrons flowing per unit time, i.e., the current (**Fig. 3-15**). In general, both V and I vary with time, and the **electrical work** done during a time interval Δt is expressed as:

$W_e = \int_1^2 VI\, dt$ **(kJ)** (3-4)

If both V and I remain constant during the time interval Δt, this equation will reduce to:

$W_e = VI\Delta t$ **(kJ)** (3-5)

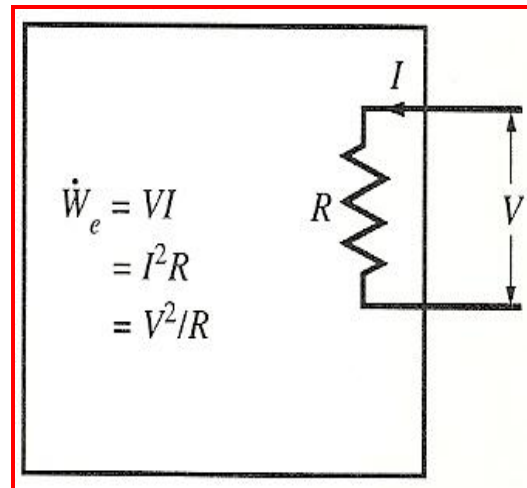$$\dot{W}_e = VI$$
$$= I^2 R$$
$$= V^2/R$$

Figure 3-15. Electrical power in terms of resistance R, current I, and potential difference V.

3-4. Mechanical forms of work

In elementary mechanics, the work W done by aconstant force F on a body which is displaced distance s in the direction of the force (**Fig. 3-17**) is given by:

$W = Fs$ **(kJ)** (3-6)

If the force F is not constant, the **work** done is obtained by adding (i.e., integrating) the differential amount of work (force times the differential displacement ds):

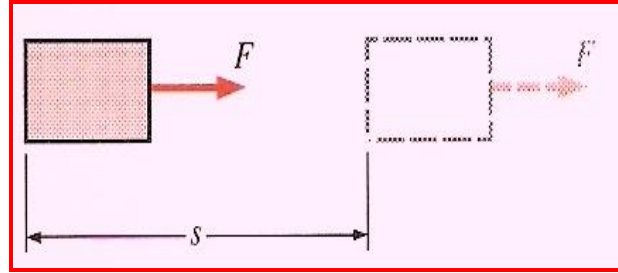$$W = \int_1^2 F\, ds \qquad \text{(kJ)} \qquad (3\text{-}7)$$

Figure 3-17. The work done is proportional to the force applied (F) and the distance traveled (s).

In many thermodynamic problems, **mechanical work** is the only form of work involved. It is **associated** with the **movement** of the **boundary** of a system **or** with the **movement** of the **entire system** as a whole. Some common forms of mechanical work are discussed below.

1. Moving boundary work

One form of mechanical work frequently encountered in practice is associated with the **expansion** or **compression** of a gas in a piston-cylinder device. The expansion and compression work is often called **moving boundary work**, or simply **boundary work (Fig. 3-19)**.
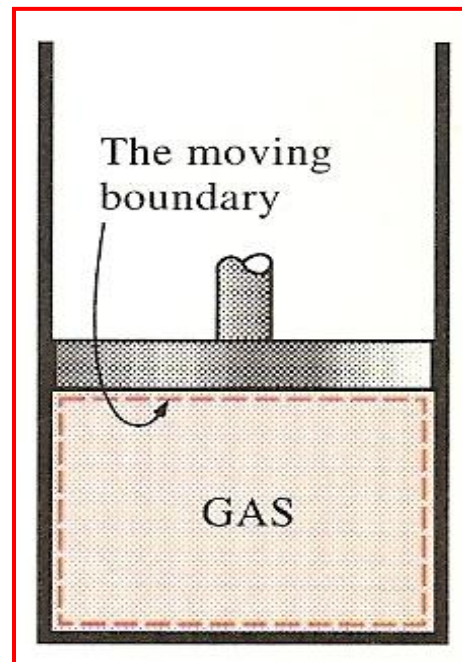

Figure 3-19. The work associated with a moving boundary is called *boundary work*.

Consider the gas enclosed in the piston-cylinder arrangement shown in **Fig. 3-20**. The initial pressure of the gas is P, the total volume is V and the cross-sectional area of the piston is A. If the piston is allowed to move a distance ds in a quasi-equilibrium manner, the differential work done during this process is:
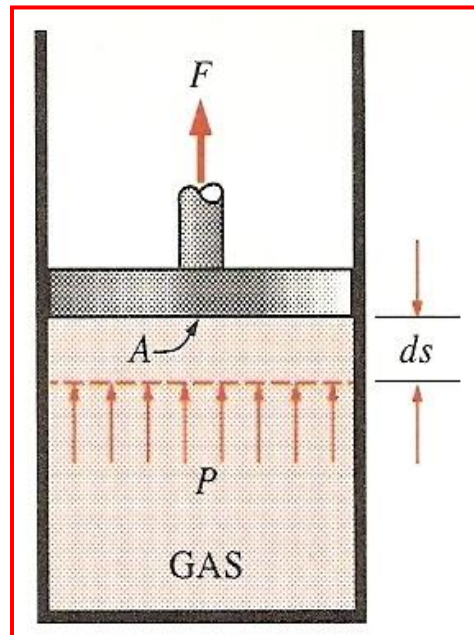
$$\delta W_b = F\ ds = PA\ ds = P\ dV \qquad\qquad (3\text{-}8)$$

Figure 3-20. A gas dose a differential amount of work δW_b as itforces the piston to move by differential amount ds.

The **total boundary work** done during the entire process as the piston moves is obtained by adding all the differential works from the initial state to the final state:

$$W_b = \int_1^2 P\ dV \qquad \textbf{(kJ)} \qquad\qquad (3\text{-}9)$$

That is, $P = f(V)$ showed be a available. Note that $P = f(V)$ is simply the equation of the process path on a **P-V** diagram.

The quasi-equilibrium expansion process described above is shown on a **P-V** diagram in **Fig.3-21**. On this diagram, the differential area dA is equal to $P\ dV$, which is the differential

work. The total area A under the process curve 1-2 is obtained by adding these differential areas:

$$\text{Area} = A = \int_1^2 dA = \int_1^2 P\, dV$$

Figure 3-21. The area under the process curve on a *P-V* diagram represents the boundary work.
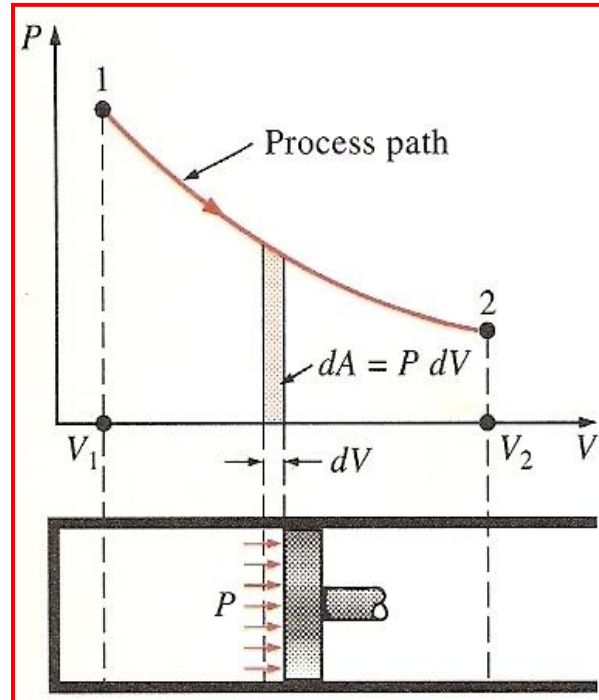
A comparison of this equation with **Eq. 3-9** reveals that *the area under the process curve on a P-V diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.* (On the $P-v$ diagram, it represents the **boundary work** done per **unit mass**).

A gas can follow several different paths as it expands from **state 1** to **state 2**. In general, each path will have a different area underneath it, and since this area represents the magnitude of the work, the work done will be different for each process (**Fig.3-22**). This is expected since work is path function (i.e., it depends on the path followed as well as the end states).
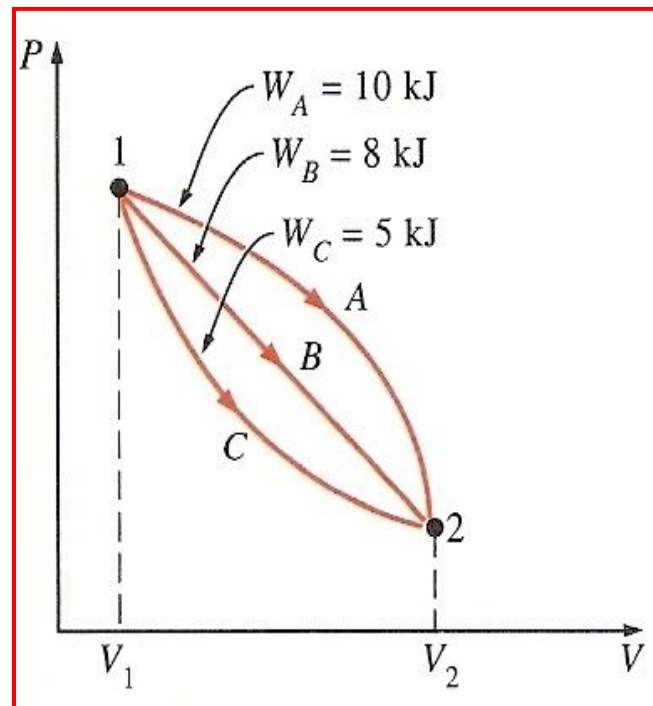
Figure 3-22. The boundary work done during a process depends on the path followed as well as the end states.

The cycle shown in **Fig. 3-23** produces a **net work** output because the work done by the system during **expansion process** (area under path A) is greater than the work done on the system during the compression part of the cycle (area under path B), and the difference between these two is the **net work** done during the cycle (**the color area**).

The use of the boundary work relation (**Eq. 3-9**) is not limited to the quasi-equilibrium processes of **gas only**. It can also be **used** for **solids** and **liquids**.
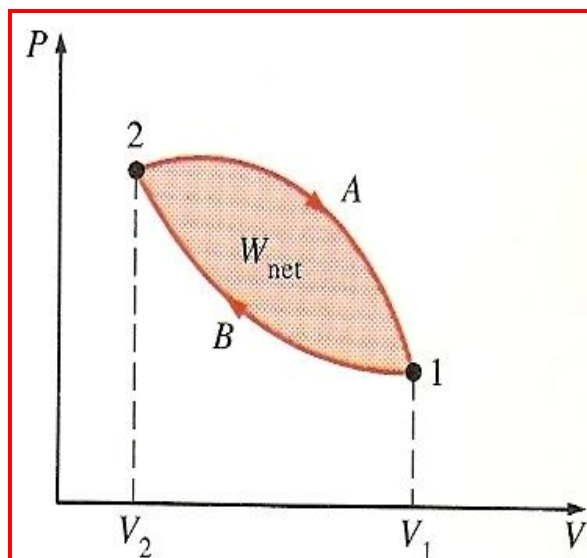

Figure 3-23. The net work done during a cycle is difference between the work done by the system and the work done on the system.

Example 3-6. (3-7)

A rigid tank contains air at **500 kPa** and **150**℃. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to **65**℃ and **400 kPa**, respectively. **Determine** the **boundary work** done during this process.

Solution

A sketch of the system and the *P-V* diagram of the process are shown in **Fig. 3-24**. Assuming the process to be quasi-equilibrium, the **boundary work** can be determined from **Eq. 3-9**:

$$W_b = \int_1^2 P \, dV = 0$$
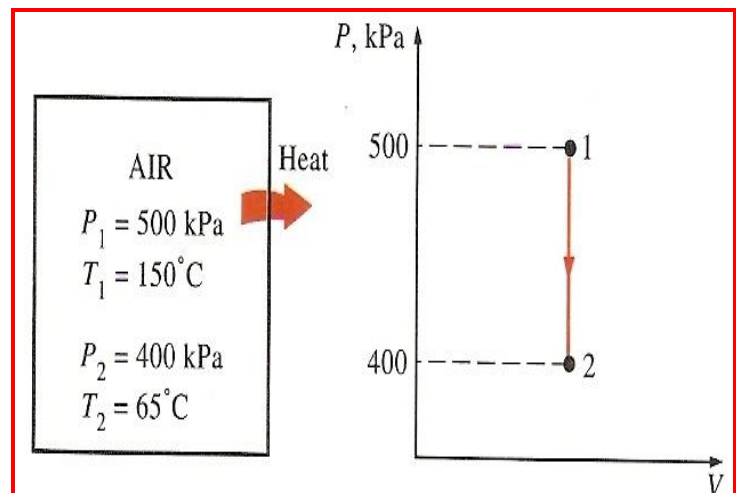
Figure 3-24. Schematic and *P-V* diagram for Example 3-6.

This is expected since a rigid tank has a constant volume and **dV = 0** in the above equation. Therefore, there is no boundary done during this process. That is, the **boundary work** done during a **constant-volume** process is **always zero**. This is also evident from the *P-V* diagramof the process (the area under the process curve is **zero**).

Example 3-7. (3-8)

A frictionless piston-cylinder device contains **10 lbm** of water vapor at **60 psia** and **320**°F. Heat is now added to the steam until the temperature reaches **400**°F. If the piston is not attached to a

shaft and its mass is constant, determine the work done by the steam during this process.

Solution

A sketch of the system and the *P-V* diagram of the process are shown in Fig. 3-25. Even though it is not explicitly stated, the pressure of the steam within the cylinder remains constant during this process since both the atmospheric pressure and the weight of the piston remain constant. Therefore, this is a constant-pressure process, and from Eq. 3-9:
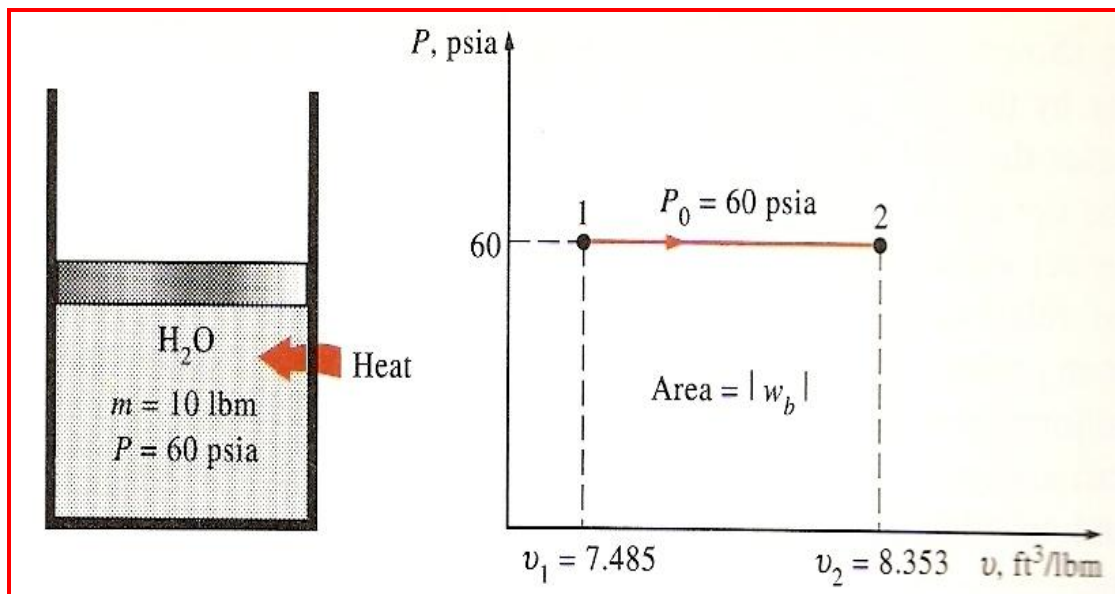
Figure 3-25. Schematic and *P-V* diagram for Example 3-7.

$$W_b = \int_1^2 P \, dV = P_0 \int_1^2 dV = P_0 (V_2 - V_1)$$

or $\qquad W_b = mP_0(v_2 - v_1)$ $\qquad\qquad\qquad$ (3-10)

since $V = mv$. From the superheated vapor table (Table A-6E), the specific volumes are determined to be v_1 = 7.485 ft³/lbm at state 1 (60 psia, 320°F) and v_2 = 8.353 ft³/lbm at state 2 (60 psia, 400°F). Substituting these values yields:

$$W_b = \text{(10 lbm)(60 psia) [(8.353 -7.485) ft}^3\text{/lbm)] } \left(\frac{1 \text{ Btu}}{5.404 \text{ psia.ft}^3} \right)$$

= **96.4 Btu**

The **positive sign** indicates that the work is done by the system. That is, the steam used **96.4 Btu** of its energy to do this work. The magnitude of this work could also be determined by **calculating the area** under the **process curve** on the *P-V* diagram, which is $P_0 \Delta V$ for this case.

Example 3-8. (3-9)

A piston-cylinder device initially contains **0.4 m³** of air at **100 kPa** and **80°C**. The air is now compressed to **0.1 m³** in such a way that the temperature inside the cylinder remains constant. **Determine** the **work** done during this process.

Solution

A sketch of the system and the *P-V* diagram of the process are shown in **Fig. 3-26**. At the specified conditions, air can be considered to be an ideal gas since it is at a high temperature and low pressure relative to its critical-point values ($T_{cr} = $ **−147°C**, $P_{cr} = $ **3390 kPa** for nitrogen, the main constituent of air). For an ideal gas at constant temperature T_0,
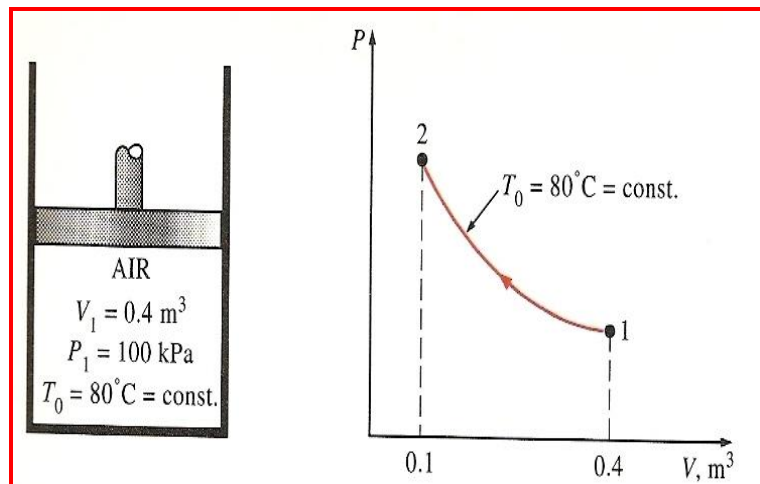

Figure 3-26. Schematic and *P-V* diagram for Example 3-8.

$$PV = mRT_0 = C \qquad \text{or} \qquad P = \frac{C}{V}$$

where C is a constant. Substituting this into **Eq. 3-9**, we have:

$$W_b = \int_1^2 P \, dV = \int_1^2 \frac{C}{V} \, dV = C \int_1^2 \frac{dV}{V} = C \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{V_2}{V_1} \qquad \text{(3-11)}$$

In the above equation, $P_1 V_1$ can be replace by $P_2 V_2$ or mRT_0. Also, V_2/V_1 can be replaced by P_2/P_1 for this case since $P_1 V_1 = P_2 V_2$. Substituting the numerical values into the above equation yields:

$$W_b = (100 \text{ kPa})(0.4 \text{ m}^3)\left(\ln \frac{0.1}{0.4} \right)\left(\frac{1 \text{kJ}}{1 \text{kPa.m}^3} \right) = -\ 55.45 \text{ kJ}$$

The **negative sign** indicates that this work is done on the system, which is always the case for compression process.

Example 3-9.

During expansion and compression processes of real gases, pressure and volume are often related by $PV^n = C$, where n and C are constants. Aprocess of this kind is called a polytropic process. Develop a general expression for the work done during a polytropic process.

Solution

A sketch of the system and the P-V diagram of the process are shown in **Fig. 3-27**. The pressure for a polytropic process can be expressed as:

$$PV^n = C$$
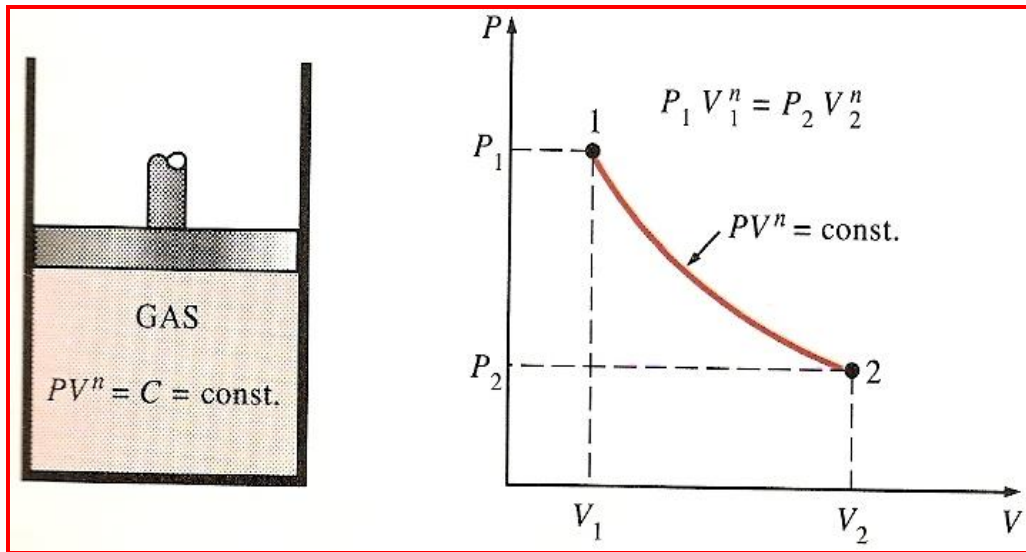
Substitution this relation into **Eq. 3-9**, we obtain:

Figure. 3-27. Schematic and *P-V* diagram for Example 3-9.

$$W_b = \int_1^2 PdV = \int_1^2 CV^{-n}\, dV = C\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2 V_2 - P_1 V_1}{1-n} \quad (3\text{-}12)$$

Since $C = P_1 V_1^n = P_2 V_2^n$. For an ideal gas ($PV = mRT$), this equation can also be written as:

$$W_b = \frac{mR(T_2 - T_1)}{1-n} \quad \text{(kJ)} \qquad\qquad (3\text{-}13)$$

The special case of $n = 1$ is equivalent to the isothermal process discussed in the previous example.

2 Gravitational work

The **gravitational work** can be defined as the work done by or against a gravitational force field. In a gravitational field, the **force** acting on a body is:

$$F = mg$$

Where m is mass of thebody and g is the acceleration of gravity which is assumed to be constant. Then the work required to raise this body from level z_1 to level z_2.

$$W_g = \int_1^2 F\,dz = mg \int_1^2 dz = mg\,(z_2 - z_1) \qquad \text{(kJ)} \qquad (3\text{-}14)$$

where $z_2 - z_1$ is the vertical distance traveled (**Fig. 3-28**). This expression is easily recognized as the *change in potential energy*. We conclude from **Eq. 3-14** that the gravitational work depends on the **end states only** and is **independent** of the path followed.
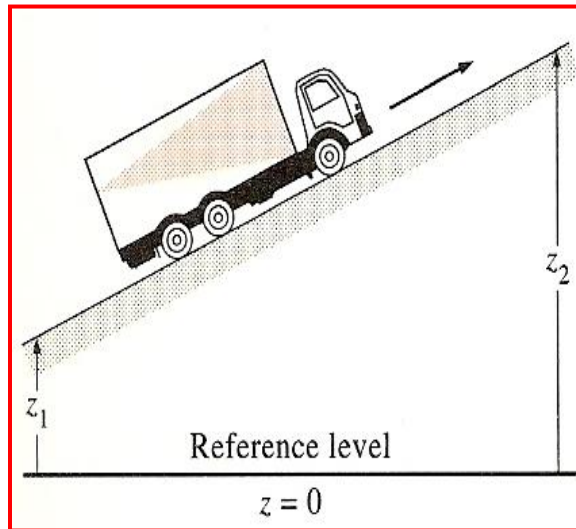
Figure 3-28. Vehicles required more power (gravitational work per unit time) as they climb a hill.

Example 3-10.

Determine the **work** done by a person to lift a **50-lbm** suitcase shown in **Fig.3-29** by **1 ft**.

Solution

By assuming a standard gravitation and using **Eq. 3-14**, the work done is:

$$W_g = mg\,(z_2 - z_1)$$

$$W_g = (50\ \text{lbm})(32.174\ \text{ft/s}^2)(1\ \text{ft})\left(\frac{1\ \text{Btu}}{25.037\ \text{ft}^2/\text{s}^2}\right) = 0.064\ \text{Btu}$$

That is, **0.064 Btu** of work is needed to perform this task. The potential energy of the system (the suitcase) increases by **0.064 Btu** during this process.
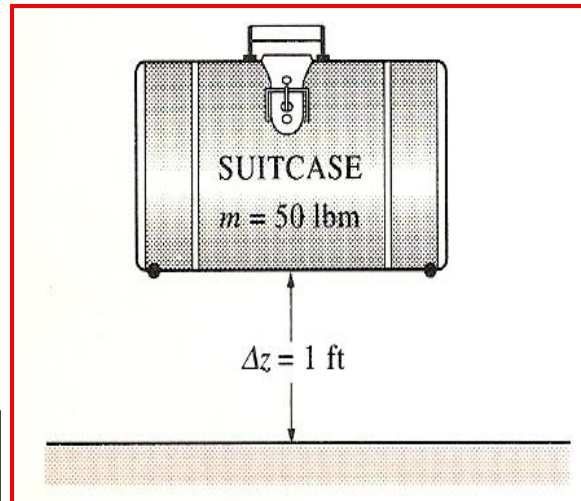

Figure 3-29. Schematic for Example 3-10.

3 Accelerational work

The **work** associated with a change in velocity of a system is defined as the **accelerational work**. The accelerational work required to accelerate a body of mass m from an initial velocity of V_1 to final velocity V_2 (**Fig. 3-30**) is determined from the definition of acceleration and Newton's second law:

$$\left.\begin{array}{c} F = ma \\ a = \dfrac{dV}{dt} \end{array}\right\} F = m\dfrac{dV}{dt}$$

The differential displacement ds is related to velocity V

by: $V = \dfrac{ds}{dt} \longrightarrow ds = V\,dt$

Substituting the F and ds relations into the expression (**Eq. 3-7**), we obtain:

$$W_a = \int_1^2 F\,ds = \int_1^2 \left(m \frac{dV}{dt}\right)(V dt)$$

$$= m \int_1^2 V\,dV = \frac{1}{2}m\,(V_2^2 - V_1^2) \qquad \text{(kJ)} \qquad\qquad \text{(3-15)}$$

The work done to accelerate a body is independent of path and is equivalent to the *change in the kinetic energy* **of the body.**

The sign of accelerational work is determined by inspection: *positive* **if done by the system and** *negative* **if done on the system.**
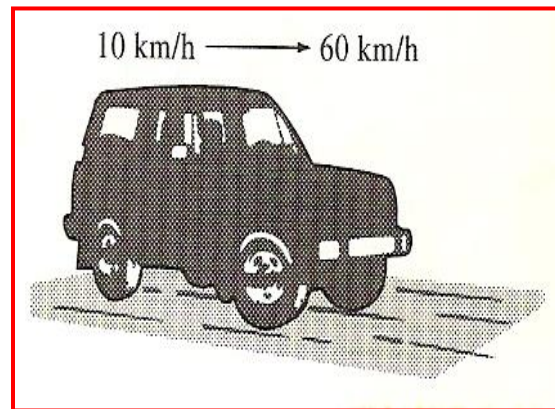

Figure 3-30. Vehicles require more power (accelerational work per unit time) as they accelerate.

Example 3-11.

Determine the power required to accelerate a 900kg car shown in Fig. 3-31 from rest to a velocity of 80 km/h in 20 s on a level road.
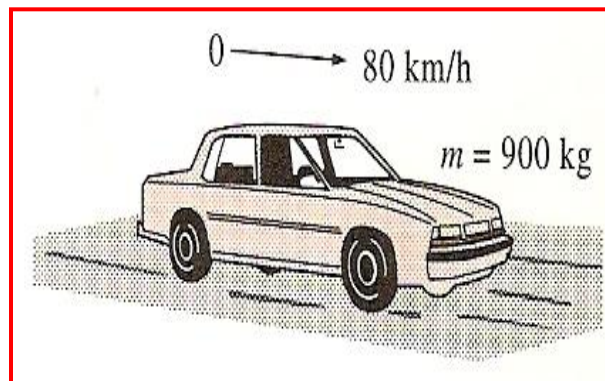

Figure 3-31. Schematic for Example 3-11.

Solution

The accelerational work is determined from **Eq. 3-15** to be:

$$W_a = \frac{1}{2} m(V_2^2 - V_1^2)$$

$$W_a = \frac{1}{2} (900 \text{ kg}) [(\frac{80000 \text{ m}}{3600 \text{ s}})^2 - 0^2] (\frac{1 \text{ kJ}}{1000 \text{ kg.m}^2/\text{s}^2})$$

$$= 222.2 \text{ kJ}$$

The average power is determined from:

$$\dot{W}_a = \frac{W_a}{\Delta t} = \frac{222.2 \text{ kJ}}{20 \text{ s}} = 11.1 \text{ kW} \quad \text{(or } 14.9 \text{ hp)}$$

Where, **kW = 1.341 hp**

4 Shaft work

Energy transmission with a rotating shaft is very common in engineering practice (**Fig. 3-32**). Often the torque τ applied to the shaft is constant, which means that the force F applied is also constant. For a specified constant torque, the work done during n revolutions is determined as follows: A force F acting through a moment arm r generates a torque τ (**Fig. 3-33**) which is determined from:

$$\tau = Fr \longrightarrow F = \frac{\tau}{r}$$

This force acts through a distance s which is related to the radius r by:

$$s = (2\pi r) n$$

Then the shaft work is determined from **Eq. 3-6**:

$$W_{sh} = Fs = \left(\frac{\tau}{r}\right)(2\pi r n) = 2\pi n \tau \qquad \text{(kJ)} \qquad\qquad \text{(3-16)}$$

The power transmitted through the shat is the **work** done per unit time, which can be expressed as:

$$\dot{W}_{sh} = 2\pi \dot{n} \tau \qquad \text{(kW)} \qquad\qquad \text{(3-17)}$$

Where $\dot{n}$ is the number of revolutions per unit time.

The sign of the shaft work is also determined by inspection: *positive* if done by the system and *negative* if done on the system.

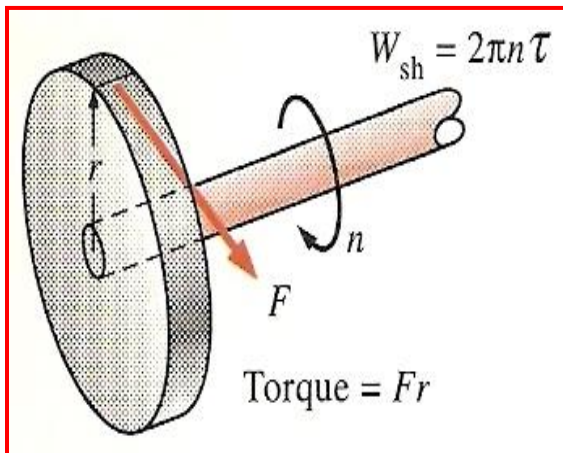**Figure 3-33**. Shaft work is proportional to the torque applied and the number of revolutions of the shaft.
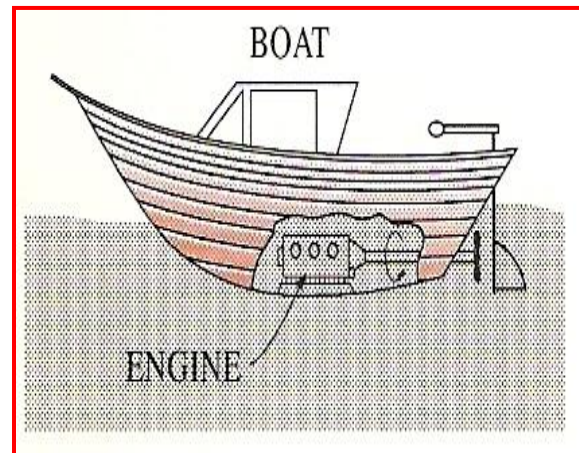

Figure 3-32. Energy transmission through rotating shafts is commonly encountered in practice

Example 3-12.

Determine the **power** transmitted through the shaft of a car when the **torque** applied is **200 N.m** and the shaft rotates at a rate of **4000** revolutions per minute (**rpm**).

Solution

A sketch of the car is given in **Fig. 3-34**. The shaft power is determined from **Eq. 3-17**:
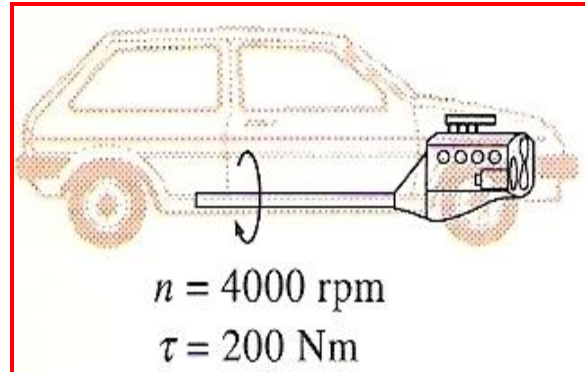
Figure 3-34. Schematic for Example 3-12.

$$W_{sh}^{\cdot} = 2\pi \dot{n}\tau = (2\,\pi)(4000\,\frac{1}{min})(200N.m)\left(\frac{1\,min}{60\,s}\right)\left(\frac{1\,kJ}{1000\,N.m}\right)$$

= 83.7 kW   (or 112.2 hp)

5 Spring work

It is common knowledge that when a force is applied on a spring, the length of the spring changes (**Fig. 3-35**). When the length of the spring changes by a differential amount dx under the influence of force F, the work done is:

$$\delta W_{spring} = F \tag{3-18}$$

For liner elastic springs, the displacement x is proportional to the force F applied (**Fig. 3-36**). That is,

$$F = kx \qquad (kN) \tag{3-19}$$

Where,

k = spring constant (kN/m), x = displacement of the spring (m)

x = 0 when F = 0

Substituting Eq. 3-18 and integrating yield:

$$W_{spring} = \frac{1}{2} k(x_2^2 - x_1^2)$$ (3-20)

Where,

x_1 = initial displacement of the spring

x_2 = final displacement of the spring
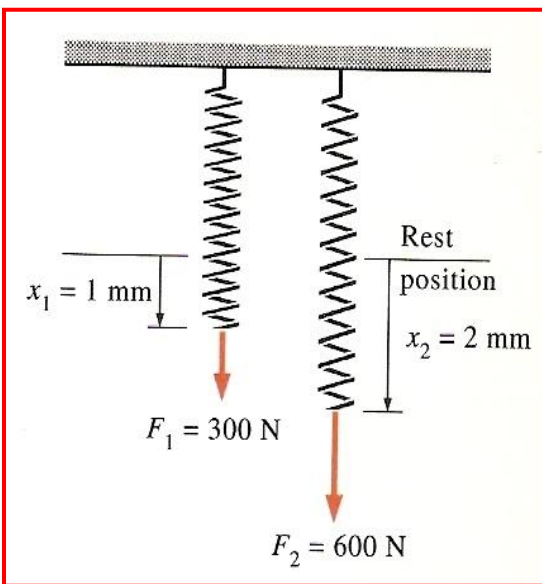
Figure 3-36. The displacement of a linear spring doubles when the force is doubled.
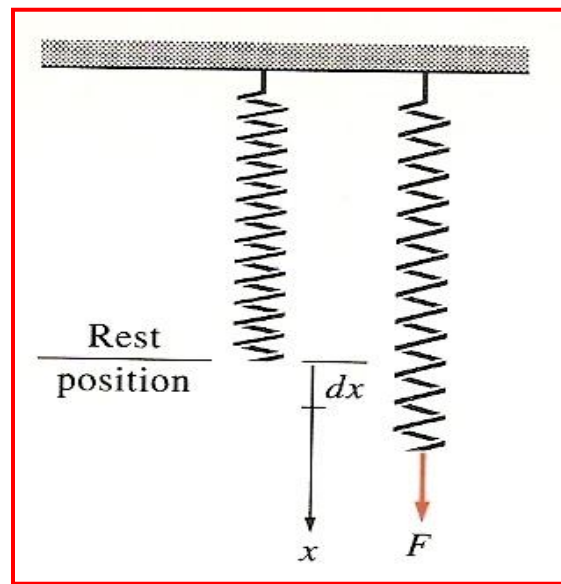
Figure 3-35. Elongation of a spring under the influence of a force.

Example 3-13.

A piston-cylinder device contains **0.05 m³** of a gas initially at **200kPa**. At this state a linear spring which has a spring constant of **150 kN/m** is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is **0.25 m²**,

determine (**a**) the final pressure inside the cylinder, (**b**) the total work done by the gas, and (**c**) the fraction of this work done against the spring to compress it.

Solution

(**a**) A sketch of the system and the **P-V** diagram of the process are shown in **Fig. 3-37**. The enclosed volume at the final state is:

$$V_2 = 2\,V_1 = (2)(0.05\ \text{m}^3)$$

$$= 0.1\ \text{m}^3$$

Then the displacement of the piston (and the spring) becomes:
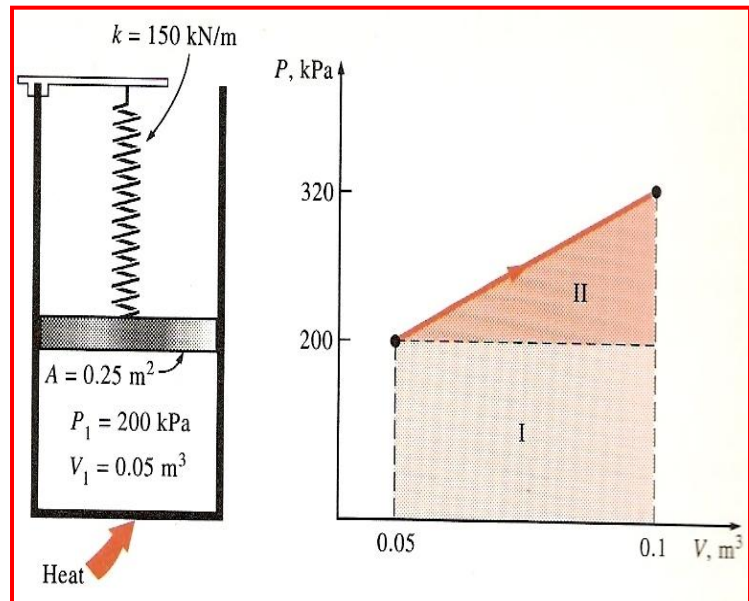
Figure 3-37. Schematic and **P-V** diagram for Example 3-13.

$$x = \frac{\Delta V}{A} = \frac{(0.1-0.05)\,\text{m}^3}{0.25\ \text{m}^2} = 0.2\ \text{m}$$

The force applied by the linear spring at the final state is determined from **Eq. 3-19** to be:

F = **kx** = (150 kN/m)(0.2 m) = 30 kN

The additional pressure applied by the spring on the gas at this state is:

$$P = \frac{F}{A} = \frac{30\ \text{kN}}{0.25\ \text{m}^2} = 120\ \text{kPa}$$

Under the effect of the spring, the pressure rises linearly from **200 kPa** to:

Final pressure = (200 + 120) kPa = 320 kPa

(**b**) From **Fig. 3-37**, the area under the process curve is determined to be:

$$|W| = \text{area} = \frac{(200+320)\text{kPa}}{2} \, [(0.1 - 0.05) \text{ m}^3]\left(\frac{1 \text{ kJ}}{1 \text{ kPa.m}^3}\right) = 13 \text{ kJ}$$

The sign of the work is *positive*, mean that work is done by the system.

(**c**) The work represented by the rectangular area (**region I**) is done against the piston and the atmosphere, and the work represent by the triangular area (**region II**) is done against the spring. Thus:

$$W_{\text{spring}} = \frac{1}{2}\,[(320 - 200) \text{ kPa}] \, (0.05 \text{ m}^3)\left(\frac{1 \text{ kJ}}{1 \text{ kPa.m}^3}\right) = 3 \text{ kJ}$$

or this result could also be obtained from **Eq. 3-20**:

$$W_{\text{spring}} = \frac{1}{2}\,k(\, x_2^2 - x_1^2)$$

$$= \frac{1}{2}\,(150 \text{ kN/m})[(0.2 \text{ m})^2 - 0^2]\left(\frac{1 \text{ kJ}}{1 \text{ kN.m}}\right) = 3 \text{ kJ}$$

3-5. The first law of thermodynamics

The *first law of thermodynamics*, also **known** as *the conservation of energy principle*. Based on experimental observations, the first law of thermodynamics states that *energy can be neither created nor destroyed; it can only change forms*.

We all know that a rock at some elevation possesses some potential energy, and part of this potential energy is converted to kinetic energy as the rock falls (**Fig. 3-39**).

Consider first some processes that involve **heat transfer** but no **work interactions**. As a result of heat transfer to the potato as shown in (**Fig. 3-40**), the energy of the potato will increase. If we

disregard any mass transfer (moisture loss from the potato), the **increase in the total energy** of the potato becomes **equal to the amount of heat transfer**. Therefore, the conservation of energy principle for this case can be expressed as:

$Q = \Delta E$.

$Q = 5$ kJ
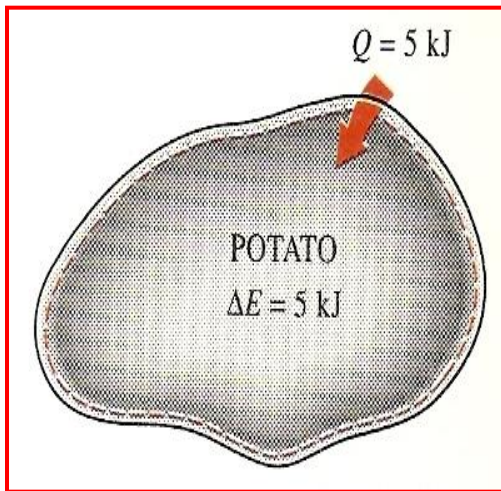
POTATO
$\Delta E = 5$ kJ

Figure 3-40. The increase in the energy of a potato in an oven is equal to the amount of heat transferred to it.

m $PE_1 = 10$ kJ
$KE_1 = 0$

Δz

$PE_2 = 7$ kJ
m $KE_2 = 3$ kJ
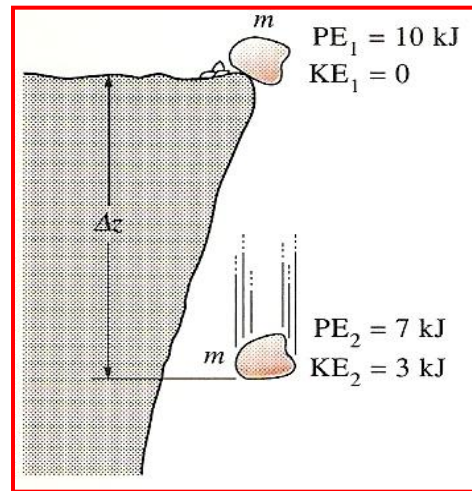
Figure 3-39. Energy cannot be created or destroyed; it can only change forms.

Consider the heating of water in a pan on top of a range (**Fig. 3-41**). If **15kJ** of heat is transferred to the water from the heating element and **3 kJ** of it is lost from the water to the surrounding air, the **increase** in **energy** of the water will be **equal** to the **net heat transfer** to water, which is **12 kJ**. That is:

$Q = Q_{net} = \Delta E$

$Q_2 = -3$ kJ

$\Delta E = Q_{net} = 12$ kJ

$Q_1 = 15$ kJ

Figure 3-41. In the absence of any work interactions, energy change of a system is equal to the net heat transfer.

The above **conclusions** can be summarized as follows: *In the absence of any work interactions between a system and its surroundings, the amount of net heat transfer is equal to the change in energy of a closed system.* That is:

$Q = \Delta E$ when $W = 0$ (3-21)

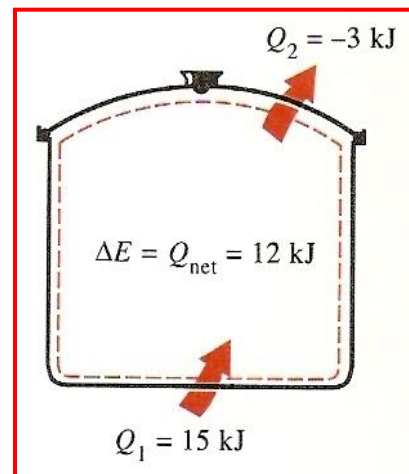Now consider a **well-insulated** (i.e., adiabatic) room heated by an electric heater as our system (**Fig.3-42**). The **conservation of energy** principle dictates that the **electrical work** done on the system must **equal the increase** in **energy of the system**. That is:

     $- W_e = \Delta E$.   when $Q = 0$

The **negative** sign is due to the Sign convention that work done on the system is *negative*.

Figure 3-42. The work (electrical) done on an adiabatic system is equal to the increase in the energy of the system.

Now let us replace the electric heater with a paddle wheel (**Fig. 3-43**). Again since there is no heat interaction between the system and its surroundings ($Q = 0$), the **paddle-wheel work** done on the **system** must show up as an **increase** in the **energy** of the **system**. That is:
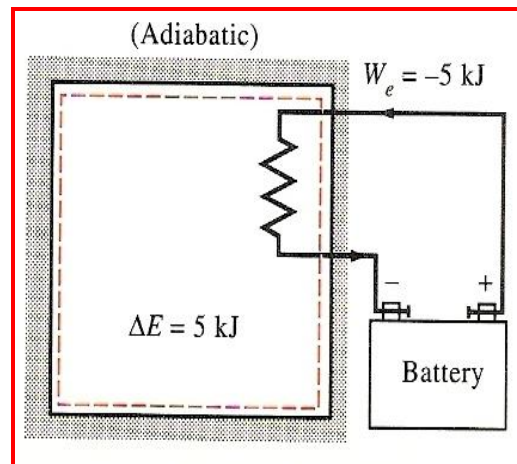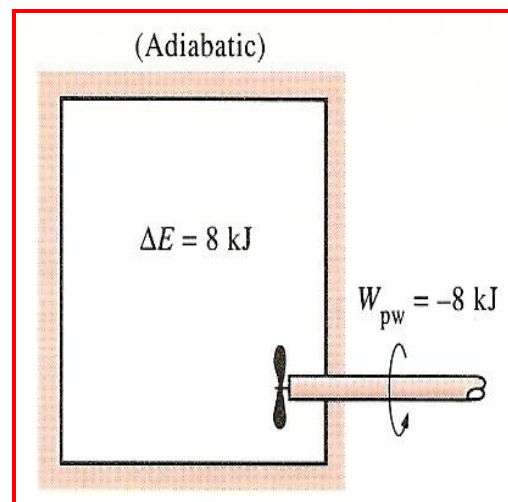
     $- W_{pw} = \Delta E$.

**Figure 3-43.** The work (shaft) done on an adiabatic system is equal to the

Many of you have probably noticed that the temperature of air rises when it is compressed (Fig. 3-44). This is because energy is added to the air in the form of boundary work. In the absence of any heat transfer (Q = 0), the entire boundary work will be stored in the air as part of its total energy. The conservation of energy principle again requires that:

$-W_b = \Delta E$.

It is clear from the foregoing examples that for adiabatic processes, the amount of work done is equal to the change in the energy of a closed system. That is:

$-W = \Delta E$ when $Q = 0$ (3-22)

$W_b = -10 \text{ kJ}$

$\Delta E = 10 \text{ kJ}$

(Adiabatic)

Figure 3-44. The work (boundary) done on an adiabatic system is equal to the increase in the energy of the system.
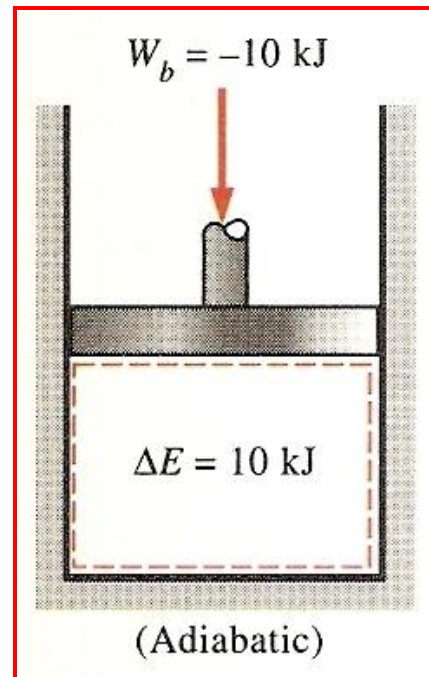
To generalize our conclusions, the first law of thermodynamics, or the conservation of energy principle for a closed system or a fixed mass, may be expressed as follows:

$$\begin{bmatrix} \text{Net energy tansfer} \\ \text{to (or from)the system} \\ \text{as heat and work} \end{bmatrix} = \begin{bmatrix} \text{Net incease (or decrease)} \\ \text{in the total energy} \\ \text{of the system} \end{bmatrix}$$

or $Q - W = \Delta E$ (3-23)

where

Q = net heat transfer across system boundaries ($= \sum Q_{in} - \sum Q_{out}$)

W = net work done in all forms (= $\sum W_{out} - \sum W_{in}$)

ΔE = net change in total energy of system, $E_2 - E_1$

The total energy E of a system is considered to consist of three parts: internal energy U, kinetic energy KE, and potential energy PE.

Then the **change in total energy** of a system during a process can be expressed:

$$\Delta E = \Delta U + \Delta KE + \Delta PE \qquad \text{(kJ)} \qquad\qquad \text{(3-24)}$$

Substituting this relation into **Eq. 3-23**, we obtain:

$$Q - W = \Delta U + \Delta KE + \Delta PE \qquad \text{(kJ)} \qquad\qquad \text{(3-25)}$$

Where $\Delta U = m(u_2 - u_1)$

$$\Delta KE = \frac{1}{2} m(V_2^2 - V_1^2)$$

$$\Delta PE = mg(z_2 - z_1)$$

Most **closed systems** encountered in practice are **stationary**, i.e., they **do not** involve any **changes** in their **velocity** or the **elevation** of their center of gravity during a process (**Fig. 3-46**). Thus for stationary closed systems, the change in kinetic and potential energies are negligible (that $\Delta KE = \Delta PE = 0$), and the **first-law** relation reduces to:

$$Q - W = \Delta U \qquad \text{(kJ)} \qquad \text{(3-26)}$$

Stationary Systems

$z_1 = z_2 \rightarrow \Delta PE = 0$

$V_1 = V_2 \rightarrow \Delta KE = 0$

$\Delta E = \Delta U$

Figure 3-46. For stationary systems, $\Delta KE = \Delta PE = 0$; thus $\Delta E = \Delta U$.

Sometimes it is convenient to consider the **work term** in two parts: $Q - W_{other}$, where W_{other} represents **all** forms of work **except** the **boundary work**. The first law takes the following form:

$$Q - W_{other} - W_b = \Delta E \qquad \text{(kJ)} \qquad \qquad (3\text{-}27)$$

Other forms of the first-law relation

The **first-law** relation for **closed systems** can be written in various forms (**Fig. 3-47**). Dividing **Eq. 3-23** by the **mass** of the system gives:

$$q - w = \Delta e \qquad \text{(kJ/kg)} \qquad \qquad (3\text{-}28)$$

The **rate form** of the **first-law** is obtained by dividing **Eq. 3-23** by the **time** interval Δt and taking the limit as $\Delta t \longrightarrow 0$. It yields:

$$\dot{Q} - \dot{W} = \frac{dE}{dt} \qquad \text{(kW)} \qquad \qquad (3\text{-}29)$$

Where, $\dot{Q}$ is the **rate** of net **heat** transfer, $\dot{W}$ is the **power**, and dE/dt is the rate of change of **total energy**.

Equation 3-23 can be expressed in the **differential form** as:

$$\delta Q - \delta W = dE \qquad \text{(kJ)} \qquad \qquad (3\text{-}30)$$

or $\qquad \delta q - \delta w = de \qquad \text{(kJ/kg)} \qquad \qquad (3\text{-}31)$

General $Q - W = \Delta E$

Stationary systems $Q - W = \Delta U$

Per unit mass $q - w = \Delta e$

Differential form $\delta q - \delta w = de$

Figure 3-47. Various forms of the first-law relation for closed systems.
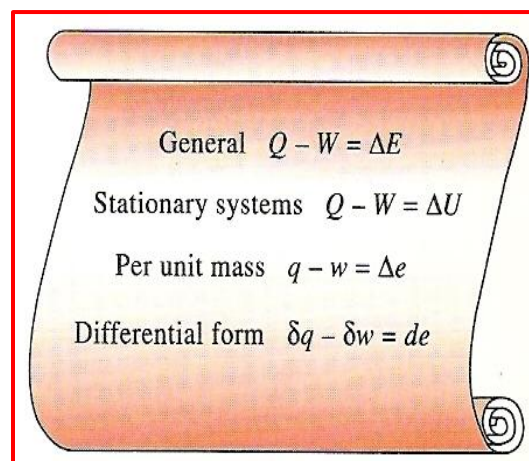
For a **cyclic process**, the initial and final states are identical, and therefore $\Delta E = E_2 - E_1 = 0$. Then the first-law relation for a cycle simplifies to:

$$Q - W = 0 \quad \text{(kJ)} \qquad\qquad (3\text{-}32)$$

That is, the **net heat transfer** and the **net work done** during a cycle must be equal (**Fig. 3-48**).
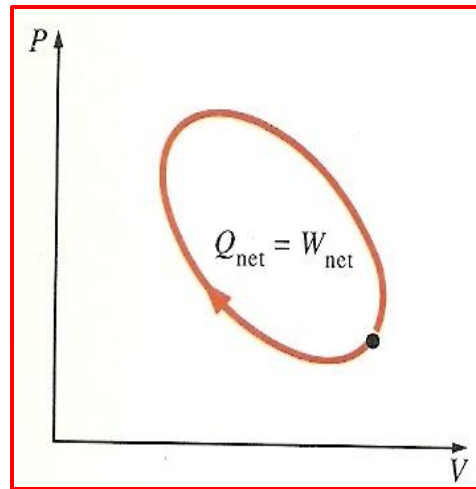
Figure 3-48. For a cycle $\Delta E = 0$, thus $Q = W$.

Example 3-14.

A rigid tank contains a hot fluid which is cooled while being stirred by a **paddle wheel**. Initially, the **internal energy** of the fluid is **800 kJ**. During the cooling process, the fluid loses **500 kJ** of heat, and the paddle wheel does **100 kJ** of work on the fluid. Determine the final internal energy of the fluid. Neglect the energy stored in the paddle wheel.

Solution

We choose the fluid in the tank as our system. The system boundaries are indicated in **Fig. 3-49**. By applying the conservation of energy principle as given by **Eq. 3-25** to this process, U_2 is determined to be:

$$Q - W = \Delta U + \Delta \cancel{KE}^{\,0} + \Delta \cancel{PE}^{\,0}$$

$$= \Delta U = U_2 - U_1$$

$$-\,500 \text{ kJ} - (-\,100 \text{ kJ}) = U_2 - 800 \text{ kJ}$$
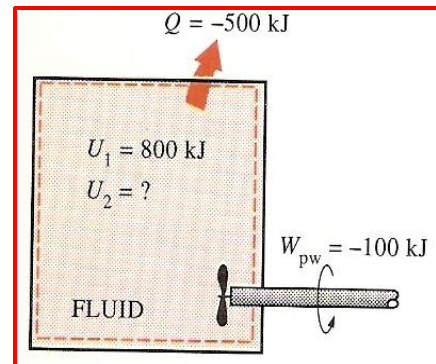
$$U_2 = 400 \text{ kJ}$$

Figure 3-49. Schematic for Example 3-14.

3-6. A systematic approach to problem solving

Thermodynamic problems, particularly the complicated ones, also require a systematic approach. By using a step-by-step approach, an engineer can solve a series of simple problems instead of one large, formidable problem (Fig. 3-50).

The proper approach to solving thermodynamic problems is illustrated below with the help of a sample problem.
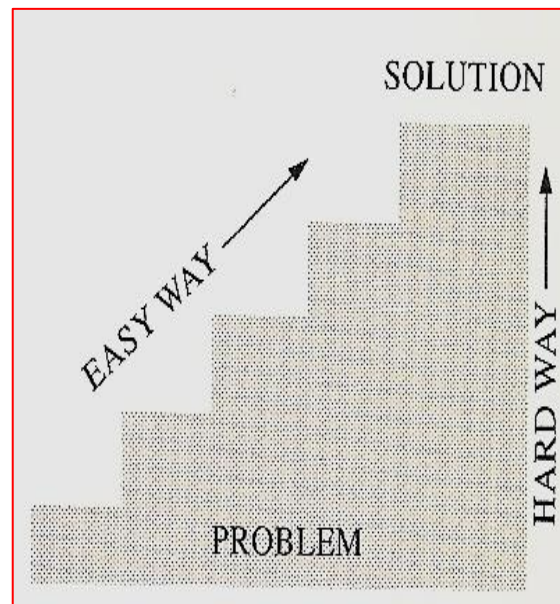

Figure 3-50. A step-by-step approach can greatly simplify problem solving.

Sample problem

A 0.1 m³ rigid tank contains steam initially at 500 kPa and 200℃. The steam is now allowed to cool until its temperature drops to 50℃. Determine the amount of heat transfer during this process and the final pressure in the tank.

Step 1: Draw a sketch and identify the system (Fig. 3-51):

Figure 3-51. Draw a sketch of the system and system boundaries.

SYSTEM: Steam

System boundaries
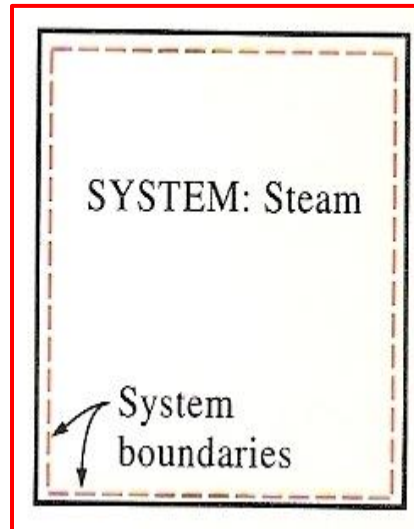
Step 2: List the given information on the sketch (Fig. 3.52):

$V = 0.1 \text{ m}^3$
$P_1 = 500 \text{ kPa}$
$T_1 = 200°C$
$T_2 = 50°C$

Heat

Figure 3-52. List the given information on the sketch.
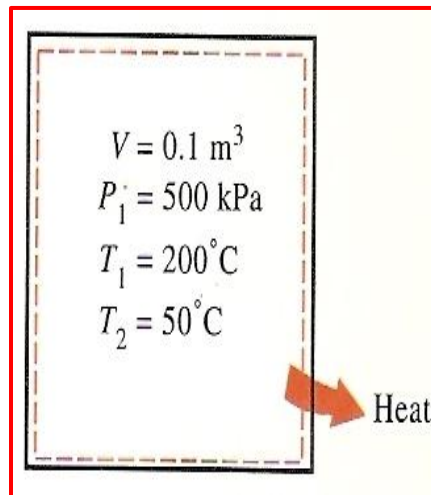
Step 3: Check for special processes (Fig. 3-53):

In our case, both the temperature and the pressure vary but the specific volume remains constant since **rigid tanks** have a **fixed volume (V = constant), and the mass** is **fixed (m = constant).**

$v_1 = v_2$

$W = 0$

Figure 3-53. Look for simplifications.

Step 4: State any assumptions (Fig. 3-54):

The system in our case can be assumed **to be stationary** since there is no indication to the contrary. Thus the **changes** in **kinetic** and **potential energies** can be **neglected**.

Figure 3-54. Make realistic assumptions, if necessary.

Assume:

(1) $\Delta PE = 0$ (since there is no mention of elevation change).

(2) $\Delta KE = 0$ (since there is no mention of velocity change).
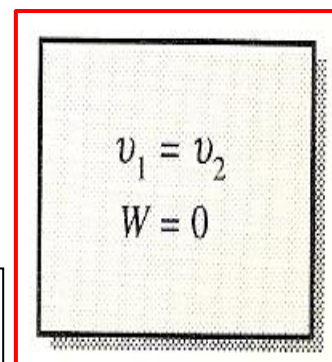
Step 5: Apply the conservation equations (Fig. 3-55):

Figure 3-55. Apply relevant conservation equations and simplify them.

Mass: $m_2 = m_1 = m$

Energy: $Q - W = \Delta U + \Delta KE^{0} + \Delta PE^{0}$

$$Q = m(u_2 - u_1)$$

Step 6: Draw a process diagram (Fig. 3-56):

Process diagrams, such as the $P\text{-}v$ or $T\text{-}v$ diagrams, are extremely helpful in **visualizing the initial** and **final states** of a system and the **path** of **the process**.

Figure 3-56. Show the process on a property diagram.

T, °C

200

50

1

2

0.4249 v, m^3/kg

Steam exists as a superheated vapor at the **state 1** since the temperature is greater the saturation temperature a the given pressure [that is, **200°C** > $T_{\text{sat @ 500 kPa}}$ = **151.9°C** (**Table A-5**)].

State 1: $P_1 = 500\ \text{kPa}$ ⎱ $v_1 = 0.4249\ \text{m}^3/\text{kg}$
$T_1 = 200°\text{C}$ ⎰ $u_1 = 2642.9\ \text{kJ/kg}$ **(Table A-6)**

State 2: $v_2 = v_1 = 0.4269\ \text{m}^3/\text{kg}$
$T_2 = 50°\text{C} \rightarrow v_f = 0.001\ \text{m}^3/\text{kg}$
$v_g = 12.03\ \text{m}^3/\text{kg}$
$u_f = 209.32\ \text{kJ/kg}$
$u_g = 2443.5\ \text{kJ/kg}$

(Table A-4)

$P_2 = P_{\text{sat @ 50°C}}$ **= 12.349 kPa**
$v_2 = v_f + x_2 v_{fg}$
$0.4249 = 0.001 + x_2(12.03 - 0.001)$

$x_2 = 0.0352$
$u_2 = u_f + x_2 u_{fg}$
$= 209.32 + (0.0352)(2443.5 - 209.32)$
$= 288.0\ \text{kJ/kg}$

$m = \dfrac{V}{v} = \dfrac{0.1\ \text{m}^3}{0.4249\ \text{m}^3/\text{kg}} = 0.235\ \text{kg}$

$Q = m(u_2 - u_1)$
$= (0.235\ \text{kg})(288 - 2642.9\ \text{kJ/kg})$

= − 553.4 kJ

Figure 3-57. Determine the required properties, and solve the problem.

Example 3-15.

A piston-cylinder device contains **25 g** of saturated water vapor which is maintained at a **constant** pressure of **300 kPa**. A resistance heater within the cylinder is turned on and passes a current of **0.2 A** for **5 min** from a **120 V** source. AT the same time, a heat loss of **307 kJ** occurs. (**a**) Show that for a closed system the boundary work W_b and the change in internal energy ΔU in the first-law relation can be combined into one term, ΔH, for a

constant-pressure process. (_b_) determine the final temperature of the steam.

Solution
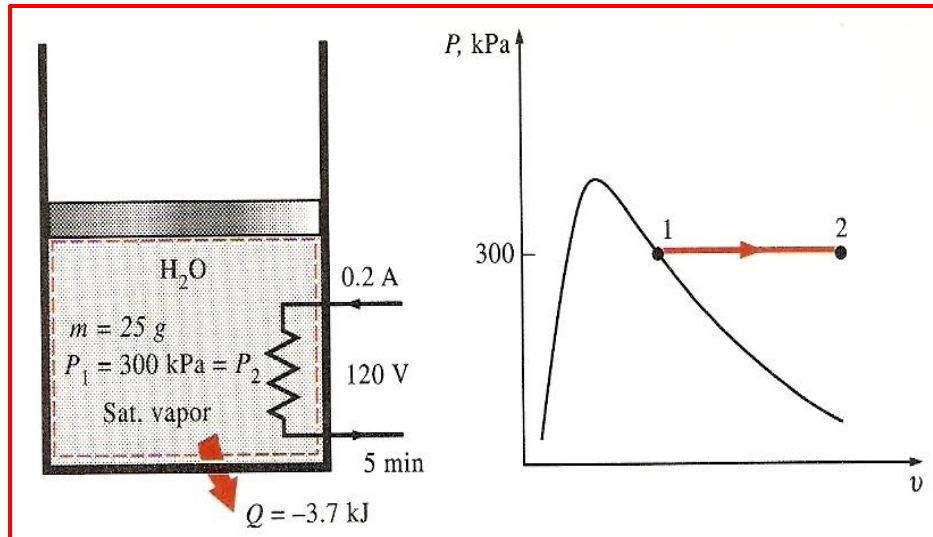
Figure 3-58. Schematic and diagram for Example 3-15.

(_a_) **Neglecting the changes in kinetic and potential energies and expressing the work as the sum of boundary and other forms of work, Eq.3-25 simplifies to:**

$$\overset{0}{\qquad}\overset{0}{\qquad}$$

$$Q - W = \Delta U + \Delta KE + \Delta PE \qquad \text{when} \quad W = W_{other} + W_b \quad \text{then:}$$

$$Q - W_{other} - W_b = \Delta U = U_2 - U_1$$

For a constant-pressure process, the boundary work is given by Eq. 3-10 as:

$W_b = P_0(V_2 - V_1)$. **Substituting this into the above relation gives:**

$$Q - W_{other} - P_0(V_2 - V_1) = U_2 - U_1$$

But

$$P_0 = P_2 = P_1 \longrightarrow Q - W_{other} = (U_2 + P_2 V_2) - (U_1 + P_1 V_1)$$

Also $H = U + PV$, and thus:

$$Q - W_{other} = H_2 - H_1 \qquad \text{(kJ)} \qquad\qquad (3\text{-}33)$$

which is the desired relation (Fig. 3-59):

Figure 3-59.For a closed system undergoing a quasi-equilibrium P = constant process, $\Delta U + W_b = \Delta H$
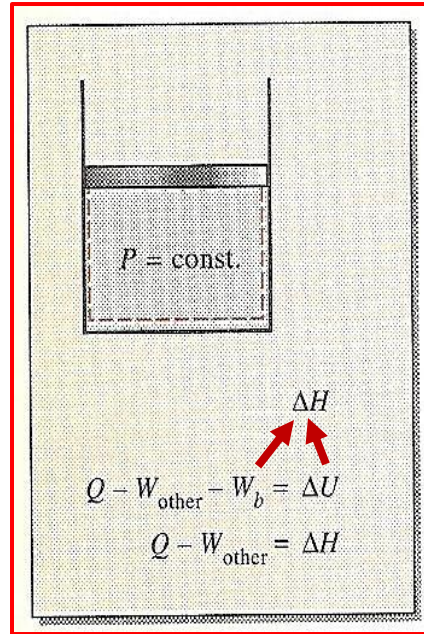
(**b**) **For our case, the only other form of work is the electrical work which, which can be determined from Eq. 3-5:**

$$W_e = VI\Delta t = (120 \text{ V})(0.2 \text{ A})(300 \text{ s})\left(\frac{1 \text{ kJ}}{1000 \text{ VA.s}}\right) = 7.2 \text{ kJ}$$

State 1: $\left.\begin{array}{l} P_1 = 300 \text{ kPa} \\ \text{sat. vapor} \end{array}\right\}$ $h_1 = h_{g \,@\, 300KPa}$ = 2725.3 kJ/kg (Table A-5)

The enthalpy at the final state can be determined from the conservation of energy relation for closed systems undergoing a constant-pressure process (Eq. 3-33):

$$Q - W_e = m(h_2 - h_1)$$

– 3.7 kJ – (–7.2 kJ) = (0.025 kg)(h_2 – 2725.3 kJ/kg)

$$h_2 = \textbf{2865.3 kJ/kg}$$

sine $h_2 > h_1$, the *state 2* is superheated water

State 2: $\left.\begin{array}{l} P_2 = 300 \text{ kPa} \\ h_2 = 2865.3 \text{ kJ/kg} \end{array}\right\}$ T_2 = **200**℃   (Table A- 6)

Example 3-16.

A rigid tank with a volume of **3 ft³** is initially filled with refrigerant-12 at **120 psia** and **140**°F. The refrigerant is now cooled to **20**°F. Determine (*a*) the mass of the refrigerant, (*b*) the final pressure in the tank, and (*c*) the heat transferred from the refrigerant.

Solution.

A sketch of the system and the *T-v* diagram of the process are given in **Fig. 3-60**.

(*a*) To find the mass, we need to know the specific volume of the refrigerant at the initial state, which is determined from **Table A-13E**:

State 1: P_1 = **120 psia** v_1= **0.389 ft³/lbm**

           T_1 = **140**°F u_1 = **86.098 Btu/lbm**

Thus, $m = \dfrac{V}{v_1} = \dfrac{3 \text{ ft}^3}{0.389 \text{ ft}^3/\text{Ibm}}$ = **7.71 lbm**

(*b*) This is a constant-volume process, and therefore $v_2 = v_1 =$ **0.389 ft³/lbm.**

At **20**°F:

$\left.\begin{array}{ll} v_f = \textbf{0.01130 ft}^3\textbf{/lbm} & u_f = \textbf{12.79 Btu/lbm} \\ \\ v_g = \textbf{1.0988 ft}^3\textbf{/lbm} & u_g = \textbf{72.12 But/lbm} \end{array}\right\}$ **(Table A-11E)**

The refrigerant is a saturated liquid-vapor mixture at the final state since $v_f < v_2 < v_g$. Therefore, the pressure must be the saturation pressure at **20°F**:

$$P_2 = P_{sat\ @\ 20F} = \textbf{35.736 psia} \qquad \textbf{(Table A-11E)}$$

(*c*) Assuming $\Delta KE = \Delta PE = 0$ and realizing that there are no boundary or other forms of work interactions, we see that the first-law relation (**Eq. 3-25**) simplifies to:

$$\overset{0}{Q} - \overset{0}{W} = \Delta U + \overset{0}{\Delta KE} + \overset{0}{\Delta PE}$$

$$Q = \Delta U = m(u_2 - u_1)$$

To determine u_2, we first need to know the quality x_2 at the final state. It is determined from:

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{[(0.389 - 0.0113)\text{m}^3/\text{kg}]}{[(1.0988 - 0.0113)\text{m}^3/\text{kg}]} = \textbf{0.348}$$
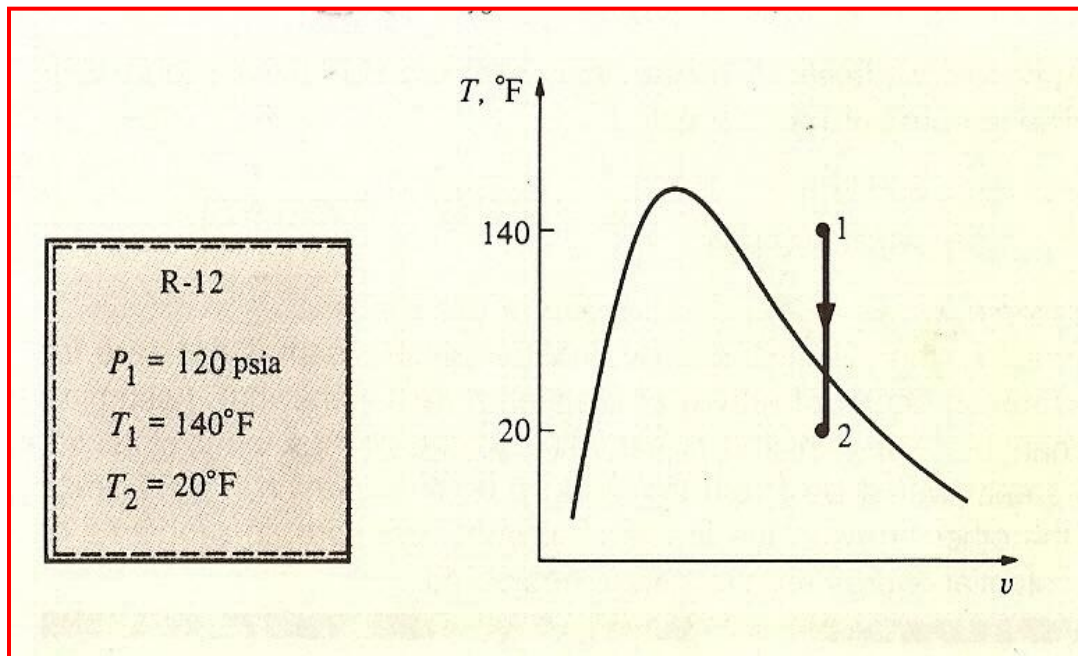

Figure 3-60. Schematic and *T-v* diagram for Example 3-16.

That is, **34.8 percent** of the refrigerant is in the vapor form at the final state. Then,

$$u_2 = u_f + x_2 u_{fg}$$

$$= 12.79 \text{ Btu/lbm} + (0.348)[(72.12 - 12.79) \text{ Btu/lbm}]$$

$$= 33.44 \text{ Btu/lbm}$$

Finally, substituting these values into **the first-law relation** will give us the **heat transfer**:

$$Q = (7.71 \text{ lbm})[(33.44 - 86.098) \text{ Btu/lbm}]$$

$$= - 406.0 \text{ Btu}$$

3-7. Specific heats

We know from experience that it takes different amounts of energy to raise the temperature of identical masses of different substances by one degree (**Fig. 3-62**). Therefore, it is desirable to have a property that will enable us to compare the energy storage capability of various substances. This property is the specific heat.

The **specific heat** is **defined** as *the energy required to raise the temperature of a unit mass of a substance by one degree* (**Fig. 3-63**). In thermodynamics, we are interested in **two kinds** of specific heats: specific heat at constant volume C_v and specific heat at costant pressure C_p.
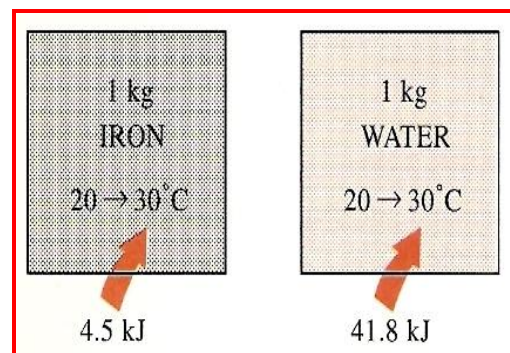

Figure 3-62. It takes different amounts of energy to raise the temperature of different substances by the same amount.

The **specific heat** at constant pressure C_p is always **greater** than C_v because at constant pressure the system is allowed to expand and the energy for this expansion work must also be supplied to the system as shown in (**Fig. 3-64**).

From the definition of C_v, this energy must be equal to C_v dT, where dT is the differential change in temperature. Thus,

C_v $dT = du$ at constant volume

$$C_v = \left(\frac{\partial u}{\partial T}\right)_v \qquad \text{(3-34)}$$

Figure 3-63. Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specific way.

Similarly, an expression for the specific heat at constant pressure C_p can be obtained by considering a constant-pressure process (w_b + Δu = Δh).

It yields

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p \qquad \text{(3-35)}$$

A common **unit** for **specific heats** is:

   kJ/(kg.°C)   or   **kJ/(kg.K)**,

these two units are identical

since ΔT (°C) = ΔT (K)

Figure 3-64. Constant-volume and constant-pressure specific heats C_v and C_p (values given are for helium gas).

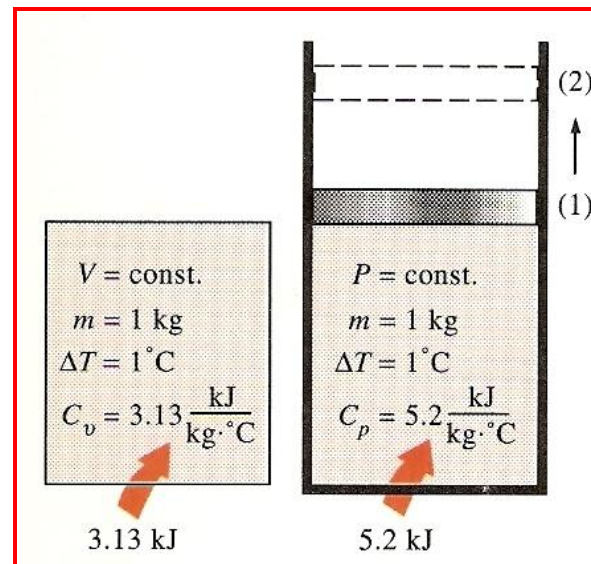Equations **3-34** and **3-35** are the defining equations for C_v and C_p, and their interpretation is given in **Fig. 3-65**.

The energy required to raise the temperatures of substance by one degree will be different at different temperatures and pressures (**Fig. 3-66**). But the difference usually not large.

$$C_v = \left(\frac{\partial u}{\partial T}\right)_v$$
= the change in internal energy with temperature at constant volume

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p$$
= the change in enthalpy with temperature at constant pressure

AIR

$m = 1$ kg

$300 \rightarrow 301$ K

0.718 kJ

AIR

$m = 1$ kg

$1000 \rightarrow 1001$ K

0.855 kJ
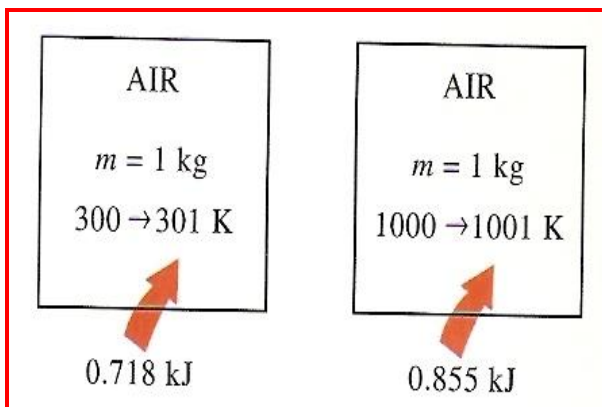
Figure 3-66. The specific heat of a substance changes with temperature.
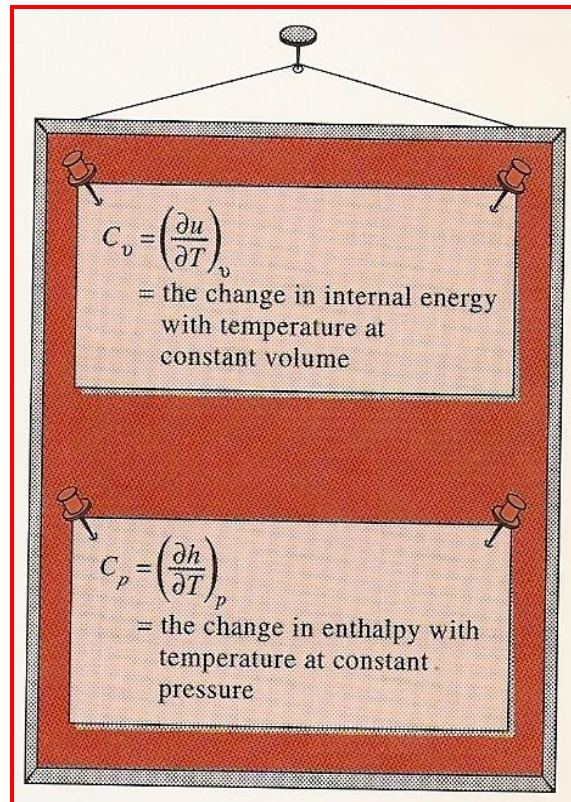
Figure 3-65. Mathematical definitions of C_p and C_v.

The **specific heats** are sometimes given on a **molar basis**. They are denoted by $\overline{C}_v$ and $\overline{C}_p$ and have the units **kJ/(kmol.$°C$)** or **kJ/(kmol.K)**.

3-8. Internal energy, enthalpy and specific heats of ideal gases

We defines an **ideal gas** as a substance whose temperature, pressure, and specific volume are related by:

$$Pv = RT$$

For ideal gas the internal energy is a function of the temperature only. That is,

$$u = u(T) \tag{3-36}$$

In the Joule's experiment, Joule submerged two tanks connected with a pipe and a valve in a water bath, as shown in Fig. 3-67. Since there was also no work done, he concluded that the internal energy of the air did not change even though the volume and the pressure changed. Therefore, he reasoned, the internal energy is a function of temperature only and not a function of pressure or specific volume.


Using the definition of enthalpy and the equation of state of an ideal gas, we have:
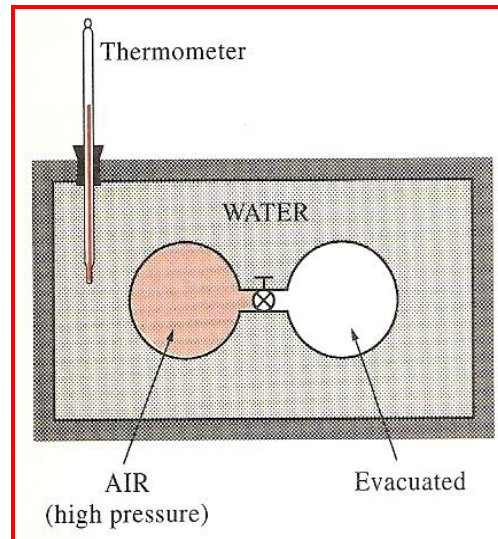
Figure 3-67. Schematic of the experimental apparatus used by Joule.

$$\left.\begin{array}{l} h = u + Pv \\ Pv = RT \end{array}\right\} \quad h = u + RT$$

Since R is constant and $u = u(T)$, it follows that the enthalpy of an ideal gas is also a function of temperature only:

$$h = h(T) \tag{3-37}$$

Since u and h depend only on temperature for an ideal gas, the specific heats C_v and C_p also depend, at most, on temperature only. Therefore, *at a given temperature u, h, C_v, and C_p of an ideal gas will have fixed values regardless of the specific volume or pressure* (Fig. 3-68).

Then the **differential changes** in the internal energy and enthalpy of an **ideal gas** can be expressed as:

$$du = C_v(T)\ dT \qquad (3\text{-}38)$$

and $dh = C_p(T)\ dT$ (3-39)

The change in **internal energy** or enthalpy for an ideal gas during a

Process from **state 1** to **state 2** is

Determined by integration these equations:

$$\Delta u = u_2 - u_1 = \int_1^2 C_v(T)\ dT \qquad \text{(kJ/kg)} \qquad (3\text{-}40)$$

and $\Delta h = h_2 - h_1 = \int_1^2 C_p(T)\ dT$ (kJ/kg) (3-41)

At **low pressures** all **real gases** approach **ideal-gas** behavior, and therefore their specific heats **depend** on **temperature** only. The specific heats of real gases at low pressures are called *ideal-gas specific heats* or *zero pressure specific heats*, and are often denoted C_{p0} and C_{v0} (**Table A-2** a, b, c). A plot of $\overline{C}_{p0}(T)$ data for some gases is given in **Fig. 3-69**.

Figure 3-69. Ideal-gas constant – pressure specific heats for some gases (see Table A-2c for $\overline{C}_{p0}$ equations).
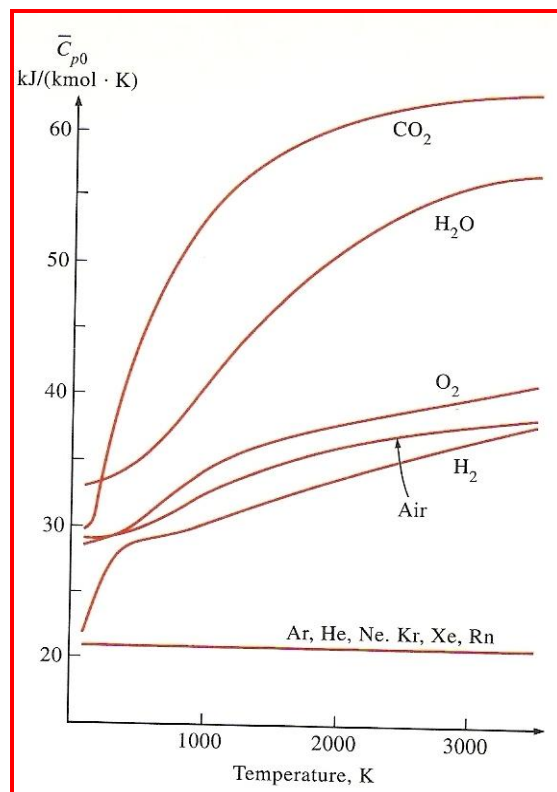
Some observation can made from **Fig. 3-69**. First, the **specific heats of gases** with **complex** molecules (**molecules with two or more atoms**) are **higher** and **increase** with **temperature**. Also the variation of specific heats with temperature is smooth and may be approximated as linear over small temperature intervals (**a few hundred or less**). Then the specific heat functions in **Eqs. 3-40** and **3-41** can be replaced by **the constant average specific heat values**. Now the integration in these equations can be performed, yielding:

$$u_2 - u_1 = C_{v,\text{av}}(T_2 - T_1) \qquad \text{(kJ/kg)} \qquad\qquad (3\text{-}42)$$

and $\qquad h_2 - h_1 = C_{p,\text{av}}(T_2 - T_1) \qquad \text{(kJ/kg)} \qquad\qquad (3\text{-}43)$

The specific heat values for some common gases are listed as a function of temperature in **Table A-2***b*. The **average specific heats** $C_{p,\text{av}}$ and $C_{v,\text{av}}$ are evaluated from this table at the average temperature $(T_2 + T_1)/2$, as shown in **Fig. 3-71**.

Another way of determining the average specific heats is to evaluate them at T_1 and T_2 and then take **their average**. Usually both methods give reasonably **good results**, and one is not necessarily better than the other.

Another observation that can be made from **Fig. 3-69** is that the **ideal-gas specific heats** of *monatomic gases* such as **argon**, **neon**, and **helium** remain **constant** over the entire **temperature** range. Thus Δu and Δh of *monatomic gases* can easily evaluated from **Eqs. 3-42** and **3-43.**
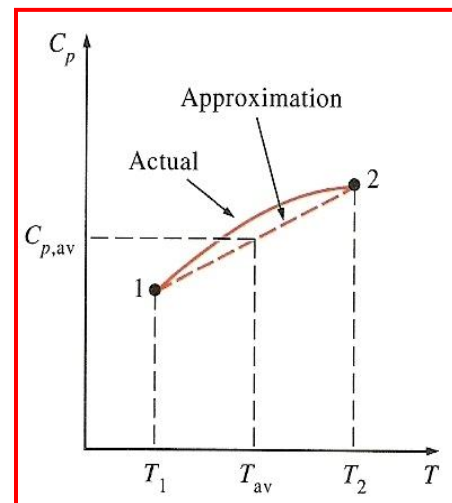

Figure 3-71. For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.

Specific-heat relations of ideal gases

A special relationship between C_v and C_p for ideal gases can be obtained by differentiating the relation $h = u + RT$, which yields:

$$dh = du + RdT$$

Replacing dh by $C_p \, dT$ and du by $C_v \, dT$ and dividing the resultant expression by dT, we obtain:

$$C_p = C_v + R \qquad \text{[kJ/(kg.K)]} \qquad \text{(3-44)}$$

This is an important relationship for ideal gases since it enables us to determine C_v from a knowledge of C_p and the gas constant R.

When the specific heats are given on a molar basis, R in the above equation should be replaced by the universal gas constant R_u (**Fig. 3-74**). That is,

$$\overline{C}_p = \overline{C}_v + R_u \qquad \text{[kJ/(kmol.K)]} \qquad \text{(3-45)}$$

At this point, we introduce another ideal-gas property called the specific heat ratio k, defined as:

$$k = \frac{C_p}{C_v} \qquad \text{(3-46)}$$

Figure 3-74. The C_p of an ideal gas can be determine from a knowledge of C_v and R.

AIR at 300 K

$$\left. \begin{array}{l} C_v = 0.718 \text{ kJ/(kg} \cdot \text{K)} \\ R = 0.287 \text{ kJ/(kg} \cdot \text{K)} \end{array} \right\} C_p = 1.005 \text{ kJ/(kg} \cdot \text{K)}$$

or,

$$\left. \begin{array}{l} \overline{C}_v = 20.80 \text{ kJ/(kmol} \cdot \text{K)} \\ R_u = 8.314 \text{ kJ/(kmol} \cdot \text{K)} \end{array} \right\} \overline{C}_p = 29.114 \text{ kJ/(kmol} \cdot \text{K)}$$
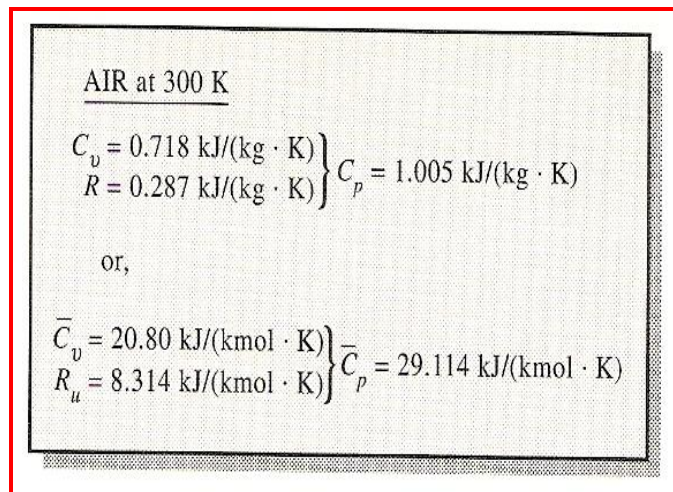
The **specific heat ratio (k)** *also* varies **with temperature**, but this variation is **very small**. For **monatomic** gases, its value is essentially constant at k = **1.667**. Many **diatomic** gases, including **air**, have specific heat ratio of about k = **1.4** at room temperature.

Example 3-18.

Air at **300 K** and **200 kPa** is heated at constant pressure to **600 K**. **Determine** the change in **internal energy** of air per unit mass(Δu), using (**a**) data from the air table (**Table A-17**), (**b**) the functional form of the specific heat (**Table A-2c**), and (**c**) the average specific heat value (**Table A-2b**).

Solution.

Air can be considered to be an **ideal gas** since it is at a **high temperature** and **low pressure relative** to its **critical-point** values (T_{cr} = **-147**℃, P_{cr} = **3390 kPa** for nitrogen, the main constituent of air). Δu of ideal gases depends on the **initial** and **final** **temperatures only**, and not on the type of process.

(**a**)

$$u_1 = u_{@\ 300\ K} = \textbf{214.07 kJ/kg}$$

$$u_2 = u_{@\ 600\ K} = \textbf{434.78 kJ/kg}$$

(**Table A-17**)

Thus, $\Delta u = u_2 - u_1$ = (434.78 − 214.07) kJ/kg = **220.71 kJ/kg**

(**b**)

The $\overline{C}_p(T)$ of air is given in **Table A-2c** in the form of a third-degree polynomial expressed as:

$$\overline{C}_p(T) = a + bT + cT^2 + dT^3$$

Where a = 28.11, b = 0.1967 x 10^{-2}, c = 0.4802 x 10^{-5}, d = -1.966 x 10^{-9}. From **Eq. 3-45**,

$$\overline{C}_v(T) = \overline{C}_p - R_u = (a - R_u) + bT + cT^2 + dT^3$$

From **Eq. 3-40**,

$$\Delta \overline{u} = \int_1^2 \overline{C}_v(T)\, dT$$

$$= \int_{T_1}^{T_2}[(a - R_u) + bT + cT^2 + dT^3]\, dT$$

Performing the **integration** and **substitution the values**, we obtain:

$$\Delta \overline{u} = \text{6447.15 kJ/kmol}$$

The change in the internal energy on a unit-mass basis (Δu) is determined by **dividing** this value by the **molar mass** of **air (Table A-1)**:

$$\Delta u = \frac{\Delta \overline{u}}{M} = \frac{6447.15 \text{ kJ/kmol}}{28.97 \text{ kg/kmol}} = \text{222.55 kJ/kg}$$

This differs from the **exact** result by **0.8 percent**:

$$\% \text{ error} = \frac{(222.55 - 220.71) \text{ kJ/kg}}{(220.71) \text{ kJ/kg}} \text{ x } 100 \cong \textbf{0.8 \%}$$

(**c**) The average value of the constant-volume specific heat $C_{v,av}$ is determined from **Table A-2b** at the average temperature($T_2 + T_1$)/2 = **(300 + 600) K/2 = 450 K** to be:

$$C_{v,av} = C_{v \text{ @ 450 K}} = \text{0.733 kJ/(kg.K)} \qquad \textbf{(Table A-2}\textbf{b)}$$

Thus $\qquad \Delta u = C_{v,av}(T_2 - T_1) = [\text{0.733 kJ/(kg.K)}][(600 - 300) \text{ K}]$

$$= \text{219.9 kJ/kg}$$

This answer differs the **exact** result (**220.71 kJ/kg**) by only **0.4 percent**:

$$\% \text{ error} = \frac{(220.71 - 219.9) \text{ kJ/kg}}{(220.71) \text{ kJ/kg}} \text{ x } 100 \cong 0.4 \%$$

Example 3-19.

An insulted rigid tank initially contains **1.5 lbm** of helium at **80°F** and **50 psia**. A paddle wheel with a power rating of **0.02 hp** is operated within the tank for **30 min**. Determine (**a**) the final temperature and (**b**) the final pressure of the helium gas.

Solution.

A sketch of the system and the **P-v** diagram of the process are shown in **Fig. 3-75**. The helium gas at the specified conditions can be considered to be an **ideal gas** since it is at a very high temperature relative to its critical-point temperature (T_{cr} = - 451°F for helium **Table A-1E**).

(**a**) The amount of paddle-wheel work done on the system is:

$$W_{pw} = \dot{W}_{pw} \, \Delta t = (-0.02 \text{ hp})(0.5 \text{ h}) \left(\frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right) = -25.45 \text{ Btu}$$

Since, $W_b = 0$ (rigid tank have not moving boundaries)

$Q = 0$ (insulated tank)

$\Delta KE = \Delta PE = 0$ (The system is **assumed** to be **stationary**)

Then **the conservation of energy equation** for this closed system reduces to:

$$\overset{0}{\cancel{Q}} - W_{pw} - \overset{0}{\cancel{W_b}} = \Delta U + \overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}}$$

$$- W_{pw} = \Delta U = m(u_2 - u_1) \cong mC_{v,av}(T_2 - T_1)$$

As we pointed out earlier, the ideal-gas specific heats of monatomic gases (helium being one of them) are constant. The C_v value of helium is determined from Table A-2E*a* to be C_v = 0.753 Btu/(lbm.°F). Substituting this and other known quantities into above energy equation, we obtain:

$$- (- 25.45 \text{ Btu}) = (1.5 \text{ lbm})(0.753 \text{ Btu/(lbm.°F)})(T_2 - 80°F)$$

$$T_2 = 102.5°F$$

(*b*) The final pressure is determined from the ideal-gas relation:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Where V_1 and V_2 are identical and cancel. Then the final pressure becomes:

$$\frac{50 \text{ psia}}{(80+460) \text{ R}} = \frac{P_2}{(102.5+460)\text{R}}$$
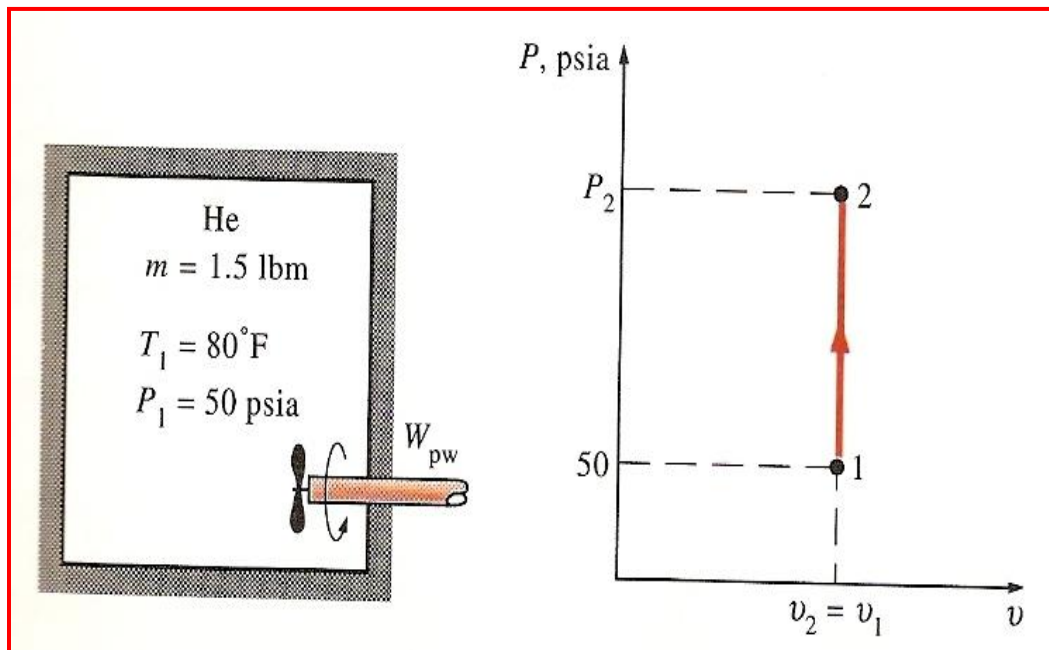
$$P_2 = 53.1 \text{ psia}$$

Figure 3-75. Schematic and *P-v* diagram for Example 3-19.

Example 3-20.

A piston-cylinder device initially contains 0.5 m³ of nitrogen gas at 400 kPa and 27℃. An electric heater within the device is turned on and is allowed to pass a current of 2 A for 5 min from a 120 V source. Nitrogen expands at constant pressure, and a heat loss of 2800 J occurs during the process. Determine the final temperature of the nitrogen, using data from the nitrogen table (Table A-18).

Solution.

A sketch of the system and the *P-V* diagram of the process are given in Fig. 3-76.
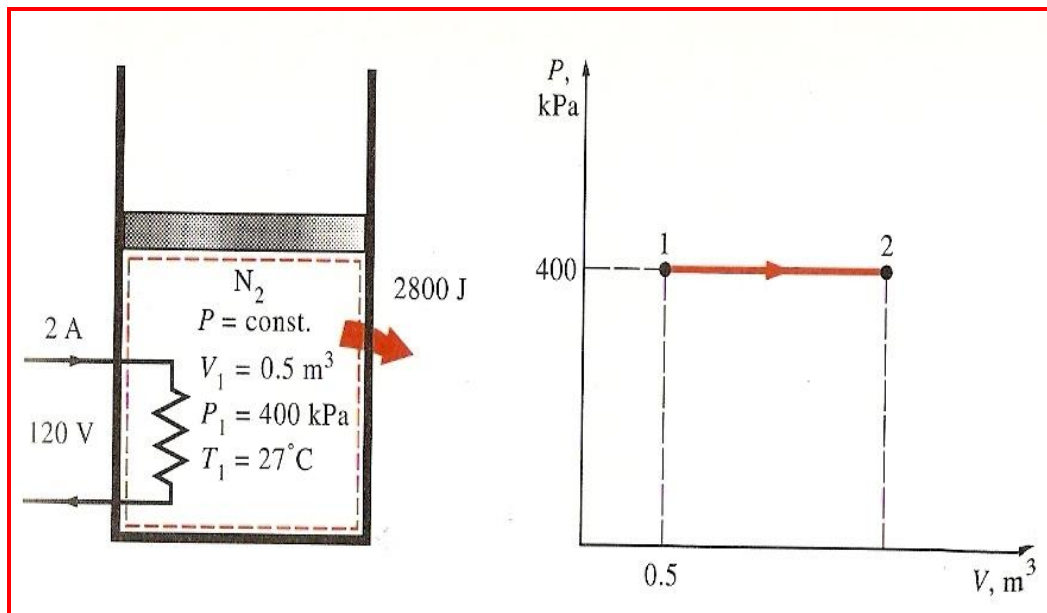

Figure 3-76. Schematic and *P-V* diagram for Example 3-20.

At the specified conditions, the nitrogen gas can be considered to be an ideal gas since it is at a high temperature and low

pressure relative to its critical-point values (T_{cr} = **−147**°C, P_{cr} = **3390 kPa**, Table A-1).

First, let us determine the **electrical work** done on the nitrogen:

$$W_e = VI\,\Delta t = (120\text{ V})(2\text{ A})(5 \times 60\text{ s})\left(\frac{1\text{ kJ}}{1000\text{ VA.}s}\right) = -\ 72\text{ kJ}$$

The **negative sign** is added because the work is done on the system.

The **number of moles** of nitrogen is determined from the ideal-gas relation:

$$N = \frac{P_1 V_1}{R_u T_1} = \frac{(400\text{ kPa})(0.5\text{ m}^3)}{[8.314\text{ kJ/(kmol.K)}](300\text{ K})} = 0.080\text{ kmol}$$

When gases other than air are involved, it is more convenient to work with mole numbers instead of masses since all the u and h data are given on a mole basis (**Fig. 3-77**). Assuming **no change** in **kinetic** and **potential** energies (Δ**KE** = Δ**PE** = 0), the conservation of energy equation for this closed system can be written as:

$$Q - W_e - W_b = \Delta U$$

For the constant- pressure process of a closed system, ΔU + W_b is equivalent to ΔH. Thus,

$$Q - W_e = \Delta H = m(h_2 - h_1) = N(\bar{h}_2 - \bar{h}_1)$$

From the nitrogen table, $\bar{h}_1 = \bar{h}_{@\ 300\ K}$ = 8723 kJ/kmol. The only unknown quantity in the above equation is $\bar{h}_2$, and it is found to be:

$$-\ 2.8\text{ kJ} - (-\ 72\text{ kJ}) = (0.08\text{ kmol})(\ \bar{h}_2 - 8723\text{ kJ/kmol})$$

$$\bar{h}_2 =\ 9588\text{ kJ/kmol}$$

The temperature corresponding to this enthalpy value is:

T_2 = 329.7 K = 56.7℃   (Table A-18)

$$\Delta H = m(h_2 - h_1) \quad (kJ)$$

$$\Delta H = N(\bar{h}_2 - \bar{h}_1) \quad (kJ)$$

Figure 3-77. Two equivalent ways of determining the total enthalpy change ΔH.

3-9. Internal energy, enthalpy, and specific heats of solids and liquids

A substance whose specific volume (or density) is constant is called an *incompressible substance*. The specific volumes of solids and liquids essentially remain constant during a process (Fig. 3-79). Therefore, liquids and solids can be approximated as incompressible substances without sacrificing much in accuracy.

The constant-volume and constant-pressure specific heats are identical for incompressible substances (Fig. 3-80).
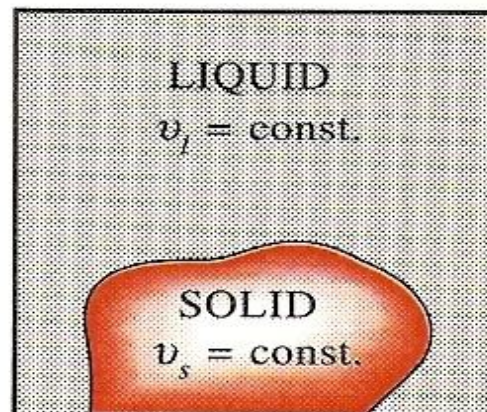

Figure 3-79. The specific volumes of incompressible substances remain constant during a process.

Therefore, for **solids** and **liquids** the subscripts on C_p and C_v can be **dropped**, and both specific heats can be represented by a single symbol C. That is,

$$C_p = C_v = C \qquad \text{(3-47)}$$

Specific heat values for several common liquids and solids are given in **Table A-3**.

Like those of ideal gases, the specific heats of incompressible substances depends on temperature only. Thus the partial differentials in the defining equation of C_v (**Eq. 3-34**) can be replaced by ordinary differentials, which yields:

Figure 3-80. The C_v and C_p values of incompressible substances are identical and are denoted by C.

$$du = C_v\, dT = C(T)\, dT \qquad \text{(3-48)}$$

The **change in internal energy** between state **1** and **2** is then obtained by integration:

$$\Delta u = u_2 - u_1 = \int_1^2 C(T)\, dT \qquad \text{(kJ/kg)} \qquad \text{(3-49)}$$

For **small temperature intervals**, a C values at the average temperature can be used and treated as a constant, yielding:

$$\Delta u \cong C_{\text{av}}(T_2 - T_1) \qquad \text{(kJ/kg)} \qquad \text{(3-50)}$$

For enthalpy change of **solids** or **liquids** during process **1-2** can be determined from the definition of enthalpy ($h = u + Pv$) to be:

$$h_2 - h_1 = (u_2 - u_1) + v(P_2 - P_1) \qquad \text{(3-51)}$$

since $v_1 = v_2 = v$. It can also be expressed in a compact form as:

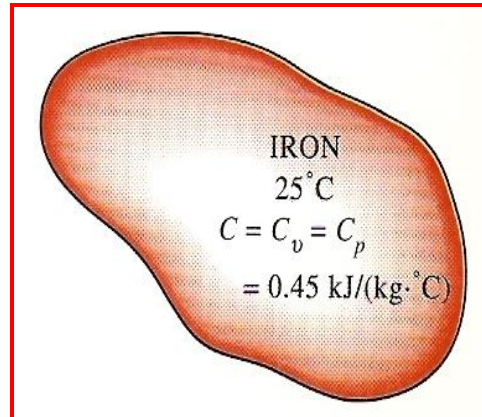$$\Delta h = \Delta u + v\, \Delta P \qquad \text{(kJ/kg)} \qquad \text{(3-52)}$$

The second term ($v \, \Delta P$) in **Eq.3-52** is often **small** compared with the first term (Δu) and can be neglected without significant loss in accuracy.

For a constant-temperature process ($\Delta T = 0$) , the internal energy change of an incompressible substance is ($\Delta u = 0$). The from **Eq. 3-51** the change in enthalpy change will be:

$$h_2 - h_1 = v(P_2 - P_1)$$

By taking **state 2** as the **compressed liquid** state and **state 1** as the **saturated liquid** state at the **same temperature**, the **enthalpy of the compressed liquid** at a given P and T can be determined from:

$$h_{@ \, P,T} \cong h_{f \, @ \, T} + v_{f \, @ \, T}(P - P_{\text{sat}}) \tag{3-53}$$

where P_{sat} is the **saturation pressure** at the given **temperature**.

This is an improvement over the assumption that the **enthalpy of the compressed liquid** could be taken as h_f at the given temperature (that is, $h_{@ \, P,T} \cong h_{f \, @ \, T}$). However, the contribution of the **last term** is often **very small**, so it is usually **neglected**.

Example 3-22.

A **50 kg** iron block at **80°C** is dropped into an insulated tank which contains **0.5 m³** of liquid water at **25°C**. Determine the temperature when thermal equilibrium is reached.

Solution.

We take the iron block and water as our system. The **inner surfaces of the tank walls** form the **system boundary**, as shown in **Fig. 3-81**.

Since the tank is insulated, no heat will cross these boundaries ($Q = 0$). Also since there is no movement of the boundary ($W_b =$

0) and no indication of other forms of work (W_{other} = 0), the work for this process is zero (W = 0). Then the conservation of energy equation for this process will reduce to:

$$\overset{0}{\cancel{Q}} - \overset{0}{\cancel{W}_{other}} - \overset{0}{\cancel{W}_b} = \Delta U + \overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} \quad \text{or} \quad \Delta U = 0$$

The total energy **U** is an extensive property, and therefore it can be expressed as the sum of the internal energies of the parts of the system. Then the total internal energy change of the system is:

$$\Delta U_{system} = \Delta U_{iron} + \Delta U_{water} = 0$$

$$[mC(T_2 - T_1)]_{iron} + [mC(T_2 - T_1)]_{water} = 0$$

The specific volume of liquid water at or about room temperature can be

Taken to be v_{water} = 0.001 m³/kg (**Table A-4**).

Figure 3-81.Schematic for Example 3-22.

Then mass of the water is:

$$m_{water} = \frac{V}{v} = \frac{0.5 \text{ m}^3}{0.001 \text{ m}^3/\text{kg}} = 500 \text{ kg}$$

The specific heats of iron and liquid water are determined:

C_{iron} = 0.45 kJ/(kg.℃)

C_{water} = 4.184 kJ/(kg.℃)

(**Table A-3 a**) at **25℃**

Subsisting these values into the energy equation, we obtain:

(50 kg)[0.45 kj/(kg. ℃)](T_2 − 80℃)

+ (500 kg)[4.184 kj/(kg. ℃)](T_2 −25℃) = 0, T_2 = 25.6℃

Example 3-23.

Determine the **enthalpy** of liquid water at **100°C** and **15 MPa** (**a**) from the compressed liquid tables, (**b**) by approximating it as saturated liquid, and (**c**) by using the correction given by **Eq. 3-53**.

Solution.

At **100°C**, the saturation pressure of water P_{sat} = **101.35 kPa** from (**Table A-4**), and since $P > P_{sat}$, the water exists as a **compressed** liquid at the specified sate.

(**a**) From the **compressed liquid tables**, we read:

$$\left. \begin{array}{l} P = 15 \text{ MPa} \\ T = 100°C \end{array} \right\} \quad h = 430.28 \text{ kJ/kg} \quad \text{(Table A-7)}$$

This is the **exact value.**

(**b**) Approximating the **compressed liquid** as a saturated liquid at **100°C**, as is commonly done, we obtain:

$$h \cong h_{f @ 100 C} = 419.04 \text{ kJkg} \qquad \text{(Table A-4)}$$

This value is in **error** by about **2.6 percent.**

(**c**) From **Eq. 3-53**:

$$h_{@ P,T} \cong h_{f @ T} + v_{f @ T}(P - P_{sat})$$

$$\cong (419.04 \text{ kJ/kg}) + (0.001 \text{ m}^3\text{/kg})[(15000 - 101.35)\text{kPa}]^*$$

$$*\left(\frac{1 \text{ kJ}}{1 \text{ kPa.m}^3} \right)$$

$$\cong 434.60 \text{ kJ/kg}$$

The correction term **reduced error** from **2.6** to about **1 percent**. But this improvement in accuracy is often not worth the extra effort involved.