# . Free Radical Copolymerization

### > Radical copolymerization

Regular copolymer CCC



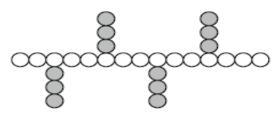
Random copolymer



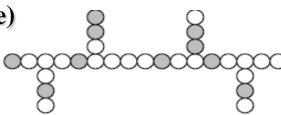
**Block copolymer** 



**Graft copolymer** 



**Actual copolymer (case)** 





# **Copolymer Equation**

#### **Only Binary Case**

Two Monomers; M1 + M2

$$M_1 \cdot + M_1 \xrightarrow{k_{11}} \longrightarrow M_1 \cdot M_1$$
 $M_1 \cdot + M_2 \xrightarrow{k_{12}} \longrightarrow M_2 \cdot M_2$ 

#### **Steady State Assumption**

$$-\frac{d[M_1\cdot]}{dt} = -\frac{d[M_2\cdot]}{dt} = 0$$

 $-\frac{d[M_1\cdot]}{dt} = -\frac{d[M_2\cdot]}{dt} = 0$  and chain transfer & termination compared w/ propagation

$$k_{12}[M_1 \cdot][M_2] = k_{21}[M_2 \cdot][M_1]$$

$$\frac{[M_1 \cdot]}{[M_2 \cdot]} = \frac{k_{21}[M_1]}{k_{12}[M_2]} \qquad \dots .....$$



### **Copolymer Equation**

$$\begin{split} &-\frac{d[M_1]}{dt} = k_{11}[M_1 \cdot][M_1] + k_{21}[M_2 \cdot][M_1] \\ &-\frac{d[M_2]}{dt} = k_{12}[M_1 \cdot][M_2] + k_{22}[M_2 \cdot][M_2] \\ &\frac{d[M_1]}{d[M_2]} = \frac{[M_1]}{[M_2]} \frac{k_{11}[M_1 \cdot] + k_{21}[M_2 \cdot][M_1]}{k_{12}[M_1 \cdot] + k_{21}[M_2 \cdot][M_2]} \qquad ..... \textcircled{2} \end{split}$$

Instantaneous ratio of monomers in copolymer

#### From (1) and (2)

$$\frac{d[M_1]}{d[M_2]} = \frac{1 + r_1 \frac{[M_1]}{[M_2]}}{1 + r_2 \frac{[M_2]}{[M_1]}} \quad \text{where} \quad r_2 = \frac{k_{22}}{k_{21}}, \quad r_1 = \frac{k_{11}}{k_{12}} \quad \text{monomer reactivity ratio}$$

$$r_2 = \frac{k_{22}}{k_{21}}$$
 ,  $r_1 = \frac{k_{11}}{k_{12}}$ 

Copolymer Eq.



# Meaning of r & Definition of f1, F1

#### Meaning of r

 $r_1$  characterizes the reactivity of the 1 radical with respect to the two monomers, 1 and 2

 $r_1 > 1$  then homopolymerization growth is preferred

 $r_1 = 0$  then only reaction with 2 will occur

#### Define f1, F1

 $f_1, f_2$ : mole fractions of monomers in feed

 $\overline{F}_1$ ,  $\overline{F}_2$ : mole fractions of monomers in polymer

$$f_1 = 1 - f_2 = \frac{[M_1]}{[M_1 + M_2]}$$
 ..... 3  $F_1 = 1 - F_2 = \frac{d[M_1]}{d[M_1] + d[M_2]}$  ..... 4

From ③, ④ 
$$\mathbf{F}_{1} = \frac{\mathbf{r}_{1}\mathbf{f}_{1}^{2} + \mathbf{f}_{1}\mathbf{f}_{2}}{\mathbf{r}_{1}\mathbf{f}_{1}^{2} + 2\mathbf{f}_{1}\mathbf{f}_{2} + \mathbf{r}_{2}\mathbf{f}_{2}^{2}} \quad \dots \dots 5$$



# **Ideal Copolymerization**

#### **Ideal Copolymerization**

$$\frac{d[M_1]}{d[M_2]} = \frac{[M_1]}{[M_2]} \cdot \frac{r_1[M_1] + [M_2]}{[M_1] + r_2[M_2]} \qquad \text{where} \quad r_1 \cdot r_2 = 1 \implies r_2 = \frac{1}{r_1}$$

$$= r_1 \frac{[M_1]}{[M_2]} \cdot \frac{r_1[M_1] + [M_2]}{r_1[M_1] + [M_2]}$$

$$= r_1 \frac{[M_1]}{[M_2]}$$

$$F_1 = \frac{r_1 f_1}{r_1 f_1 + f_2} \qquad \frac{k_{11}}{k_{12}} \cdot \frac{k_{22}}{k_{21}} = 1 \qquad \frac{k_{11}}{k_{12}} = \frac{k_{21}}{k_{22}}$$

Most ionic copolymerizations are characterizes by the ideal type of behavior

When  $r_1=1=r_2$  , the two monomers show equal reactivity toward both propagating species random copolymer

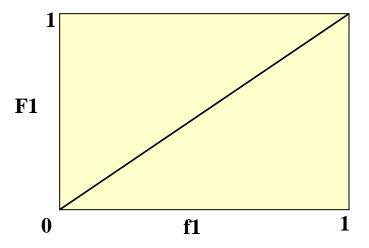


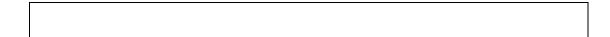
# **Ideal Copolymerization**

 $r_1 > 1$   $r_2 < 1$  or  $r_1 < 1$  One of the monomer us nire reactive than

The other toward both propagating spices. The copolymer will contain a larger proportion of

the more reactive monomer in random placement

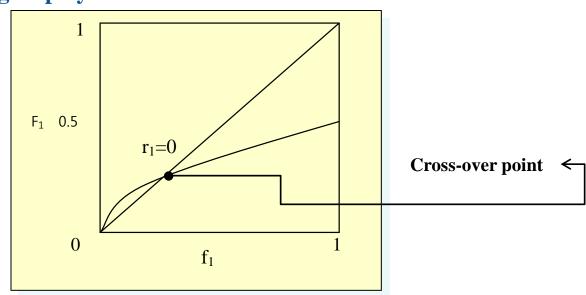






#### **Alternating Copolymerization**

$$r_1 = r_2 = 0$$
or
$$r_1 r_2 = 0$$



As  $r_1$ ,  $r_2$  approach to zero, alternating tendency can be observed

If  $r_1 = r_2 = 0$   $\longrightarrow$  perfect alternation!

$$\frac{d[M_1]}{d[M_2]} = 1$$

$$\frac{d[M_1]}{d[M_2]} = 1 \qquad F_1 = 0.5 = \frac{d[M_1]}{d[M_1] + d[M_2]}$$

If  $r_1 < 1$ ,  $r_2 < 1$   $F_1 / f_1$  plots cross the line representing  $F_1 = f_1$ 

If  $r_1 = r_2 = \infty$ , then, become a homopolymer



#### **Mean of Cross-over Point**

$$F_1 = f_1$$

At these crossover points the copolymer and feed compositions are the same and copolymerization occurs without a change in the feed composition

Such copolymerizaions are termed Azeotropic copolymerizaions.

#### **Condition of Azeotropic copolymeriztion**

$$\frac{d[M_1]}{d[M_2]} = \frac{[M_1]}{[M_2]}$$
 and  $\frac{[M_1]}{[M_2]} = \frac{(r_2 - 1)}{(r_1 - 1)}$ 

$$F_1 = \frac{d[M_1]}{d[M_1] + d[M_2]} = \frac{[M_1]}{[M_1] + [M_2]} = f_1$$



$$\frac{d[M_1]}{d[M_2]} = \frac{[M_1]}{[M_2]} \cdot \frac{r_1[M_1] + [M_2]}{[M_1] + r_2[M_2]}$$

$$r_1[M_1] + [M_2] = [M_1] + r_2[M_2]$$

$$r_1 \frac{[M_1]}{[M_2]} + 1 = \frac{[M_1]}{[M_2]} + r_2$$

$$\frac{[M_1]}{[M_2]} = \frac{r_2 - 1}{r_1 - 1} \qquad f_1 = \frac{[M_1]}{[M_1] + [M_2]}$$

$$\frac{1}{f_1} = 1 + \frac{[M_2]}{[M_1]} = 1 + \frac{r_1 - 1}{r_2 - 1} = \frac{r_2 + r_1 - 2}{r_2 - 1}$$

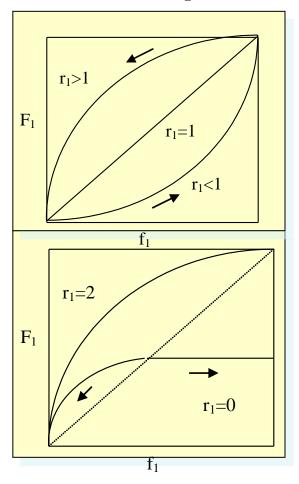
$$f_1 = \frac{r_2 - 1}{r_2 + r_1 - 2} = \frac{1 - r_2}{2 - r_1 - r_2}$$



$$r_1 >> r_2$$
 (  $r_1 >> 1$  and  $r_1 << 1$ )

Both types of propagating species preferentially add monomer  $\mathbf{M}_1$ , there is a tendency toward consecutive Homopolymerization of the two monomers.

And then monomer M<sub>2</sub> will subsequently homopolymerize.



In the result  $r_1r_2=1$  ideal or random

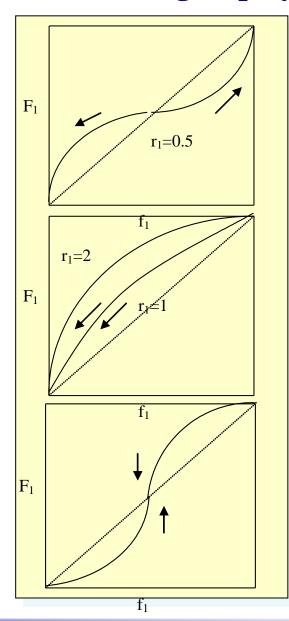
$$fix r_2 = 0.5$$

$$r_1 = 0 = \frac{k_{11}}{k_{12}}$$
 Addition monimer A and A\*

Can not prepare copolymer

It's alternating up to 0.5, and above 0.5 there's no formation of copolymer





$$r_2$$
=0.5  
azeotropic comp  
alternating

$$r2=0.5$$

$$r_1=1$$
 no azeotrope

$$r_1, r_2 > 1$$
 Case

tend to be a block copolymerization

Drift: 
$$r_1, r_2 > 1$$
 block azeotrope  $r_1, r_2 < 1$  alternating

$$r_1 > 1$$
  $r_2 > 1$  finally  $r_1 r_2 > 1$  Block COPOLYMERIZATION



### **Experimental Determination of r1 & r2**

### **Experimental Determination of r1 & r2**

1.Mayo and Lewis rearrange copolymer eq. and can get

$$r_2 = \frac{[M_1]}{[M_2]} \cdot \left\{ \frac{d[M_2]}{d[M_1]} \left[ 1 + \frac{r_1[M_1]}{[M_2]} \right] - 1 \right\}$$

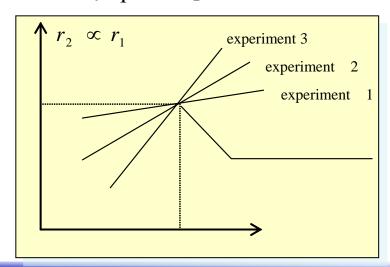
#### monomer comp

$$[M_1]/[M_2]$$

#### copolymer comp.

$$d[M_1]/d[M_2] \\ \vdots \\$$

#### then vary $r_1$ value (put) and iterate



This  $\Delta$  is more for smaller,  $r_1$ ,  $r_2$  value calculate with accuracy

Hanyang Univ.

# **Experimental Determination of r1 & r2**

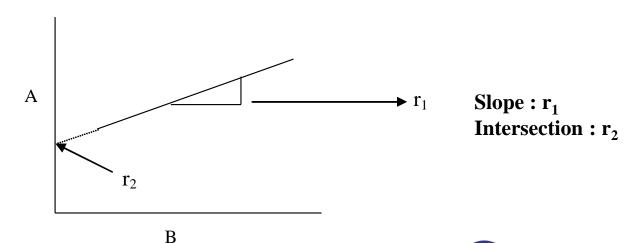
#### 2. Finemann and Ross

Recall

$$F_1 = \frac{r_1 f_1^2 + f_1 f_2}{r_1 f_1^2 + 2f_1 f_2 + r_2 f_2^2}$$

$$\frac{f_{1}(1-2F_{1})}{F_{1}(1-f_{1})} = r_{2} + \frac{f_{1}^{2}(F_{1}-1)}{F_{1}(1-f_{1})^{2}} \cdot r_{1}$$
**A B const. const.**

#### at low conversion



### Relationship Between ξand F1, f1

#### Relationship Between ξ and F1, f1

**Material Balance for M**<sub>1</sub>

$$-d([M]f_1) = -d[M]F_1$$
 where [M] = total # of moles of monomers

decrease of M1 monomer

$$+d([M]f_1) = \{d[M]\} \cdot F_1 = f d[M] + [M]df_1$$

$$(F_1 - f_1)d[M] = [M]df_1$$

$$\frac{d[M]}{[M]} = \frac{df_1}{F_1 - f_1}$$

$$\ln \frac{[M]}{[M_0]} = \int_{f_{1,0}}^f \frac{df_1}{F_1 - f_1}$$

$$\xi = 1 - \frac{[M]}{[M]_0} = 1 - e^{\int_{f_{1,0}}^{f} \frac{df_1}{F_1 - f_1}}$$



### **Effect of Reaction Condition**

#### **Reaction medium**

Depend on Solubility, PH, Viscosity, and Polarity

#### **Temperature**

$$r_1 = \frac{k_{11}}{k_{12}} = \frac{A_{11}}{A_{12}} \exp\left[\frac{(E_{12} - E_{11})}{RT}\right]$$

But the effect of temperature on r is not large

#### **Pressure**

$$\frac{d \ln r_1}{dP} = \frac{-\left(df_1 \Delta V_{11} - \Delta V_{12}\right)}{RT}$$

But the effect of pressure on r is not large

### Reactivity

Next page



### **Effect of Reaction Condition**

### **Structure and Reactivity**

\_ I. Resonance Stabilization

**II. Polar Effects** 

**III. Steric Effects** 

#### **I.Resonance Stabilization**

Substituent	Relative Reactivity	Stabilization Energy, kcal/mole			
on Double Bond	of Monomer	Olefine	Radical		
-H, -OCH <sub>3</sub>	1	0	0		
-OAc, -CH <sub>3</sub>	1.5-5	2.5	4		
-CI	3-20	-	6		
-COO, -COOH	20-60	2.5	-		
-CN, -COR	30-60	0	-		
-C <sub>2</sub> H <sub>3</sub> , -C <sub>6</sub> H <sub>5</sub>	50-100	3-4	25		

<sup>\*</sup> Walling's "Free Radicals in Solution"



#### **Define**

 $r_A$ ,  $r_B$ : monomer reactivity ratios

 $R_A$ ,  $R_B$ : active center reactivity ratios

$$r_A = \frac{k_{AA}}{k_{AB}} \qquad \qquad r_B = \frac{k_{BB}}{k_{BA}}$$

$$R_A = \frac{k_{AA}}{k_{BA}} \qquad R_B = \frac{k_{BB}}{k_{AB}}$$

$$R_A = \frac{k_{AA}}{k_{BB}} \cdot r_B = \frac{k_{AA}}{k_{BB}} \frac{k_{BB}}{k_{BA}}$$

$$R_B = \frac{k_{BB}}{k_{AA}} \cdot r_A$$

TABLE I. Propagation Rate Constants, Monomer Reactivity Ratios, and Active Center Reactivity Ratios for Radical Chain-Growth Polymerizations<sup>1</sup>

Mon A <sup>2</sup>	Mon B <sup>2</sup>	K <sub>AA</sub> x10 <sup>-3</sup>	K <sub>BB</sub> x10 <sup>-3</sup>	r <sub>A</sub>	r <sub>B</sub>	R <sub>A</sub>	R <sub>B</sub>
AN	MA	1.96	2.09	1.26	0.67	6.28x10 <sup>-1</sup>	1.34 x10 <sup>0</sup>
AN	MMA	1.96	0.515	0.15	1.20	4.57x10 <sup>0</sup>	3.94 x10 <sup>-2</sup>
AN	STY	1.96	0.165	0.04	0.40	4.8 x10 <sup>0</sup>	3.4 x10 <sup>-3</sup>
AN	VA	1.96	2.30	5.4	0.050	4.3 x10 <sup>-2</sup>	6.3 x10 <sup>0</sup>
MA	MMA	2.09	0.515	0.25	3.22	1.31x10 <sup>1</sup>	6.16 x10 <sup>-2</sup>
MA	STY	2.09	0.165	0.20	0.75	9.5 x10 <sup>0</sup>	1.6 x10 <sup>-2</sup>
MA	VA	2.09	2.30	9.	0.1	9. x10 <sup>-2</sup>	1. x10 <sup>1</sup>
MMA	STY	0.515	0.165	0.46	0.52	1.6 x10 <sup>0</sup>	1.5 x10 <sup>-1</sup>
MMA	VA	0.515	2.30	20	0.015	3.4 x10 <sup>-3</sup>	8.9 x10 <sup>1</sup>
STY	VA	0.515	2.30	55	0.01	7. x10 <sup>-4</sup>	8. x10 <sup>2</sup>

1All values are based on data collected at 60°C

2AN=acrylonitrile; MA=methylacrylate; MMA=methylmetacrylate;

STY=styrene; VA=vinyl acetate



**Active Center Reactivity Ratios vs. Monomer Reactivity Ratios** 

$$\frac{d[A]}{d[B]} = \frac{[A^*]}{[B^*]} \left( \frac{R_A[A^*] + [B^*]}{[A^*] + R_B[B^*]} \right)$$

$$R_A = \frac{k_{AA}}{k_{BA}} \qquad R_B = \frac{k_{BB}}{k_{AB}}$$

when 
$$R_A > R_B$$
 then  $r_A < r_B$ 

$$\frac{R_A}{R_B} > \frac{r_B}{r_A} \quad \text{when} \quad R_A > R_B$$

The effect in relative reactivity of the active center is more stronger than that of monomer.

The monomer reactivity gets affect in opposite way comparing to the active center reactivity.



Odian Table 6-3. Relative Reactivities(1/r) of Monomers

	Polymer Radical										
Monomer	Butadiene	Sty	VAc	VC	MMA	MA	AN				
Butadiene		1.7		29	4	20	50				
Styrene	0.7		100	50	2.2	5.0	25				
Methyl Metacrylate	1.3	1.9	67	10		2	6.7				
Methyl Vinyl Ketones		3.4	20	10			1.7				
Acrylonitrile	3.3	2.5	20	25	0.82	1.2					
Methyl Acrylate	1.3	1.3	10	17	0.52		0.67				
Vinylidene Chloride		0.54	10		0.39		1.1				
Vinyl Chloride	0.11	0.059	4.4		0.10	0.25	0.37				
Vinyl Acetate		0.019		0.59	0.05	0.11	0.24				



#### **@ Substituent Effects**

$$\Phi$$
, CH2=CH- > -C $\equiv$ N, -COR > -COOH, -COOR > -Cl > -O-COR, -R > -OR, -H

monomers increase relative reactivity by resonance stabilization.

The resonance stability of the monomer increases the reactivity of the monomer.

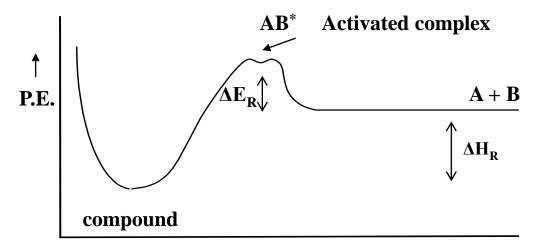
The resonance stability of the radical is weakened reactivity of the radical.

<u>Table 6-4 Rate Constant(k<sub>12</sub>) for Radical-Monomer Reactions</u>

Monomer(M₁)	Buta-	Sty-	Methyl	lymer Rad Acrylo	Methyl	Vinyl	Vinyl	Q <sub>1</sub>	e <sub>1</sub>
\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	diene	rene	Metacrylate	-nirile	Acrylate	Acetate	Chloride		'
Butadiene	100	280	2,060	98,000	41,800	230,000	319,000	2.39	-1.05
Styrene	70	165	1,130	49,000	10,045	154,000	550,000	1.00	-0.80
Methyl							110,000	0.74	0.40
methacrylate	130	314	515	13,100	4,180	46,000	225,000	0.60	1.20
Acrylonitrile	330	413	422	1,960	2,510	23,000	187,000	0.42	0.60
Methyl acrylate				·			11,000	0.044	0.20
Vinyl chloride	130	215	268	1,310	2,090	0,100	6,490	0.026	-0.22
Vinyl acetate	11	9.7	52	720	520	2,300			
-		3.4	26	230	530	Man of the second	nvena Tiniv		

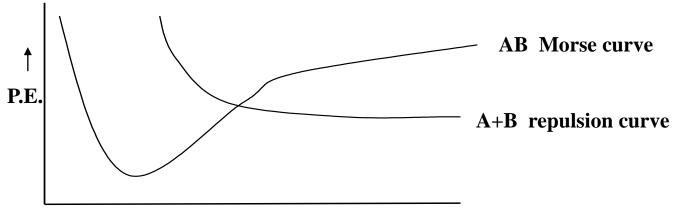
**Resonance stabilization of Active Center** 

**Transition State Theory** 

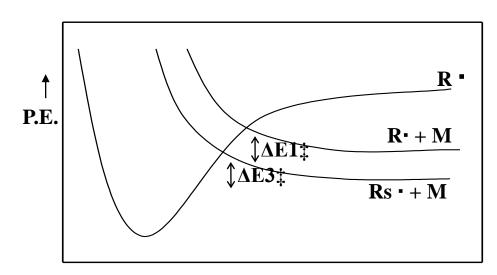


**Increasing Separation of A & B** →





**Increasing Separation of A & B**→



**Increasing Separation of Reactant** →

 $\Delta E1\ddagger > \Delta E3\ddagger$  then Rxn III is less stable than Rxn I  $\Delta HR3\ddagger < \Delta HR1\ddagger$ 

Reaction I → R •

Reaction III → R •



Resonance stability of Active Center becomes primary Resonance stability of Monomer becomes secondary

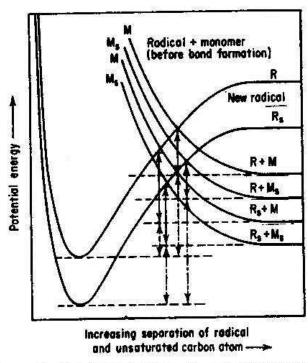


Fig. 6-12 Reaction coordination diagram for the reaction of a polymer radical with a monomer. The dependence of the potential energy of the system (radical + monomer) on the separation between the radical and the unsaturated carbon atom of the monomer is shown. The subscript s indicates the presence of a substituent that is capable of resonance stabilization. Activation energies are represented by the solid-line arrows; heats of reaction by the broken-line arrows. After Walling [1957] (by permission of Wiley, New York).



	-ΔΗ	ΔE‡
$R \cdot + M_S \rightarrow R_S \cdot$	1.20	0.40
$R \cdot + M \rightarrow R \cdot$	0.95	0.50
$R_S$ ·+ $M_S$ $\rightarrow$ $R_S$ ·	0.70	0.70
$R_S$ ·+ $M \rightarrow R$ ·	0.40	0.80

$$|-\Delta H|$$
 (1) > (2) > (3) > (4)

$$\Delta E^{\ddagger} \quad \text{(4)} > \text{(3)} > \text{(2)} > \text{(1)}$$



#### **II. Polar Effects**

TABLE 6-6	Values of r <sub>1</sub> r <sub>2</sub> in Radical Copolymerization <sup>c</sup>	
-----------	--	--

n-Butyl vinyl ether (+1.50)										
1	Butadiene (-	-0,50)								
	0.78	Styrene (-0.	80)							
		(8)	Vinyl							
l l		0.55	acetate (-0.8	88)						
3			ACCOMMONDATION OF THE PARTY OF	Vinyl						
	0.31	0.34	0.39	chloride (0.1	16)					
		200		8	Methyl					
	0.19	0.24	0.30	1.0	methacrylate	e (0.40)				
				100	grand brondered	Vinylidene				
Ó	<0.1	0.16	0.6	0.96	0.61	chloride (0.	34)			
	\$200,00000	5,772,572,07	-12/1000	CHIDWELLO	g societies	H-BAS CO-SOURCE	Methyl viny	l		
		0.10	0.35	0.83		0.99	ketone (1.06	)		
0.0004	0.006	0.016	0.21	0.11	0.18	0.34	1.1	Acrylonitrile	(1.23) Diethyl	
~0		0.021	0.0049	0.056		0.56	-	ē.	fumarate (2.	26)
11139				enwikana.			9	ē		Maleic
~0.002		0.006	0.00017	0.0024	0.13				}	anhydride (3.69)

<sup>&</sup>quot;r<sub>1</sub>r<sub>2</sub>, values are calculated from data in Table 6-2 Greeley [1989a].

### Tend to cause alternation in a copolymerization

i.e.

$$r_A \cdot r_B < 1$$

for polar effects

e: tendency to give monomer a polar effects

Alfrey-Price Q,e scheme (Polarity Values)



<sup>&</sup>quot;values are shown in parentheses after each monomer.

### @ Q-e scheme of Alfred Price

 $r_A$  ,  $r_B$  forecast randomness of copolymerization

As know r, prediction is possible to polar, resonance effect Guidance to chemists

$$k_{ij} = P_i Q_j \exp(-e_i e_j)$$

where P: active center reactivity

Q: monomer reactivity

i, j: active center, monomer, respectively

$$r_i = \frac{k_{ii}}{k_{ij}} = \frac{P_i Q_i \exp(-e_i e_i)}{P_i Q_j \exp(-e_i e_j)}$$

$$\mathbf{r_i} = \frac{\mathbf{Q_i}}{\mathbf{Q_j}} \exp[-\mathbf{e_i}(\mathbf{e_i} - \mathbf{e_j})]$$

So that, this equation forecasts  $r_i$ Base materials use styrene: Q=1(arbitrary) e=-0.8

fair results, but not absolute in predicting r using Q-e scheme.

$$r_{j} = \frac{k_{jj}}{k_{ji}} = \frac{P_{j}Q_{j} \exp(-e_{j}e_{j})}{P_{i}Q_{j} \exp(-e_{j}e_{i})} = \frac{Q_{j}}{Q_{i}} \exp[-e_{j}(e_{j} - e_{i})]$$

$$r_i \cdot r_j = \frac{\exp[-e_i(e_i - e_j)]}{\exp[-e_j(e_j - e_i)]} = \exp[-(e_i - e_j)^2]$$

$$r_i \cdot r_j = \exp[-(e_i - e_j)^2] < 1$$

$$-(e_i - e_j)^2 < 0 \Longrightarrow (e_i - e_j)^2 > 0$$

∴ alternating tendency is correct



#### **Active Center Reacting Ratios**

$$\frac{d[A]}{d[B]} = \frac{[A^*]}{[B^*]} \cdot \frac{R_A[A^*] + [B^*]}{[A^*] + R_B[B^*]}$$

#### P-e scheme

$$R_i = \frac{P_i Q_i \exp(-e_i e_i)}{P_j Q_i \exp(-e_j e_i)} = \frac{k_{ii}}{k_{ji}}$$

$$R_i = \frac{P_i}{P_j} \exp[-e_i(e_i - e_j)]$$

	Р	е	Q	Е
St	1	-0.8	1	-0.8
AN	58.23	1.233	0.4	1.2
MA	21.03	0.577	0.42	1.2
MMA VAc	2.413 751.3	0.397 -0.027	0.24 0.024	0.4 -0.22

#### The criticism against Q-e scheme

Reference state arbitrarily set.

Alternating effect was observed due to fixed charges not due to the induced dipole

**Exercise**)

**How to indicate Randomness of Copolymer with Q-e scheme?** 

We can forecast alternation or randomness through Q-e scheme, however, why can not forecast Blockcopolymerization? (algebraic standpoint)



#### III. Steric Effects

1) 1,2-disubstituted ethylene do not homopolymerize readily

$$C=C$$

2) 1,1-disubstituted ethylene

#### **II. Polar Effects**

planar conformation tetrahedral conformation more reactive



#### 3) Cis-trans Effect

The trans is stabilized the cis than thermodynamics(Heat of Hydrogenation)

Planarity! Easier for trans than cis Steric Effect!

Table 6-5 Rate Constants  $(k_{12})$  for Radical-Monomer Reactions<sup>a</sup>

Monomer	Polymer Radical					
	Vinyl Acetate	Styrene	Acrylonitrile			
Vinyl chloride	10,000	9.7	725			
Vinylidene chloride	23,000	89	2,150			
Cis-1,2-Dichloroethylene	365	0.79				
Trans-1,2-Dichloroethylene	2,320	4.5				
Trichloroethylene	3,480	10.3	29			
Tetrachloroethylene	338	0.83	4.2			

 $<sup>^{\</sup>rm a}k_{12}$  Values were calculated from data in Table 3-11 and 6-2 and [66]

