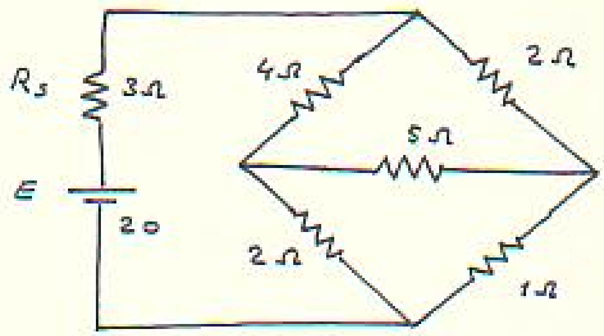
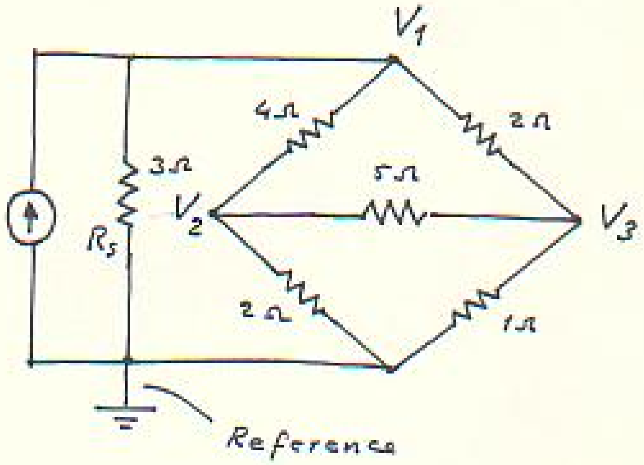
Example:

For the bridge cct. of the previous example, find the current in $R_S = 3\Omega$, using the nodal voltage method.

Solution:



change the voltage source into a current source, then



For node 1:
$$\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{2}\right)V_1 - \left(\frac{1}{4}\right)V_2 - \left(\frac{1}{2}\right)V_3 = \frac{20}{3}$$

For node 2:
$$\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{5}\right)V_2 - \left(\frac{1}{4}\right)V_1 - \left(\frac{1}{5}\right)V_3 = 0$$

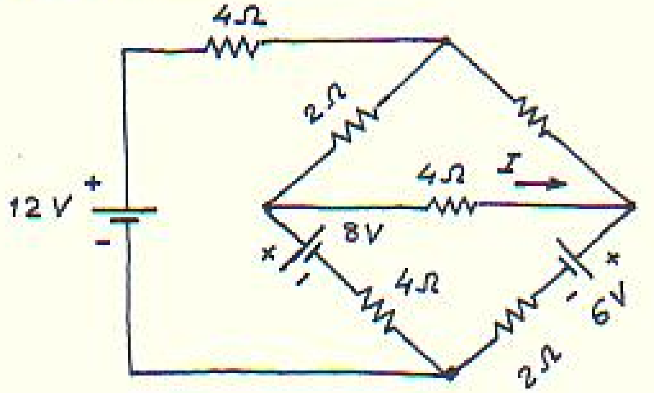
For node 3:
$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{1}\right)V_3 - \left(\frac{1}{2}\right)V_1 - \left(\frac{1}{5}\right)V_3 = 0$$

Rearrange, and solve using determinants, then

$V_1 = 8\ V$ $\Leftarrow$ only V_1 is needed to find I_{R_S}

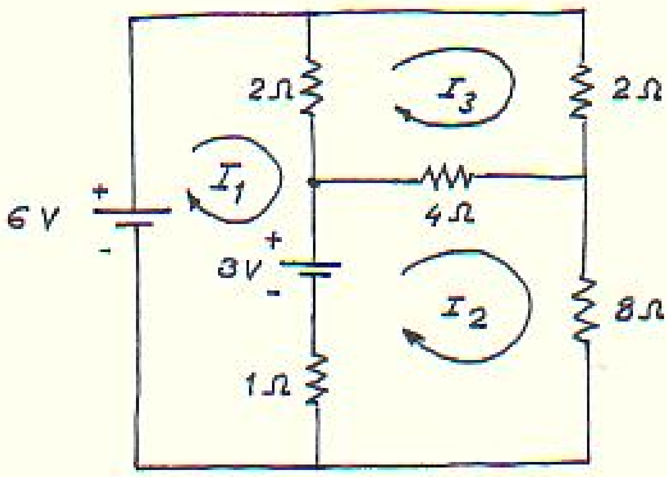
$\therefore$ No need to find V_2 or V_3

Practice Problem: Using the nodal voltage method, determine the current I in the circuit shown.

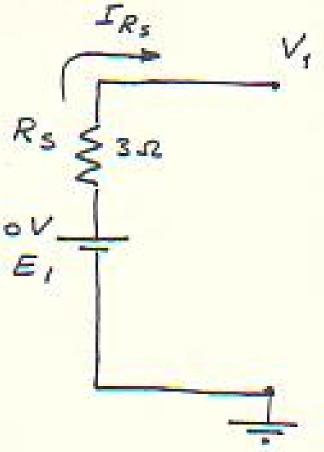


Answer: $I = 0.25$ A

Practice Problem: Use the loop current analysis to determine the currents I_1, I_2 and I_3 in the circuit shown;



To find the current in R_S, return to the original circuit,



$$I_{R_S} = \frac{E - V_1}{R_S}$$

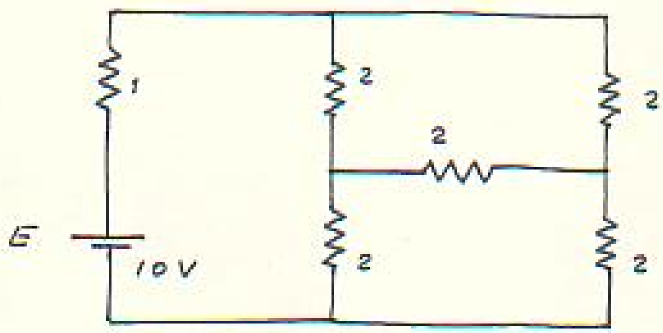
$$= \frac{20 - 8}{3} = \frac{12}{3}$$

$$= 4\ A.$$

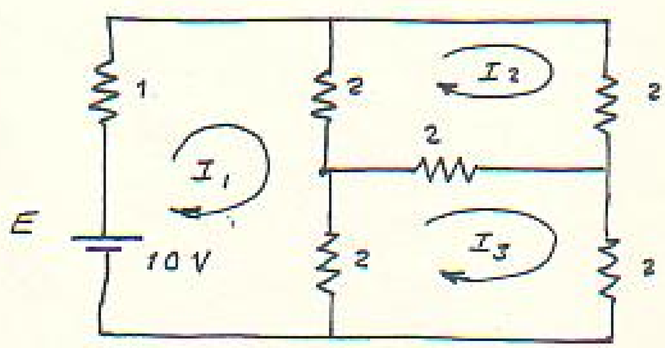
Note that the same result is obtained as that in the previous example.

<u>ملاحظة</u> : يمكنك المحاولة لإيجاد (I_{R_S}) للمثال أعلاه من خلال
إيجاد (R_T) للدائرة ومن ثم فإن $I_{R_S} = \dfrac{E}{R_T}$
وقد تم الحصول على النتيجة ذاتها كما في المثالين السابقين .

<u>Example</u>: Using the loop current method find the current through the dc supply in the network shown; all resistors are in ohms.



<u>Solution</u>:



TS3

Loop 1

$$10 = (1+2+2)\,I_1 - (2)\,I_2 - (2)\,I_3$$

Loop 2: $0 = (2+2+2)\,I_2 - (2)\,I_1 - (2)\,I_3$

Loop 3: $0 = (2+2+2)\,I_3 - (2)\,I_2 - (2)\,I_3$

Rearrange, then we have:

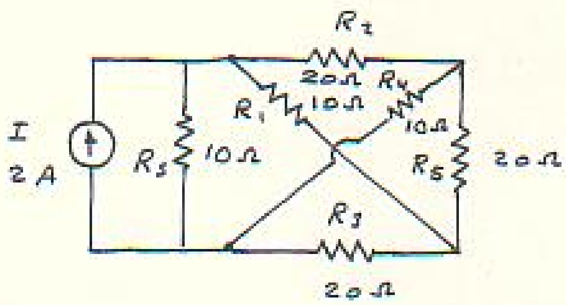
$$\begin{aligned} 5I_1 - 2I_2 - 2I_3 &= 10 \\ -2I_1 + 6I_2 - 2I_3 &= 0 \\ -2I_1 - 2I_2 + 6I_3 &= 0 \end{aligned}$$

Solving using determinants, then:

$$I_1 = 3.33 \text{ A}$$

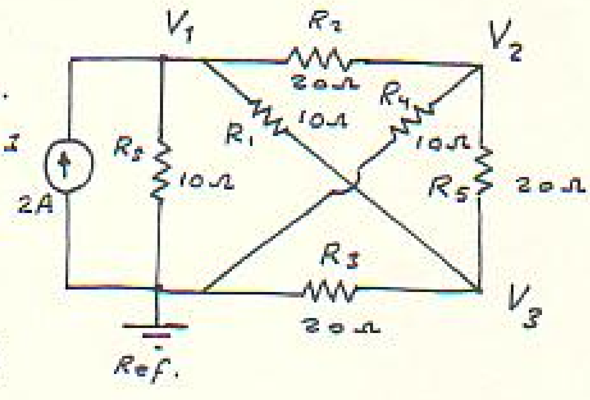
* Solve the example using the nodal voltage method.

Example: For the circuit shown, write the nodal equation



Solution:

We have 3 independent nodes and a reference node as shown.



Node 1:

$$2 = V_1\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{20}\right) - V_2\left(\frac{1}{20}\right) - V_3\left(\frac{1}{10}\right)$$

Node 2:

$$0 = V_2\left(\frac{1}{20}+\frac{1}{10}+\frac{1}{20}\right) - V_1\left(\frac{1}{20}\right) - V_3\left(\frac{1}{20}\right)$$

Node 3:

$$0 = V_3\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{20}\right) - V_2\left(\frac{1}{20}\right) - V_1\left(\frac{1}{10}\right)$$

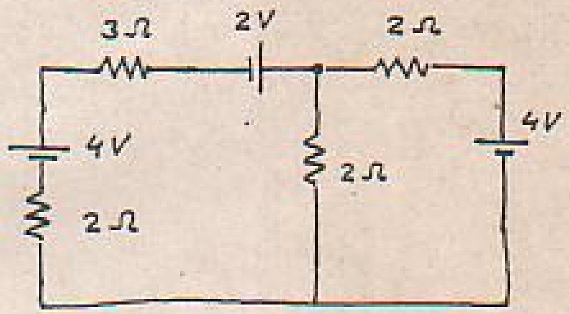
Rearrange and solve.

TS3

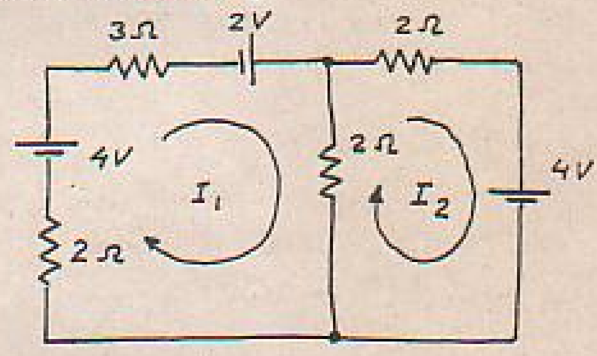
Tutorial Sheet № 3

ملاحظة : جميع اسئلة الكتاب المنهجي والاسئلة المحلولة فيه مطلوبة

Example (1) : For the circuit shown, find the current in the 3 Ω resistor using : (a) loop current method, (b) nodal voltage method.



Solution : (a) Using loop current method;



Loop 1

$$4+2 = I_1(2+3+2) - I_2(2)$$

$$\Rightarrow \quad 7I_1 - 2I_2 = 6$$

Loop 2

$$-4 = I_2(2+2) - I_1(2)$$

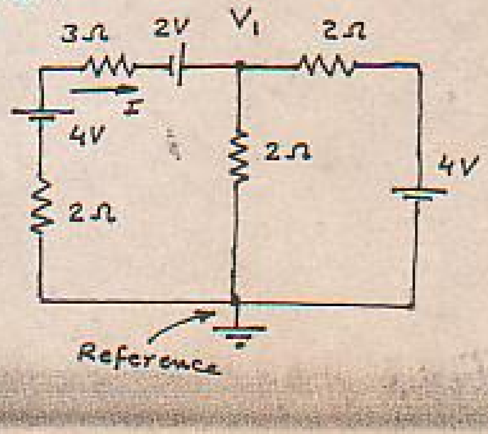
$$\Rightarrow \quad -2I_1 + 4I_2 = -4$$

$$\therefore \quad I_1 = \frac{\begin{vmatrix} 6 & -2 \\ -4 & 4 \end{vmatrix}}{\begin{vmatrix} 7 & -2 \\ -2 & 4 \end{vmatrix}} = \frac{(6)(4)-(-2)(-4)}{(7)(4)-(-2)(-2)} = \frac{24-8}{28-4} = \frac{16}{24}$$

$$\therefore \quad I_1 = \frac{2}{3}\ A$$

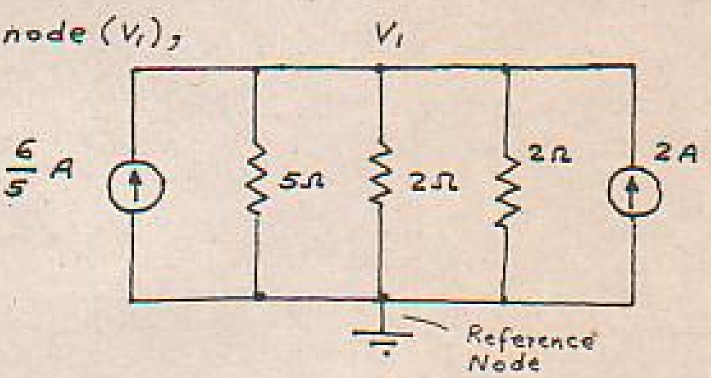
(b) using nodal voltage method :

* There is one independent node and a reference node as shown;



TS3

* Converting the voltage sources to current sources as shown;
* We have only ONE independent node (V_1), So we have one equation to find V_1



$$\therefore \quad V_1\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{2}\right)=\frac{6}{5}+2$$

Simplifying, we get;

$$\Rightarrow \quad V_1 = \frac{8}{3} \ V$$

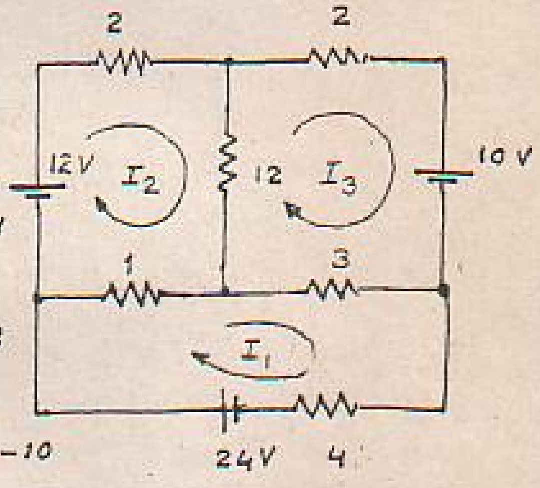
Returning to the original circuit, the current through the 3Ω resistor is:

$$I_1 = \frac{4+2-V_1}{3+2} = \frac{6-(8/3)}{5}$$

$$\therefore \ I = \frac{2}{3} \ A$$

Example: Determine the current in the 4Ω resistor for the circuit shown, using loop current method. All resistor values are in Ohms.

Solution:



Loop 1

$$I_1(4+1+3)-I_2(1)-I_3(3)=24$$

Loop 2

$$I_2(1+2+12)-I_1(1)-I_3(12)=12$$

Loop 3

$$I_3(3+12+2)-I_2(12)-I_1(3)=-10$$

Rearrange, we get

$$8I_1 - I_2 - 3I_3 = 24$$

$$-I_1 + 15I_2 - 12I_3 = 12$$

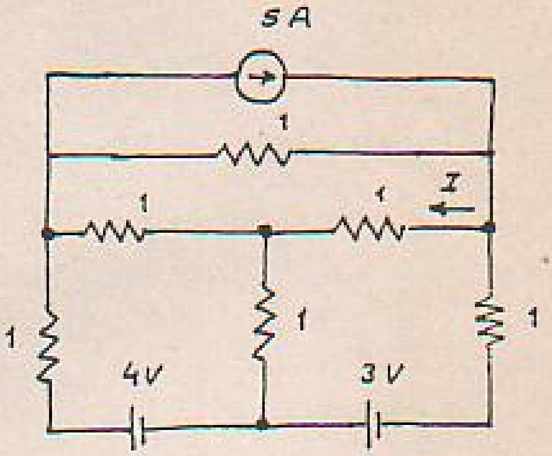
$$-3I_1 - 12I_2 + 17I_3 = -10$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 24 & -1 & -3 \\ 12 & 15 & -12 \\ -10 & -12 & 17 \end{vmatrix}}{\begin{vmatrix} 8 & -1 & -3 \\ -1 & 15 & -12 \\ -3 & -12 & 17 \end{vmatrix}}$$

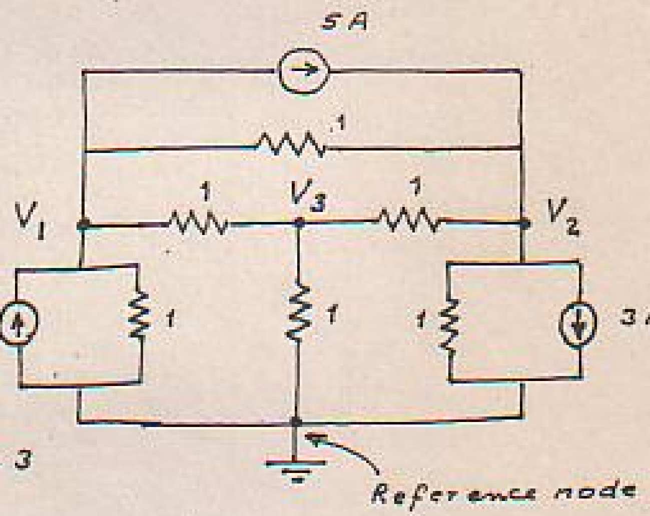
$$\Rightarrow I_1 = \frac{2730}{664} = \underline{4.1 \ A}$$

T.S 3

Example: Using the nodal voltage method, find, the current I in the circuit shown. All resistors are in Ohms.



Solution: * First Convert voltage sources to current sources.
There are three independent nodes and a reference node as shown.



For node 1

$$V_1(1+1+1) - V_2(1) - V_3(1) = 4-5$$

$$\rightarrow 3V_1 - V_2 - V_3 = -1$$

For node 2

$$V_2(1+1+1) - V_2(1) - V_3(1) = 5-3$$

$$\rightarrow -V_1 + 3V_2 - V_3 = 2$$

For node 3

$$V_3(1+1+1) - V_2(1) - V_1(1) = 0$$

$$\rightarrow -V_1 - V_2 + 3V_3 = 0$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & 0 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}} = \frac{12}{16} = \frac{3}{4}\ V$$

$$\therefore V_2 = \frac{3}{4}\ V$$

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & 2 \\ -1 & -1 & 0 \end{vmatrix}}{16} = \frac{4}{16} = \frac{1}{4}\ V$$

$$\therefore V_3 = \frac{1}{4}\ V$$

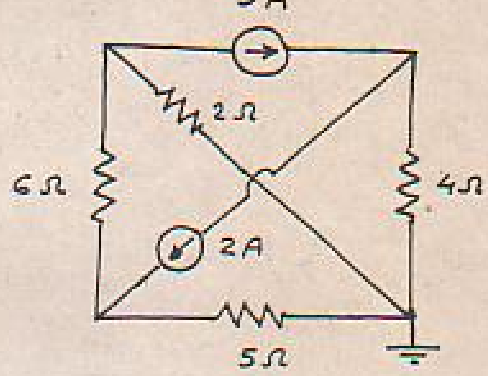
$$I = \frac{V_2 - V_3}{1\,\Omega} = \frac{\frac{3}{4} - \frac{1}{4}}{1} = \frac{1}{2}\ A$$

$$\therefore I = 0.5\ A$$

ملاحظة : أعد الحل باستخدام طريقة
Loop current method
لابد من الحصول على النتيجة نفسها

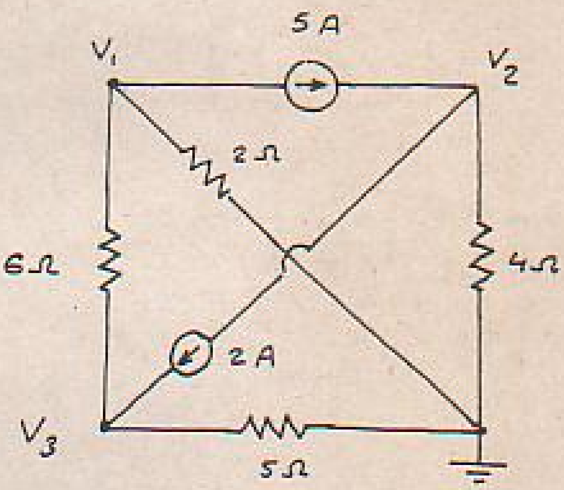
TS3

Example: For the network shown, write the nodal equations and solve for the nodal voltages.



Solution: There are 3 independent nodes and a reference node as shown;

* The independent nodes are V_1, V_2, and V_3



for node 1:

$$V_1\left(\frac{1}{2}+\frac{1}{6}\right)-V_3\left(\frac{1}{6}\right)=-5$$

for node 2:

$$V_2\left(\frac{1}{4}\right)=5-2=3$$

for node 3:

$$V_3\left(\frac{1}{5}+\frac{1}{6}\right)-V_1\left(\frac{1}{6}\right)=2$$

Solving the three equations results in:

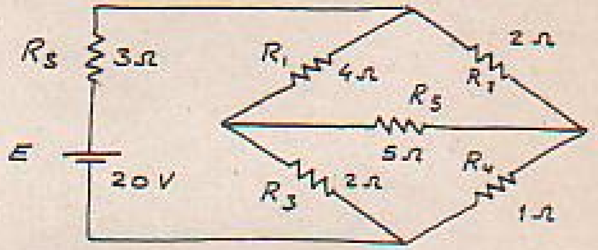
$$V_1=-6.917\ V$$

$$V_2=12\ V$$

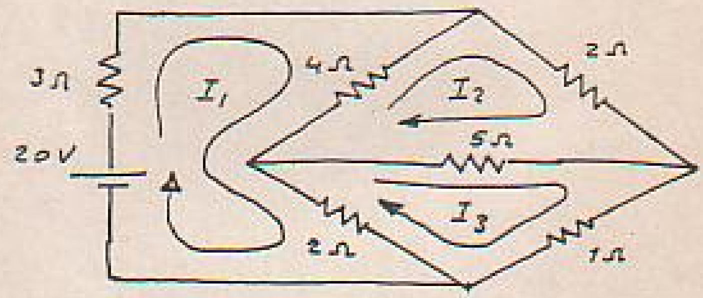
$$V_3=2.3\ V$$

T33

Example : For the bridge network shown, using the loop current method find the current in R_5.



Solution :



Loop1 :

$$I_1(3+4+2) - (4)I_2 - (2)I_3 = 20$$

Loop2 :

$$I_2(4+2+5) - (4)I_1 - (5)I_3 = 0$$

Loop3 :

$$I_3(2+5+1) - (2)I_1 - (5)I_2 = 0$$

Rearrange, we have :

$$\begin{aligned} 9I_1 - 4I_2 - 2I_3 &= 20 \\ -4I_1 + 11I_2 - 5I_3 &= 0 \\ -2I_1 - 5I_2 + 8I_3 &= 0 \end{aligned}$$

Solving using determinants, we have

$I_1 = 4\ A$

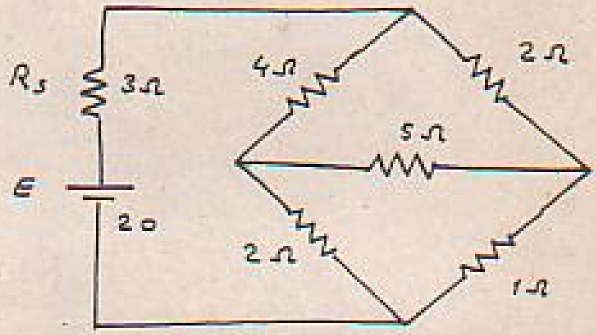
$I_2 = 2.67\ A$ $\Rightarrow \therefore I_{R_5} = I_2 - I_3$

$I_3 = 2.67\ A$ $= 2.67 - 2.67$

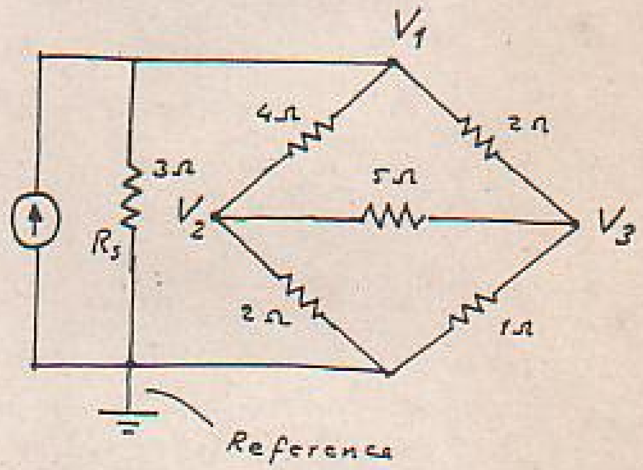
$= \underline{\text{zero}}$

Example: For the bridge cct. of the previous example, find the current in $R_S = 3\,\Omega$, using the nodal voltage method.

Solution:



change the voltage source into a current source, then



For node 1:
$$\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{2}\right)V_1 - \left(\frac{1}{4}\right)V_2 - \left(\frac{1}{2}\right)V_3 = \frac{20}{3}$$

For node 2:
$$\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{5}\right)V_2 - \left(\frac{1}{4}\right)V_1 - \left(\frac{1}{5}\right)V_3 = 0$$

For node 3:
$$\left(\frac{1}{5}+\frac{1}{2}+\frac{1}{1}\right)V_3 - \left(\frac{1}{2}\right)V_1 - \left(\frac{1}{5}\right)V_3 = 0$$

Rearrange, and solve using determinants, then

$V_1 = 8\text{ V}$ $\Leftarrow$ only V_1 is needed to find I_{R_S}

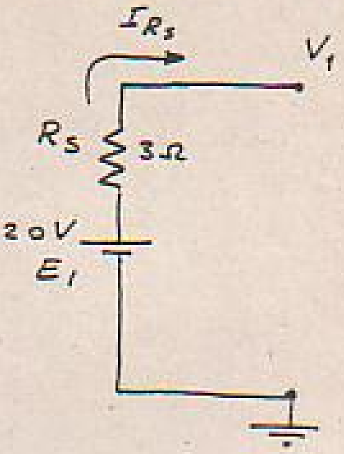
$\therefore$ No need to find V_2 or V_3

To find the current in R_s , return to the original circuit,

$$I_{R_s} = \frac{E - V_1}{R_s}$$

$$= \frac{20 - 8}{3} = \frac{12}{3}$$

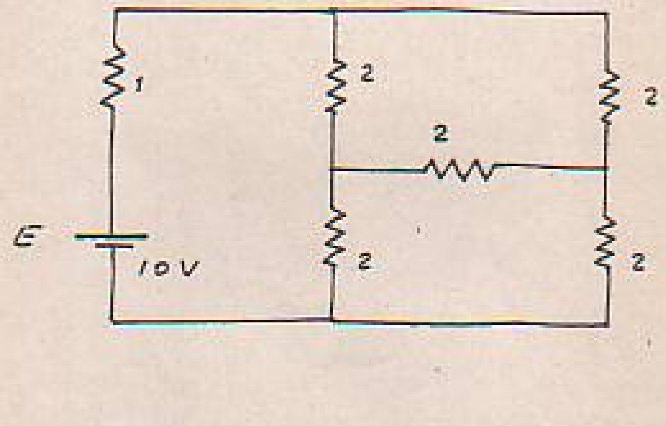
$$= 4\ A .$$



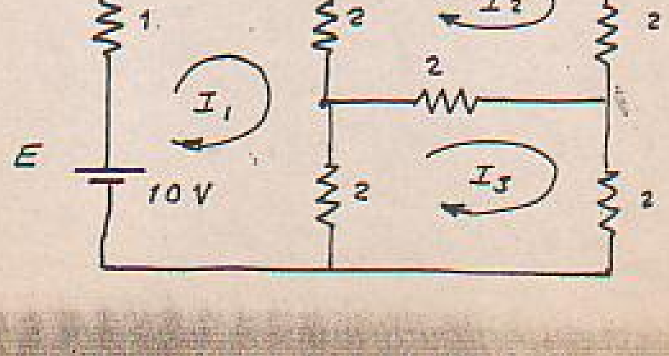
Note that the same result is obtained as that in the previous example.

ملاحظة : يمكنك المحاولة لإيجاد (I_{R_s}) للمثال أعلاه من خلال
إيجاد (R_T) للدائرة ومن ثم ثانياً $I_{R_s} = \frac{E}{R_T}$
وتدرب من الحصول على النتيجة ذاتها كما في المثالين السابقين.

Example : Using the loop current method find the current through the dc supply in the network shown; all resistors are in ohms.



Solution :



TS3

Loop 1

$$10 = (1+2+2)I_1 - (2)I_2 - (2)I_3$$

Loop 2: $0 = (2+2+2)I_2 - (2)I_1 - (2)I_3$

Loop 3: $0 = (2+2+2)I_3 - (2)I_2 - (2)I_3$

Rearrange, then we have:

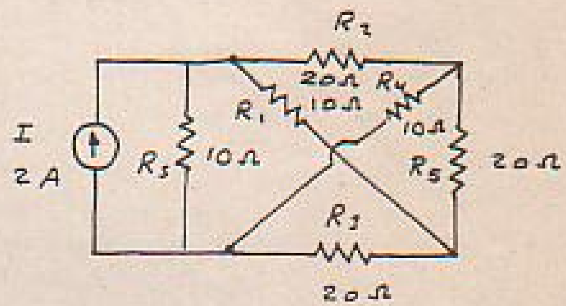
$$\begin{aligned} 5I_1 - 2I_2 - 2I_3 &= 10 \\ -2I_1 + 6I_2 - 2I_3 &= 0 \\ -2I_1 - 2I_2 + 6I_3 &= 0 \end{aligned}$$

Solving using determinants, then:

$$I_1 = 3.33\ A$$

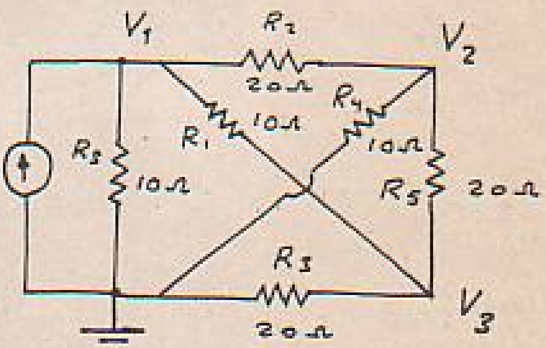
* Solve the example using the nodal voltage method.

Example: For the circuit shown, write the nodal equation



Solution:

We have 3 independent nodes and a reference node as shown.



Node 1:

$$2 = V_1\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{20}\right) - V_2\left(\frac{1}{20}\right) - V_3\left(\frac{1}{10}\right)$$

Node 2:

$$0 = V_2\left(\frac{1}{20}+\frac{1}{10}+\frac{1}{20}\right) - V_1\left(\frac{1}{20}\right) - V_3\left(\frac{1}{20}\right)$$

Node 3:

$$0 = V_3\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{20}\right) - V_2\left(\frac{1}{20}\right) - V_1\left(\frac{1}{10}\right)$$

Rearrange and solve.

4. Circuit Theorems

EE4

3.1 Superposition Theorem

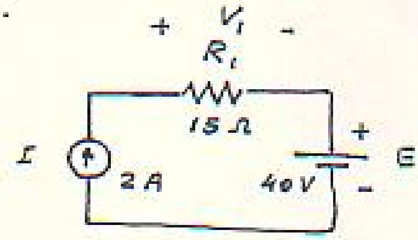
: The theorem states that: "the current through (or the voltage across) an element in a linear bilateral network is equal to the algebraic sum of the currents (or voltages) produced independently by each source.

* To apply this theorem to find the current (or voltage) in a certain part of a network, remove the sources of the network and find the current (or voltage) in the existance of only one source each time. The resultant current (or voltage) will be the algebraic sum of currents (or voltages) due to all sources when acting independently once a time.

* Removing the sources means: SHORT CIRCUITING the voltage source and OPEN CIRCUITING the current source.

Example

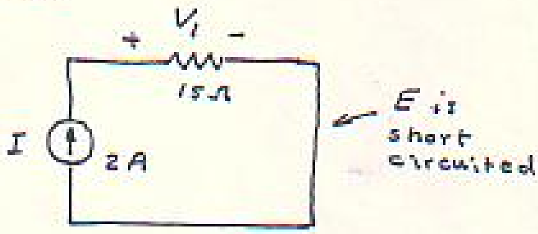
: Using the superposition theorem, determine V_1 for the network shown.



Solution

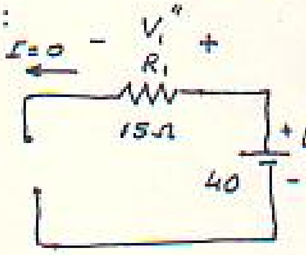
: * Due to the current source:

$$V_1' = IR_1$$
$$= (2)(15)$$
$$= 30\ V$$



* Due to the voltage source:

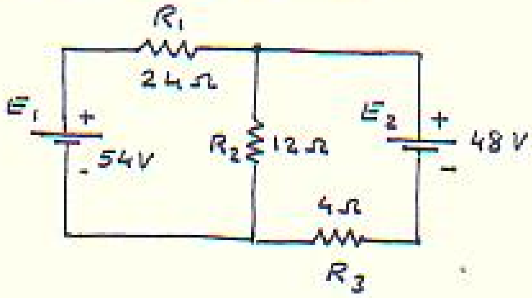
$$V_1'' = I_1 R_1$$
$$= (0)(15)$$
$$= 0\ V$$



$$\therefore V_1 = V_1' + V_1''$$
$$= 30 - 0 = 30\ V$$

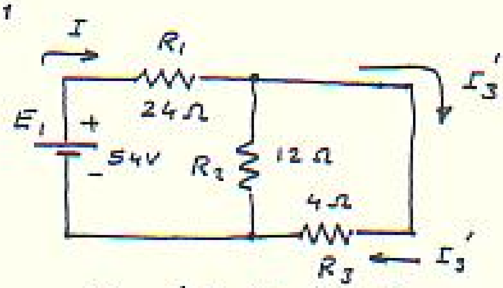
EE4

Example: Using the superposition theorem, determine the current through the $4\text{-}\Omega$ resistor for the network shown.



Solution: Consider the effect of E_1

$$I = \frac{E_1}{R_T} = \frac{54}{27} = 2\ A$$



$R_T = (R_2 // R_3) + R_1$

$(12//4) + 24 = 3 + 24$

$= 27\ \Omega$

Using the current division rule ∴

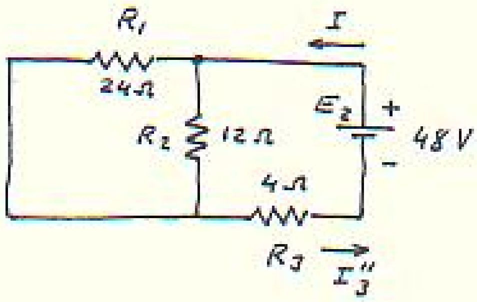
$$\therefore I_3' = I \frac{R_2}{R_2 + R_3}$$

$$= 2 \frac{12}{12+4} = \underline{1.5\ A}$$

* Consider the effect of E_2:

$$I = I_3'' = \frac{E_2}{R_T}$$

$$R_T = (24//12) + 4$$
$$= 8 + 4$$
$$= 12\ \Omega$$

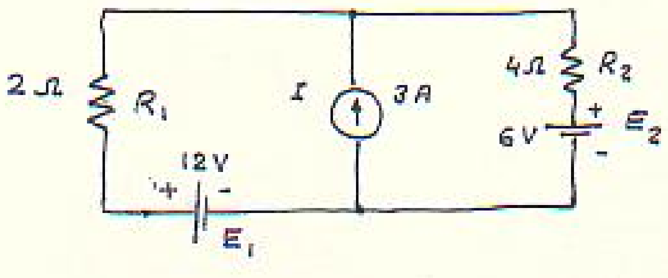


$$\therefore I_3'' = \frac{48}{12} = \underline{4\ A}$$

$$\therefore I_3 = I_3'' - I_3'$$

$$= 4 - 1.5 = \underline{2.5\ A}$$ (in the direction of I_3'').

Example: Using the superposition theorem, find the current through the $2\text{-}\Omega$ resistor of the network shown.

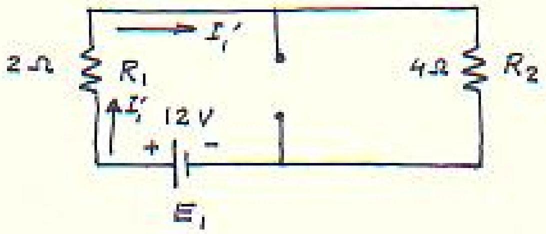


EE4

Solution:

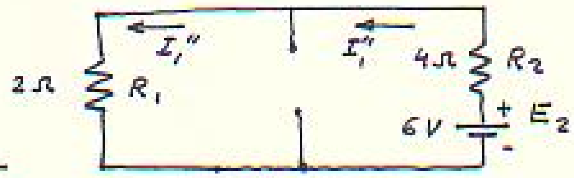
* **The effect of E_1**

Remove the voltage source E_2 (short circuited) and the current source I (open circuited); the network will be as shown:



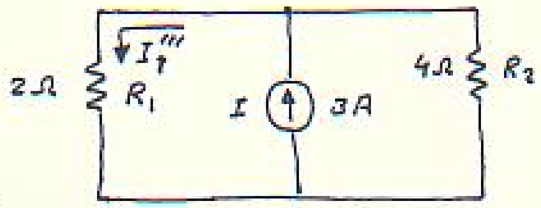
$$\therefore \quad I_1' = \frac{E_1}{R_T} = \frac{12}{2+4} = 6\ A$$

* **The effect of E_2**: removing E_1 & I, the network will be as shown:



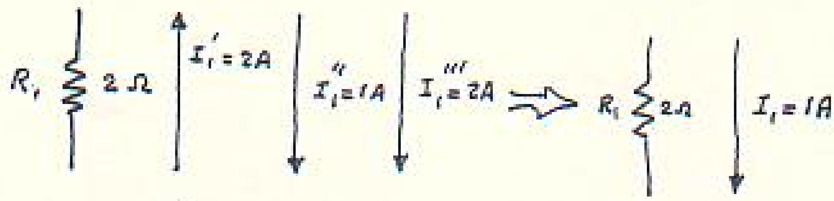
$$\therefore \quad I_1'' = \frac{E_2}{R_T} = \frac{6}{2+4} = 1\ A$$

* **The effect of I**: removing E_1 and E_2, the network will be as shown:



$$\therefore \quad I_1''' = I \frac{R_2}{R_1+R_2} = (3)\frac{4}{4+2} = 2\ A$$

$$\therefore \quad I_1 = \underbrace{I_1'' + I_1'''}_{\text{same direction}} - \underbrace{I_1'}_{\text{opposite direction}} \Rightarrow I_1 = 1+2-1 = \underline{1\ A}$$



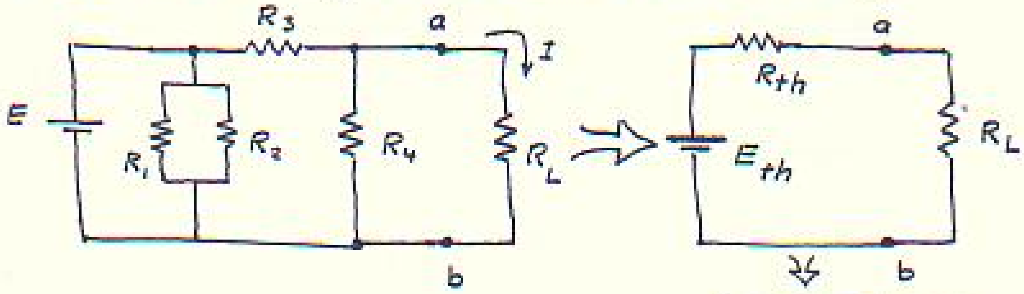
Resulting current in R_2

3.2 Thevenin's Theorem

EE4

: Thevenins theorem states that " Any two-terminal linear biletaral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

Consider the network shown, it can be replaced by the voltage source E_{th} and the series resistor R_{th} :



* To find I through the resistance R_L $\Rightarrow$ $\boxed{I = \dfrac{E_{th}}{R_{th}+R_L}}$

* Steps to find E_{th} and R_{th} :

STEP 1 : Remove that portion of the network across which the Thevenins equivalent circuit is to be found.

STEP 2 : Mark the terminals of the remaining two-terminal network.

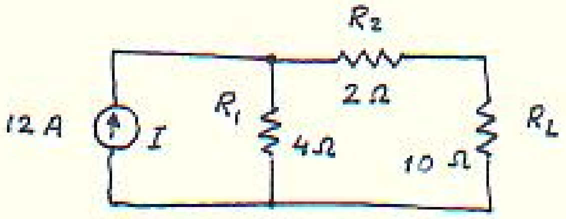
STEP 3 (R_{th}) : Calculate R_{th} by first setting all sources to zero (voltage sources are replaced by short circuits and current sources are replaced by open circuits), and finding the resultant resistance between the two marked terminals.

STEP 4 (E_{th}) : Calculate E_{th} by first returning all sources to their original positions and finding the open circuit voltage between the marked terminals

STEP 5 : Draw the Thevenins equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

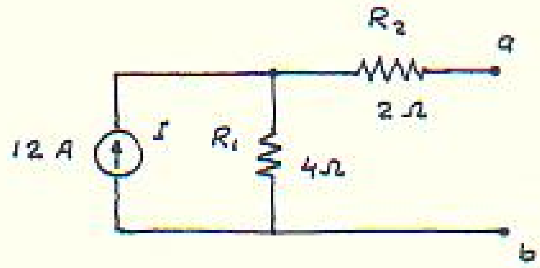
EE4

Example: Using Thevenins theorem, find the current in the $R_L = 10\,\Omega$ of the network shown.



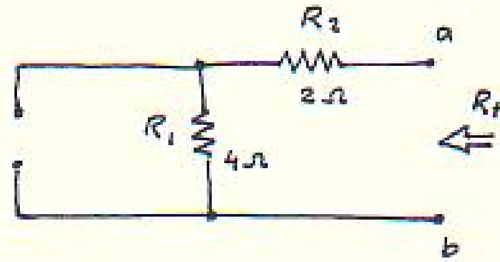
Solution:

steps 1 and 2:



step 3: $R_{th} = ?$

Remove the current source I, then calculate R_{th} between the terminals a and b;



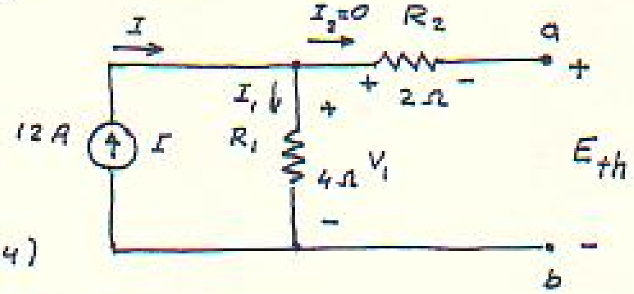
$$\therefore\ R_{th} = R_1 + R_2 = 4 + 2 = 6\,\Omega$$

step 4: $E_{th} = ?$

Return the current source to its original position then determine E_{th} across the open circuit terminals a and b.

$I_2 = 0$

$\Rightarrow I_2 R_2 = 0$



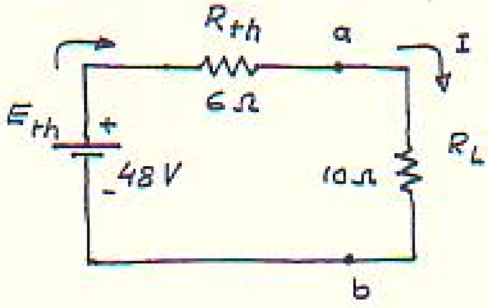
Then

$$E_{th} = I_1 R_1 - I_2 R_2$$

$$= I_1 R_1 = 12(4)$$

$$= 48\ V$$

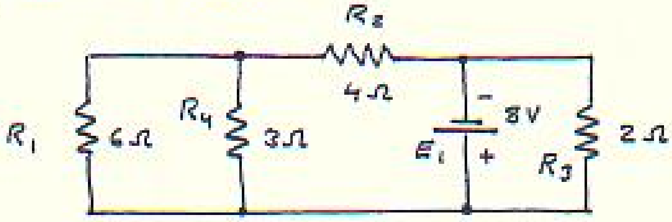
step 5: Draw the Thevenins equivalent circuit representing the network between points a and b with R_L added.



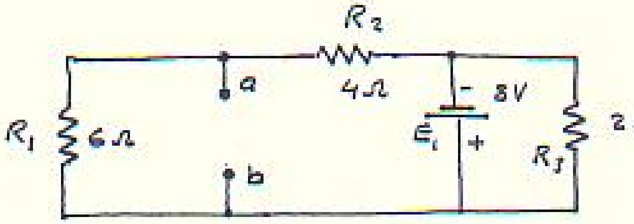
$$\therefore\ I = \frac{E_{th}}{R_{th}+R_L}$$

$$= \frac{48}{6+10} = 3\ A$$

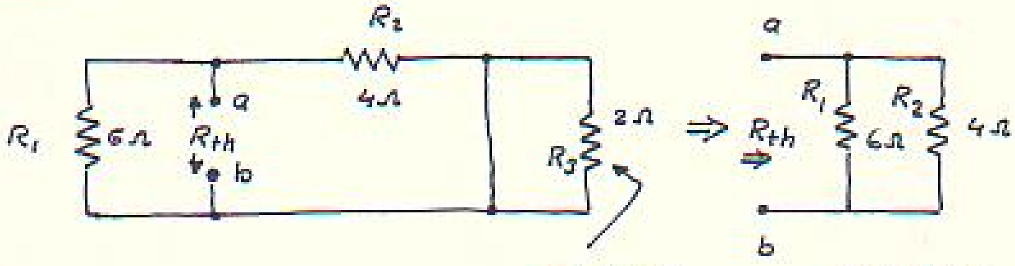
Example: For the circuit shown, find the current in the $3\text{-}\Omega$ resistor using Thevenin's theorem.



Solution: steps 1 and 2



step 3: $R_{th} = ?$

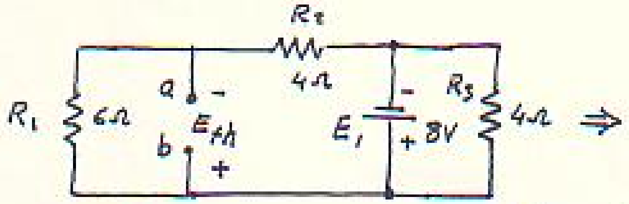


R_3 short circuited

$$\therefore\ R_{th} = \frac{6(4)}{6+4} = 2.4\,\Omega$$

$R_{th} = 6 /\!/ 4$
$= 2.4\,\Omega$

step 4: $E_{th} = ?$

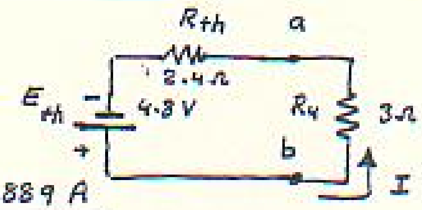


using the voltage divider rule,

$$E_{th} = E_1 \frac{R_1}{R_1+R_2} = 8\ \frac{6}{6+4}$$

$$= 4.8\ V$$

step 5



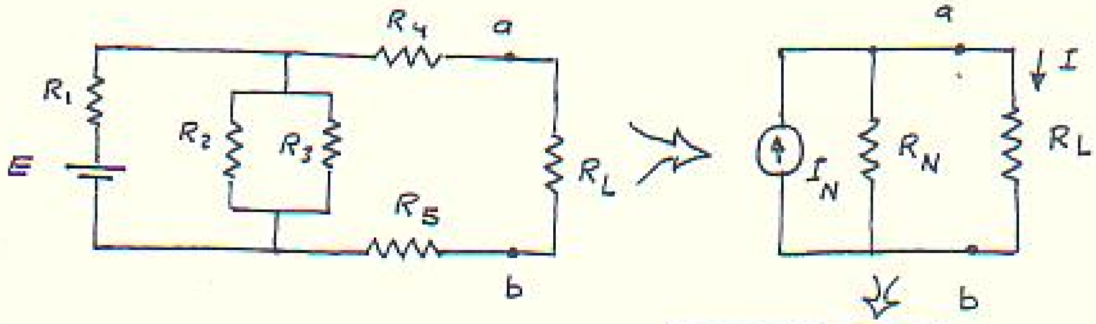
$$\therefore\ I = \frac{E_{th}}{R_{th}+R_4}$$

$$= \frac{4.8}{2.4+3} = \underline{0.889\ A}$$

3.3. Norton's Theorem

: Norton's theorem states that " Any two terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor."

Consider the network shown, it can be replaced by the current source I_N and the parallel resistor R_N;



To find the current through R_L $\Rightarrow$ $\boxed{I = \dfrac{I_N R_N}{R_N + R_L}}$

How to find I_N and R_N

STEP 1

: Remove that portion of the network across which the Norton equivalent circuit is found.

STEP 2

: Mark the terminals of the remaining two-terminal network.

STEP 3 (R_N)

: Calculate R_N by first removing all the sources (voltage sources replaced by short circuits and current sources replaced by open circuits) and then finding the resultant resistance between the two marked terminals.

STEP 4 (I_N)

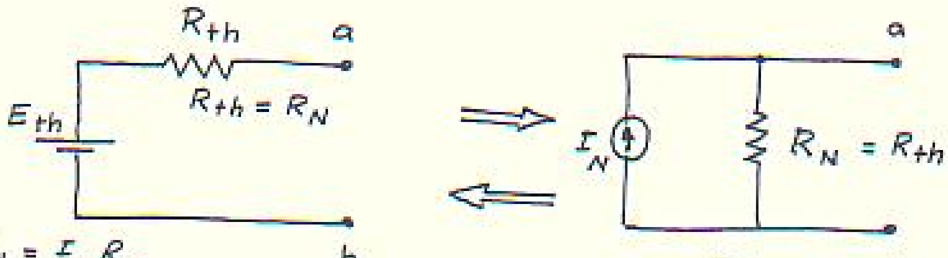
: Calculate I_N by first returning all sources to their original position and then finding the short circuit current between the marked terminals

STEP 5

: Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals ~~of~~ the equivalent circuit.

Relation between Norton equivalent circuit and Thevenin's equivalent circuit

: The Norton and Thevenin equivalent circuits can also be found from each other by using the source transformation previously discussed, as shown ;

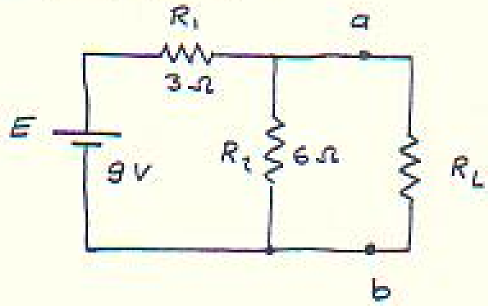


$E_{th} = I_N R_N$

$I_N = \dfrac{E_{th}}{R_{th}}$

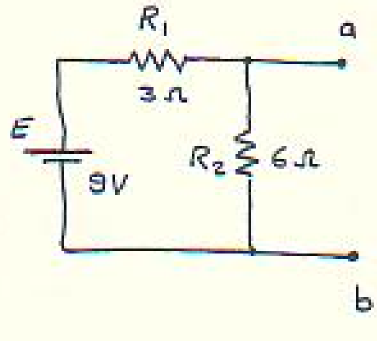
Example

: For the circuit shown, find the Norton equivalent circuit for the network to the left of (a-b).

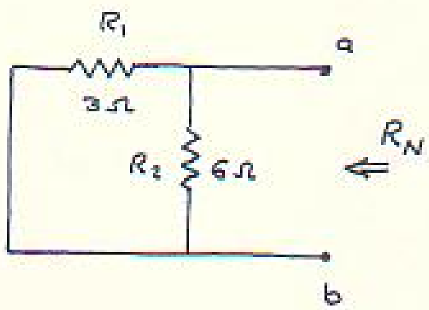


Solution

: steps 1 and 2

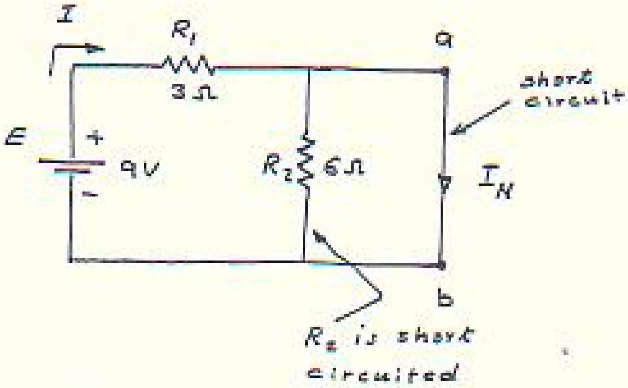


step 3 : $R_N = ?$



$$R_N = R_1 // R_2 = \frac{3(6)}{3+6} = 2\Omega$$

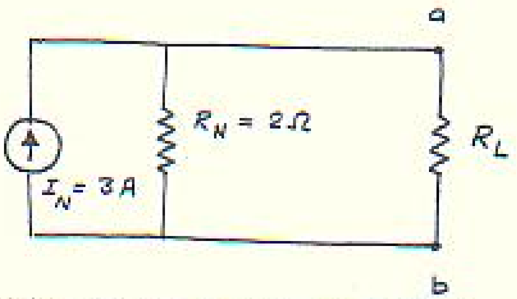
STEP 4 $I_N = ?$



$$I_N = I = \frac{E}{R_1} = \frac{9}{3}$$

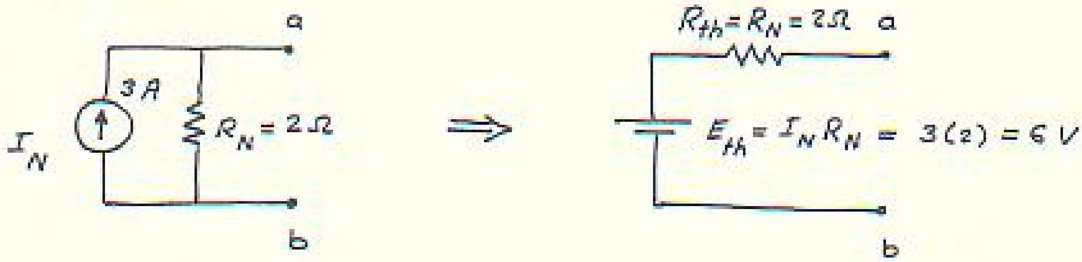
$$= 3\ A$$

∴ step 5 :

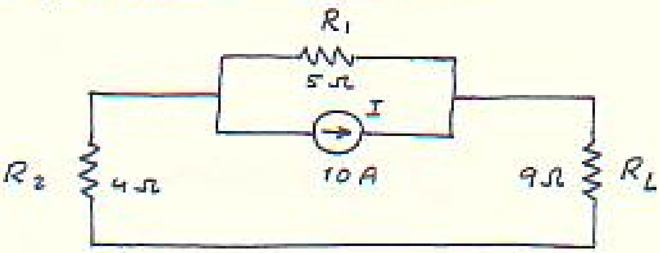


which is the Norton equivalent circuit of the network.

Note : Thevenin's theorem can be determined by Norton's theorem as shown :

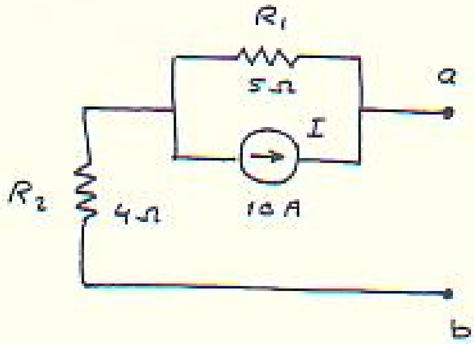


Example : Using Norton theorem find the current through the load resistor R_L in the network shown.

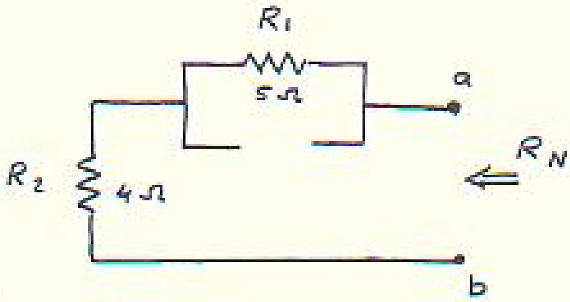


Solution

step1 and 2

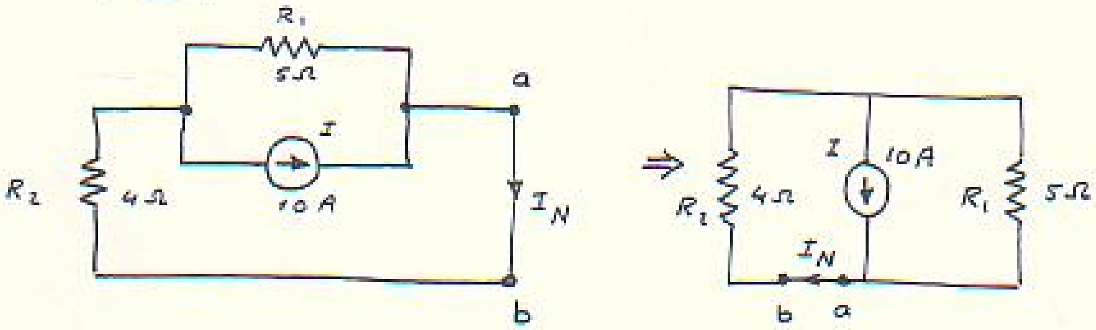


step 3 : $R_N = ?$



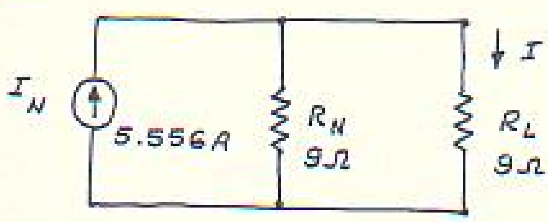
$$R_N = R_1 + R_2 = 5 + 4 = 9\ \Omega$$

step 4 $I_N = ?$



$$\therefore\ I_N = I \cdot \frac{R_1}{R_1 + R_2} = 10\,\frac{5}{5+4} = 5.556\ A$$

* step 5

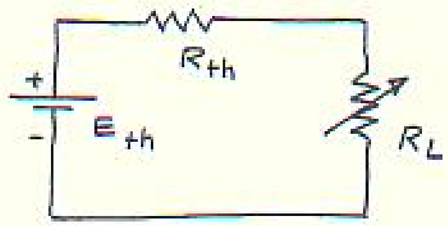


$$\therefore\ I = \frac{I_N}{2} = 2.778\ A$$

3.4 Maximum Power Transfer Theorem

: The maximum power transfer theorem states the following:

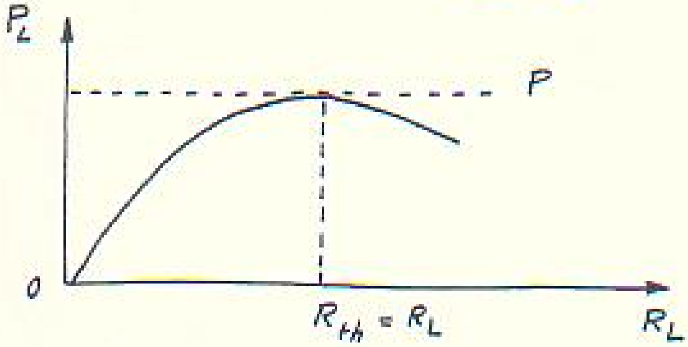
" A load will recieve maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thevenin resistance of the network as <u>seen</u> by the load."



For maximum power transfer $\Rightarrow$ $\boxed{R_{th} = R_L}$

$$I = \frac{E_{th}}{R_{th} + R_L}$$

$$P_L = I^2 R_L = \left(\frac{E_{th}}{R_{th} + R_L}\right)^2 R_L$$



$\therefore$ $P_{L\,max}$ $\Rightarrow$ at $R_{th} = R_L$

$$P_{L\,max} = \left(\frac{E_{th}}{2R_{th}}\right)^2 R_{th} = \frac{E_{th}^2}{4R_{th}}$$

$$\therefore \boxed{P_{L\,max} = \frac{E_{th}^2}{4R_{th}}}$$

* When dealing with Norton equivalent circuit, maximum power transfer takes place when:

$$\boxed{R_N = R_L}$$

That is ;

Max. power transfer at

$$R_N = R_L$$

$$P_L = I^2 R_L$$

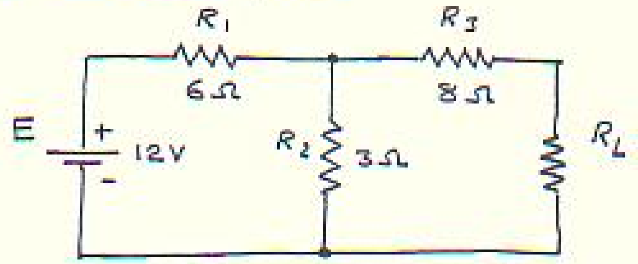
$$= \left(I_N \cdot \frac{R_N}{R_N + R_L}\right) \cdot R_L$$

$$\therefore P_{L_{max}} = \left(I_N \cdot \frac{R_N}{2R_N}\right)^2 R_N$$

$$= \frac{I_N^2 R_N}{4}$$

$$\boxed{P_{L_{max}} = \frac{I_N^2 R_N}{4}}$$

Example : For the network shown, determine the value of R_L for maximum power transfer, and calculate the power delivered under these conditions.

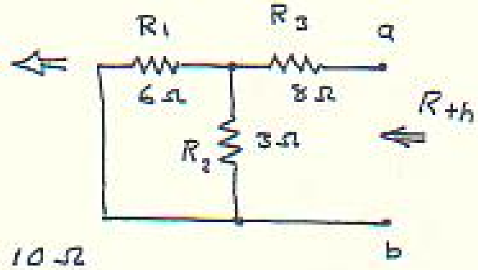


Solution :

$$* \quad R_{th} = (R_1 // R_2) + R_3$$

$$= \frac{6(3)}{6+3} + 8$$

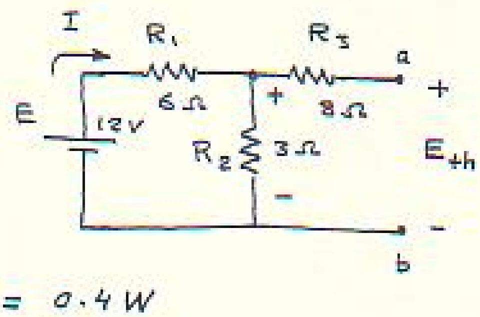
$$= 10\ \Omega$$



$\therefore$ For max. power the value of $R_L = R_{th} = 10\ \Omega$

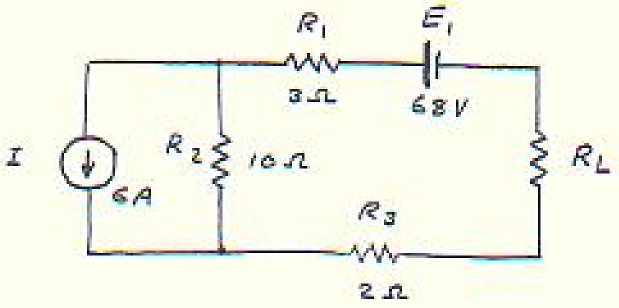
$$* \quad E_{th} = \frac{E \cdot R_2}{R_1 + R_2}$$

$$= \frac{12(3)}{6+3} = 4V$$



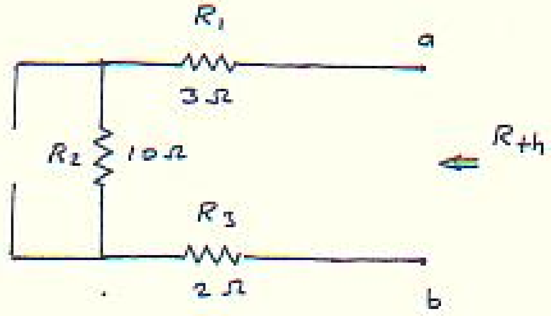
$$\therefore P_{L_{max}} = \frac{E_{th}^2}{4R_{th}} = \frac{(4)^2}{4(10)} = \underline{0.4\ W}$$

Example: Find the value of R_L in the network shown, for maximum power to R_L and determine the maximum power.



Solution:

* R_{th}:

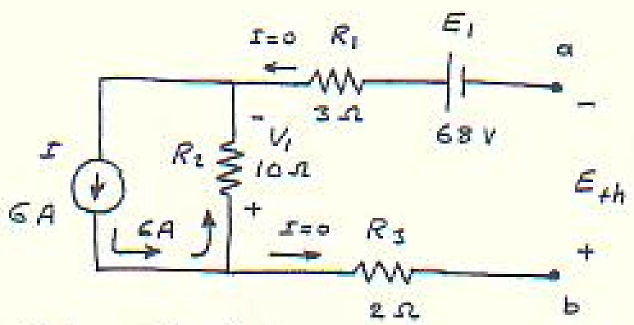


$R_{th} = 3 + 10 + 2$
$= 15\ \Omega$

∴ For max power transfer

$$R_L = R_{th} = 15\ \Omega$$

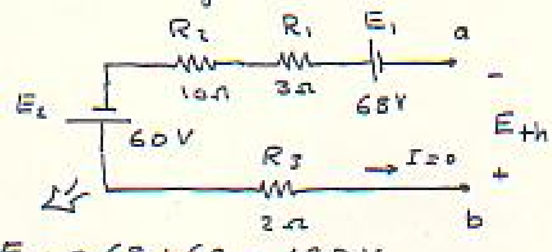
* E_{th}:



$E_{th} = E_1 + V_1$
$= E_1 + I R_1 = 68 + 6(10)$
$= 128\ V$

$$\therefore\ P_{L_{max}} = \frac{E_{th}^2}{4R_{th}} = \frac{(128)^2}{4(15)} = 273.07\ W$$

The current source can be converted into a voltage source:



$E_{th} = 68 + 60 = 128\ V$

OR

$R_{th} = R_1 + R_2 + R_3$
$= 3 + 10 + 2$
$= 15\ \Omega$

$R_L = R_{th} = 15\ \Omega$ for max. power

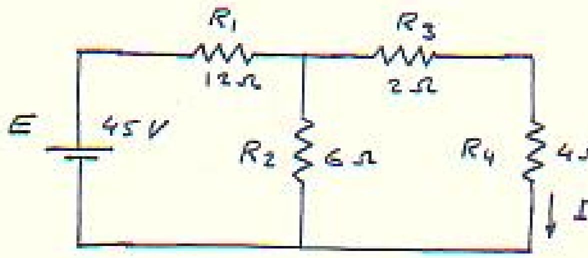
3.5 Reciprocity Theorem

: The reciprocity theorem is applicable only to a single-source networks. The theorem states that:

> "The current I in any branch of a network due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured."

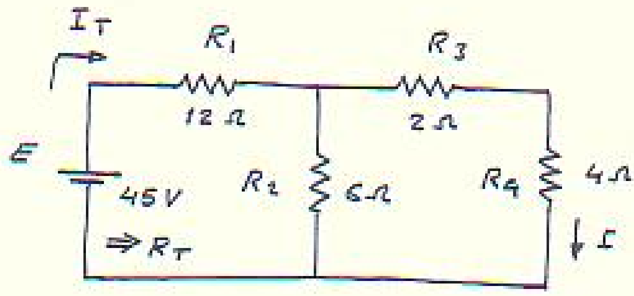
Example

: For the network shown, determine the current I. Is the reciprocity theorem satisfied?



Solution

:



$$I_T = \frac{E}{R_T}$$

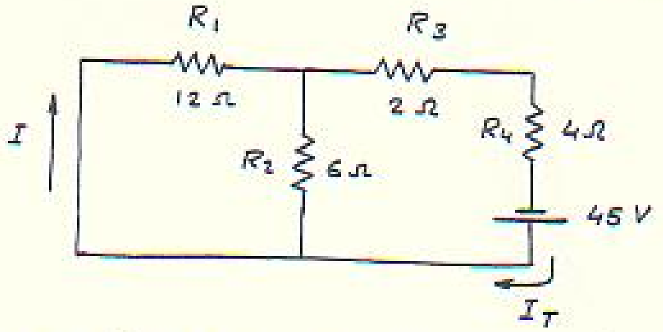
$$= \frac{45}{15} = \underline{3\ A}$$

$$R_T = [\underbrace{(R_3 + R_4)}_{\substack{2+4\\6}} // R_2] + R_1$$

$$= 3 + 12 = 15\ \Omega$$

$$\boxed{\therefore I = \frac{3}{2} = 1.5\ A}$$

To check the reciprocity, place E in the branch of the current I, and calculate the current in the branch where E was originally exist.



$$I_T = \frac{E}{R_T}$$

$$= \frac{45}{10}$$

$\therefore\ I_T = 4.5\ A$

$$R_T = (R_1 // R_2) + R_3 + R_4$$

$$= \frac{12(6)}{12+6} + 2 + 4 = 4 + 2 + 4$$

$\therefore\ R_T = 10\ \Omega$

Finding I ?

$$I = I_T \frac{R_2}{R_1 + R_2} = 4.5 \frac{6}{12+6}$$

$\therefore\ I = 1.5\ A$

Since $I = 1.5\ A$

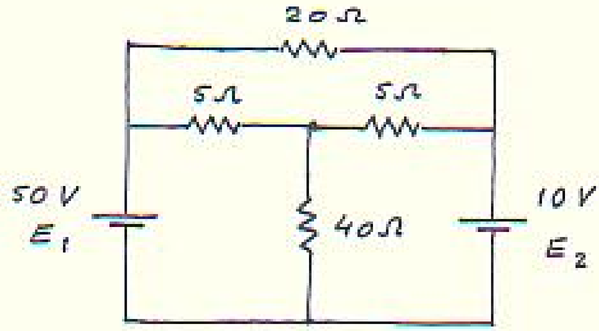
$\therefore$ The reciprocity theorm is satisfied.

Tutorial Sheet No 4

TS4

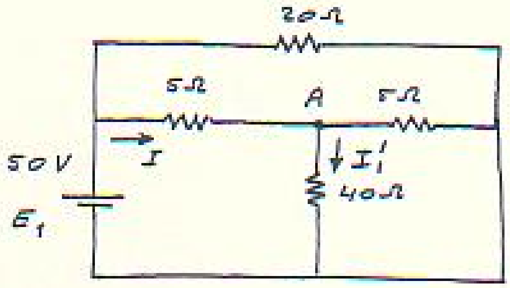
ملاحظة : جميع امثلة الكتاب المنهجي واسئلته مطلوبة .

Example : Use the superposition theorem, find the current in the $40\,\Omega$ resistor of the circuit shown.

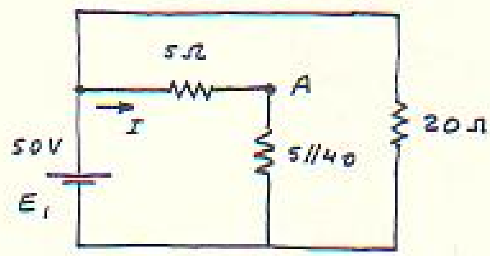


Solution :

* The effect of E_1



$\Rightarrow$

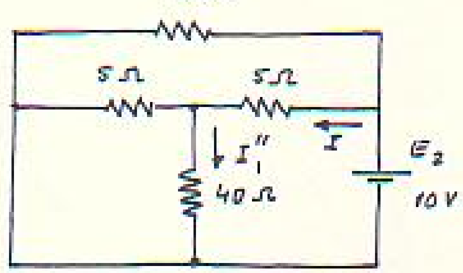


$5//40$ $\qquad$ $5//40 = 4.44\,\Omega$

$$\therefore\ I = \frac{50}{5+4.44} = 5.296\ \text{A}$$

$$\therefore\ I_1' = I\,\frac{5}{5+40} = 5.296\ \frac{5}{45} = 0.589\ \text{A}$$

* The effect of E_2



$$I = \frac{10}{(5//40)+5} = 1.059\ \text{A}$$

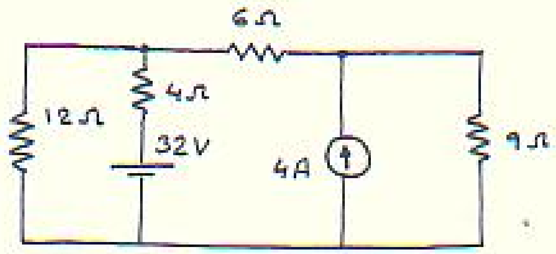
$$I_1'' = I\,\frac{5}{40+5} = 1.059\ \frac{5}{45}$$

$$= 0.118\ \text{A}$$

$$\therefore\ I_1 = I_1' + I_1'' = 0.589 + 0.118 = 0.707\ \text{A}\ .$$

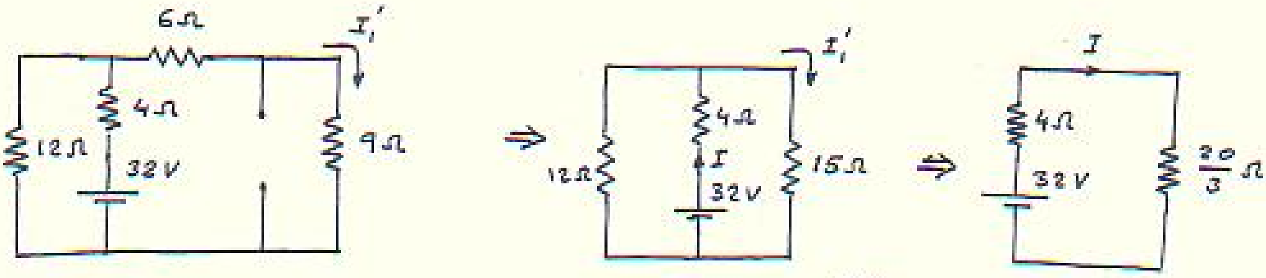
Example: TS4

For the circuits shown, calculate the current through the 9 Ω resistor using the superposition theorem.



Solution:

* **The effect of the voltage source**



$6+9=15\Omega$

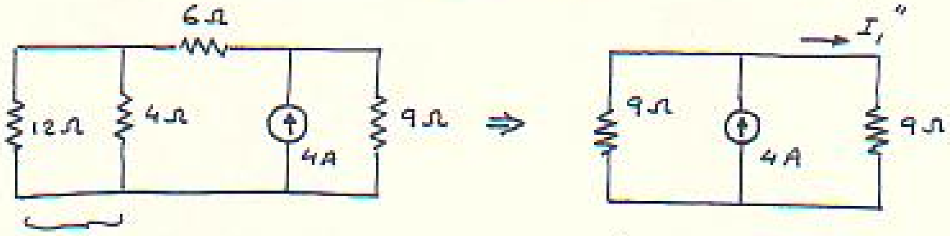
$12 // 15 = \frac{20}{3}\,\Omega$

$$R_T = 4 + \frac{20}{3} = \frac{32}{3}\,\Omega$$

$$\therefore\ I = \frac{E}{R_T} = \frac{32}{(32/3)} = 3\ A$$

$$\therefore\ I_1' = I\,\frac{12}{12+15} = \frac{4}{3}\ A$$

* **The effect of the current source**:



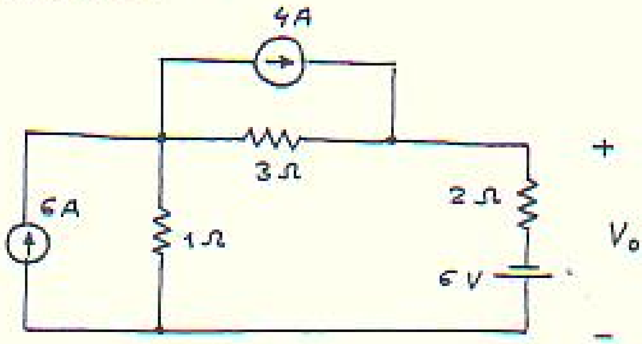
$12 // 4 = 3\,\Omega$

$$\therefore\ I_1'' = \frac{4}{2} = 2\ A$$

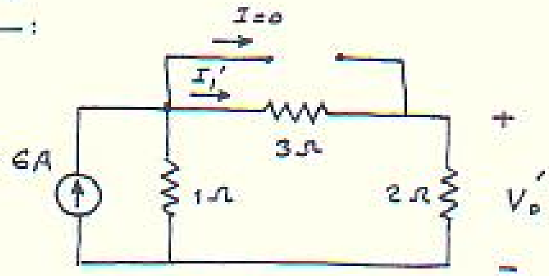
$$\therefore\ I_1 = I_1' + I_1'' = \frac{4}{3} + 2 = \frac{10}{3}\ A.$$

TS4

Example: Using the superposition theorem, find the value of the output voltage V_o in the circuit shown.



Solution: + **Effect 6A source**:



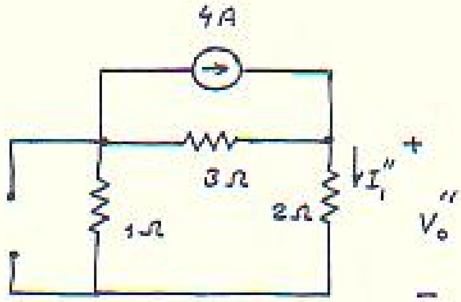
Using the current divider rule;

$$I_1' = 6 \left(\frac{1}{(1+2+3)} \right)$$

$$= 1\ A$$

$$\therefore V_o' = I_1'(2) = 2\ V$$

+ **Effect of 4A source**



current divider rule

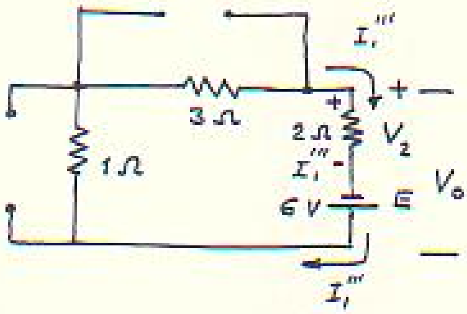
$$\therefore I_1'' = 4 \frac{3}{(1+2)+3}$$

$$= 2\ A$$

$$\therefore V_o'' = I_1''(2)$$

$$= 4\ V$$

+ **Effect of 6V-source**:



$$I_1''' = \frac{6}{1+3+2} = 1\ A$$

$$\therefore V_2 = I_1'''(2) = 2\ V$$

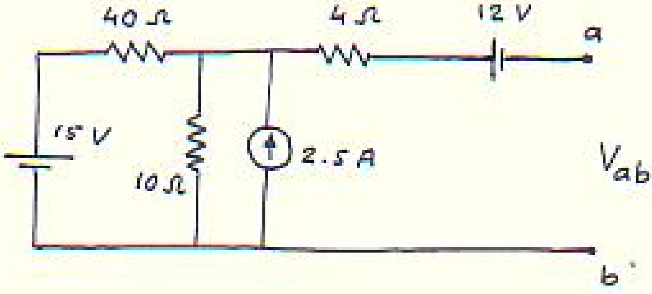
$$\therefore V_o''' = E - V_2 = -6+2 = -4\ V$$

$$\therefore V_o = V_o' + V_o'' - V_o''' = 2+4-4$$

$$= 2\ V$$

Example TS4

Use the superposition theorem to find the voltage V_{ab} in the circuit shown.

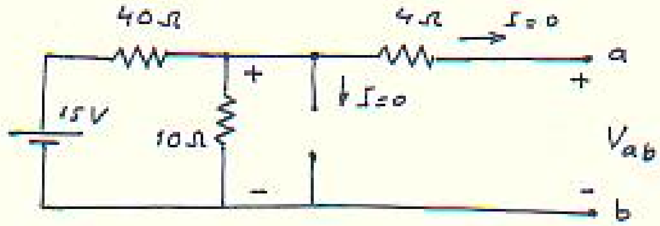


ملاحظة: V_{ab} ربما تكون هي E_{th} في ثفنن .

Solution:

* The effect of the 15 V - source:

$$\therefore \quad V'_{ab} = 15 \frac{10}{10+40} = 3\ V$$

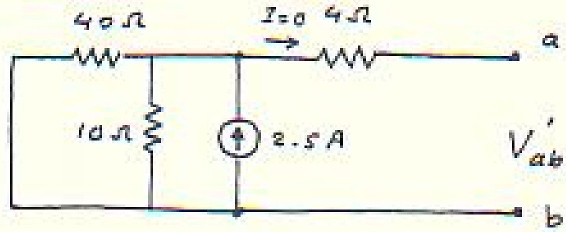


V_{ab} = voltage across 10 Ω resister

* The effect of 2.5 A - source:

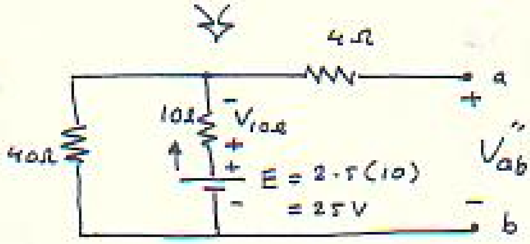
$40 // 10 = 8\ \Omega$

$\therefore \quad V_{ab} = 8 \times 2.5 = 20 V$



or convert the current source to a voltage source then:

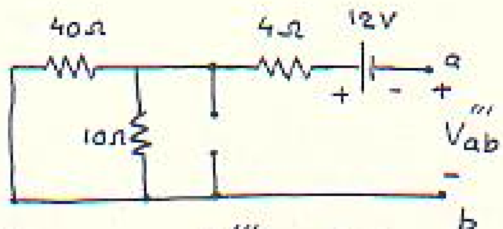
$$V_{ab} = E - V_{10\Omega}$$



$$V_{10\Omega} = E \frac{10}{10+40} = 25 \frac{10}{50} = 5 V$$

$$\therefore \quad V_{ab} = 25 - 5 = 20\ V$$

* The effect of 12V - source:

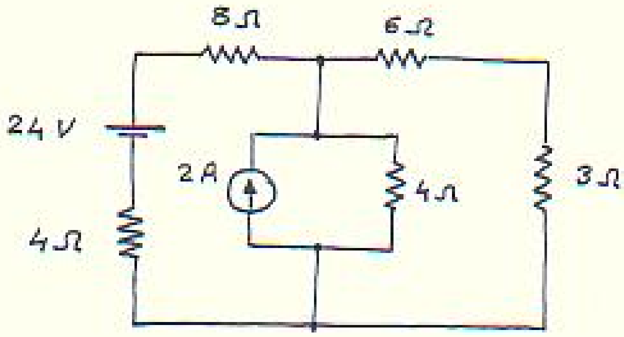


$V'''_{ab} = -12\ V$

$$\therefore \quad V_{ab} = V'_{ab} + V''_{ab} - V'''_{ab}$$
$$= 3 + 20 - 12 = 11\ V$$ with point a (+ve).

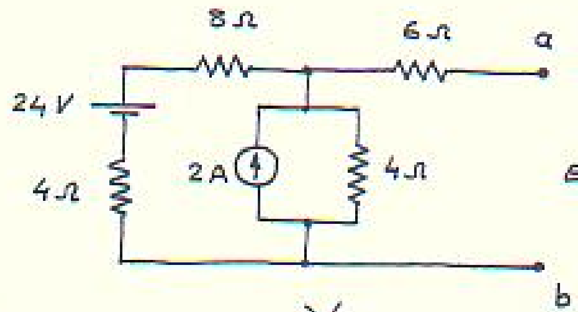
Example TS4

: Use the Thevenin's theorem to find the current in the $3\,\Omega$ resistor in the network shown.



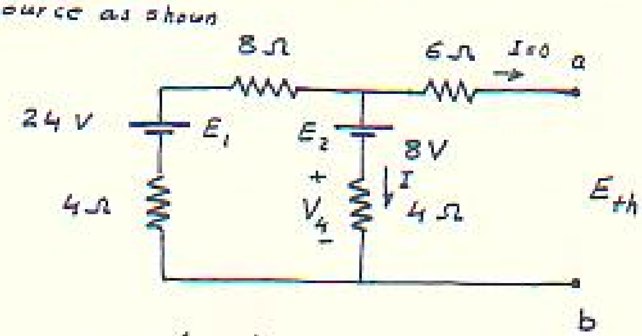
Solution :

$E_{th} = ?$



$E_{th} = V_{ab}$

* convert the current source to voltage source as shown



$\therefore\ E_{th} = E_2 + V_4$

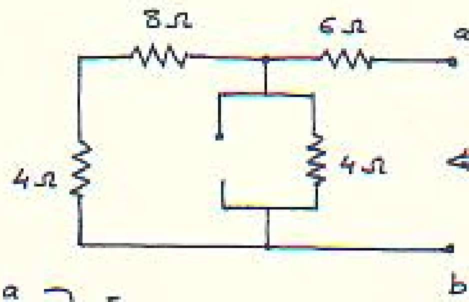
$$V_4 = I(4\,\Omega)$$

$$I = \frac{E_1 - E_2}{4+8+4} = \frac{24-8}{16} = 1\ A$$

$$\therefore\ V_4 = (1)(4) = 4\ V$$

$$\therefore\ E_{th} = 8 + 4 = 12\ V$$

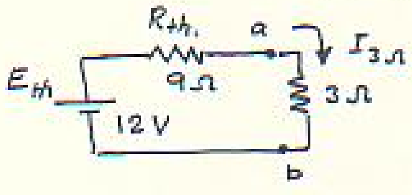
$R_{th} = ?$



$R_{th} = R_{ab}$

$$R_{th} = \left[(8+4)//4\right] + 6$$

$$= 9\,\Omega$$



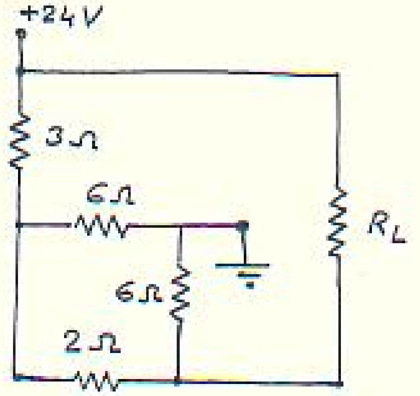
$$\therefore\ I_{3\Omega} = \frac{12}{9+3}$$

$$= 1\ A$$

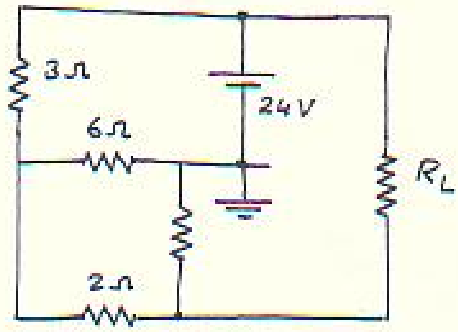
Note : Repeat this example to find the value of R_L for max. power transfer and compute $P_{L_{max}}$

TS4

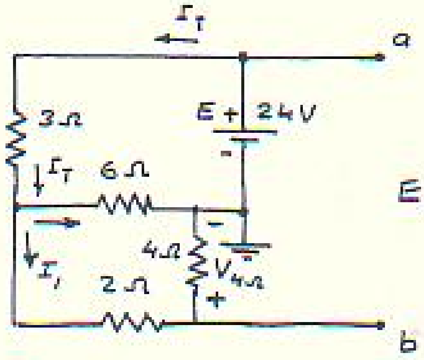
Example : For the network shown, what is the value of R_L for maximum power transfer condition? Calculate this power.



Solution : The cct is redrawn to be as shown;



$E_{th} = ?$:



$E_{th} = V_{ab}$

$$E_{th} = 24 - V_{4\Omega}$$
$$= 24 - I_1(4\Omega)$$

$$I_T = \frac{E}{R_T}$$

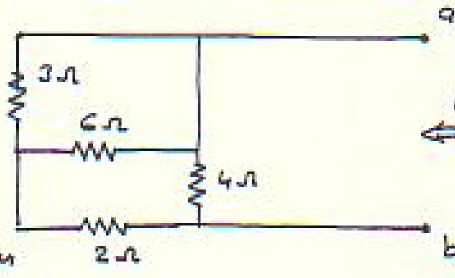
$$R_T = [\underbrace{(2+4)//6}_{3}] + 3 = 6\,\Omega$$

$$\therefore I_T = \frac{24}{6} = 4\,A \Rightarrow I_1 = \frac{4}{2} = 2\,A$$

$$\therefore E_{th} = 24 - 2(4) = 16\,V$$

* **$R_{th} = ?$**

$$R_{th} = [(3//6) + 2] // 4$$
$$= 2\,\Omega$$



$R_{th} = R_{ab}$

$\boxed{R_L = R_{th}}$ for maximum power transfer

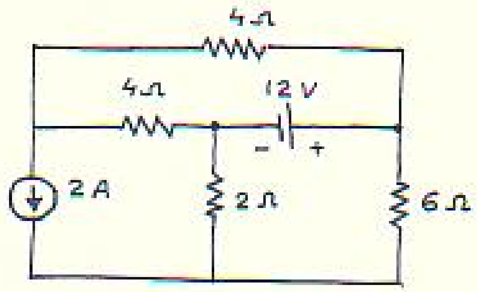
$$\therefore R_L = 2\,\Omega$$

$P_{L_{max}}$ = max. power transferred to the load resistance R_L $\boxed{TS4}$

$$P_{L_{max}} = \frac{E_{th}^2}{4R_{th}}$$

$$= \frac{(16)^2}{4(2)} = 32 \quad W$$

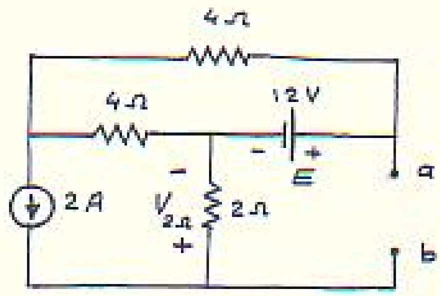
Example: Use the Thevenin's theorem to find the current flowing through the $6\,\Omega$ resistor in the network shown.



Solution: $E_{th} = ?$:

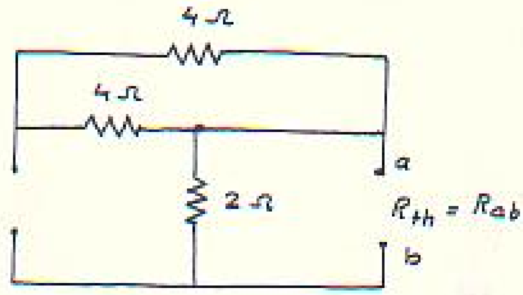
$$E_{th} = E - V_{2\Omega}$$

$$= 12 - (2)(2) = 8\,V$$



$R_{th} = ?$:

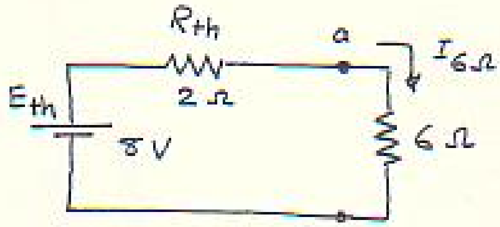
$$R_{th} = 2\,\Omega$$



$$\therefore I_{6\Omega} = \frac{E_{th}}{R_{th} + 6}$$

$$= \frac{8}{2+6}$$

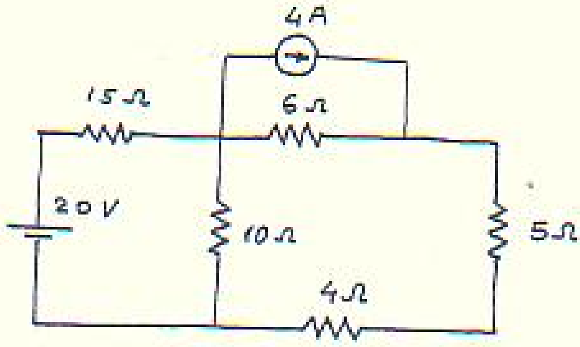
$$\therefore I_{6\Omega} = 1\ A$$



Example

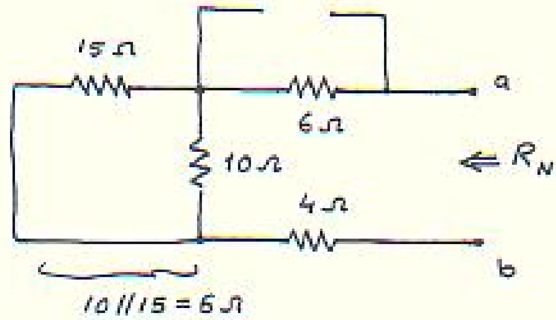
TS 4

For the circuit shown, find the value of the current passing through the 5 Ω resistor using Norton's theorem. Calculate the power absorbed by this resistor.



Solution:

$R_N = ?$

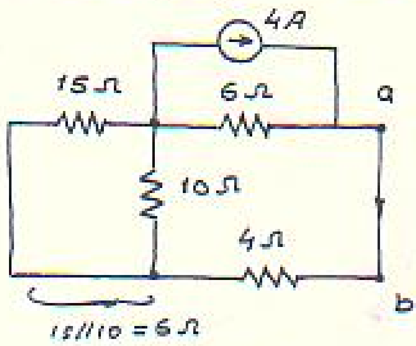


$$R_N = (15//10) + 6 + 4$$
$$= 16\ \Omega$$

$I_{sc} = ?$

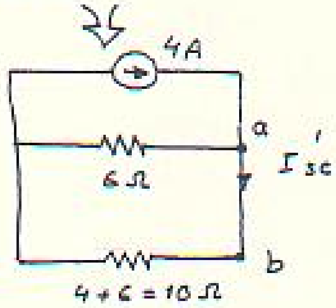
We have two sources; we can use superposition theorem to find the resulting I_{sc}.

Effect of 4A source



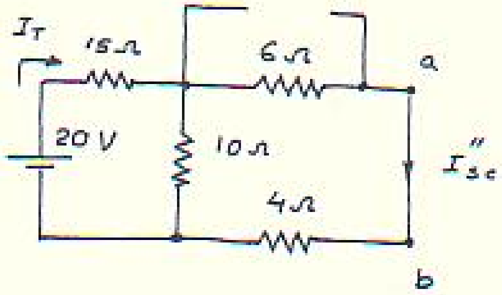
current divider rule

$$I'_{sc} = 4\,\frac{6}{6+10} = \frac{4(6)}{16}$$
$$= \frac{3}{2} = 1.5\ A$$



TS4

Effect of 20V source



$$I_T = \frac{E}{R_T}$$

$$R_T = [(\underbrace{6+4}_{10})//10] + 15$$

$$= 20\ \Omega$$

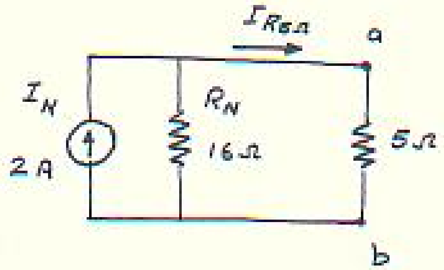
$$\therefore I_T = \frac{20}{20} = 1\ A$$

$$\therefore I''_{sc} = \frac{I_T}{2} = \frac{1}{2} = 0.5\ A$$

$$\therefore I_N = I'_{sc} + I''_{sc} = 1.5 + 0.5 = \underline{2\ A}$$

$$\therefore I_{R_{5\Omega}} = I_N \frac{R_N}{R_N + R_{5\Omega}}$$

$$= 2\ \frac{16}{16+5} = 1.52\ A$$

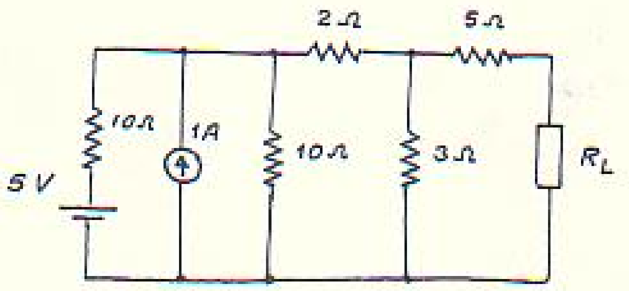


and the power $P = I^2_{R_{5\Omega}} \cdot R_{5\Omega}$

$$= (1.52)^2 \cdot (5) = 11.6\ W$$

ملاحظة : من الممكن استخراج E_{th} ومن ثم تحويلها الى دائرة نورتن المكافئة ومن ثم الحصول على التيار .

Example: For the circuit shown, obtain the condition for power transfer to the load R_L. Hence determine the maximum power transferred.

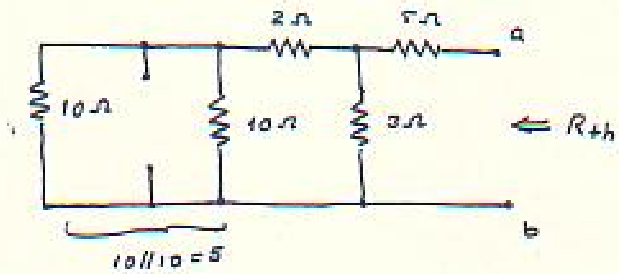


Solution: The condition for max. power transfer is $R_L = R_{th}$ of the cct.

$R_{th} = ?$

$R_{th} = 7.1\ \Omega$ ⇒

$\therefore R_L = 7.1\ \Omega$ for max. power transfer

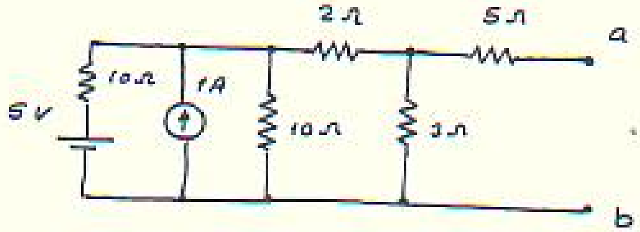


To calculate $P_{L_{max}}$ (max. power transfer), E_{th} must be determined.

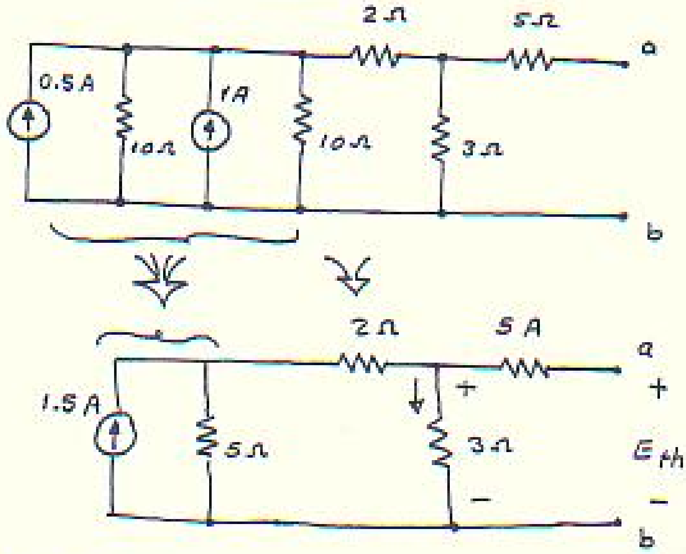
TS 4

$$P_{L_{max}} = \frac{E_{th}^2}{4R_{th}}$$

$\therefore E_{th} = ?$



convert the voltage source into a current source



$$\therefore I_{R_{2\Omega}} = 1.5 \; \frac{5}{2+3}$$

$$= 0.75 \text{ A}$$

$$\therefore E_{th} = 0.75\,(3)$$

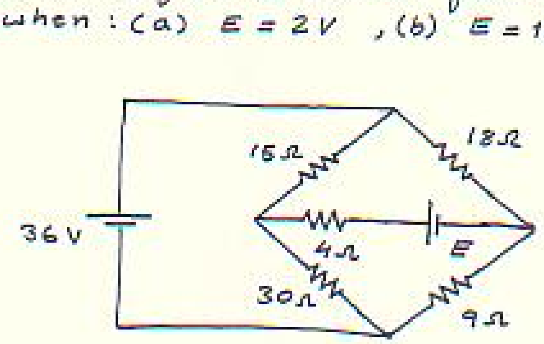
$$= \underline{2.25 \text{ V}}$$

$$\therefore P_{L_{max}} = \frac{(2.25)^2}{4\,(7.1)} = 0.178 \text{ W}$$

$$= 178 \text{ mW}$$

Example

: For the circuit shown, find the current flowing through the 4 Ω resistor when : (a) E = 2 V , (b) E = 12 V , (c) E = 20 V

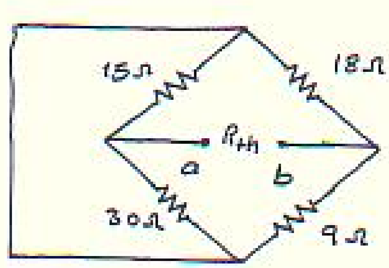


Solution

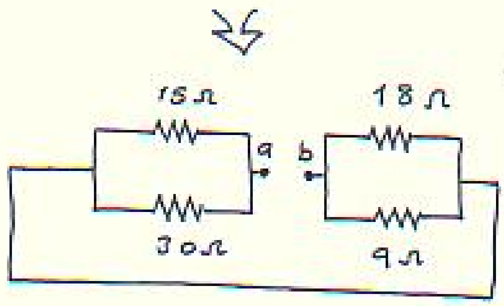
Since it is required to determine the current in the same branch many times, it convenient to use Thevenin's theorem.

TS4

<u>$R_{th} = ?$</u> :

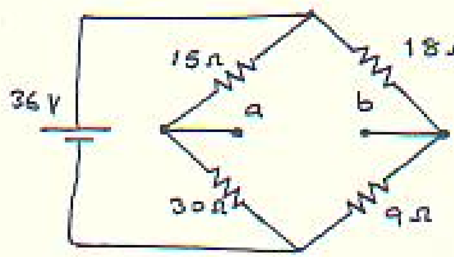


$$R_{th} = (15 // 30) + (18 // 9)$$
$$= 16\ \Omega$$



<u>$E_{th} = ?$</u>

$$E_{th} = V_{ab} = V_a - V_b$$



using voltage divider rule

$$V_b = 36 \frac{9}{9+18}$$
$$= 12\ V$$

also,

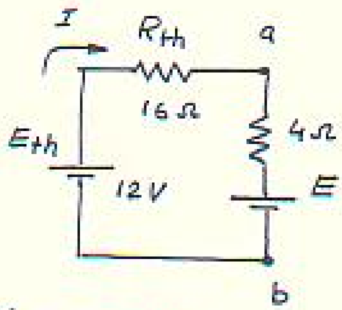
$$V_a = 36 \frac{30}{30+15}$$
$$= 24\ V$$

then;

$$\therefore E_{th} = V_a - V_b = 24 - 12 = \underline{12 V}$$

∴ Thevenin's equivalent circuit of the network is :

$$\therefore I = \frac{E_{th} - E}{R_{th} + 4}$$
$$= \frac{12 - E}{16 + 4} = \frac{12 - E}{20}$$



∴ I = ∗ For $E = 2V$

$$\therefore I = \frac{12-2}{20} = 0.5\ A$$

∗ For $E = 12V$

$$\therefore I = \frac{12-12}{20} = 0\ A$$

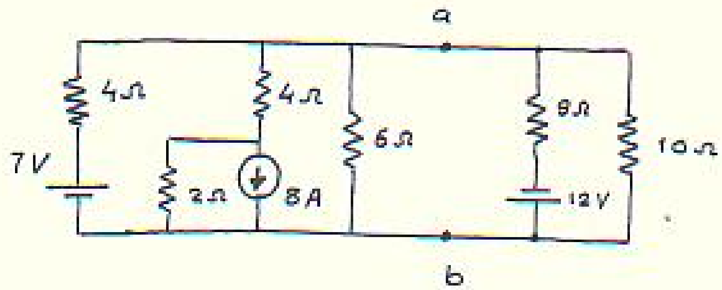
∗ For $E = 20V$

$$\therefore I = \frac{12-20}{20} = -0.4\ A$$ (in the reversed direction.)

Example

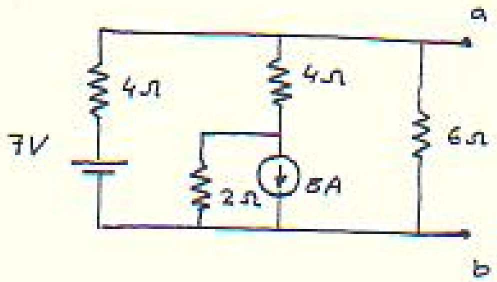
TS4

: Find the Norton equivalent circuit for the portion of the network to the left of (a – b) in the circuit shown.



Solution

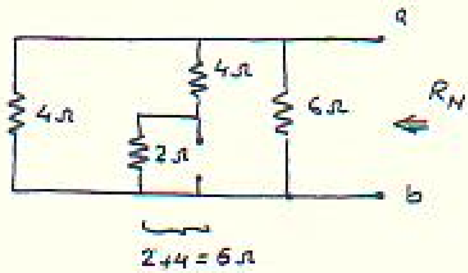
:



$R_N = ?$:

$R_N = 6 // 6 // 4$

$= 1.714\ \Omega$

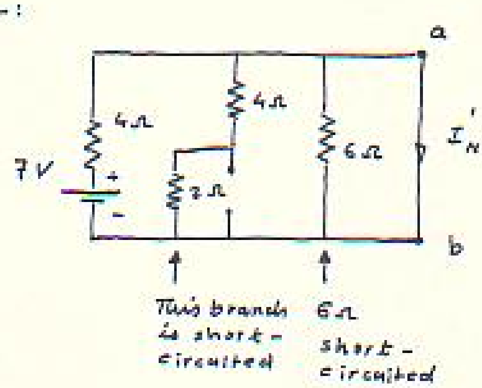


$I_N = ?$: We have 2 sources, it is recommended to use the superposition theorem to find I_N

[illegible] I_N

– Effect of 7 V source :

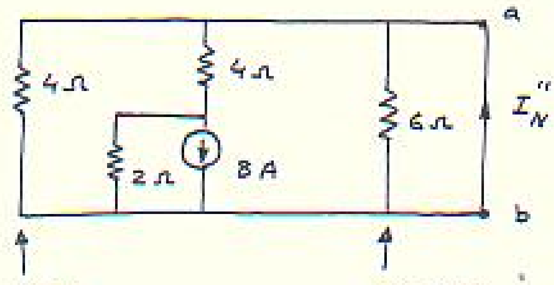
$\therefore\ I_N' = \frac{7}{4} = 1.75\ A$



- Effect of 8 A source :

current divider rule

$$\therefore I_N'' = 8\,\frac{2}{2+4} = 2.667\ A$$

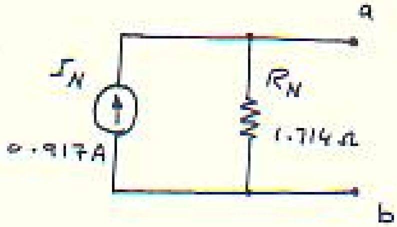


short cct short cct.

$$\therefore I_N = I_N' - I_N'' = 2.667 - 1.75$$

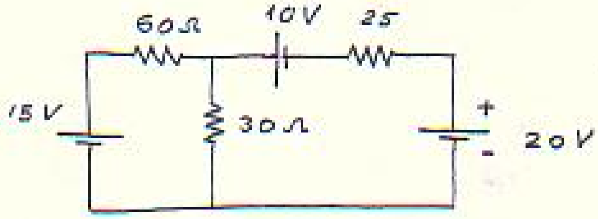
$$= 0.917\ A \quad (\text{in the direction of } I_N')\,.$$

∴ The Norton equivalent cct of the portion of the network to the left of (a-b) is :

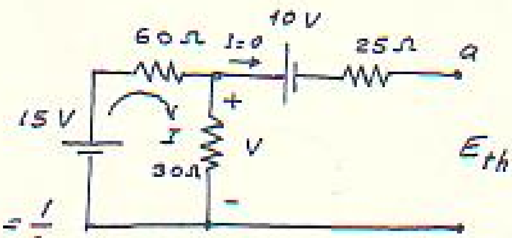


ملاحظة : من الممكن الحصول على دائرة نورتن من دائرة (Thevenin) .

Example : For the circuit shown, find the current through the 20 V voltage source using Thevenin's theorem.



Solution : $E_{th} = ?$:



$$E_{th} = I(30) - 10$$

$$I = \frac{15}{60+30} = \frac{1}{6}$$

$$\therefore E_{th} = 5 - 10 = -5\ V$$

or $E_{th} = V - 10$

voltage divider rule $\quad V = 15\,\frac{30}{30+60} = 5\ V$

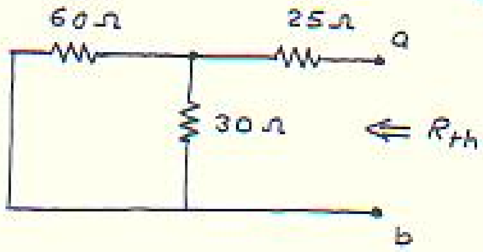
$$\therefore E_{th} = 5 - 10 = -5\ V$$

TS4

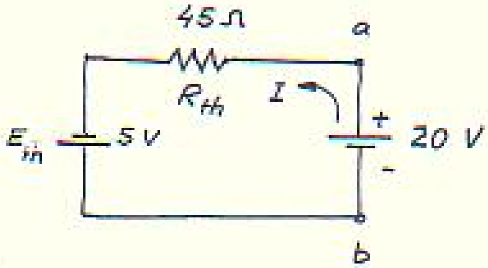
$\underline{R_{th} = ?}$:

$$R_{th} = (60 // 30) + 25$$

$$= 45\ \Omega$$

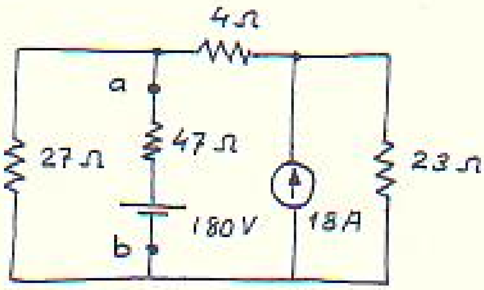


∴ Thevenin equivalent circuit of the network is :

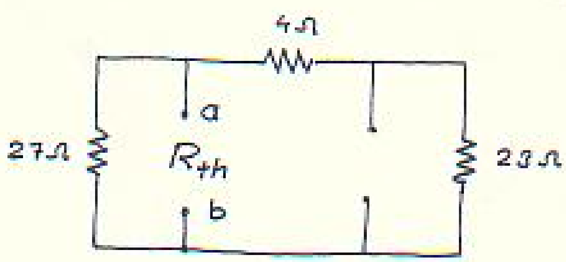


$$\therefore I = \frac{20+5}{45} = \frac{25}{45} = \underline{0.56\ A}$$

$\underline{\text{Example}}$: In the cct. shown, find the current through the branch a-b using Thevenin's theorem.



$\underline{\text{Solution}}$: $\underline{R_{th} = ?}$:

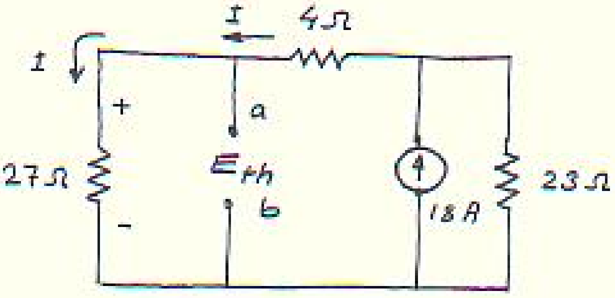


$$R_{th} = (\underbrace{4+23}_{27}) // 27$$

$$\therefore R_{th} = \frac{27}{2} = 13.5\ \Omega$$

TS4

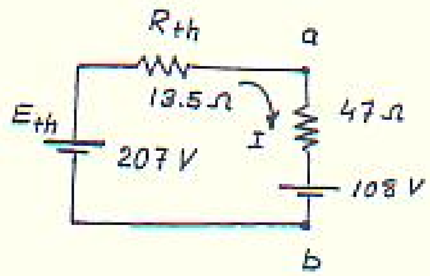
$E_{th} = ?$:



$$E_{th} = I\,(27\,\Omega)$$
$$= 7.67\,(27)$$
$$= 207\ V$$

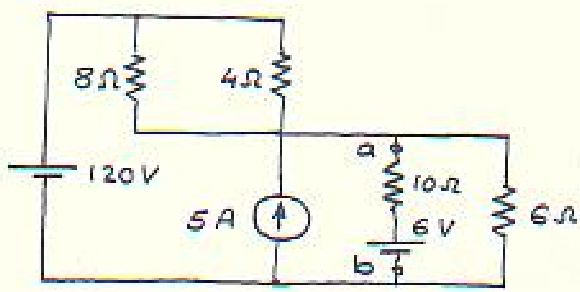
$$I = 18\,\frac{23}{(4+27)+23} = 18\,\frac{23}{54}$$
$$= 7.67\ A$$

∴ Thevenin equivalent circuit is:



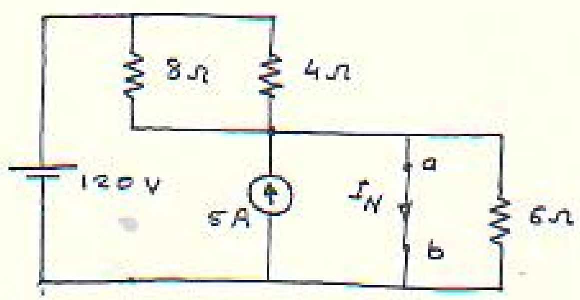
$$\therefore\ I = \frac{E_{th} - 108}{R_{th} + 47}$$
$$= \frac{207 - 108}{13.5 + 47}$$
$$\Rightarrow\ I = 1.64\ A$$

Example : Using Norton's theorem, find the current through the branch a-b in the circuit shown.



Solution : $I_N = ?$

* We have two source in the cct show, so we can use the superposition theorem to determine I_N

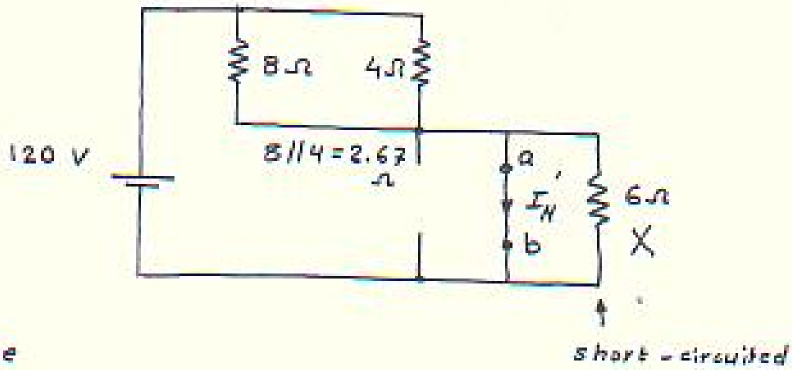


TS4

* Effect of 18 V source

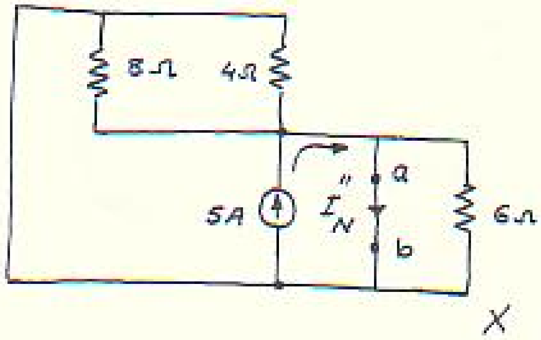
$$I_N' = \frac{120}{2.67}$$

$$= 45 \text{ A}$$



* Effect of 5A source

$$\therefore I_N'' = 5 \text{ A}$$



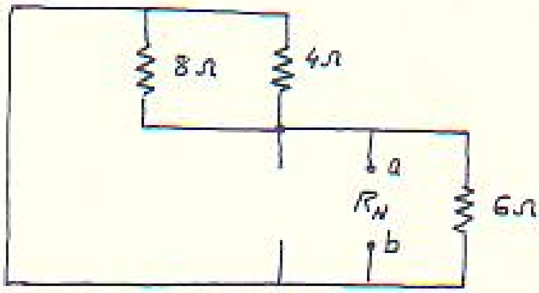
$$\therefore I_N = \underbrace{I_N' + I_N''}_{\text{in the same direction}}$$

$$= 45 + 5 = \underline{50 \text{ A}}$$

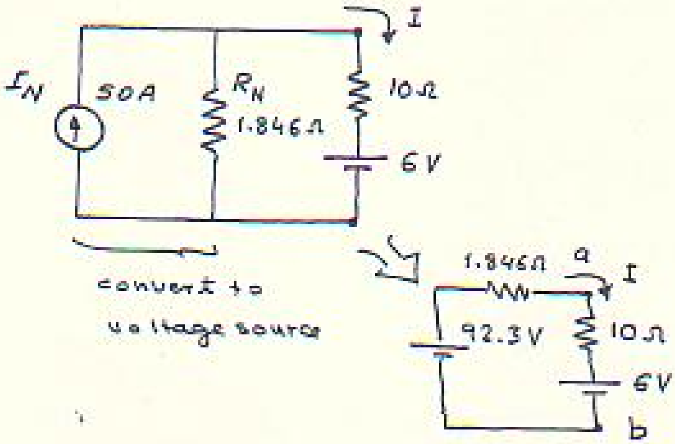
$R_N = ?$

$$R_N = (8//4)//6$$

$$= 1.846 \ \Omega$$



$\therefore$ Norton equivalent circuit is:

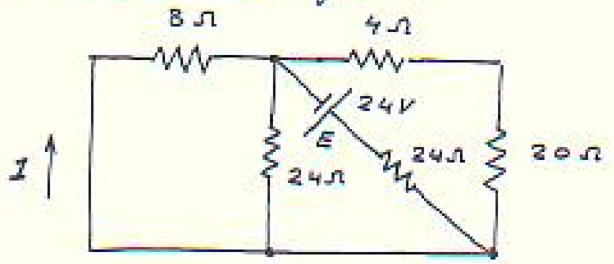


$$\therefore I = \frac{92.3 - 6}{1.846 + 10}$$

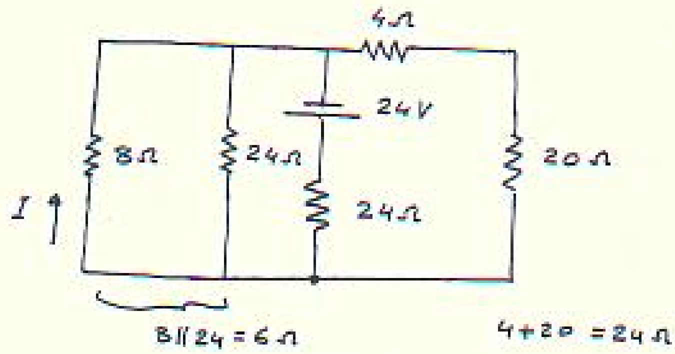
$$= \underline{7.29 \text{ A}}$$

TS4

Example: For the network shown, determine the current I. Is the reciprocity theorem satisfied?

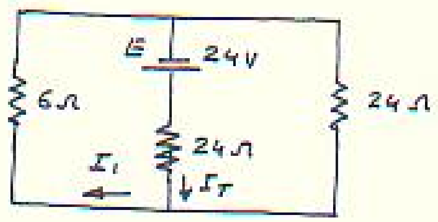


Solution: The circuit is redraw to be as shown:



$R_T = (6 \| 24) + 24$

$= 28.8\ \Omega$



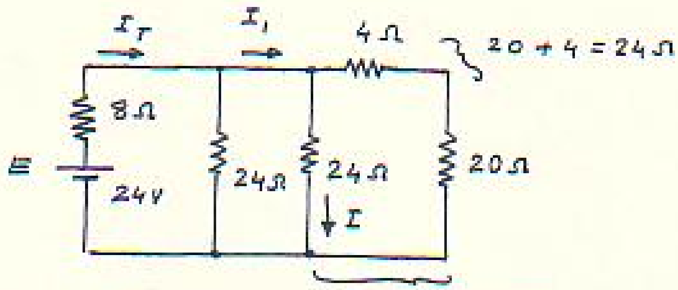
$\therefore\ I_T = \dfrac{E}{R_T} = \dfrac{24}{28.8}$

$= 0.833\ A$

$\therefore\ I_1 = I_T \dfrac{24}{24+6} = 0.833\ \dfrac{24}{30} = 0.666\ A$

$\Rightarrow\ I = I_1 \dfrac{24}{8+24} = 0.666\ \dfrac{24}{32} = \underline{0.5\ A}$

Reciprocity!



$\therefore\ I_T = \dfrac{E}{R_T} = \dfrac{24}{16} = 1.5\ A$

and $I_1 = I_T \dfrac{24}{24+12} = 1\ A$

$\therefore\ I = \dfrac{I_1}{2} = \underline{0.5\ A}$

$24 \| 24 = 12\ \Omega$

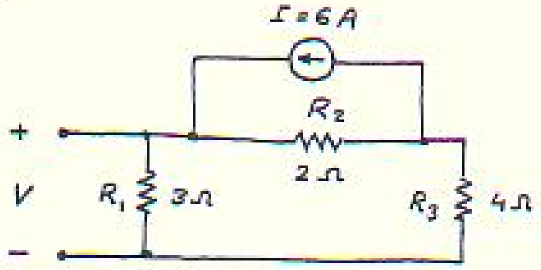
$R_T = 8 + 8 = 16\ \Omega$

$24 \| 12 = 8\ \Omega$

Hence, the reciprocity theorem is satisfied

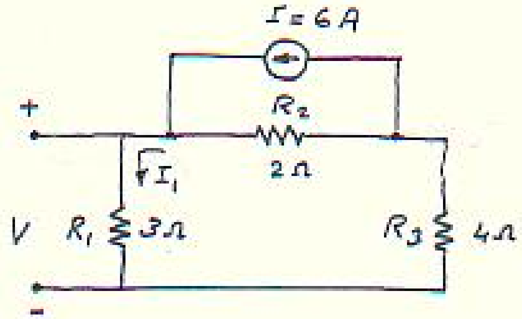
Example: For the circuit shown, determine the voltage V. Is the reciprocity theorem satisfied ?

TS4



Solution:

$V = ?$



$$V = I_1 R_1$$

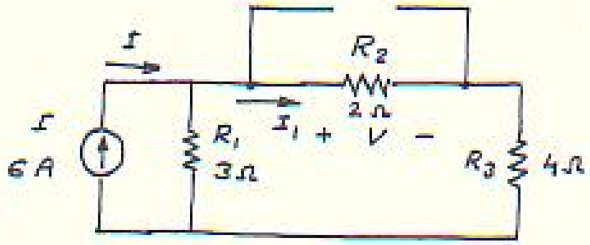
$$I_1 = \frac{I(2)}{(3+4)+2}$$

$$\Rightarrow I_1 = \frac{6 \times 2}{9} = \frac{12}{9} = 1.333\ A$$

$$\therefore V = (1.333)(3)$$

$$= \underline{4V}$$

Reciprocity !



$$I_1 = I \frac{R_1}{R_1 + (R_2 + R_3)}$$

$$= 6 \frac{3}{3 + (2+4)} = 2\ A$$

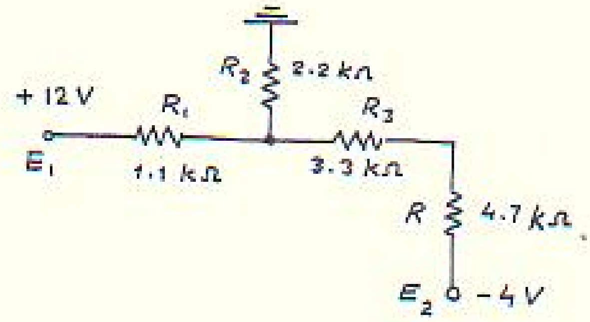
$$\therefore V = I_1 R_2 = 2(2) = \underline{4V}$$

$\therefore$ Reciprocity theorem is satisfied.

TS4

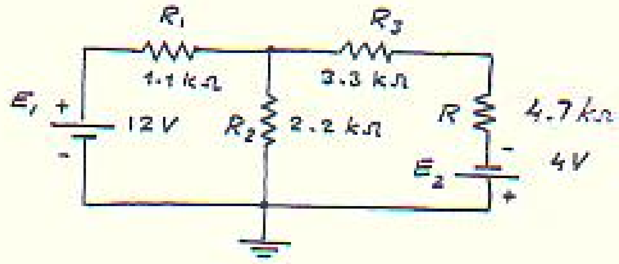
Example

: For the network shown determine the value of R_L to acheive maximum power transfer condition to R_L. Calculate this maximum power.



Solution

: The circuit is redrawn to be as shown;



For max. power transfer

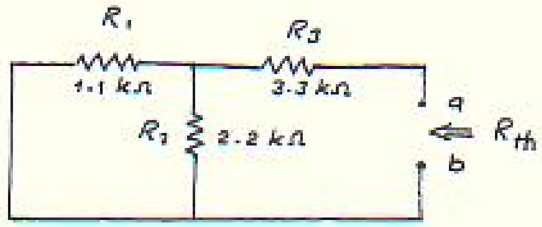
$$\boxed{R_L = R_{th}}$$

R_{th}

:

$$R_{th} = (R_1 // R_2) + R_3$$
$$= 4.033 \text{ k}\Omega$$

$\therefore R_L = 4.033$ kΩ for max power transfer



E_{th}

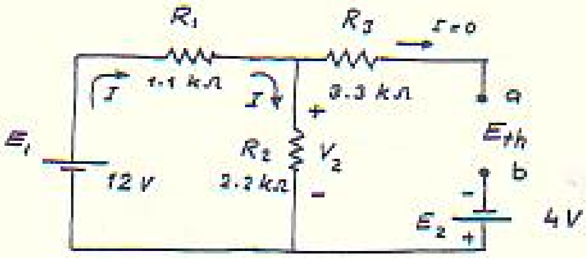
:

$$E_{th} = E_2 + V_2$$

$$V_2 = 12 \frac{2.2 \times 10^3}{(1.1 + 2.2) \times 10^3}$$

$$\therefore V_2 = 8 \text{ V}$$

$$\therefore E_{th} = 4 + 8 = 12 \text{ V}$$

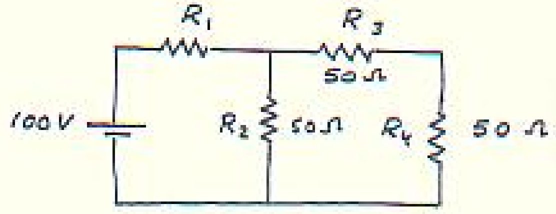


$$\therefore P_{L_{max}} = \frac{E_{th}^2}{4R_{th}} = \frac{(12)^2}{4(4.033) \times 10^3}$$

$$= 9.93 \times 10^{-3} \text{ W}$$
$$= 9.93 \text{ mW}$$

Example: TS4

For the circuit shown, find the value of the resistor R_1 such that the resistor R_4 will recieve maximum power.



Solution: Since R_4 recieve maximum power, then:

$$\therefore \quad R_4 = R_{th}$$

$$\therefore \quad R_{th} = 50\ \Omega$$

$$R_{th} = (R_1 // R_2) + R_3$$

$$= \frac{R_1 * R_2}{R_1 + R_2} + R_3$$

$$\therefore \quad 50 = \frac{R_1 (50)}{R_1 + 50} + 50$$

$$\frac{50 R_1}{R_1 + 50} = 0 \quad \Rightarrow \quad 50 R_1 = 0$$

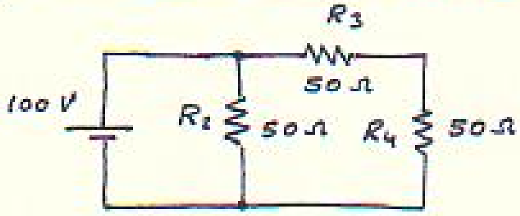
$$\therefore \quad R_1 = 0$$

or

$$R_1 + 50 = 0 \quad \Rightarrow \quad R_1 = -50$$

$\therefore \quad R_1 = 0$ (short circuit) —ve sign neglected

$\therefore$ For maximum power transfer, the cct. will be as shown:

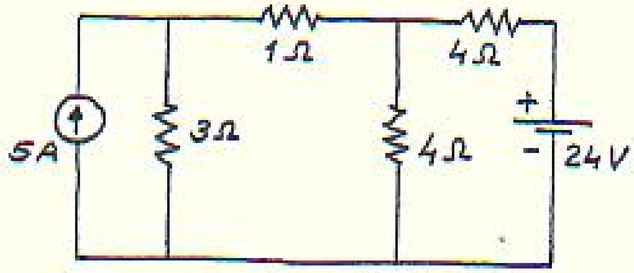


TS4

Practice Problem

: For the circuit shown, find the current through the 1Ω resistor using;

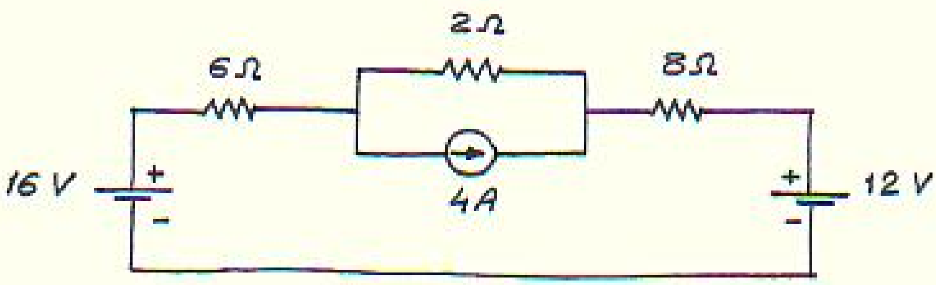
(a). the superposition theorem,
(b). the nodal voltage method,
(c). the loop current method.
(d). Thevenin's theorem.
(e). Norton's theorem.



Answer: $\underset{\rightarrow}{I_{1\Omega}} = 0.5\ A$

Practice Problem

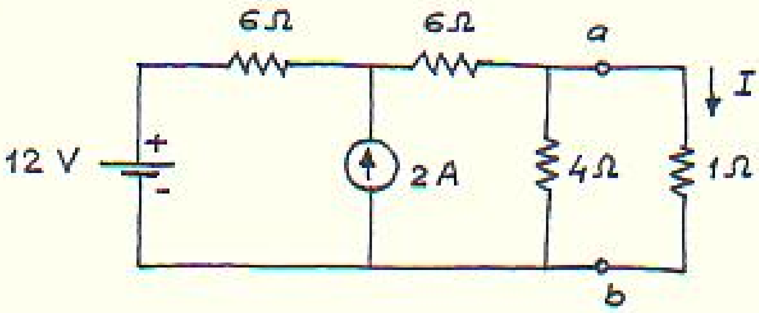
: Find the current through the 8Ω resistor in the circuit shown, using the superposition theorem.



Answer: $\underset{\rightarrow}{I} = 0.75\ A$

Practice Problem

: Using Thevenin's theorem, find the equivalent equivalent circuit to the left of the terminals in the circuit shown, then find the current I.

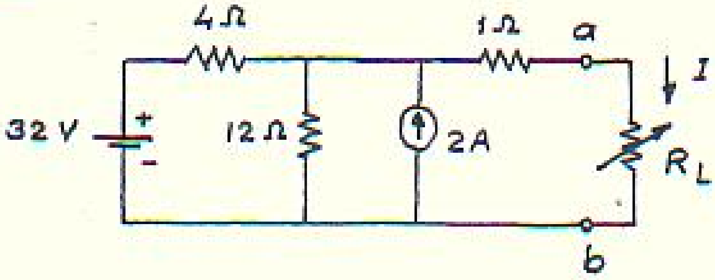


Answer: $\underline{V_{th} = 6V}$, $\underline{R_{th} = 3\Omega}$ $\Rightarrow$ $\underline{I = 1.5\ A}$

TS4

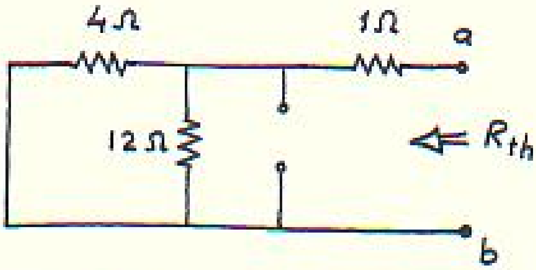
Example

Find the Thevenin's equivalent circuit of the circuit shown to the left of the terminals a - b. Then find the current through $R_L = 6, 16$, and $36\ \Omega$.



Solution:

⊛ $R_{th} = ?$:

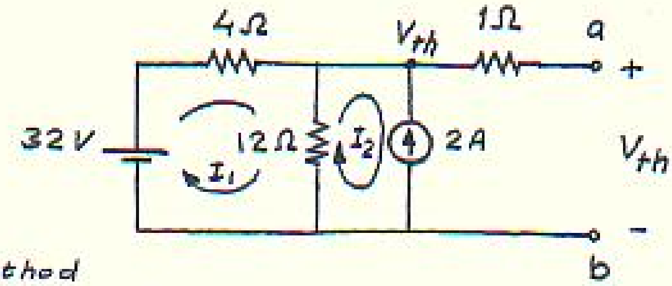


$$R_{th} = (4 // 12) + 1$$

$$= \frac{4 \times 12}{4 + 12} + 1 = \frac{48}{16} + 1 = \underline{4\ \Omega}$$

⊛ $V_{th} = ?$:

هنالك عدة طرق لاستخراج
قيمة V_{th} والطريقة (او الطرق)
المذكورة هنا ليست الوحيدة.



* using loop method

Loop 1: $32 = I_1(4+12) - I_2(12)$

Loop 2: $I_2 = -2\ A$

$$\therefore\ 32 = I_1(16) - (-2)(12)$$

$$\Rightarrow I_1 = \frac{32 - 24}{16} = \frac{8}{16} = 0.5 A$$

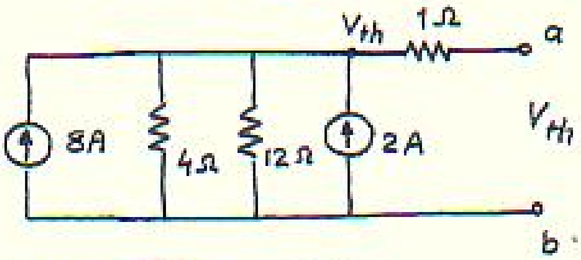
$$\therefore\ V_{th} = (I_1 - I_2)(12\Omega)$$
$$= (0.5 + 2)(12) = \underline{\underline{30\ V}}$$

→

T54

* من الممكن كذلك استخدام الـ (Nodal method)، حيث بعد تحويل مصدر الفولتية الى مصدر تيار نحصل على الدائرة الآتية :

∴ There is one node, which is V_{th}, then



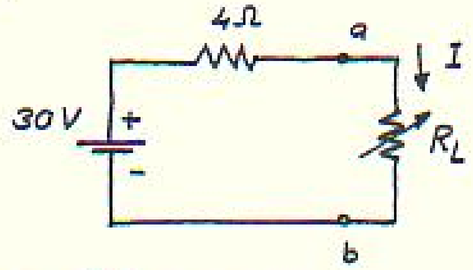
يلاحظ تم إهمال المقاومة (1Ω) لأنه لا يمر فيها تيار .

$$V_{th}\left(\frac{1}{4}+\frac{1}{12}\right) = 8+2$$

$$\therefore V_{th}(3+1) = 120$$

$$\Rightarrow V_{th} = \frac{120}{4} = 30\ V$$

∴ The Thevenin's equivalent cct. is :



$$\therefore I = \frac{V_{th}}{R_{th}+R_L}$$

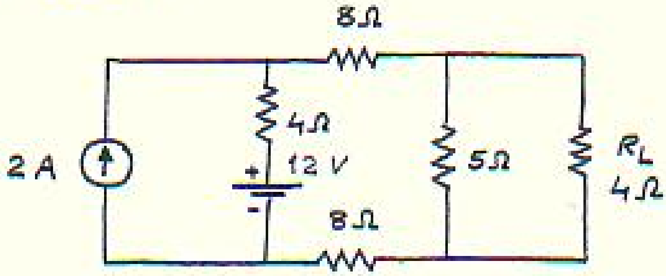
So;

When $R_L = 6\,\Omega$ $\Rightarrow I = \frac{30}{4+6} = \underline{3\ A}$

When $R_L = 16\,\Omega$ $\Rightarrow I = \frac{30}{4+16} = \underline{1.5\ A}$

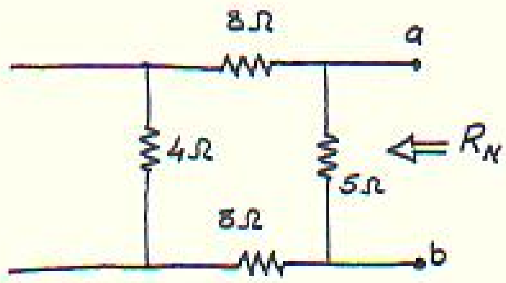
When $R_L = 36\,\Omega$ $\Rightarrow I = \frac{30}{4+36} = \underline{0.75\ A}$

__Example__ : Using Norton's theorem, find the current through R_L in the circuit shown.



__Solution__ :

* $\underline{R_N = ?}$



$$\therefore R_N = 5 // (8+4+8)$$

$$= \frac{5\times 20}{5+20} = \underline{\underline{4\,\Omega}}$$