Chapter 2

2-1. Pressure:

Pressure is defined as a *normal force exerted by fluid per unit area.* We speak of pressure only when we deal with a gas or liquid. The counterpart of pressure in solids is *normal stress*. Since pressure is defined as force per unit area, it has the unit of newtons per square meter (N/m^2), which is called a Pascal (Pa), that is:

$$1\ Pa = 1\ N/m^2$$

There are other pressure units commonly used in practice, especially in Europe, are *bar, standard atmosphere,* and *kilogram-force per square centimeter*:

$$1\ bar = 10^5\ Pa = 0.1\ M\ Pa = 100\ k\ Pa$$

$$1\ atm = 101{,}325\ Pa = 101.325\ kPa = 1.01325\ bars$$

$$1\ kg/cm^2 = 9.807\ N/cm^2 = 9.807 \times 10^4\ N/m^2 = 9.807 \times 10^4\ Pa$$

$$= 0.9807\ bar = 0.9679\ atm$$

The actual pressure at a given position is called the absolute pressure, and it is measured relative to absolute vacuum (i.e., absolute zero pressure). Most pressure-measuring devices, however, are calibrated to read zero in the atmospheric (Figure 2-1), and so they indicate the difference between the absolute pressure and the local atmospheric pressure. This difference is called the gage pressure. P_{gage} can be positive or negative, but pressures below atmospheric are sometimes called and are measured by by vacuum gages that indicate the difference between the atmospheric pressure and the absolute pressure. Absolute, gage, and vacuum pressures are related to each other by:

$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$

This is illustrated in Figure 2-2.

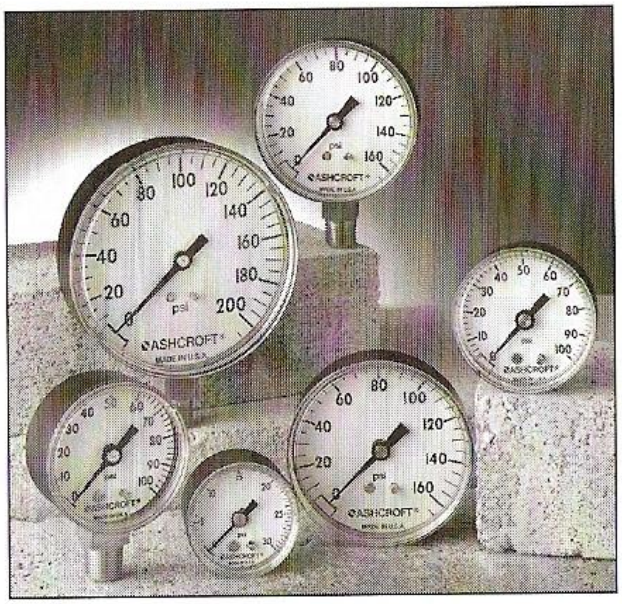


Figure 2-1. Some basic pressure gages.

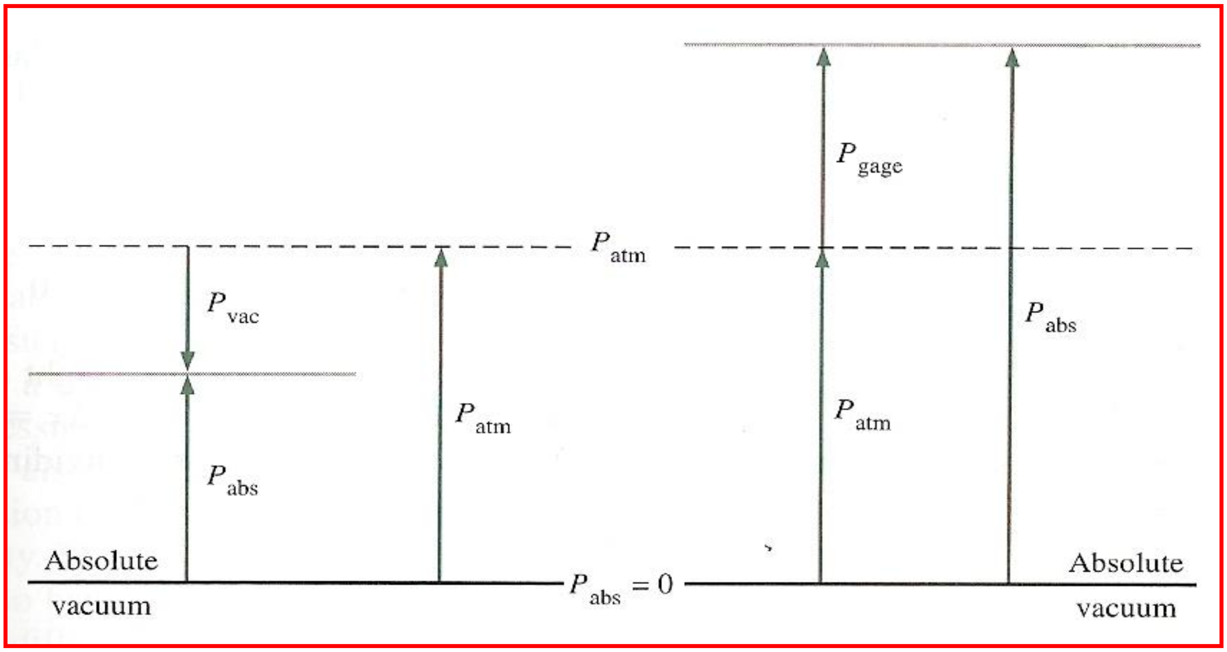


Figure 2-2. Absolute, gages, and vacuum pressures.

The gage used to measure the air in an automobile tire reads the gage pressure. Often the letters "a" (for absolute pressure) and "g" (for gage pressure) are added to pressure units (such as psia and psig).

Example 2-1:

A vacuum gage connected to a chamber reads 40 kPa at a location where atmospheric is 100 kPa. Determine the absolute pressure in the chamber.

Solution:

The gage pressure of an vacuum chamber is given. The absolute pressure in the chamfer is be determine.

***Analysis*: the absolute pressure is easily determined from Equation 2 to be:**

$P_{abs} = P_{atm} - P_{vac} = 100 - 40 = 60$ kPa

***Discussion*: Note that the *local* value of the atmospheric pressure is used when determining the absolute pressure.**

2-2. Pressure measurement Devices:

2-2-1. The Barometer:

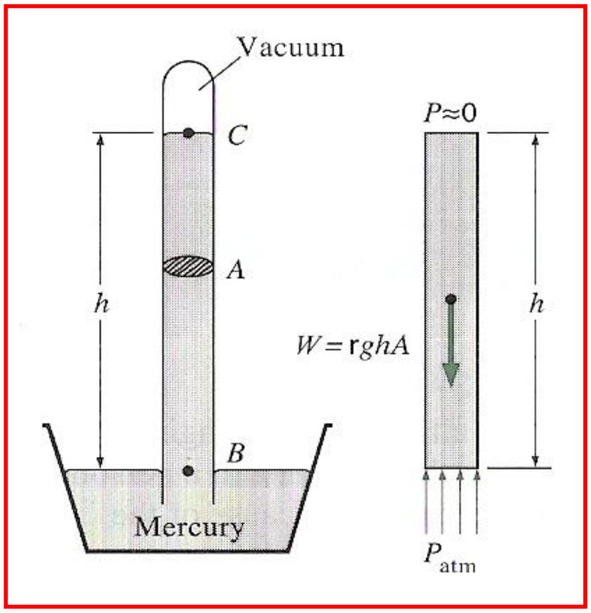
Atmospheric pressure is measured by a device called a barometer, thus, the atmospheric pressure is often referred to as the *barometric pressure.*

The Italian Evangelista Torricelli (1608-1647) was the first to conclusively prove that the atmospheric pressure can be measured by inverting a mercury-filled tube into a mercury container that is open to the atmosphere, as shown in Figure 2-3. The pressure at point B is equal to the atmospheric pressure, and the pressure at C can be taken to be zero since there is only mercury vapor above point C and the pressure is very low relative to P_{atm} and can be neglected to an excellent approximation. Writing a force balance in the vertical direction gives:

$$P_{atm} = \rho g h \qquad (2.1)$$

Where ρ is the density of mercury, g is the local gravitational acceleration, and h is the height of the mercury column above the free surface. Note that

the length and cross-sectional area of the tube have no effect on the height of the fluid column of a barometer (Figure 2-4).



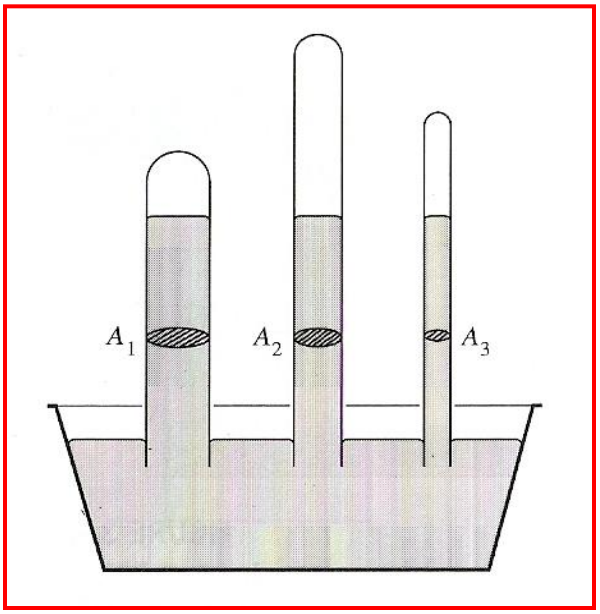


Figure 2-3. The basic barometer. **Figure 2-4.**

A frequently used pressure unit is the *standard atmospheric*, which is defined as the pressure produced by a column of mercury 760 mm in height at 0℃ (ρ_{Hg} = 13595 kg/m^3) under standard gravitational acceleration (g = 9.807 m/s^2). If water instead of mercury were used to measure the standard atmospheric pressure, a water column of about 10.3 m would be needed. Pressure is sometimes expressed (especially by weather forecasters) in terms of the height of the mercury column. The standard atmospheric pressure, for example, is 760 mmHg at 0℃. The unit mmHg is also called the torr in honor of Torricelli. Therefore, 1 atm = 760 torr and 1 torr = 133.3 Pa.

Example 2-2:

Determine the atmospheric pressure at a location where the barometric reading is 740 mm Hg and the gravitational acceleration is g = 9.805 m/s^2. Assume the temperature of mercury to be 10℃, at which its density is 13570 kg/m^3.

Solution:

Assumptions: **The temperature of mercury is assumed to be 10℃.**

Properties: **The density of mercury is given to be 13570 kg/m^3.**

Analysis: **The atmospheric pressure is determined to be:**

$$P_{atm} = \rho g h$$

$$= (13570\ \text{kg/m}^3)(9.805\ \text{m/s}^2)(0.740\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg.m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$

$$= 98.5\ \text{kPa}$$

2-2-2. The Manometer:

Consider the manometer shown in Figure 2-5 that is used to measured the pressure in the tank. Since the gravitational effects of gases are negligible, the pressure anywhere in the tank and position 1 has the same value. Furthermore, since pressure in a fluid does not vary in the horizontal direction within a fluid, the pressure at point 2 is the same as the pressure at point 1, $P_2 = P_1$.

The differential fluid column of height h is in static equilibrium, and it is open to the atmosphere. Then the pressure at point 2 is determined directly from the flowing equation:

$$P_2 = P_{atm} + \rho g h \qquad (2.2)$$

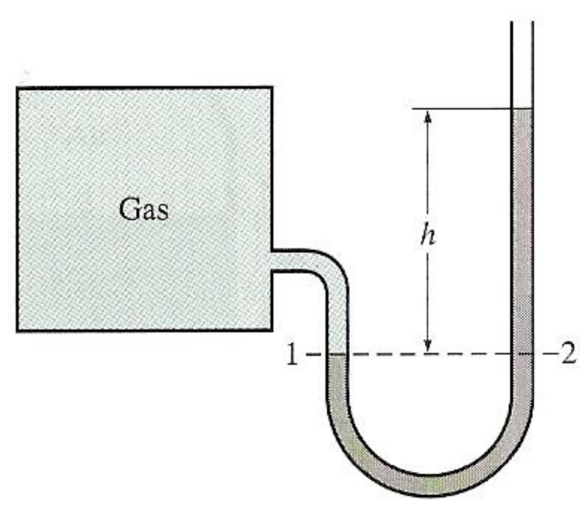


Figure 2-5. The basic manometer.

Example 2-3:

A manometer is used to measure the pressure of a gas in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm, as shown in Figure 2-6. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank?

Solution:

The reading of a manometer attached to a tank and the atmospheric pressure are given. The absolute pressure in the tank is to be determined.

Assumption: **the density of the gas in the tank is much lower than the density of the manometer fluid.**

Properties: **The specific gravity of the manometer fluid is given to be 0.85. We take the standard density of water to be 1000 kg/m^3.**

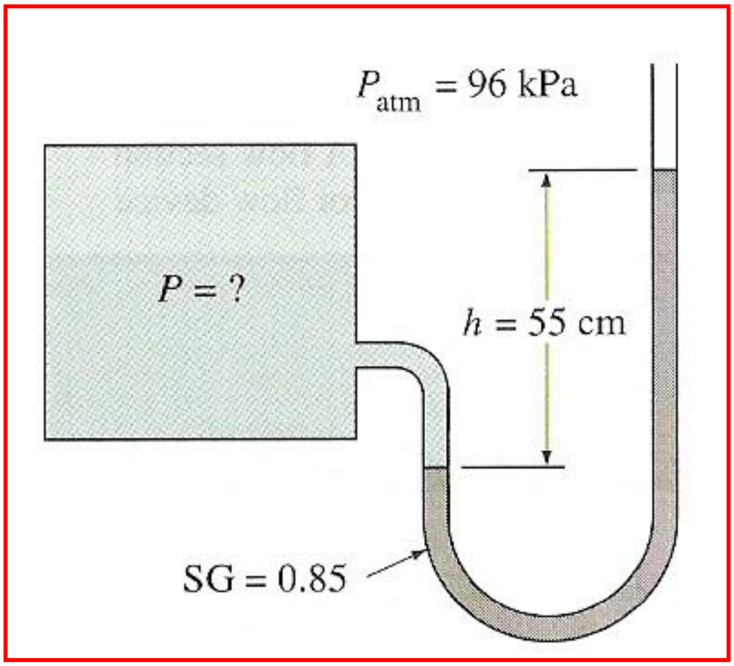


Figure 2-6. Schematic for Example 2-3.

Analysis: **The density of the fluid is obtained by multiplying its specific gravity by the density of water.**

ρ = SG (ρ_{H2O}) = (0.85)(1000 kg/m^3) = 850 kg/m^3

Then from Equation 2.2:

$$P = P_{atm} + \rho g h$$

$$= 96\ \text{kPa} + (850\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.55\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg.m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$

$$= 100.6\ \text{kPa}$$

Discussion: **Note that the gage pressure in the tank is 4.6 kPa**

The last principles, which is a result of *Pascal's law*, allows us to "jump" from one fluid column to next in manometers without worrying about pressure change as long as we stay in the same continuous fluid and the fluid is at rest. Then the pressure at any point can be determined by starting with a point of known pressure and adding or subtracting ρgh terms as we advance toward the point of interest. For example, the pressure at the bottom of the tank in Figure 2-7 can be determine by starting at the free surface where the pressure is P_{atm} moving downward until we reach point 1 at the bottom, and setting the result equal to P_1. It gives:

$$P_{atm} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = P_1$$

In the special case of all fluids having the same density, this relation reduces to $P_{atm} + \rho g\,(h_1 + h_2 + h_3) = P_1$.

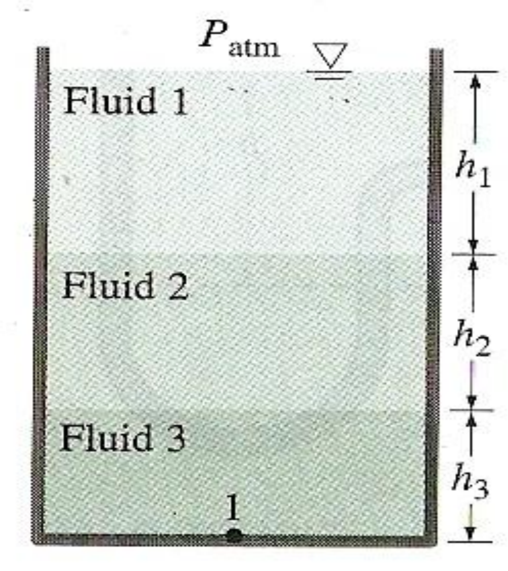


Figure 2-7.

Figure 2-7. In stacked-up fluid layers at rest, the pressure change across each fluid layer density and height h is ρgh.

Manometers are particularly well-suited to measure pressure drops across a horizontal flow section between two specified points due to the presence of a device such as a valve or heat exchanger or any resistance to flow. This is done by connecting the two legs of the manometer to these two points, as shown in Figure 2-8.

The working fluid can be either a gas or a liquid whose density is ρ_1. The density of the manometer fluid is ρ_2, and the differential fluid height is h. the two fluids must be immiscible, and ρ_2 must be greater than ρ_1. A relation for the pressure difference $P_1 - P_2$ can be obtained by starting at point 1 with P_1, moving along the tube by adding or subtracting the $\rho g h$ terms until we reach point 2, and setting the results equal to P_2:

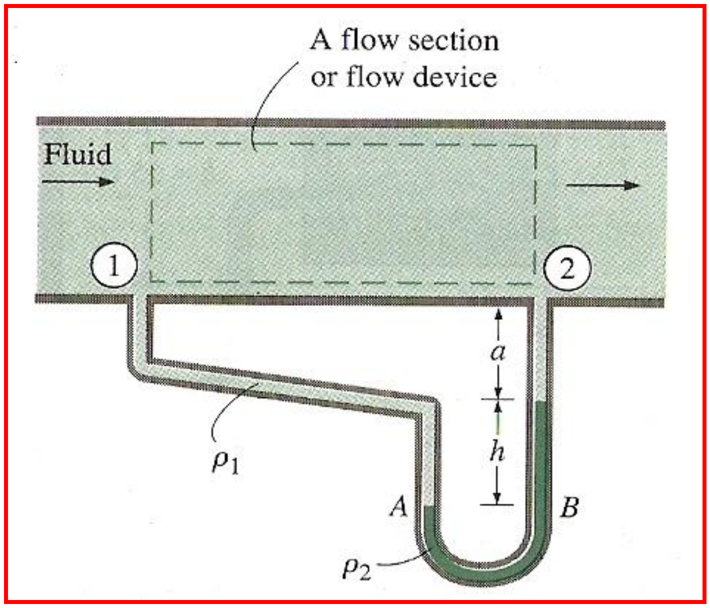


Figure 2-8.

$$P_1 + \rho_1 g\,(a + h) - \rho_2 g h - \rho_1 g a = P_2 \qquad (2.3)$$

Note that we jumped from point A horizontally to point B and ignored the part underneath since the pressure at both points is the same. Simplifying,

$$P_1 - P_2 = (\rho_2 - \rho_1)\, g h \qquad (2.4)$$

Note that the distance a must be included in the analysis even though it has no effect on the result. Also, when the fluid flowing in the pipe is a gas, then $\rho_1 \ll \rho_2$ and the relation in Equation 2.4 simplifies to $P_1 - P_2 \cong \rho_2 g h$.

2-3. Flow of incompressible fluid:

2-3-1. Fiction factor and pressure drop in pipes:

Consider steady, fully developed flow in a straight pipe of length L and internal diameter d_i. As shown in Figure 2-9 a force balance on a cylindrical element of the fluid can be written as:

$$A\,\Delta P_f = A_s\,\tau_w \qquad (2.5)$$

$$\frac{\pi}{4}\, d_i^2 \Delta P_f = \pi\, d_i L\, \tau_w$$

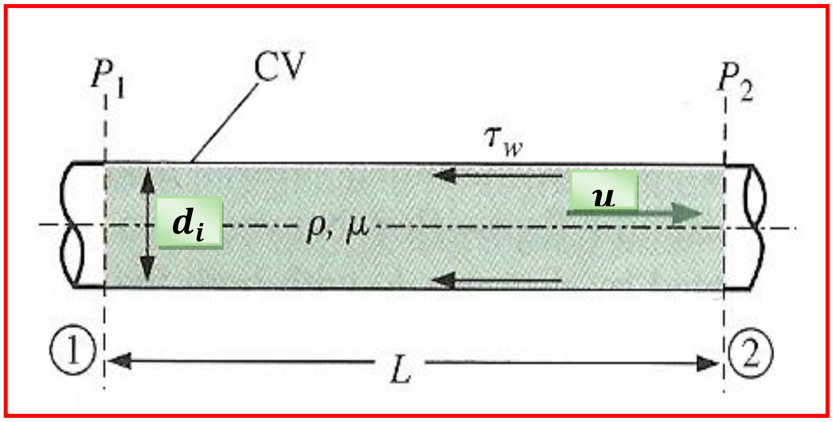


Figure 2-9.

$$\Delta P_f = 4\, \tau_w \frac{L}{d_i} \tag{2.6}$$

It was noted that the friction pressure drop for turbulent flow in a pipe varies as the square of the flow rate at very high values of Re. At lower values of Re the pressure drop varies with flow rate, and therefore with Re, to a slightly lower power which gradually increases to the value 2 as Re increases. The pressure drop in turbulent flow is also proportional to the density of the fluid. This suggests writing Equation 2.6 in the form:

$$\Delta P_f = 4 \frac{L}{d_i} \left(\frac{\tau_w}{\rho u^2} \right) \rho u^2 \tag{2.7}$$

In the range where ΔP_f varies exactly as u^2 the quantity $\tau_w/\rho u^2$ must be constant, while at lower values of Re the value of $\tau_w/\rho u^2$ will not quite be constant but will decrease slowly with increasing Re. Consequently, $\tau_w/\rho u^2$ is a useful quantity with which to correct pressure drop data. A slightly different form of Equation 2.7 is obtained by replacing the two occurrences of ρu^2 with $\frac{1}{2}\rho u^2$:

$$\Delta P_f = 4 \frac{L}{d_i} \left(\frac{\tau_w}{\frac{1}{2}\rho u^2} \right) \frac{1}{2}\rho u^2 \tag{2.8}$$

The quantity $\frac{1}{2}\rho u^2$ will be recognized as the kinetic energy per unit volume of the fluid.

The term $\tau_w/(\frac{1}{2}\rho u^2)$ in Equation 2.7 defines a quantity known as the Fanning friction factor f, thus:

$$f = \frac{\tau_w}{\frac{1}{2}\rho u^2} \qquad (2.9)$$

It will be appreciated that the factor of $\frac{1}{2}$ in Equation 2.9 is arbitrary and various other friction factors are in use. For example, in the first the basic friction factor denoted by j_f was used. This is defined by:

$$j_f = \frac{\tau_w}{\rho u^2} \qquad (2.10)$$

Thus

$$j_f = f/2 \qquad (2.11)$$

When using j_f, the pressure drop is given by Equation 2.7. using the fanning friction factor, which is defined by Equation 2.10, Equation 2.8 may be written as:

$$\Delta P_f = 4f\left(\frac{L}{d_i}\right)\frac{\rho u^2}{2} = \frac{2fL\rho u^2}{d_i} \qquad (2.12)$$

This is the basic equation from which the frictional pressure drop may be calculated. It is valid for all types of fluid and for both laminar and turbulent flow. However, the value of f to be used does depend on these conditions. For laminar flow of a Newtonian fluid in pipe, the Fanning factor is given by:

$$f = \frac{16}{Re} \qquad (2.13)$$

For turbulent flow of a Newtonian fluid, f decreases gradually with Re, which must be the case in view of the fact that the pressure drop varies with flow rate to a power slightly lower than 2.0. It is also found with turbulent flow that the value of f depends on the relative roughness of the pipe wall. The relative roughness is equal to e/d_i where e is the absolute

roughness and d_i the internal diameter of the pipe. Values of absolute roughness for various kinds of pipes and ducts are given in Table 2.1.

Table 2.1.

Material	*Absolute roughness e (in m)*
Drawn tubing	0.0000015
Commercial steel and wrought iron	0.000045
Asphalted cast iron	0.00012
Galvanized iron	0.00015
Cast iron	0.00026
Wood stave	0.00018–0.0009
Concrete	0.00030–0.0030
Riveted steel	0.0009–0.009

Values of the friction factor are traditionally presented on a friction factor chart such as that shown in Figure 2-10. It will be noted that the greater the relative roughness, the higher the value of f for a given value of Re. At higher values of Re, the friction factor becomes independent of Re; this is true for the region of the chart above and to the right of the broken line. In the region of transition between laminar and turbulent flow, the flow is rather unpredicted and caution should be exercised in relying on the values of f used.

Considerable effort has been expended in trying to find algebraic expression to relate f to Re and e/d_i. For turbulent flow in smooth pipes, the simplest expression is the Blasius equation:

$$f = 0.079\ Re^{-0.25} \qquad (2.14)$$

The equation is valid for the range of Re from 3000 to 1×10^5 .

Similarly, the Drew equation:

$$f = 0.00140 + 0.125\ Re^{-32} \qquad (2.15)$$

Is good for Re from 3000 to at least 3 x 10^6.

The most widely accepted relationship for turbulent flow in smooth pipes is the von Karman equation:

$$\frac{1}{f^{1/2}} = 4.0 \log (f^{1/2}\, Re) - 0.40 \qquad (2.16)$$

This equation is very accurate but has disadvantage of being implicit in f.

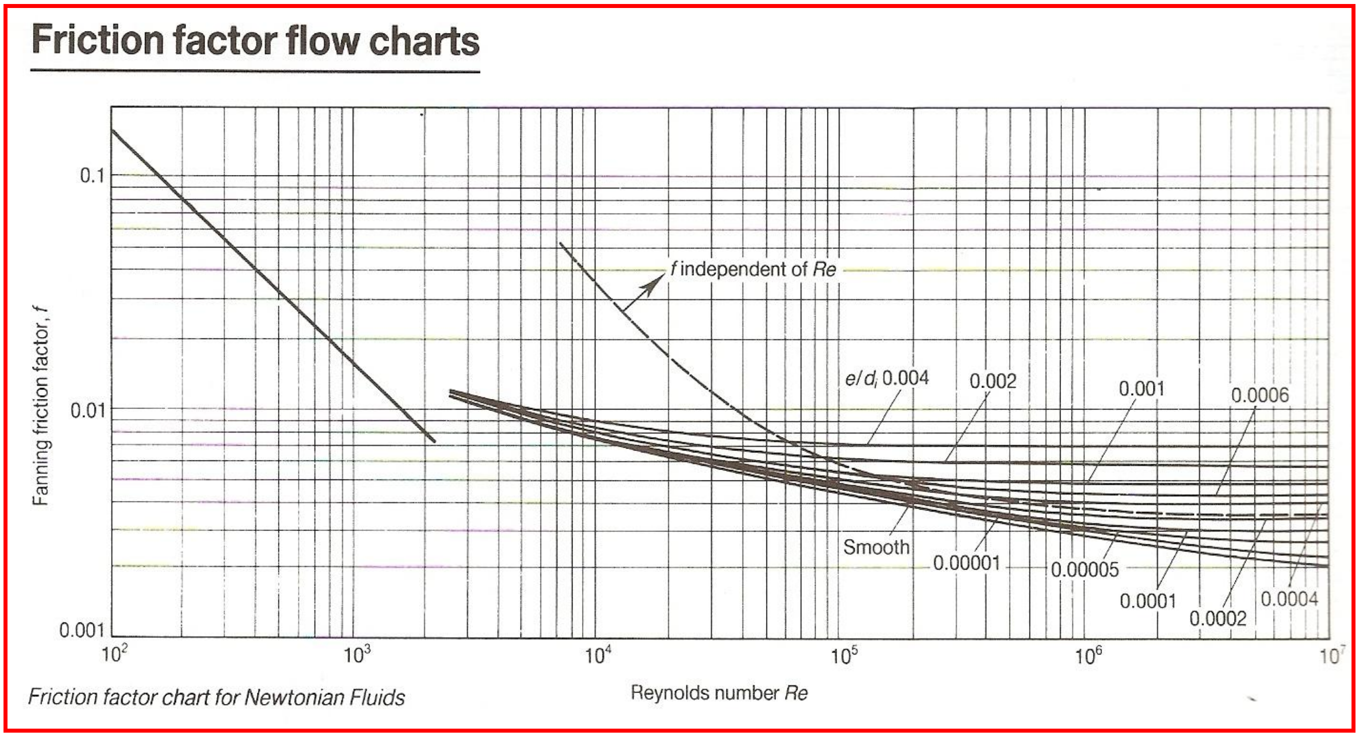


Figure 2-10.

Example 2-4:

Calculate the frictional pressure drop for a commercial steel pipe with the following characteristics:

Length L = 30.48 m

Inside diameter d_I = 0.0526 m

Pipe roughness e = 0.000045 m

Steady liquid flow rate Q = 9.085 m^3/h

Liquid dynamic viscosity μ = 0.01 Pa s

Liquid density ρ = 1200 kg/m^3

Solution:

$$\text{Mean velocity } u = \frac{Q}{\pi d_i^2/4}$$

From the given values:

$$\frac{\pi d_i^2}{4} = \frac{(3.142)(0.0526\ \text{m})^2}{4} = 0.002173\ \text{m}^2$$

$$Q = \frac{9.085\ \text{m}^3/\text{h}}{3600\ \text{s/h}} = 0.002524\ \text{m}^3/\text{s}$$

Therefore

$$u = \frac{0.002\,524\ \text{m}^3/\text{s}}{0.002173\ \text{m}^2} = 1.160\ \text{m/s}$$

The Reynolds number is given by:

$$Re = \frac{\rho u d_i}{\mu}$$

Substituting the given values:

$$Re = \frac{\left(1200\frac{\text{kg}}{\text{m}^3}\right)\left(1.160\frac{\text{m}}{\text{s}}\right)(0.0526\ \text{m})}{0.01\text{Pa.s}} = 7322$$

Relative roughness is given by:

$$\frac{e}{d_i} = \frac{0.000045\ \text{m}}{0.0526\ \text{m}} = 0.000856$$

From the graph of f against Re in Figure 2-10, f = 0.0084 for Re = 7322 and e/d_i= 0.000856.

The frictional pressure drop is given by:

$$\Delta P_f = 4f\left(\frac{L}{d_i}\right)\frac{\rho u^2}{2}$$

From the given values:

$$\left(\frac{L}{d_i}\right) = \frac{30.48\ m}{0.0526\ m} = 579.5$$

And

$$\frac{\rho u^2}{2} = \frac{\left(1200\frac{kg}{m^3}\right)(1.160\frac{m}{s})^2}{2} = 807.4\ \text{N/m}^2$$

Therefore

$$\Delta P_f = 4\ (0.0084)(579.5)(807.4\ \text{N/m}^2)$$

$$= 15720\ \text{N/m}^2$$

2-4. Pumps:

Pumps are devices for supplying energy or head to a flowing liquid in order to overcome head losses due to friction and also, if necessary, to raise the liquid to a higher level. In general, pumps added energy to the fluid-they do work on the fluid; turbines extract energy from the fluid-the fluid does work on them as shown in Figure 2-11. The term "pump" will be used to generically refer to all pumping machines, including *pumps*, *fans*, *blowers*, and *compressors*. Fluid machines can be divided into two main categories: *positive displacement machines* (denoted as the static type) and *turbomachines* (denoted as the dynamic type).

Some fundamental parameters are used to analyzed the performance of a pump. The mass flow rate $\dot{m}$ of fluid through the pump is an obvious primary pump performance parameter. For incompressible flow, it is more common to use volume flow rate $\dot{V}$ rather than mass flow rate. In the turbomachinery industry, volume flow rate is called capacity and is simply mass flow rate divided by fluid density,

***Volume flow rate* (*capacity*):** $\dot{V} = \frac{\dot{m}}{\rho}$ **(2.17)**

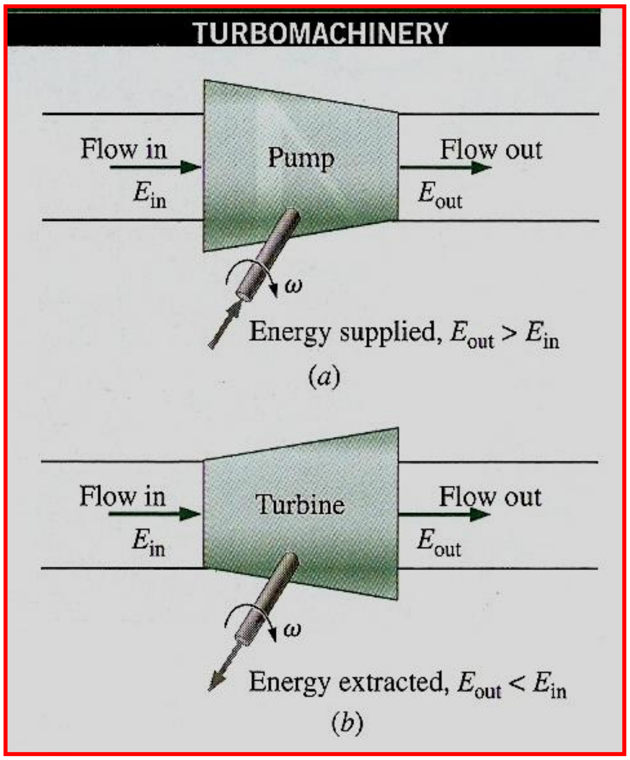


Figure 2-11.

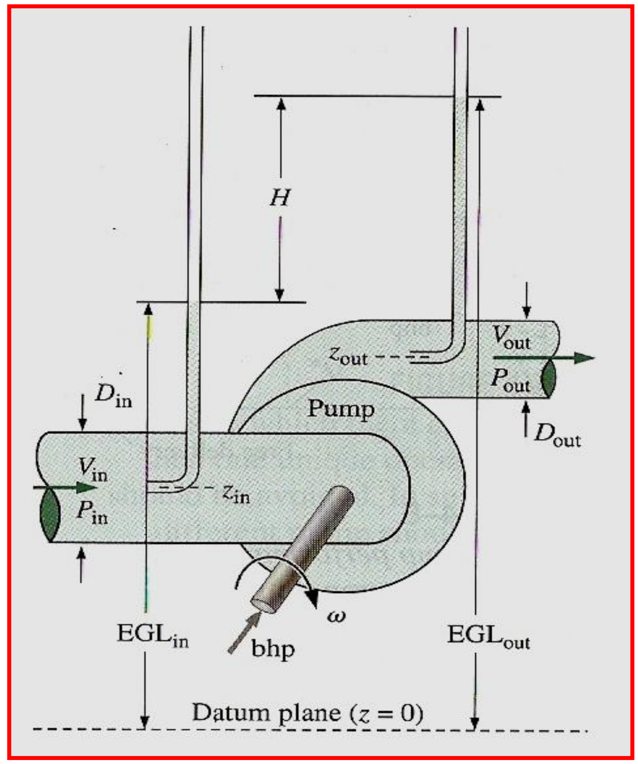


Figure 2-12.

The performance of a pump is characterized additionally by its net head H, defined as the change Bernoulli head between the inlet and the outlet of the pump,

***Net head*:** $$H = \left(\frac{P}{\rho g} + \frac{V^2}{2g} + z\right)_{out} - \left(\frac{P}{\rho g} + \frac{V^2}{2g} + z\right)_{in} \qquad (2.18)$$

The dimension of net head is length, and it is often listed as an equivalent column height or water, even for a pump that is not pumping water.

For the case in which a *liquid* is being pumped, the Bernoulli head at the inlet equivalent to the energy grade line at the inlet, EGL_{in} obtained by aligning a Pitot probe in the center of the flow as illustrated in Figure 2-12. The energy grade line at the outlet EGL_{out} is obtained in same manner, as also illustrated in the figure. In the general case, the outlet of the pump may be at a different elevation than the inlet, and its diameter and average

speed may not be the same as those at the inlet. Regardless of these differences, net head H is equal to the difference between EGL_{out} and EGL_{in},

Net head for a liquid pump: $H = \mathrm{EGL}_{out} - \mathrm{EGL}_{in}$

Consider the special case of **incompressible flow** through a pump in which the inlet and outlet diameters are identical, and there is no change in elevation. **Equation 2.18** reduces to:

Special case with $D_{out} = D_{in}$ and $z_{out} = z_{in}$: $$H = \frac{P_{out} - P_{in}}{\rho g}$$

For this simplified case, net heat is simply the pressure rise across the pump expressed as a head (column height of the fluid).

Net head is proportional to the useful power actually delivered to the fluid. It is traditional to call this power the water horsepower, even if the fluid being pumped is not water, and even if the power is not measured in units of horsepower. By dimensional reasoning, we must multiply the net head of **Equation 2.18** by mass flow rate and gravitational acceleration to obtain dimensions of power. Thus,

Water horsepower: $$\dot{W}_{\text{water horsepower}} = \dot{m}gH = \rho g \dot{V} H \quad (2.19)$$

All pumps suffer from irreversible losses due to friction, internal leakage, flow separation on blade surfaces, turbulent dissipation, etc. Therefore, the mechanical energy supplied to the pump must be ***larger*** than $\dot{W}_{\text{water horsepower}}$. In pump terminology, the external power supplied to the pump is called the **brake horsepower**, which we abbreviate as **bhp**.

Brake horsepower: $$\text{bhp} = \dot{W}_{\text{shaft}} = \omega T_{\text{shat}} \quad (2.20)$$

where ω is the **rotational speed** of the shaft (**rad/s**) and T_{shat} is the **torque** supplied to the shaft. We define pump efficiency η_{pump} as the **ratio** of **useful power** to **supplied power**:

pump efficiency: $$\eta_{\text{pump}} = \frac{\dot{W}\text{water horsepower}}{\dot{W}\text{shaft}} = \frac{\dot{W}\text{water horsepower}}{\text{bhp}}$$

$$\eta_{pump} = \frac{\rho g \dot{V} H}{\omega T_{shaft}} \quad (2.21)$$

2-4-1. Pump performance curves and matching a pump to a piping system:

It is important to realize that *for steady conditions, a pump can operate only along its performance curve*. Thus, the operating point of a piping system is determined by matching system requirements (*required* net head) to pump performance (*available* net head). In a typical application, $H_{required}$ and $H_{available}$ mach at one unique value of flow rate-this is the operating point or duty point of the system.

For a given piping system with its major and minor losses, elevation changes, etc., the required net head *increases* with volume flow rate. On the other hand, the available net head of most pumps *decreases* with flow rate, as in Figure 2-13, at least over the majority of its recommended operating range. Hence, the system curve and the pump performance curve intersect as sketched in Figure 2-14, and this establishes the operating point. If we are lucky, the operating point is at or near the best efficiency point of the pump.

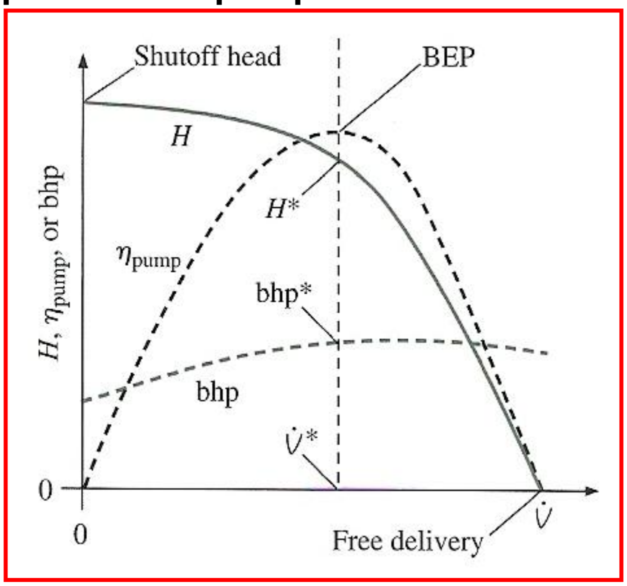


Figure 2-13.

Typical *pump performance curves* for centrifugal pump.

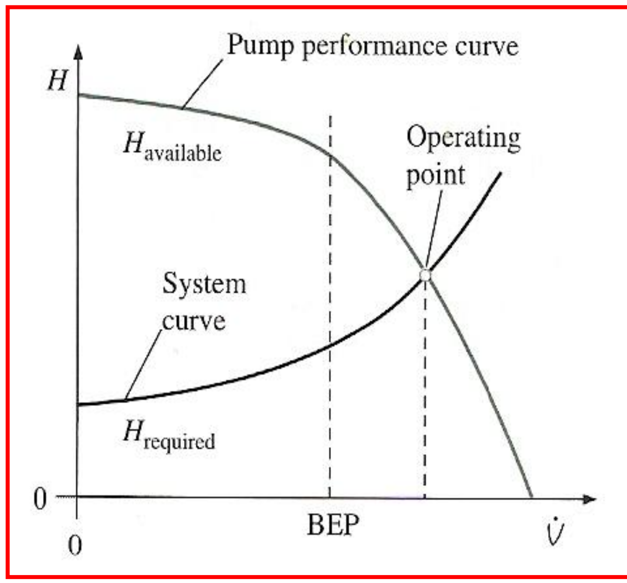


Figure 2-14.

The *operating point of a piping* system.

2-4-2. Pump cavitation and net positive suction head:

When pumping liquids, it is possible for the local pressure, P inside the pump to fall below the vapor pressure, P_v of the liquid (P_v is also called the saturation pressure P_{sat} and is listed in thermodynamics tables as a function of saturation temperature.) when $P < P_v$, vapor-filled bubbles called cavitation bubbles appear. In other words, the liquid *boils* locally, typically on the surface side of the rotating impeller blades where the pressure is lowest (Figure 2-15). It is this *collapse* of the bubbles that is undesirable, since it causes noise, vibration, reduced efficiency, and most importantly, damage to the impeller blades. To avoid cavitation, we must ensure that the local pressure everywhere inside the pump stays *above* the vapor pressure. Since pressure is most easily measured (or estimated) at the inlet of the pump, cavitation criteria are typically specified *at the pump inlet*. It is useful to employ a flow parameter called net positive suction head (NPSH), defined as *the difference between the pump's inlet stagnation pressure head and the vapor pressure head*:

***Net positive suction head*:**

$$\text{NPSH} = \left(\frac{P}{\rho g} + \frac{V^2}{2g}\right)_{\text{pump inlet}} - \frac{P_v}{\rho g} \qquad (2.22)$$

The required net positive suction head ($\text{NPSH}_{required}$), defined as the *minimum $NPSH$ necessary to avoid cavitation in the pump.* The measured value of $\text{NPSH}_{required}$ varies with volume flow rate, and therefore $\text{NPSH}_{required}$ is often plotted on the same pump performance curve as head (Figure 2-16). Since irreversible head losses through the piping system upstream of the inlet *increase* with flow rate, the pump inlet stagnation pressure head *decreases* with flow rate. Therefore, the value of NPSH *decreases* with $\dot{V}$, as sketched in Figure 2-17. By indentifying the volume flow rate at which the curves of actual NPSH and $\text{NPSH}_{required}$ intersect, we estimate the maximum volume flow rate that can be delivered by the pump without cavitation (Figure 2-17).

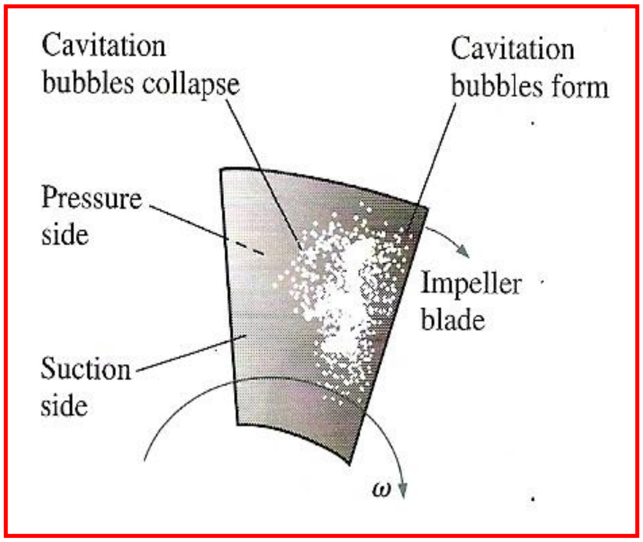


Figure 2-15.

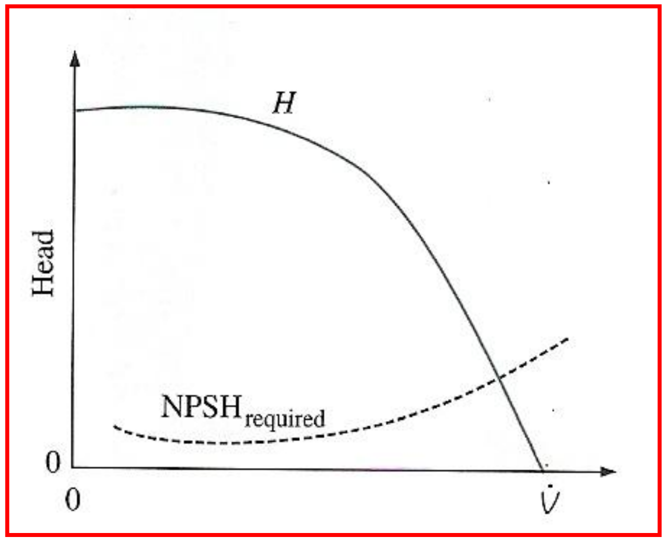


Figure 2-16.

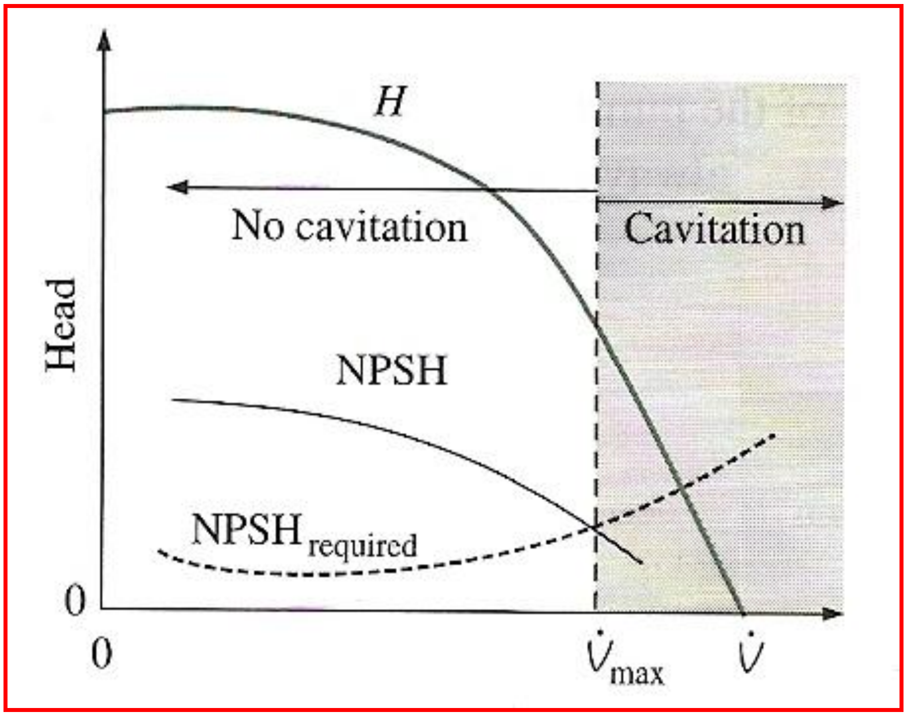


Figure 2-17.

2-4-3. Pumps in series and parallel:

When faced with the need to increase volume flow rate or pressure rise by a small amount, you might consider adding an additional smaller pump in series or in parallel with the original

pump. While series or parallel arrangement is acceptable for some applications, arranging *dissimilar* pumps in series or in parallel may lead to problems, especially if one pump is much larger than the other (Figure2-18).

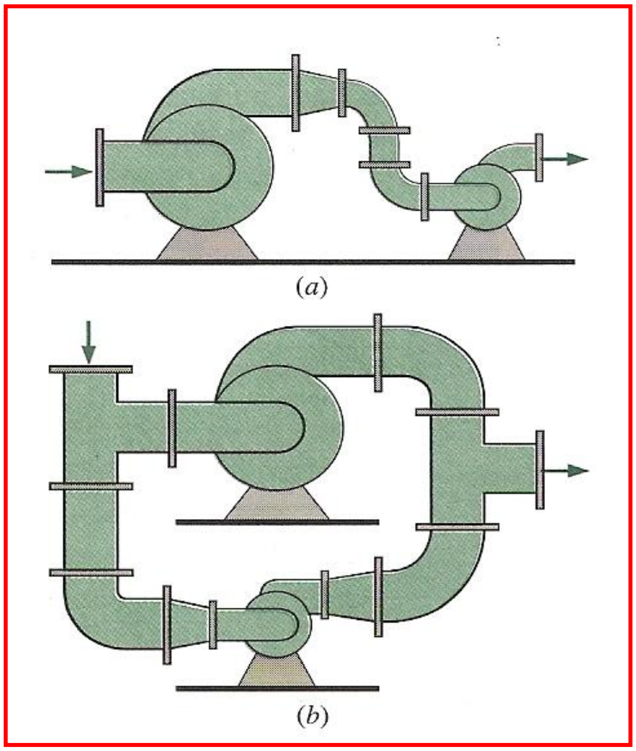
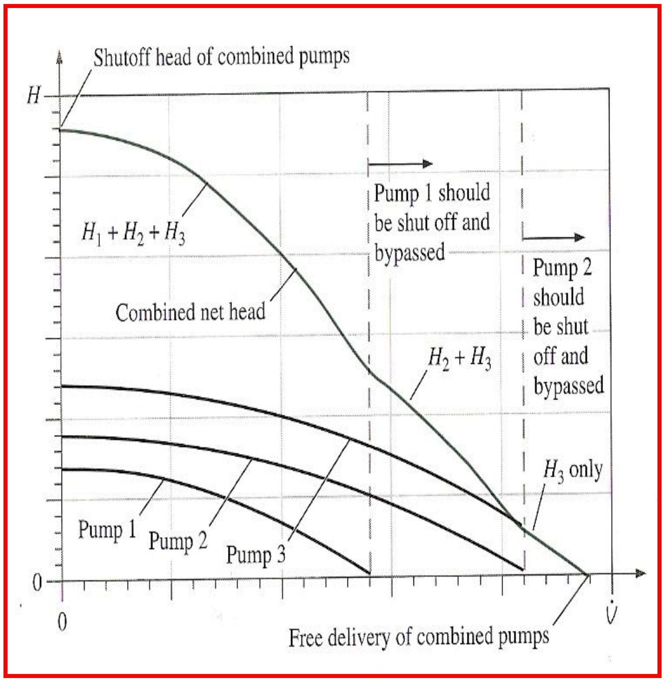


Figure 2-18.



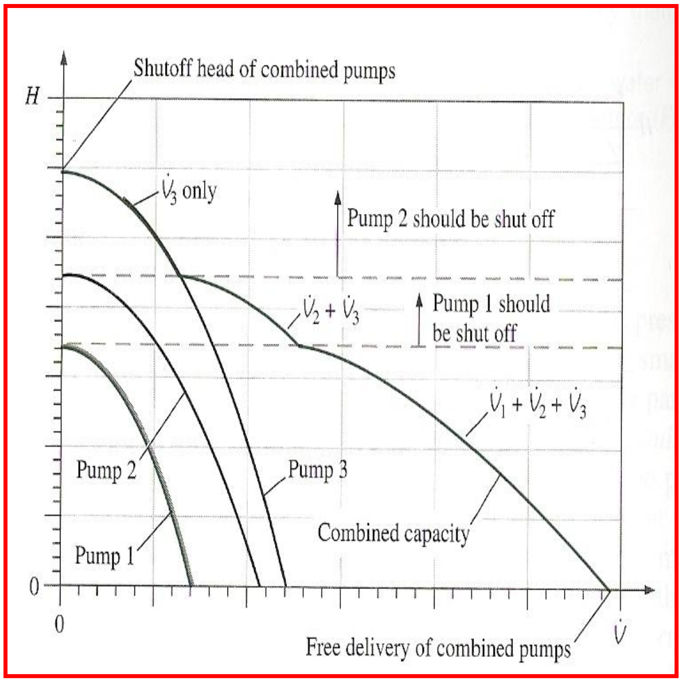


Figure 2-19.

Figure 2-20.

There are many applications where two or more similar (usually identical) pumps are operated in series or in parallel. When operated in *series,* the combined net head is simply the sum of the net heads of each pump (at a given volume flow rate):

***Combined net head for n pumps in series*: $H_{\text{combined}} = \sum_{i=1}^{n} H_i$ (2.23)**

Equation 2.23 is illustrated in Figure 2-19.

When two or more identical (or similar) pumps are operated in *parallel,* their individual volume flow rates (rather than net heads) are summed:

***Combined capacity for n pumps in parallel*: $\dot{V}_{\text{combined}} = \sum_{i=1}^{n} \dot{V}_i$ (2.24)**

As an example, consider the *same* three pumps, but arranged in *parallel,* than in series. The combined pump performance curve is shown in Figure 2-20.

2-5. Pumps classifications:

2-5-1. Positive-displacement pumps:

Positive-displacement pumps are ideal for high-pressure applications like pumping viscous liquids or thick slurries and for applications where precise amounts of liquid are to be dispensed or metered, as in medical applications. Examples of positive-displacement pumps as shown in Figure 2-21: (*a*) flexible-tube peristaltic pump, (*b*) three-lobe rotary pump, (*c*) gear pump, and (*d*) double screw pump.

In Figure 2-22 four phases (one-eighth of a turn apart) in the operation of a two-lobe rotary pump, a type of positive-displacement pump. The light blue region represents a chunk of fluid pushed through the top rotor, while the dark blue region represents a chunk of fluid pushed through the bottom rotor, which rotates in the opposite direction. Flow is left to right.

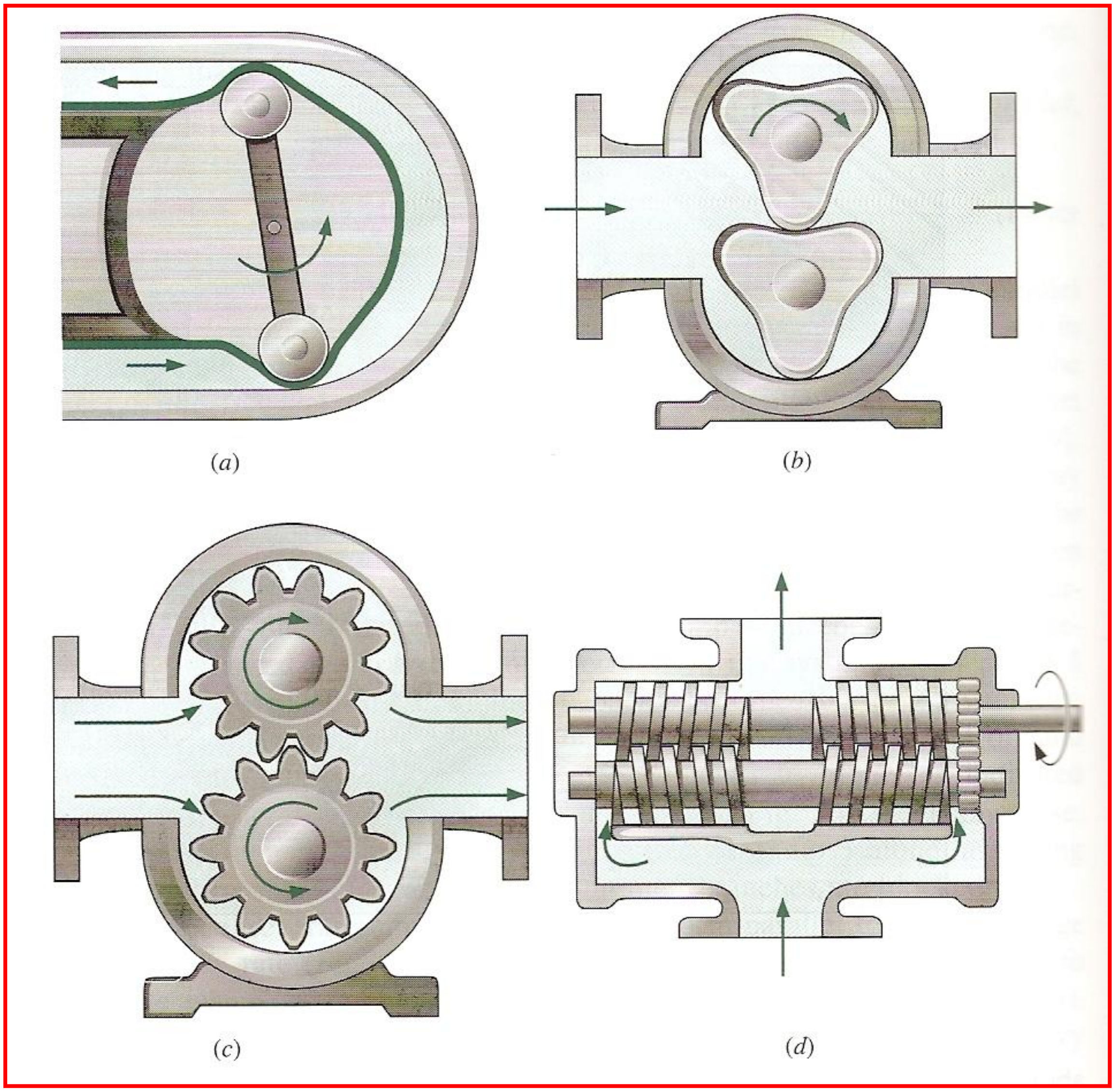


Figure 2-21.

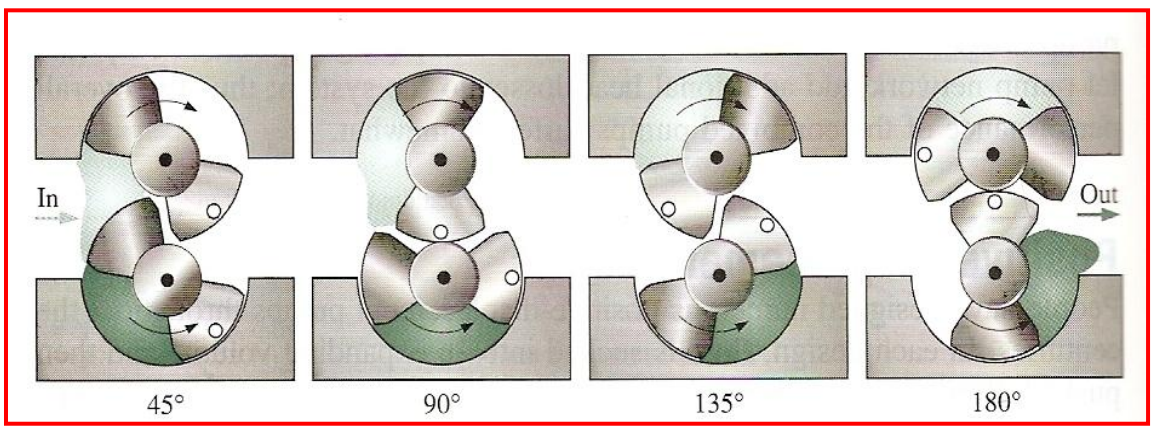


Figure 2-22.

Analysis of positive-displacement pumps is fairly straightforward. From the geometry of the pump, we calculate the closed volume (V_{closed}) that is filled (and expelled) for every n rotations of the shaft. Volume flow rate is the equal to rotation rate $\dot{n}$ times V_{closed} divided by n.

Volume flow rate positive-displacement pump: $$\dot{V} = \dot{n}\frac{V_{closed}}{n} \qquad (2.25)$$

2 -5-2. Dynamic Pumps:

There are three types of *dynamic pumps* that involve rotating blades called impeller blades or rotor blades, which impart momentum to fluid. For this reason they are sometimes called rotodynamic pumps or simply rotary pumps (not to be confused with rotary positive-displacement pumps, which use the same name). Rotary pumps are classified by the manner in which flow exits the pump as shown in Figure 2-23: (*a*) centrifugal pumps (radial-flow pumps), (*b*) mixed flow, and (*c*) axial flow. In an axial-flow pump, fluid enters and leaves axially, typically along the outer portion of the pump because of blockage by shaft, motor, hub, etc. A mixed –flow pump is intermediate between centrifugal and axial, with the flow entering axially, not necessary in the center, but leaving at some angle between radially and axially.

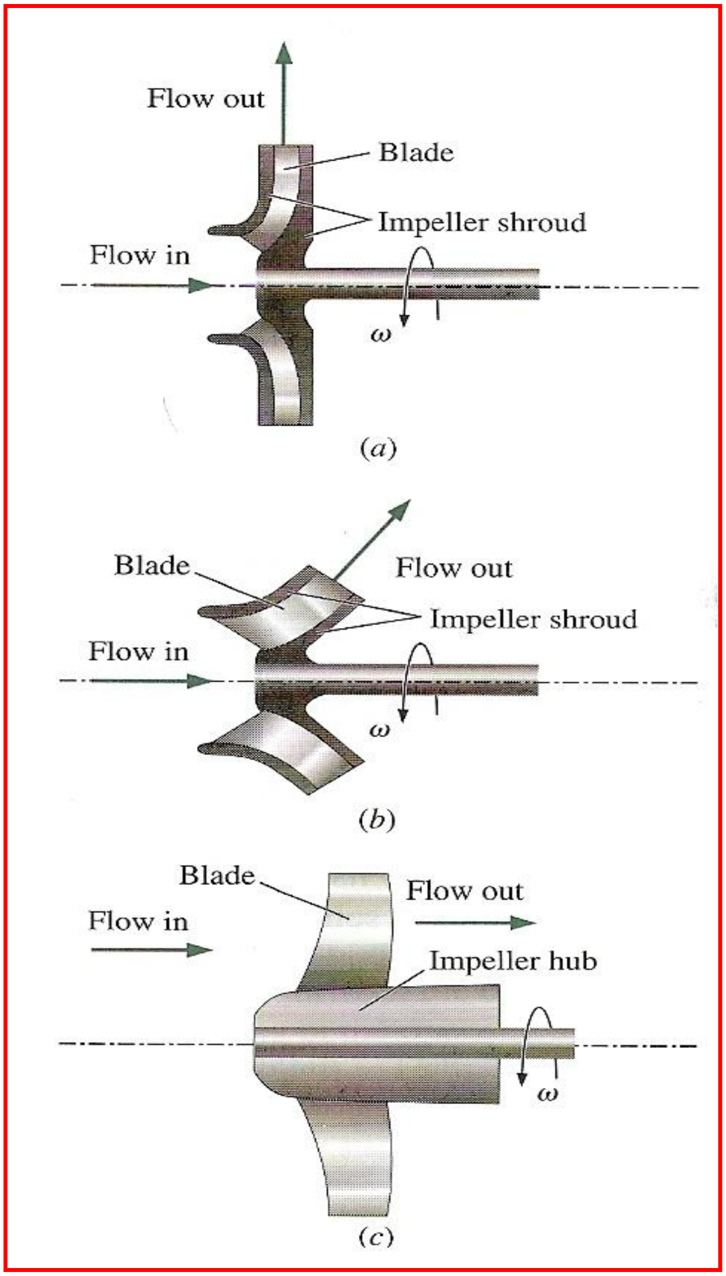


Figure 2-23.

2 -5-3. Centrifugal Pumps:

There are three types of centrifugal pump that based on the impeller blade geometry, as sketched in Figure 2-24: (*a*) backward-inclined blades, (*b*) radial blades, and (*c*) forward-inclined blades. Net head and brake horsepower performance curves for these three types of centrifugal pump are compared in Figure 2-24*d*. The curves have been adjusted such that each pump achieves the same free delivery (maximum volume flow rate at zero net head). Note that these are qualitative sketches for comparison purposes only-actual measured performance curves may differ significantly in shape, depending on details of the pump design. Centrifugal pumps with backward-inclined blades (Figure 2-24*a*) are the most common. These yield the highest efficiency of the three because fluid flows into and out of the blade passages with the least amount of turning. Sometimes the blades are airfoil shaped, yielding similar performance but even higher efficiency. The pressure rise is intermediate between the other two types of centrifugal pumps. Centrifugal pumps with radial blades (also called straight blades, Figure 2-24*b*) have the simplest geometry and produce the pressure rise of the three for a wide range of volume flow rate.

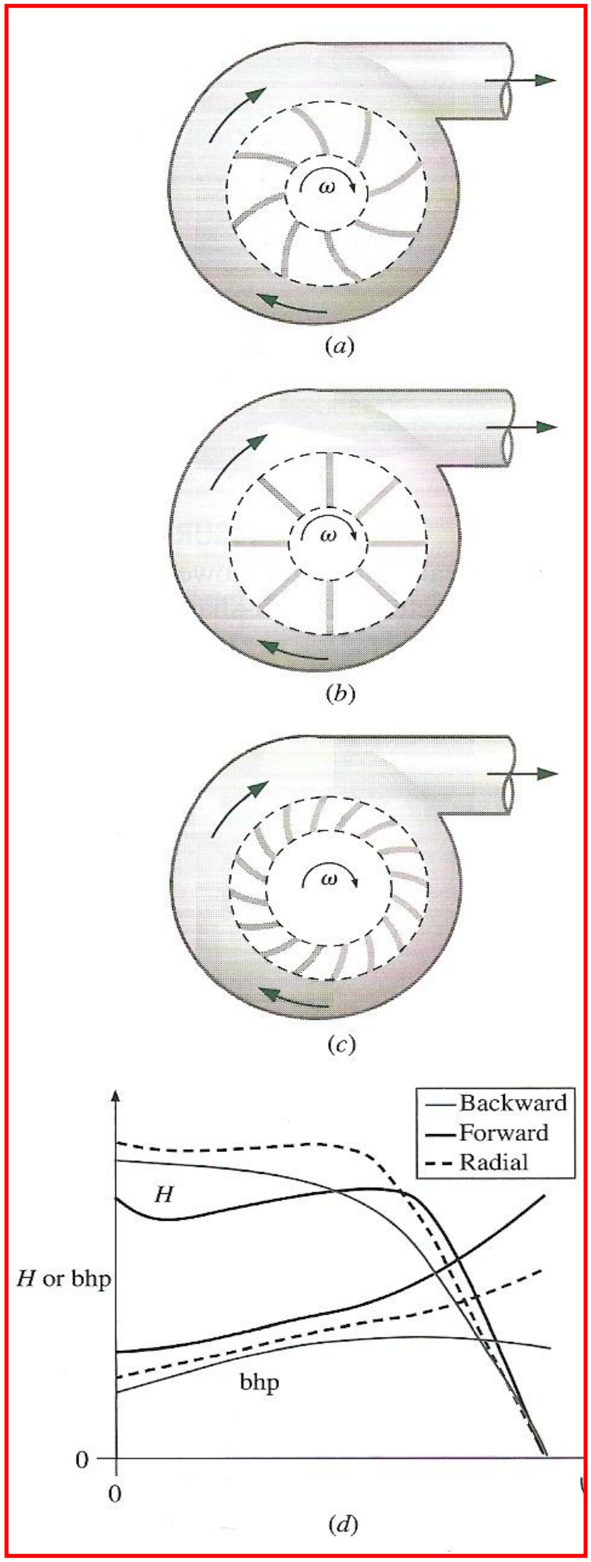


Figure 2-24.

Examples-Chapter 2

2-1 Barometer:

Example 2-1

Intravenous infusions usually are driven by gravity by hanging the fluid bottle at sufficient height to counteract the blood pressure in the vein and to force the fluid into body (Figure 2-1). The higher the bottle is raised, the higher the flow rate of the fluid will be. (a) it is observed that the fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, determine the gage pressure of the blood. (b) if the gage pressure of the fluid at the arm level needs to be 20 kPa for sufficient flow rate, determine how high the bottle must be placed. Take the density of the fluid to be 1020 kg/m^3.

solusion:

Assumption: **1. The fluid is incompressible. 2. The IV bottle is open to the atmosphere.**

Properties: **the density of the IV fluid is given to be ρ = 1020 kg/m^3.**

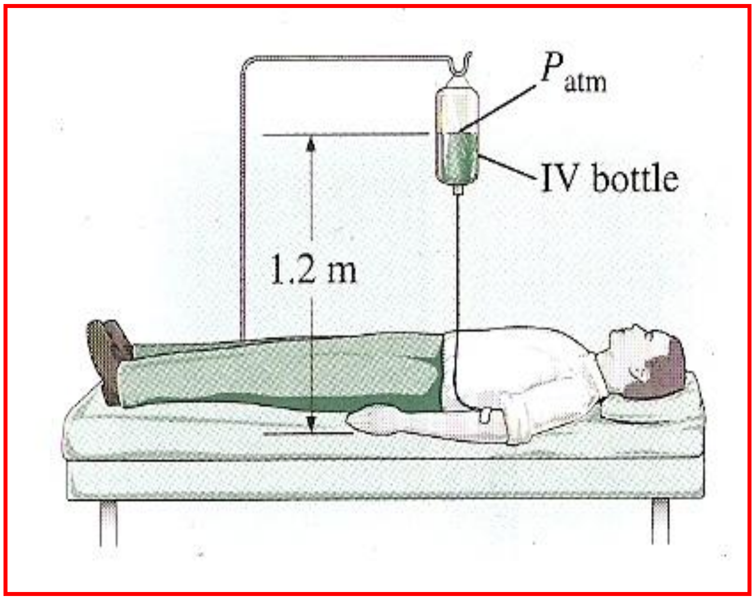


Figure 2-1.

(a) Noting that the IV fluid and the blood pressures balance each other when the bottle is 1.2 m above the arm level, the gage pressure of the blood in the arm is simply equal to gage pressure of the IV at a depth of 1.2 m:

$$P_{\text{gage, arm}} = P_{\text{abs}} - P_{\text{atm}} = \rho g h_{\text{atm-bottol}}$$

$$= (1020\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.2\ \text{m})\left(\frac{1\text{kN}}{1000\ \text{kg.m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)$$

$$= 12.0\ \text{kPa}$$

(b) to provide a gage pressure of 20 kPa at the arm level, the height of the surface of the IV fluid in the bottle from the arm level is again determined from $P_{\text{gage, arm}} = \rho g h_{\text{atm-bottol}}$ to be:

$$h_{\text{arm-bottle}} = \frac{P_{\text{gage,arm}}}{\rho g} = \frac{20 \text{ kPa}}{\left(1020 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} \left(\frac{1000 \text{ kg.m/s}^2}{1 \text{ kN}}\right)\left(\frac{1 \text{ kN/m}^2}{1 \text{ kPa}}\right)$$

$$= 2.0 \text{ m}$$

2-2 Manometer:

Example 2-2

A closed tank contains compressed air and oil (SG_{oil} = 0.90) as is shown in Figure 2-2. A U-tube manometer using mercury (SG_{Hg} = 13.6) is connected to the tank as shown. The column heights are h_1 = 36 in, h_2 = 6 in, and h_3 = 9 in. Determine the pressure reading (in psi) of the gage.

Solution:

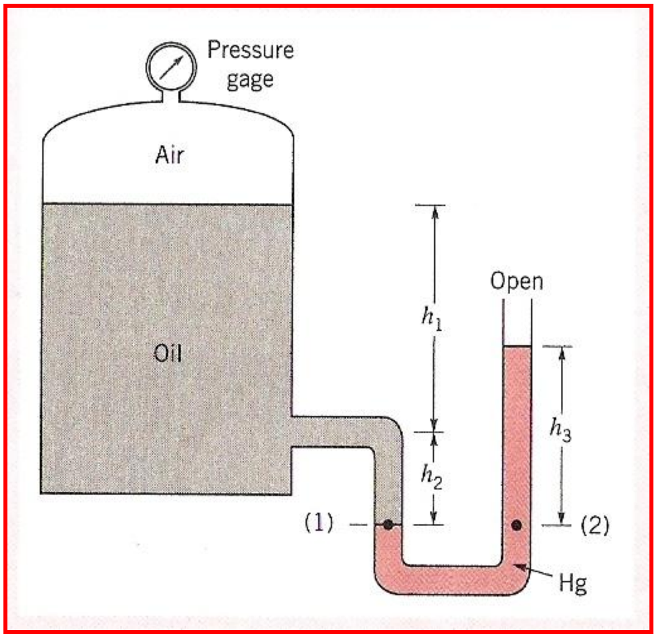
The pressure at level (1) equal to pressure at level (2):

$P_1 = P_2$

$P_1 = P_{\text{air}} + \gamma_{\text{oil}} (h_1 + h_2) = P_2 = P_{\text{atm}} + \gamma_{\text{Hg}}\, h_3$

Since, $P_{\text{atm}} = 0$

Thus, the manometer equation can be expressed as:



$P_{\text{air}} + \gamma_{\text{oil}} (h_1 + h_2) - \gamma_{\text{Hg}}\, h_3 = 0$

or

Figure 2-2.

$P_{\text{air}} + (SG_{\text{oil}})(\gamma_{H2O})(h_1 + h_2) - (SG_{\text{Hg}})(\gamma_{H2O})\, h_3 = 0$

For the value given:

$$P_{air} = -(0.9)(62.4\ \text{lb/ft}^3)\left(\frac{(36+6)\text{in}}{12\ \text{in/ft}}\right) + (13.6)(62.4\ \text{lb/ft}^3)\left(\frac{9\ \text{in}}{12\ \text{in/ft}}\right)$$

$$= 440\ \text{lb/ft}^2$$

Because the specific weight of the air above the oil is much smaller than the specific weight of the oil, the gage should read the pressure we have calculate; that is: $P_{gage} = P_{air} - P_{atm}$, since, $P_{atm} = 0$

Then, $$P_{air} = P_{gage} = \frac{440\ \text{lb/ft}^2}{144\ \text{in}^2/\text{ft}^2} = 3.06\ \text{psi}$$

Example 2-3

The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Figure 2-3. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if h_1 = 0.1 m, h_2 = 0.2 m, and h_3 = 0.35 m. take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13600 kg/m^3, respectively.

Solution:

Assumption: **The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and we can determine the pressure at the air-water interface.**

Properties: **the densities of water, oil, and mercury are given to be 1000 kg/m^3, 850 kg/m^3, and 13600 kg/m^3, respectively.**

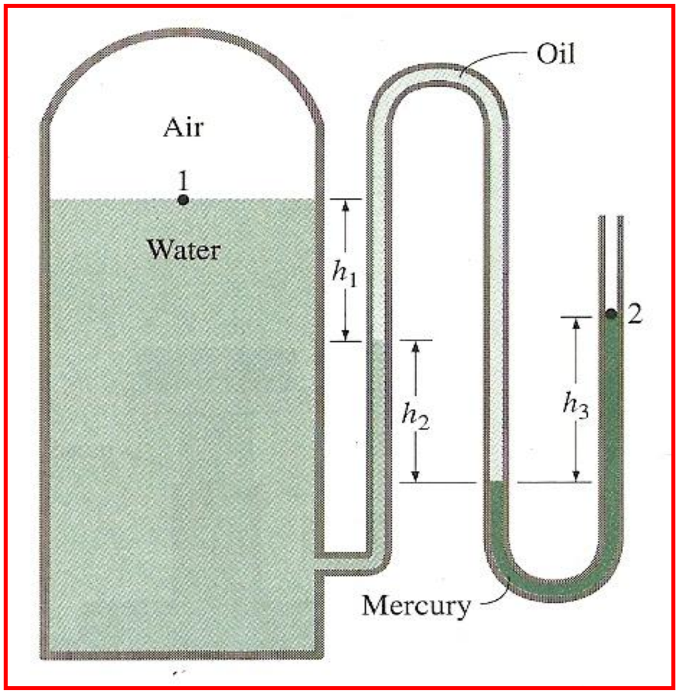


Figure 2-3.

Analysis: **starting with pressure at point 1 at the air-water interface, moving along the tube by subtracting the $\rho g h$ terms until we reach point 2, and setting the result equal to P_{atm} since**

the tube is open to the atmosphere gives:

$$P_1 + \rho_{water}\ gh_1 + \rho_{oil}gh_2 - \rho_{mercury}\ gh_3 = P_2 = P_{atm}$$

Solving for P_1 and substituting,

$$P_1 = P_{atm} - \rho_{water}\ gh_1 - \rho_{oil}gh_2 + \rho_{mercury}\ gh_3$$

$$= P_{atm} + g\ (\rho_{mercury}\ h_3 - \rho_{water}h_1 - \rho_{oil}h_2)$$

$$= 85.6\ kPa + (9.81\ m/s^2)[(13600\ kg/m^3)(0.35\ m) - (1000\ kg/m^3)(0.1\ m) -$$

$$- (850\ kg/m^3)(0.2\ m)]\left(\frac{1\ N}{1\ kg.m/s^2}\right)\left(\frac{1\ kPa}{1000\ N/m^2}\right)$$

P_1 = 130 kPa

Example 2-4

Consider a U-tube whose arms are open to the atmosphere. Now water is poured into the U-tube from one arm, and light oil (ρ = 790 kg/m^3) from the other. One arm contains 70-cm-high water, while the other arm contains both fluids with an oil-to-water height ratio of 6. Determine the height of each fluid in that arm.

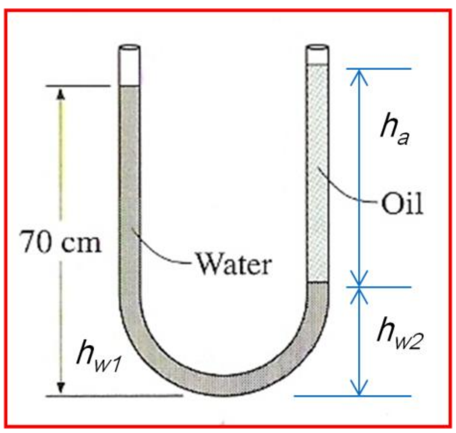


Figure 2-4.

Solution:

Water is poured into the U-tube from one arm and oil from the other arm. The water column height in one arm and the ratio of the heights of the two fluids in the other arm are given. The height of each fluid in that arm is to be determined.

Assumptions: **Both water and oil are incompressible substances.**

Properties: **The density of oil is given to be ρ_{oil} = 790 kg/m^3. We take the density of water to be ρ_w = 1000 kg/m^3.**

Analysis: **The height of water column in the left arm of the manometer is given to be h_{w1} = 0.70 m. We let the height of water and oil in the right arm to be h_{w2} and h_a, respectively. Then, $h_a = 6h_{w2}$. Noting that both arms are open to the atmosphere, the pressure at the bottom of the U-tube can be expressed as:**

$$P_{bottom} = P_{atm} + \rho_w g h_{w1} \text{ and } P_{bottom} = P_{atm} + \rho_w g h_{w2} + \rho_a g h_a$$

Setting them equal to each other and simplifying,

$$\rho_w g h_{w1} = \rho_w g h_{w2} + \rho_a g h_a \rightarrow \rho_w h_{w1} = \rho_w h_{w2} + \rho_a h_a \rightarrow h_{w1} = h_{w2} + (\rho_a / \rho_w) h_a$$

Noting that $h_a = 6h_{w2}$ and we take $\rho_a = \rho_{oil}$, the water and oil column heights in the second arm are determined to be:

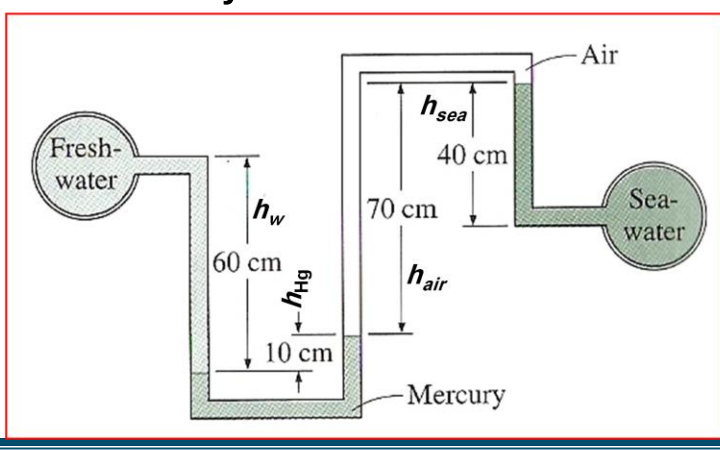
$$0.7 \text{ m} = h_{w2} + (790/1000)6h_{w2} \rightarrow h_{w2} = 0.122 \text{ m}$$

$$0.7 \text{ m} = 0.122 \text{ m} + (790/1000)h_a \rightarrow h_a = 0.732 \text{ m}$$

Example 2-5

Freshwater and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer, as shown in Fig. 2-5. Determine the pressure difference between the two pipelines. Take the density of seawater at that location to be ρ = 1035 kg/m3. Can the air column be ignored in the analysis?

Figure 2-5.



Solution:

Fresh and seawater flowing in parallel horizontal pipelines are connected to each other by a double U-tube manometer. The pressure difference between the two pipelines is to be determined.

Assumptions: **1 All the liquids are incompressible. 2 The effect of air column on pressure is negligible.**

Properties: **The densities of seawater and mercury are given to be ρ_{sea} = 1035 kg/m^3 and ρ_{Hg} = 13,600 kg/m^3. We take the density of water to be ρ_w = 1000 kg/m^3.**

Analysis: **Starting with the pressure in the fresh water pipe (point 1) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the sea water pipe (point 2), and setting the result equal to P_2 gives:**

$$P_1 + \rho_w gh_w - \rho_{Hg} gh_{Hg} - \rho_{air} gh_{air} + \rho_{sea} gh_{sea} = P_2$$

Rearranging and neglecting the effect of air column on pressure,

$$P_1 - P_2 = -\rho_w gh_w + \rho_{Hg} gh_{Hg} - \rho_{sea} gh_{sea} = g(\rho_{Hg}h_{Hg} - \rho_w h_w - \rho_{sea} h_{sea})$$

Substituting,

$$P_1 - P_2 = (9.81 \text{ m/s}^2)[(13600 \text{ kg/m}^3)(0.1 \text{ m})$$

$$- (1000 \text{ kg/m}^3)(0.6 \text{ m}) - (1035 \text{ kg/m}^3)(0.4 \text{ m})]\left(\frac{\text{kN}}{1000 \text{ kg.m/s}^2}\right)$$

$$= 3.39 \text{ kN/m}^2 = 3.39 \text{ kPa}$$

Therefore, the pressure in the fresh water pipe is 3.39 kPa higher than the pressure in the sea water pipe.

Discussion: **A 0.70-m high air column with a density of 1.2 kg/m^3 corresponds to a pressure difference of 0.008 kPa.**

Therefore, its effect on the pressure difference between the two pipes is negligible.

Example 2-6

The gage pressure of the air in the tank shown in Fig. 2-6 is measured to be 65 kPa. Determine the differential height *h* of the mercury column.

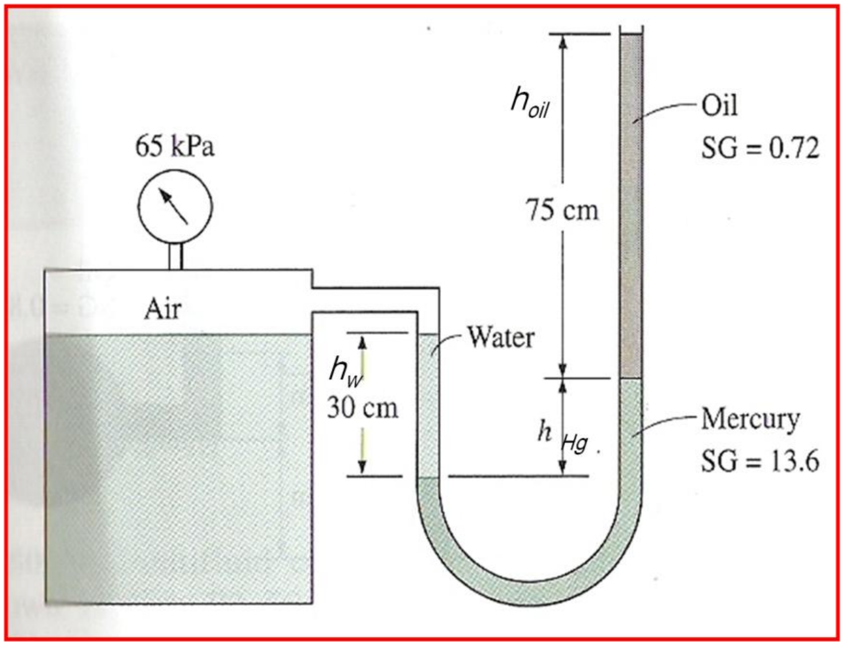


Figure 2-6.

Solution:

The gage pressure of air in a pressurized water tank is measured simultaneously by both a pressure gage and a manometer. The differential height *h* of the mercury column is to be determined.

Assumptions: **The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.**

Properties: **We take the density of water to be ρ_w =1000 kg/m^3. The specific gravities of oil and mercury are given to be 0.72 and 13.6, respectively.**

Analysis: **Starting with the pressure of air in the tank (point 1), and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface of oil where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} gives:**

$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} = P_{atm}$$

Rearranging,

$$P_1 - P_{atm} = \rho_{oil}\, g h_{oil} + \rho_{Hg}\, g h_{Hg} - \rho_w\, g h_w$$

or,

$$\left(\frac{P_{1,gage}}{\rho_w g}\right) = SG_{oil}\, h_{oil} + SG_{Hg}\, h_{Hg} - h_w$$

Substituting,

$$\left(\frac{65\ \text{kPa}}{\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)}\right)\left(\frac{1000\ \text{kg.m/s}^2}{1\ \text{kPa.m}^2}\right) = 0.72 \times (0.75\ \text{m}) + 13.6 \times h_{Hg} - 0.3\ \text{m}$$

Solving for h_{Hg} gives h_{Hg} = 0.47 m. Therefore, the differential height of the mercury column must be 47 cm.

Discussion: **Double instrumentation like this allows one to verify the measurement of one of the instruments by the measurement of another instrument.**

Example 2-7

Consider a double-fluid manometer attached to an air pipe shown in Fig. 2-7. If the specific gravity of one fluid is 13.55, determine the specific gravity of the other fluid for the indicated absolute pressure of air. Take the atmospheric pressure to be 100 kPa.

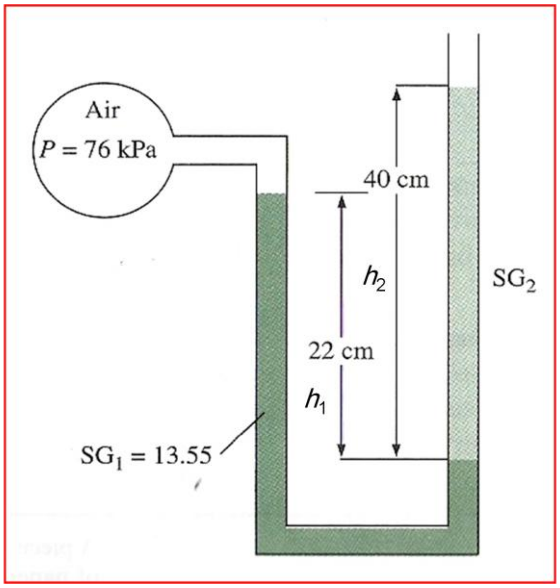


Figure 2-7.

Solution:

A double-fluid manometer attached to an air pipe is considered. The specific gravity of one fluid is known, and the specific gravity of the other fluid is to be determined.

***Assumptions:* 1 Densities of liquids are constant. 2 The air pressure in the tank is uniform (i.e., its variation with elevation is negligible due to its low density), and thus the pressure at the air-water interface is the same as the indicated gage pressure.**

***Properties:* The specific gravity of one fluid is given to be 13.55. We take the standard density of water to be 1000 kg/m^3.**

***Analysis:* Starting with the pressure of air in the tank, and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the free surface where the oil tube is exposed to the atmosphere, and setting the result equal to P_{atm} give:**

$$P_{air} + \rho_1 g h_1 - \rho_2 g h_2 = P_{atm} \rightarrow P_{air} - P_{atm} = SG_2\, \rho_w g h_2 - SG_1\, \rho_w g h_1$$

Rearranging and solving for SG_2,

$$SG_2 = SG_1 \frac{h_1}{h_2} + \frac{P_{air} - P_{atm}}{\rho_w g h_2} = 13.55\,\frac{0.22\ \text{m}}{0.40\ \text{m}} + \left(\frac{(76-100)\text{kPa}}{\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(0.40\ \text{m})}\right) * \left(\frac{1000\ \text{kg.m/s}^2}{1\ \text{kPa.m}^2}\right) = 1.34$$

***Discussion:* Note that the right fluid column is higher than the left, and this would imply above atmospheric pressure in the pipe for a single-fluid manometer.**

Example 2-8

The pressure difference between an oil pipe and water pipe is measured by a double-fluid manometer, as shown in Fig. 2-8. For the given fluid heights and specific gravities, calculate the pressure difference $\Delta P = P_B - P_A$.

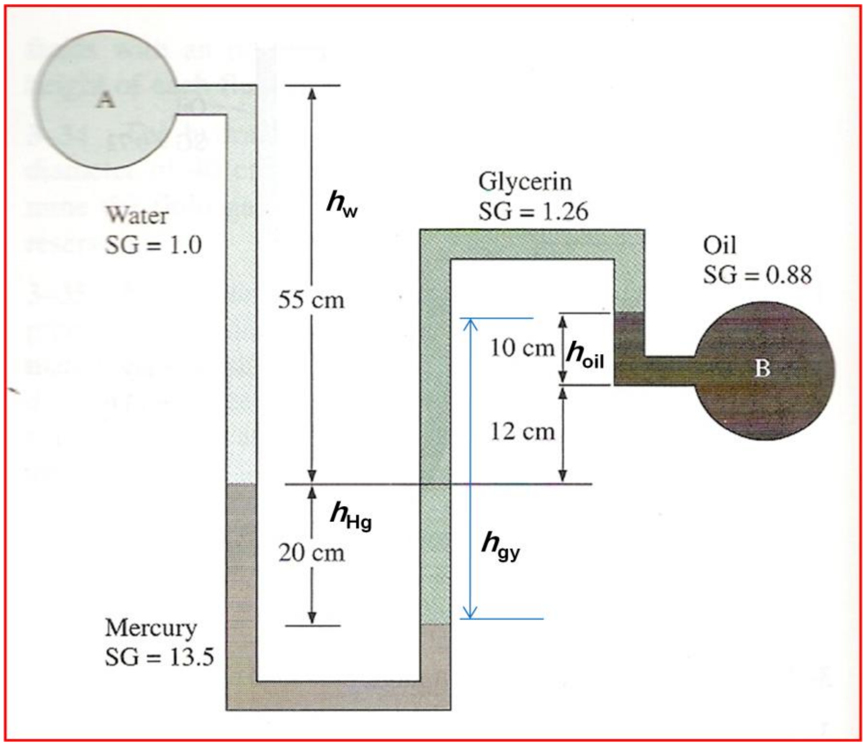


Figure 2-8.

Solution:

The pressure difference between two pipes is measured by a double-fluid manometer. For given fluid heights and specific gravities, the pressure difference between the pipes is to be calculated.

Assumptions: **All the liquids are incompressible.**

Properties: **The specific gravities are given to be 13.5 for mercury, 1.26 for glycerin, and 0.88 for oil. We take the standard density of water to be ρ_w =1000 kg/m3.**

Analysis: **Starting with the pressure in the water pipe (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the oil pipe (point B), and setting the result equal to P_B give:**

$$P_A + \rho_w gh_w + \rho_{Hg} gh_{Hg} - \rho_{gly} gh_{gly} + \rho_{oil} gh_{oil} = P_B$$

Rearranging and using the definition of specific gravity,

$$P_B - P_A = SG_w \rho_w gh_w + SG_{Hg} \rho_w gh_{Hg} - SG_{gly} \rho_w gh_{gly} + SG_{oil} \rho_w gh_{oil}$$

$$= \rho_w g (SG_w h_w + SG_{Hg} h_{Hg} - SG_{gly} h_{gly} + SG_{oil} h_{oil})$$

Substituting,

$$P_B - P_A = (1000\ \text{kg/m}^3)\ (9.81\ \text{m/s}^2)[1\ (0.55\ \text{m}) + 13.5\ (0.2\ \text{m}) - 1.26\ (0.42\ \text{m}) + 0.88\ (0.1\ \text{m})]\left(\frac{1\ \text{kN}}{1000\ \text{kg.m/s}^2}\right)$$

$$= 27.55\ \text{kN/m}^2 = 27.55\ \text{kPa}$$

Therefore, the pressure in the oil pipe is 27.7 kPa higher than the pressure in the water pipe.

Discussion: **Using a manometer between two pipes is not recommended unless the pressures in the two pipes are relatively constant. Otherwise, an over-rise of pressure in one pipe can push the manometer fluid into the other pipe, creating a short circuit.**

Example 2-9

Consider the system shown in Fig. 2-9. If a change of 0.7 kPa in the pressure of air causes the brine-mercury interface in the right column to drop by 5 mm in the brine level in the right column while the pressure in the brine pipe remains constant, determine the ratio of A_2/A_1.

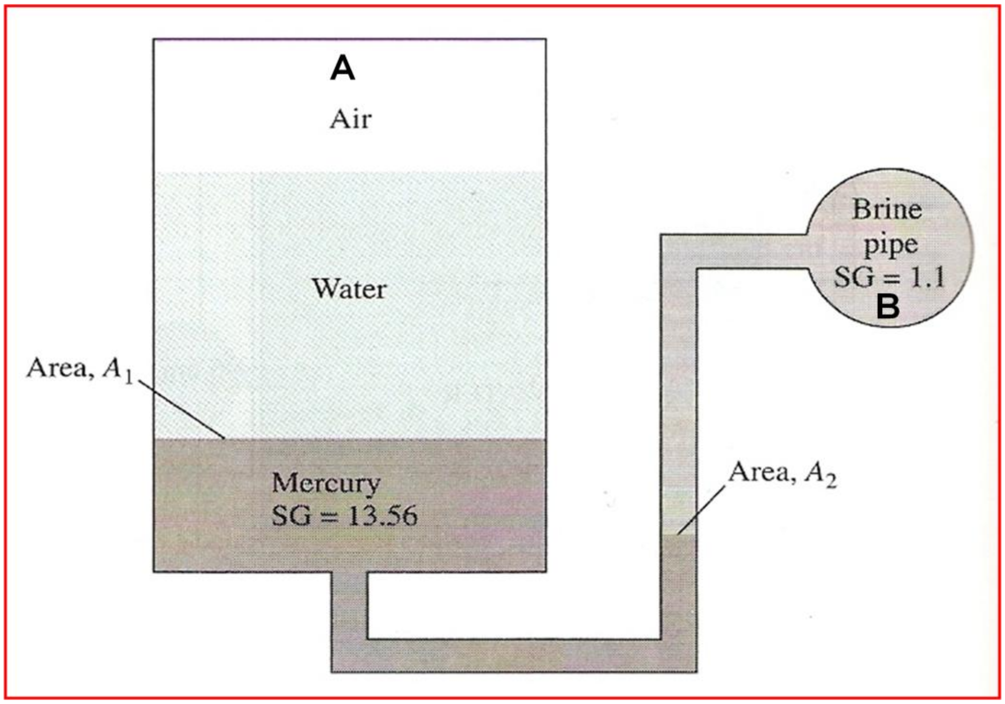


Figure 2-9.

Solution:

The fluid levels in a multi-fluid U-tube manometer change as a result of a pressure drop in the trapped air space. For a given pressure drop and brine level change, the area ratio is to be determined.

Assumptions: **1 All the liquids are incompressible. 2 Pressure in the brine pipe remains constant. 3 The variation of pressure in the trapped air space is negligible.**

Properties: **The specific gravities are given to be 13.56 for mercury and 1.1 for brine. We take the standard density of water to be ρ_w = 1000 kg/m3.**

Analysis: **It is clear from the problem statement and the figure that the brine pressure is much higher than the air pressure, and when the air pressure drops by 0.7 kPa, the pressure difference between the brine and the air space also increases by the same amount. Starting with the air pressure (point A) and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach the brine pipe (point B), and setting the result equal to P_B before and after the pressure change of air give:**

Before: $P_{A1} + \rho_w g h_w + \rho_{Hg} g h_{Hg,1} - \rho_{br} g h_{br,1} = P_B$

After: $P_{A2} + \rho_w g h_w + \rho_{Hg} g h_{Hg,2} - \rho_{br} g h_{br,2} = P_B$

Subtracting,

$$P_{A2} - P_{A1} + \rho_{Hg} g\Delta h_{Hg} - \rho_{br} g\Delta h_{br} = 0 \rightarrow \frac{P_{A1-}P_{A2}}{\rho_w\, g} = SG_{Hg}\Delta h_{Hg} - SG_{br}\Delta h_{br} = 0 \quad (1)$$

where Δh_{Hg} and Δh_{br} are the changes in the differential mercury and brine column heights, respectively, due to the drop in air pressure. Both of these are positive quantities since as the mercury-brine interface drops, the differential fluid heights for both mercury and brine increase. Noting also that the volume of mercury is constant, we have $A_1\Delta h_{Hg,left} = A_2\Delta h_{Hg,right}$ and:

$$P_{A2} - P_{A1} = -\,0.7 \text{ kPa} = -700 \text{ N/m}^2 = -700 \text{ kg/m.s}^2$$

$$\Delta h_{br} = 0.005 \text{ m}$$

$$\Delta h_{Hg} = \Delta h_{Hg,right} + \Delta h_{Hg,left} = \Delta h_{br} + \Delta h_{br} A_2/A_1 = \Delta h_{br}(1 + A_2/A_1)$$

Substituting,

$$\frac{(700\ \mathrm{kg/m.s^2})}{\left(1000\frac{\mathrm{kg}}{\mathrm{m^3}}\right)(9.81\ \mathrm{m/s^2})} = [13.56\mathrm{x}0.005\ (1 + A_2/A_1) - 1.1\mathrm{x}0.005]\ \mathrm{m}$$

It gives:

***A_2/A_1* = 0.134**

Discussion: **In addition to the equations of hydrostatics, we also utilize conservation of mass in this problem.**

Example 2-10

A multifluid container is connected to a U-tube, as shown in Fig. 2-10. For the given specific gravities and fluid column heights, determine the gage pressure at *A*. Also determine the height of a mercury column that would create the same pressure at *A*.

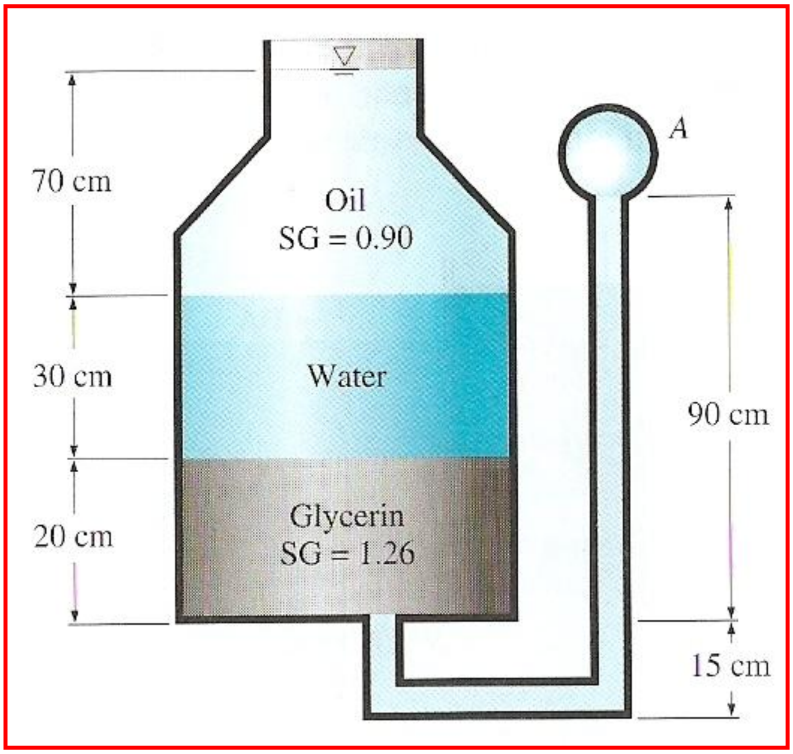


Figure 2-10.

Solution:

A multi-fluid container is connected to a U-tube. For the given specific gravities and fluid column heights, the gage pressure at *A* and the height of mercury column that would create the same pressure at *A* are to be determined.

Assumptions: **1 All the liquids are incompressible. 2 The multi-fluid container is open to the atmosphere.**

Properties: **The specific gravities are given to be 1.26 for glycerin and 0.90 for oil. We take the standard density of water to be ρ_w =1000 kg/m^3, and the specific gravity of mercury to be 13.6.**

Analysis: **Starting with the atmospheric pressure on the top surface of the container and moving along the tube by adding (as we go down) or subtracting (as we go up) the ρgh terms until we reach point *A*, and setting the result equal to P_A give:**

$$P_{atm} + \rho_{oil}\, gh_{oil} + \rho_w\, gh_w - \rho_{gly}\, gh_{gly} = P_A$$

Rearranging and using the definition of specific gravity,

$$P_A - P_{atm} = SG_{oil}\,\rho_w\, gh_{oil} + SG_w\,\rho_w\, gh_w - SG_{gly}\,\rho_w\, gh_{gly}$$

or

$$P_{A,gage} = \rho_w\, g\,(SG_{oil}\, h_{oil} + SG_w\, h_w - SG_{gly}\, h_{gly})$$

Substituting,

$$P_{A,gage} = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\,[0.90\,(0.70\ \text{m}) + 1\,(0.3\ \text{m}) - 1.26\,(0.70\ \text{m})] * \left(\frac{1\ \text{kN}}{1000\ \text{kg.m/s}^2}\right)$$

$$= 0.471\ \text{kN/m}^2 = 0.471\ \text{kPa}$$

The equivalent mercury column height is:

$$h_{Hg} = \frac{P_{A,gage}}{\rho_{Hg}\, g} = \frac{0.471\ \text{kN/m}^2}{(13.6)\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)} * \left(\frac{1000\ \text{kg.m/s}^2}{1\ \text{kN}}\right) = \mathbf{0.00353\ m}$$

Discussion: **Note that the high density of mercury makes it a very suitable fluid for measuring high pressures in manometers.**

Example 2-11

A glass tube is attached to a water pipe, as shown in Fig. 2-11. If the water pressure at the bottom of the tube is 115 kPa and the local atmospheric pressure is 92 kPa, determine how high the water will rise in the tube, in m. Assume *g* = 9.8 m/s^2 at that location and take the density of water to be 1000 kg/m^3.

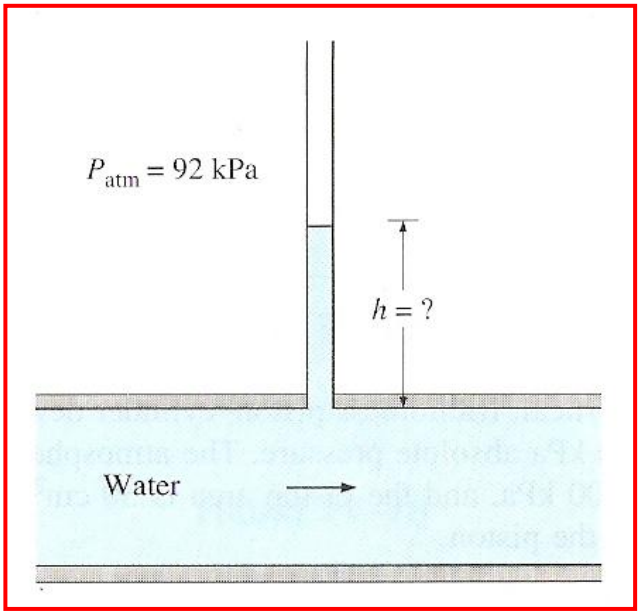


Figure 2-11.

Solution:

A glass tube open to the atmosphere is attached to a water pipe, and the pressure at the bottom of the tube is measured. It is to be determined how high the water will rise in the tube.

Properties: **The density of water is given to be ρ = 1000 kg/m^3.**

Analysis: **The pressure at the bottom of the tube can be expressed as:**

$$P = P_{atm} + (\rho g h)$$

Solving for *h*,

$$h = \frac{P - P_{atm}}{\rho\, g} = \frac{(115-95)\text{kPa}}{\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)} * \left(\frac{1000\ \text{N/m}^2}{1\ \text{kPa}}\right) * \left(\frac{1\ \text{kg.m/s}^2}{1\ \text{N}}\right) = 2.35\ \text{m}$$

Discussion: **Even though the water is flowing, the water in the tube itself is at rest. If the pressure at the tube bottom had been given in terms of gage**

pressure, we would not have had to take into account the atmospheric pressure term.

2-3 pumps:

Example 2-12

A two-lobe rotary positive-displacement pump, moves 0.45 cm^3 of SAE 30 motor oil in each lobe volume V_{lobe} as sketched in Figure 2-12. Calculate the volume flow rate of oil for the case where $\dot{n}$ = 900, and for (1) half of a rotation, and for (2) one rotation.

Solution:

We are to calculate the volume flow rate of oil through a positive-displacement pump for given values of lobe volume and rotation rate.

Assumptions: 1 The flow is steady in the mean. 2 There are no leaks in the gaps between lobes or between lobes and the casing. 3 The oil is incompressible.

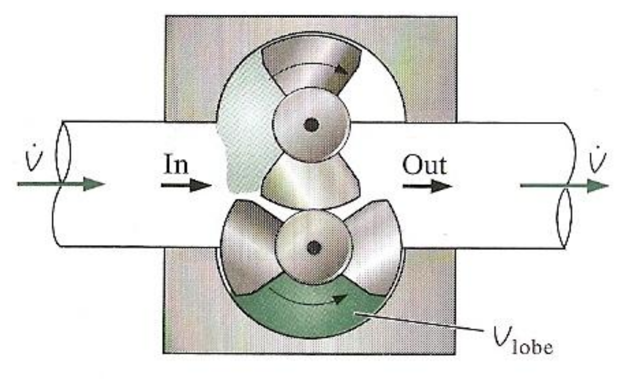


Figure 2-12.

Analysis: We see that for (1) half of a rotation (180^o for n = 0.5 rotations), and for (2) one rotation (360^o for n = 1 rotation), of the two counter-rotating shafts, the total volume of oil pumped is:

(1) V_{closed} = 2 V_{lobe}. The volume flow rate is then calculated from following equation:

$$\dot{V} = \dot{n}\frac{V_{closed}}{n} = (900\ \text{rot/min})\frac{2\ (0.45\ \text{cm}^3)}{0.5\ \text{rot}} = 1620\ \text{cm}^3/\text{min}$$

(2) V_{closed} = 4 V_{lobe}. The volume flow rate is then calculated from following equation:

$$\dot{V} = \dot{n}\frac{V_{closed}}{n} = (900\ \text{rot/min})\frac{4\ (0.45\ \text{cm}^3)}{1\ \text{rot}} = 1620\ \text{cm}^3/\text{min}$$