# PRINCIPLES OF ENVIRONMENTAL ENGINEERING AND SUSTAINABILITY

**LECTURE EIGHT: HYDRAULICS OF WATER …CONT.**
**FRICTION HEAD LOSS AND OPEN-CHANNEL FLOW**
**DEPARTMENT OF ENVIRONMENTAL ENGINEERING**

## LOSS OF HEAD FROM PIPE FRICTION

- Energy loss resulting from friction in a pipeline is commonly termed the *friction head loss* ($h_L$).
- This is the loss of head caused by pipe wall friction and the viscous dissipation in flowing water.
- Friction loss is sometimes referred to as the *major loss* because of its magnitude,
- and all other losses are referred to as *minor losses*.

# THE DARCY-WEISBACH EQUATION

- The most popular pipe flow equation was derived by *Henri Darcy* (1803 to 1858), *Julius Weisbach* (1806 to 1871 ), and others about the middle of the nineteenth century. The equation takes the following form:

$$h_f = f\left(\frac{L}{D}\right)\frac{V^2}{2g}$$

- This equation is commonly known as the *Darcy-Weisbach* equation. It is conveniently expressed in terms of the velocity head in the pipe. Moreover, it is dimensionally uniform since in engineering practice the *friction factor* (*f*) is treated as a dimensionless numerical factor; $h_L$ and $V^2/2g$ are both in units of length.

# EMPIRICAL EQUATIONS FOR FRICTION HEAD LOSS

# THE *HAZEN-WILLIAMS* EQUATION

- One of the best examples is the *Hazen- Williams* equation, which was developed for water flow· in larger pipes (D ≥ 5 cm, approximately 2 in within a moderate range of water velocity (V ≤ 3m/sec, approximately 10 ft/sec).
- This equation has been used extensively for the designing of water-supply systems in the Untied States. The *Hazen-Williams* equation, originally developed for the British measurement system, has been written in the form:

$$V = 1.318\, C_{HW} R_h^{0.63} S^{0.54}$$

## THE *HAZEN-WILLIAMS* EQUATION…CONT.

$$V = 1.318 C_{HW} R_h^{0.63} S^{0.54}$$

- where $S$ is the slope of the **energy grade** line, or the head loss per unit length of the pipe

( $S = h_f/L$), and $R_h$ is the hydraulic radius, defined as the water cross-sectional area ($A$) divided the wetted perimeter ($P$). For a circular pipe $R_h$ is given by:

$$R_h = \frac{A}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$$

## THE *HAZEN-WILLIAMS* EQUATION…CONT.

- The Hazen-Williams coefficient, $C_{HW}$, is not a function of the flow conditions (i.e., Reynolds number). Its values range from 140 for very smooth, straight pipes down to 90 or even 80 for old, unlined rough pipes. Generally, the value of 100 is taken for average conditions. The values of $C_{HW}$ for commonly used water-carrying conduits are listed in Table 3.2.

TABLE 3.2 Hazen-Williams Coefficient, $C_{HW}$, for Different Types of Pipes

| Pipe Materials | $C_{HW}$ |
|---|---|
| Brass | 130–140 |
| Cast iron (common in older water lines) | |
| New, unlined | 130 |
| 10-year-old | 107–113 |
| 20-year-old | 89–100 |
| 30-year-old | 75–90 |
| 40-year-old | 64–83 |
| Concrete or concrete lined | |
| Smooth | 140 |
| Average | 120 |
| Rough | 100 |
| Copper | 130–140 |
| Ductile iron (cement mortar lined) | 140 |
| Glass | 140 |
| High-density polyethylene (HDPE) | 150 |
| Plastic | 130–150 |
| Polyvinyl chloride (PVC) | 150 |
| Steel | |
| Commercial | 140–150 |
| Riveted | 90–110 |
| Welded (seamless) | 100 |
| Vitrified clay | 110 |

# THE *HAZEN-WILLIAMS* EQUATION IN SI UNITS

- Note that the coefficient in the Hazen-Williams equation has units of $ft^{0.37}/sec$. Therefore, Equation 3.25 is applicable *only* for the British units in which the velocity is measured in feet per second and the hydraulic radius *($R_h$)* is measured in feet.
- Because 1.318 $ft^{0.37}/sec$ = 0.849 $m^{0.37}/sec$, the *Hazen-Williams* equation in SI units may be written in the following form:

$$V = 0.849 C_{HW} R_h^{0.63} S^{0.54}$$

where the velocity is measured in meters per second and $R_h$ is measured in meters.

# EMPIRICAL EQUATIONS FOR FRICTION HEAD LOSS **THE *MANNING EQUATION* EQUATION**

- Another popular empirical equation is the *Manning equation*, which was originally developed in metric units. The Manning equation has been used **extensively for open-channel designs**. **It is also quite commonly used for pipe flows**. The Manning equation may be expressed in the following form:

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}$$

# THE *MANNING EQUATION* EQUATION…CONT.

- where the velocity is measured in meters per second and the hydraulic radius is measured in meters. Then *n* is *Manning's* coefficient of roughness, specifically known to hydraulic engineers as *Manning's n*.
- In British units, the Manning equation is written as:

$$V = \frac{1.486}{n} R_h^{2/3} S^{1/2}$$

**TABLE 3.3** Manning's Roughness Coefficient, $n$, for Pipe Flows

| Type of Pipe | Manning's $n$ | |
|---|---|---|
| | Min. | Max. |
| Brass | 0.009 | 0.013 |
| Cast iron | 0.011 | 0.015 |
| Cement mortar surfaces | 0.011 | 0.015 |
| Cement rubble surfaces | 0.017 | 0.030 |
| Clay drainage tile | 0.011 | 0.017 |
| Concrete, precast | 0.011 | 0.015 |
| Copper | 0.009 | 0.013 |
| Corrugated metal (CMP) | 0.020 | 0.024 |
| Ductile iron (cement mortar lined) | 0.011 | 0.013 |
| Glass | 0.009 | 0.013 |
| High-density polyethylene (HDPE) | 0.009 | 0.011 |
| Polyvinyl chloride (PVC) | 0.009 | 0.011 |
| Steel, commercial | 0.010 | 0.012 |
| Steel, riveted | 0.017 | 0.020 |
| Vitrified sewer pipe | 0.010 | 0.017 |
| Wrought iron | 0.012 | 0.017 |

**Example 3.8**

A horizontal pipe (old cast iron) with a 10-cm uniform diameter is 200 m long. If the measured pressure drop is 24.6 m of water, what is the discharge?

**Solution**

Area: $A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$

Wetted perimeter: $P = \pi D = 0.1\pi = 0.314 \text{ m}$

Hydraulic radius: $R_h = A/P = \frac{0.00785}{0.314} = 0.0250 \text{ m}$

Energy slope: $S = h_f/L = \frac{24.6 \text{ m}}{200 \text{ m}} = 0.123$

Manning's roughness coefficient: $n = 0.015$ (Table 3.3)

Substituting the above quantities into the Manning equation, Equation 3.28, we have

$$V = \frac{Q}{A} = \frac{1}{n}R_h^{2/3}S^{1/2}$$

$$Q = \frac{1}{0.015}(0.00785)(0.025)^{2/3}(0.123)^{1/2} = 0.0157 \text{ m}^3/\text{sec}$$

# WATER FLOW IN OPEN CHANNEL FLOW

- In Figure 6.1, open-channel flow is schematically compared to pipe flow. Figure 6.l(a) shows a pipe flow segment with two open-ended vertical tubes (piezometers) installed through the pipe wall at an upstream section, 1, and a downstream section, 2. The water level in each tube represents the pressure head (P/γ) in the pipe at the section. A line connecting the water levels in the two tubes represents the hydraulic grade line (HGL) between these sections. The velocity head at each section is represented in the familiar form, $V^2/2g$, where V is the mean velocity, V = Q/A, at the section. The total energy head at any section is equal to the sum of the elevation (potential) head (h), the pressure head (P/γ), and the velocity head ($V^2/2g$). A line connecting the total energy head at the two sections is called the energy grade line (EGL). The amount of energy lost when water flows from section 1 to section 2 is indicated by $h_L$.

## WATER FLOW IN OPEN CHANNEL FLOW

Figure 6.l(b) shows an open-channel flow segment The free water surface is subjected to only atmospheric pressure, which is commonly referred to as the zero pressure reference in hydraulic engineering practice.
The pressure distribution at any section is directly proportional to the depth measured from the free water surface. In this case, the water surface line corresponds to the hydraulic grade line in pipe flow.

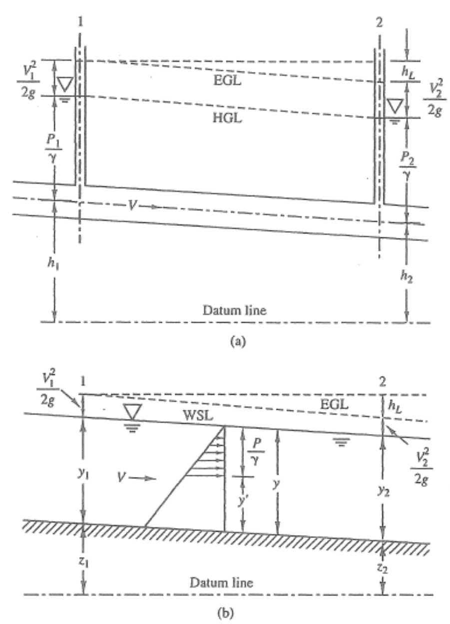



Figure 6.1 Comparison of: (a) pipe flow and (b) open-channel flow

# WATER FLOW IN OPEN CHANNEL FLOW…CONT.

- To solve open-channel flow problems, we must seek the interdependent relationships between the slope of the channel bottom, the discharge, the water depth, and other channel characteristics. The basic geometric and hydraulic definitions used to describe open-channel flow through a channel section are:

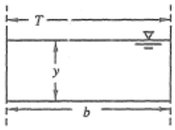
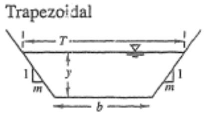
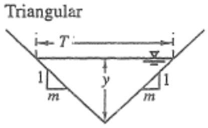
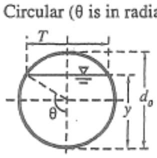
| | |
|---|---|
| Discharge ($Q$) | Volume of water passing through a flow section per unit time |
| Flow area ($A$) | Cross-sectional area of the flow |
| Average velocity ($V$) | Discharge divided by the flow area: $V = Q/A$ |
| Flow depth ($y$) | Vertical distance from the channel bottom to the free surface |
| Top width ($T$) | Width of the channel section at the free surface |

# WATER FLOW IN OPEN CHANNEL FLOW…CONT.

| | |
|---|---|
| Wetted perimeter ($P$) | Contact length of the water and the channel at a cross section |
| Hydraulic depth ($D$) | Flow area divided by the top width: $D = A/T$ |
| Hydraulic radius ($R_h$) | Flow area divided by the wetted perimeter: $R_h = A/P$ |
| Bottom slope ($S_0$) | Longitudinal slope of the channel bottom |
| Side slope ($m$) | Slope of channel sides defined as 1 vertical over $m$ horizontal |
| Bottom width ($b$) | Width of the channel section at the bottom |

Table 6.1 depicts the cross-sectional characteristics for various types of channel sections and their geometric and hydraulic relationships.

**TABLE 6.1** Cross-Sectional Relationships for Open-Channel Flow

| Section Type | Area ($A$) | Wetted perimeter ($P$) | Hydraulic Radius ($R_h$) | Top Width ($T$) | Hydraulic Depth ($D$) |
|---|---|---|---|---|---|
| Rectangular  | $by$ | $b + 2y$ | $\dfrac{by}{b + 2y}$ | $b$ | $y$ |
| Trapezoidal  | $(b + my)y$ | $b + 2y\sqrt{1 + m^2}$ | $\dfrac{(b + my)y}{b + 2y\sqrt{1 + m^2}}$ | $b + 2my$ | $\dfrac{(b + my)y}{b + 2my}$ |
| Triangular  | $my^2$ | $2y\sqrt{1 + m^2}$ | $\dfrac{my}{2\sqrt{1 + m^2}}$ | $2my$ | $\dfrac{y}{2}$ |
| Circular (θ is in radians)  | $\frac{1}{8}(2\theta - \sin 2\theta)d_0^2$ | $\theta d_0$ | $\frac{1}{4}\left(1 - \dfrac{\sin 2\theta}{2\theta}\right)d_0$ | $(\sin \theta)d_0$ or $2\sqrt{y(d_0 - y)}$ | $\frac{1}{8}\left(\dfrac{2\theta - \sin 2\theta}{\sin \theta}\right)d_0$ |

*Source:* V. T. Chow, *Open Channel Hydraulics* (New York: McGraw-Hill, 1959).

## Chezy's formula for open-channel flow

$$V = C\sqrt{R_h S_e} \qquad (6.2)$$

$$C = \frac{1}{n}R_h^{1/6} \qquad (6.3)$$

Substituting Equation 6.3 into Equation 6.2, we have *Manning's equation:*

$$V = \frac{k_M}{n}R_h^{2/3}S_e^{1/2} \qquad (6.4)$$

$$Q = AV = \frac{k_M}{n}AR_h^{2/3}S_e^{1/2} \qquad (6.5)$$

Setting $k_M = 1$ in the SI unit system, these equations become

$$V = \frac{1}{n}R_h^{2/3}S_e^{1/2} \qquad (6.4a)$$

and

$$Q = AV = \frac{1}{n}AR_h^{2/3}S_e^{1/2} \qquad (6.5a)$$

# MANNING FORMULA FOR OPEN CHANNEL FLOW

where $V$ has units of m/sec, $R_h$ is given in m, $S_e$ in m/m, A is given in $m^2$, and $Q$ is given in $m^3$/sec. On the right-hand side of this equation, the water area ($A$) and the hydraulic radius ($R_h$) are both functions of water depth ($y$), which is known as the *uniform depth* or *normal depth* ($y_n$) when the flow is uniform.

Setting $k_M = 1.49$ in the BG system, Manning's equation is written as

$$V = \frac{1.49}{n} R_h^{2/3} S_e^{1/2} \quad (6.4b)$$

or

$$Q = \frac{1.49}{n} A R_h^{2/3} S_e^{1/2} \quad (6.5b)$$

# MANNING FORMULA FOR OPEN CHANNEL FLOW…CONT.

where $V$ is in ft/sec, $Q$ is in $ft^3$/sec (or cfs), $A$ in $ft^2$, $R_h$ in ft, and $S_e$ in ft/ft. The computation of uniform flow may be performed by the use of either Equation 6.4 or Equation 6.5 and basically involves seven variables:

1. the roughness coefficient ($n$);
2. the channel slope ($S_0$) (because $S_0 = S_e$ in uniform flow);
3. the channel geometry that includes the water area ($A$) and
4. the hydraulic radius ($R_h$);
5. the normal depth ($y_n$);
6. the normal discharge ($Q$); and
7. the mean velocity ($V$).

**TABLE 6.2** Typical Values of Manning's $n$

| Channel Surface | $n$ |
|---|---|
| Glass, PVC, HDPE | 0.010 |
| Smooth steel, metals | 0.012 |
| Concrete | 0.013 |
| Asphalt | 0.015 |
| Corrugated metal | 0.024 |
| Earth excavation, clean | 0.022–0.026 |
| Earth excavation, gravel and cobbles | 0.025–0.035 |
| Earth excavation, some weeds | 0.025–0.035 |
| Natural channels, clean and straight | 0.025–0.035 |
| Natural channels, stones or weeds | 0.030–0.040 |
| Riprap lined channel | 0.035–0.045 |
| Natural channels, clean and winding | 0.035–0.045 |
| Natural channels, winding, pools, shoals | 0.045–0.055 |
| Natural channels, weeds, debris, deep pools | 0.050–0.080 |
| Mountain streams, gravel and cobbles | 0.030–0.050 |
| Mountain streams, cobbles and boulders | 0.050–0.070 |

**Example 6.1**

A 3-m-wide rectangular irrigation channel carries a discharge of 25.3 $m^3$/sec at a uniform depth of 1.2 m. Determine the slope of the channel if Manning's coefficient is $n = 0.022$.

**Solution**

For a rectangular channel, the wetted perimeter and the hydraulic radius are

$$A = by = (3)(1.2) = 3.6 \text{ m}^2$$

$$P = b + 2y = 5.4 \text{ m}$$

$$R_h = \frac{A}{P} = \frac{3.6}{5.4} = \frac{2}{3} = 0.667 \text{ m}$$

Equation 6.5a can be rewritten as

$$S_0 = S_e = \left(\frac{Qn}{AR_h^{2/3}}\right)^2 = 0.041$$

# END OF LECTURE EIGHT