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Per unit System

Ey pow 1

The solution of an interconnected power system having several different voltage levels requires the cambersome transformation of all impedance to a single voltage level. However, power system engineers have devised the per-unit system such that the various physical quantities such a power, voltage, current and impedance are expressed as a decimal fraction or multiples of base quantities.

In this system, the different voltage levels disappear, and a power network involving generators, transformer and lines (of different voltage levels) reduces a system of simple impedance. The per unit value of any quantity is defined as

$$\text{Quantity in per unit} = \frac{\text{actual quantity}}{\text{base value quantity}} \quad \text{— (1)}$$

For example : $S_{pu} = \frac{S}{S_B}$; $V_{pu} = \frac{V}{V_B}$

$I_{pu} = \frac{I}{I_B}$; $Z_{pu} = \frac{Z}{Z_B}$

where S_{pu} : per unit apparent power

S : actual apparent power

S_B : Base " "

V_{pu} : per unit voltage

V : actual voltage

V_B : Base voltage

I_{pu} : per unit current

I : actual current

I_B : Base current

Z_{pu} : per unit impedance

Z : actual impedance

Z_B : Base impedance

where the numerator (actual values) are phasor quantities or complex values and the denominators (base values) are always real number

A minimum of four base quantities are required to completely define a per unit system: volt amper, voltage, current and impedance.

Usually, the 3-φ base volt-ampere S_B or MVA_B and the line-to-line base voltage V_B or kV_B are selected.

Base current and base impedance are then dependent on S_B and V_B and must obey the circuit law.

These are given by:

$$I_B = \frac{S_B}{\sqrt{3}\, V_{B\,Line}}$$

$$Z_B = \frac{V_B/\sqrt{3}}{I_B} \Rightarrow Z_B = \frac{V_B/\sqrt{3}}{\frac{S_B}{\sqrt{3}\, V_B}}$$

$$Z_B = \frac{V_B^{\,2}}{S_B} = \frac{KV_B^{\,2}}{MVA_B}$$

Then $S_{pu} = V_{pu} I_{pu}^{*}$

$$V_{pu} = Z_{pu} I_{pu}$$

The complex Load power $S_{L(3\phi)}$ can be defined a

$$S_{L(3\phi)} = 3 V_P I_P^{*}$$

I_P : Load current per pha
V_P : Load voltage per pha

$$I_P = \frac{V_P}{Z_P}$$

$$Z_P = \frac{V_P}{I_P} = \frac{V_P}{\left(\frac{S_{L(3\phi)}}{3V_P}\right)^{*}}$$

Impedance are calculate per phase

$$Z_P = \frac{V_P}{\frac{S_L(3\varphi)^*}{3V_P^*}} = \frac{3V_P * V_P^*}{S_L(3\varphi)^*}$$

$$Z_P = \frac{3|V_P|^2}{S_L^*(3\varphi)} = \frac{V_{Line}^2}{S_L^*(3\varphi)} = \frac{V_{LL}^2}{S_L^*(3\varphi)}$$

$$\therefore Z_P = \frac{V_{L-L}^2}{S^*_L(3\varphi)}$$

$$Z_{pu} = \frac{Z_P}{Z_B} = \frac{\frac{V_{L-L}^2}{S^*_L(3\varphi)}}{\frac{V_B^2}{S_B}} =$$

$$Z_{pu} = \frac{V_{L-L}^2}{V_B^2} * \frac{S_B}{S^*_L(3\varphi)}$$

$$Z_{pu} = \frac{V_{pu}^2}{S_L^*(pu)}$$

S_L = Load apparent power

* Change of Base

For power system analysis, all impedances must be expressed in per unit on a common system base. To accomplish this, an arbitrary base for apparent power is selected; for example, 100 MVA.

Then the voltage base must be selected.

Once a voltage base has been selected for a point in a system, the remaining voltage bases are no longer independent; they are determined by the various transformer turns ratios.

For example, if on a low-voltage side of $\frac{34.5\ kV}{115\ kV}$ transformer the base voltage of $\underline{\underline{36\ kV}}$ is selected.

The base voltage on the high-voltage side must be

$$36 * \frac{115}{34.5} = 120\ kV$$

where $\frac{N_2}{N_1} = \frac{V_{B2}}{V_{B1}}$

Normaly we try to select the voltage bases that are the same as the nominal values.

Let Z_{pu}^{old} be the per-unit impedance on the power base S_B^{old} and the voltage V_B^{old}

$$Z_{pu}^{old} = \frac{Z_\Omega}{Z_B^{old}} = Z_\Omega \frac{S_B^{old}}{(V_B^{old})^2}$$

Expressing Z_Ω to anew power base and anew voltage base, results in the new perunit impedance

$$Z_{pu}^{new} = \frac{Z_\Omega}{Z_B^{new}} = Z_\Omega \frac{S_B^{new}}{(V_B^{new})^2}$$

$$Z_{pu}^{new} = \frac{Z_{pu}^{old}}{\frac{S_B^{old}}{(V_B^{old})^2}} \times \frac{S_B^{new}}{(V_B^{new})^2} =$$

$$Z_{pu}^{new} = Z_{pu}^{old} \cdot \left(\frac{S_B^{new}}{S_B^{old}}\right)\left(\frac{V_B^{old}}{V_B^{new}}\right)^2$$

* The advantage of per-unit system.

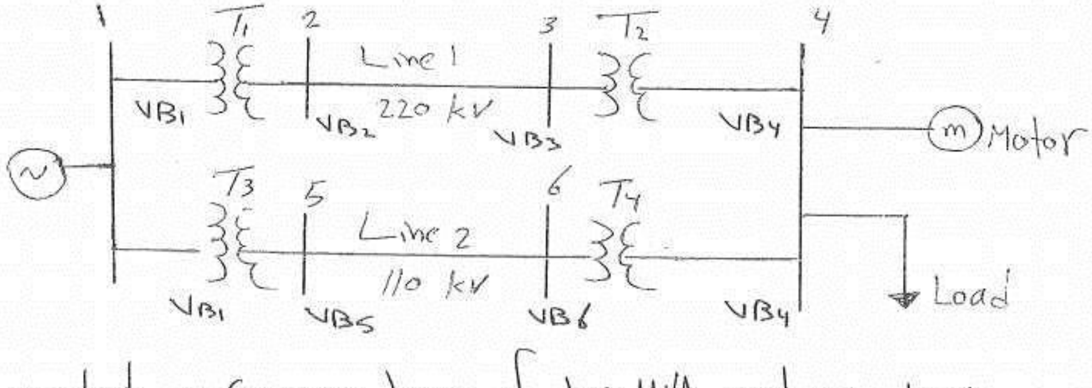
1- The per-unit system gives us a clear idea of relative magnitudes of various quantities, such as voltage, current, power and impedance.

2- The per-unit impedance of equipment of the same general type based on their own ratings fall in a narrow range regardless of rating of the equibment. Whereas their impedeance in ohms vary greatly with the rating.

3- The per-unit values of impedance, voltage and current of a transformer are the same regardless of whether they are referred to the primary or the secondary side. This is a great advantage since the different voltage levels disappear and the entire system reduces to a system of simple impedance

4- The perunit system are ideal for the computerized analysis and simulation of complex power system problem.

5- The cct law are valied in per unit system and the power and the voltage equations are simplified since the factor $\sqrt{3}$ and 3 are eliminated in per unit ex1) The one-line diagram of a 3-φ power system is shown in the fig. below:-



select a Common base of 100 MVA and 22 kV on the generator side.

Draw an impedance diagram in perunit.

The data for each device is given below:-

G :	90 MVA	22 kV	X = 18 %
T_1 :	50 MVA	22/220 kV	X = 10 %
T_2 :	40 MVA	220/11 kV	X = 6 %
T_3 :	40 MVA	22/110 kV	X = 6.4 %
T_4 :	40 MVA	110/11 kV	X = 8 %
Motor:	66.5 MVA	10.45 kV	X = 18.5 % 0.8 Lag. P.f

The 3-φ load at bus 4 obsorbs 57 MVA, 0.6 P.f Lag. at 10.45 kV. Line 1 and Line 2 have a reactance of 48.4 and 65.43 Ω respectively.

Sol.

- The voltage bases must be determine for all sections of the network.

- The generator rated voltage is given at the base voltage at bus 1

$$V_{B1} = V_{G1} = 22\ kV$$

$$V_{B2} = 22 * \left(\frac{220}{22}\right) = 220\ kV$$

where

$$\frac{N_2}{N_1} = \frac{V_{B2}}{V_{B1}}$$

$$\frac{220}{22} = \frac{V_{B2}}{22} \Longrightarrow V_{B2} = 22 * \left(\frac{220}{22}\right) = 22\ kV$$

$$V_{B2} = V_{B3} = 220 kV$$

$$V_{B4} = 220\left(\frac{11}{220}\right) = 11\ kV$$

$$V_{B3} = 22\left(\frac{110}{22}\right) = 110\ kV$$

$$V_{B6} = V_{B3} = 110\ kV$$

$$V_{B4} = 110 * \left(\frac{11}{110}\right) = 11\ kV$$

$$X_{new} = X_{old}\left(\frac{S_B^{new}}{S_B^{old}}\right)\left(\frac{V_B^{old}}{V_{B new}}\right)^2$$

$$X_{G1} = 0.18\left(\frac{100}{90}\right)\left(\frac{\cancel{22}}{22}\right)^2 = 0.2\ pu$$

$$X_{T_1} = 0.1 * \left(\frac{100}{50}\right)\left(\frac{\cancel{22}}{\cancel{22}}\right)^2 = 0.2 \text{ pu}$$ { on primary side of Transformer 1 i.e: Low voltage side.

OR

$$X_{T_1} = 0.1 * \left(\frac{100}{50}\right)\left(\frac{\cancel{220}}{\cancel{220}}\right)^2 = 0.2 \text{ pu}$$ { on secondary side of Transformer 1 i.e: High voltage side.

$$X_{T_2} = 0.6 * \left(\frac{100}{40}\right)\left(\frac{\cancel{220}}{\cancel{220}}\right)^2 = 0.15 \text{ pu}$$ { on primary side of Transformer 2 i.e: Hig voltage side.

OR

$$X_{T_2} = 0.6 * \left(\frac{100}{40}\right)\left(\frac{\cancel{11}}{\cancel{11}}\right)^2 = 0.15 \text{ pu}$$ { on secondary side of Transformer 2 i.e: Low voltage side

$$X_{T_3} = 0.064 \left(\frac{100}{40}\right)\left(\frac{22}{22}\right)^2 = 0.16 \text{ pu}$$ { on primary side of Transformer 3 i.e: Low voltage side

OR

$$X_{T_3} = 0.064 \left(\frac{100}{40}\right)\left(\frac{110}{110}\right)^2 = 0.16 \text{ pu}$$ { on secondary side of Transformer 3

$$X_{T4} = 0.08\left(\frac{100}{40}\right)\left(\frac{110}{110}\right)^2 = 0.2\ pu$$ { on primary side of Transformer 4 i.e: High Voltage side.

OR

$$X_{T4} = 0.08\left(\frac{100}{40}\right)\left(\frac{110}{110}\right)^2 = 0.2\ pu$$ { on Secondary side of Transformer 4 i.e: Low Voltage side.

$$X_{motor} = 0.185\left(\frac{100}{65.5}\right)\left(\frac{10.45}{11}\right)^2 = 0.2\,pu$$

$$Z_{B_{Line 1}} = \frac{V_{B Line 1}^2}{S_B} = \frac{(V_{B2} \text{ or } V_{B3})^2}{S_B} = \frac{220^2}{100} = 484\ \Omega$$

$$Z_{B_{Line 2}} = \frac{V_{B Line 2}^2}{S_B} = \frac{(V_{B5} \text{ or } V_{B6})^2}{S_B} = \frac{110^2}{100} = 121\ \Omega$$

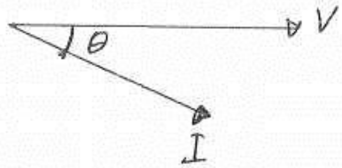
$$X_{Line 1_{pu}} = \frac{\text{actual}}{\text{Base}} = \frac{48.4}{484} = 0.1\ pu = j0.1\ pu$$

$$X_{Line 2_{pu}} = \frac{\text{actual}}{\text{Base}} = \frac{65.43}{121} = 0.54\ pu = j0.54\ pu$$

$$S_{Load_{3\varphi}} = 57\angle 53.13^\circ\ MVA \quad \{\ 0.6 \text{ Lag. P.F}\ \}$$

where :-

Lag. P.F { inductive Load }



$$V = V\angle 0 \quad ; \quad I = I\angle -\theta$$

$$S_{3\phi} = \sqrt{3}\, V_{Line} I^{*}_{Line} \quad \text{or} \quad 3 V_P I_P^{*}$$

$$S_{3\phi} = \sqrt{3}\, V\, (I\angle -\theta)^{*}$$

$$= \sqrt{3}\, V I \angle \theta = \sqrt{3}\,(VI\angle\theta)$$

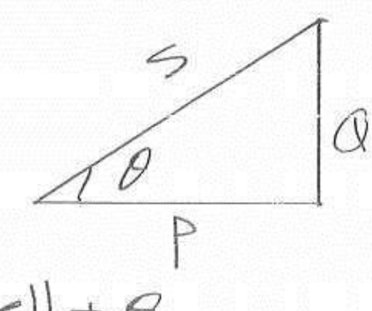
$$= \sqrt{3}\,(VI\cos\theta + j\, VI\sin\theta)$$

$$= \sqrt{3}\, VI\cos\theta + j\sqrt{3}\, VI\sin\theta$$

$$= P + jQ$$

wher $P = \sqrt{3}\, VI\cos\theta$

$Q = \sqrt{3}\, VI\sin\theta$



Lag. P.F

$$S = |S|\angle +\theta$$

Lag p.f $\Longrightarrow$

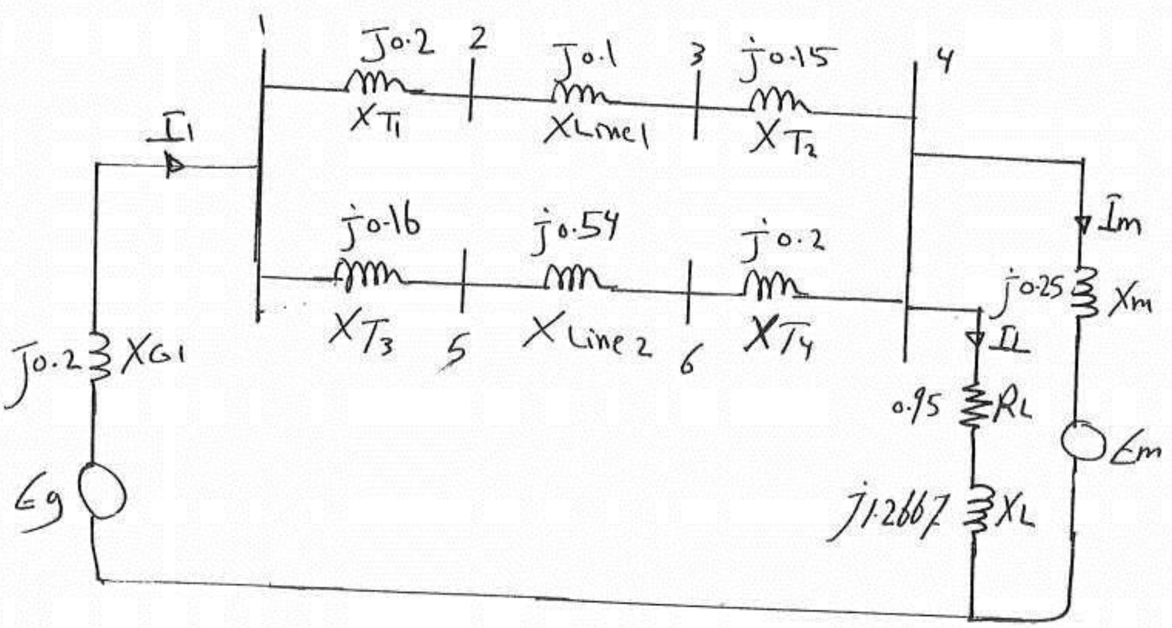
$$S_{3\phi\,\text{Load}} = 57\angle{+53.13^\circ}\ \text{MVA}$$

$$Z_{\text{Load actual}} = \frac{V_{L\text{-}L\ \text{kV}}^2}{S^*_{\text{Load}\ 3\phi\ \text{MVA}}} = \frac{10.45^2\ \text{kV}}{57\angle{-53.13}\ \text{MVA}} = 1.1495 + j1.1532\ \Omega$$

$$Z_{B\,\text{Load}} = \frac{V_{B\,\text{Load}}^2}{S_B} = \frac{11^2\ \text{kV}}{100\ \text{MVA}} = 1.21\ \Omega$$

$$Z_{\text{Load pu}} = \frac{\text{actual}}{\text{Base}} = \frac{1.1495 + j1.532}{1.21} = 0.95 + j1.2667\ \text{pu}$$

The per unit equivalent cct is shown in fig. below



ex2) If the motor in ex1 operate at full-Load 0.8 p.f Lag leading at terminal voltage of 10.45 kV.

1- Determine the voltage at the generator bus bar (bus 1)

2- Determine the generator and the motor interna emfs.

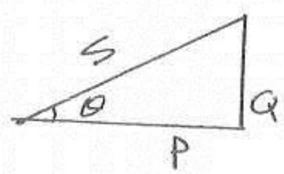
Sol)

The per-unit voltage at bus 4, taken as reference is

$$V_{4\,pu} = \frac{\text{actual}}{\text{Base}} = \frac{10.45}{11} = 0.95\angle 0 \ \text{pu}$$

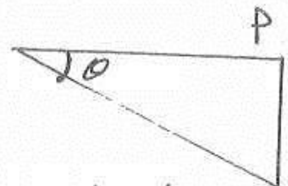
The apparent power of motor at 0.8 Lead p.f

$$S_m = \frac{\text{actual}}{\text{Base}} = \frac{66.5\angle -36.8}{100} = 0.665\angle -36.87 \ \text{pu}$$



Lag p.f

$S = S\angle +\theta$



lead p.f

$S = S\angle -\theta$

$S_{pu} = V_{pu} \, I_{pu}^{*}$

The current drawn by the motor is

$\bar{I}_m = \dfrac{S_m^{*}}{V_m^{*}}$. $\{$ where $S_m = V_m \, \bar{I}_m^{*}$

$\bar{I}_m = \dfrac{0.665 \angle +36.87}{0.95 \angle 0} = 0.56 + j0.42$ pu

The current drawn by the Load is

$\bar{I}_L = \dfrac{V_4}{Z_L} = \dfrac{0.95 \angle 0}{0.95 + j1.2667} = 0.36 - j0.48$ pu

The current drawn from bus 4

$I = \bar{I}_m + \bar{I}_L = \bar{I}_1$ $\{$ current of G $]$

$= (0.56 + j0.42) + (0.36 - j0.48) = 0.92 - j0.06$ pu

The equivalent reactance of the parallel branches is

$X = \dfrac{0.45 * 0.9}{0.45 + 0.9} = 0.3$ pu

The generator terminal voltage

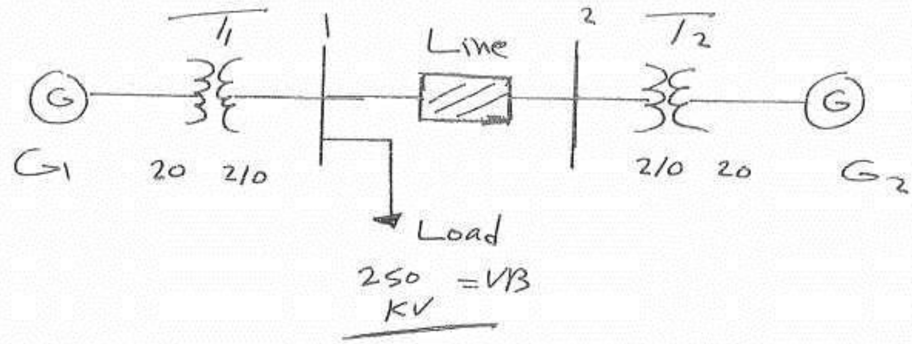
$V_1 = V_4 + ZI$

$= 0.95 \angle 0 + j0.3(0.92 - j0.06) = 0.968 + j0.276$

$= 1 \angle 15.91^\circ$

$- 1 * 22\,kV = 22 \angle 15.91^\circ\,kV$ =

ex3)



G_1	90 MVA	20 kV	$X = 0.09$ pu
T_1	80 MVA	20/210	$X = 0.16$ pu
T_2	80 MVA	210/20	$X = 0.2$ pu
G_2	90 MVA	18 kV	$X = 0.06$ pu
Line	210 kV		$X = 120\ \Omega$
Load	210 kV		$S = 48 + j64$

Choose base MVA as 100 MVA and base voltage of 250 kV at the Load bus

Sol) The base Voltage = 250 kV at the Load bus bar

$V_{B\,Load} = 250$ kV

V_{B2} at bus 2 = 250 kV

$$V_{B\,G_2} = 250 * \left(\frac{20}{210}\right) = 23.8 \text{ kV}$$

$$V_{BG1} = 250 * \left(\frac{20}{210}\right) = 23.8 \text{ kV}$$

$$X_{G1_{new}} = X_{old}\left(\frac{S_B^{new}}{S_B^{old}}\right)\left(\frac{V_B^{old}}{V_B^{new}}\right)^2$$

$$X_{G1} = 0.09\left(\frac{100}{90}\right)\left(\frac{20}{23.8}\right)^2 = 0.0706 \text{ pu}$$

$$X_{G2} = 0.06\left(\frac{100}{90}\right)\left(\frac{18}{23.8}\right)^2 = 0.038 \text{ pu}$$

$$X_{T1} = 0.16\left(\frac{100}{80}\right)\left(\frac{20}{23.8}\right)^2 = 0.141 \text{ pu} \quad \left\{\text{on primary side}\right.$$

OR

$$X_{T1} = 0.16\left(\frac{100}{80}\right)\left(\frac{210}{250}\right)^2 = 0.141 \text{pu} \quad \left\{\text{on secondar. side}\right.$$

$$X_{T2} = 0.2 * \left(\frac{100}{80}\right)\left(\frac{210}{250}\right)^2 = 0.176 \text{ pu} \quad \left\{\text{on primary side}\right\}$$

OR

$$X_{T2} = 0.2 * \left(\frac{100}{80}\right)\left(\frac{20}{23.8}\right)^2 = 0.176 \text{ pu} \quad \left\{\text{on secondary side}\right.$$

$$X_{B\,Line} = \frac{V_B^2\,Line}{S_B} = \frac{250^2}{100} = 625\ \Omega$$

$$X_{pu\,Line} = \frac{actual}{base} = \frac{120}{625} = 0.192 \text{ pu}$$

$$X_{\substack{\text{Load}\\ \text{actual}}} = \frac{V_{LL}^2}{S_{3\varphi}^*} = \frac{210^2}{48 - j64} = 378 + j504 \ \Omega$$

$$X_{\substack{\text{Load}\\ B}} = \frac{V_{B\,Load}^2}{S_B} = \frac{250^2}{100} = 625$$

$$X_{pu}\,\text{Load} = \frac{\text{actual}}{\text{Base}} = \frac{378 + j504}{625} = 0.6048 + j0.806 \text{ pu}$$