CHAPTER

# 2

# Steady-State Conduction—One Dimension

## 2-1 | INTRODUCTION

We now wish to examine the applications of Fourier's law of heat conduction to calculation of heat flow in some simple one-dimensional systems. Several different physical shapes may fall in the category of one-dimensional systems: cylindrical and spherical systems are one-dimensional when the temperature in the body is a function only of radial distance and is independent of azimuth angle or axial distance. In some two-dimensional problems the effect of a second-space coordinate may be so small as to justify its neglect, and the multidimensional heat-flow problem may be approximated with a one-dimensional analysis. In these cases the differential equations are simplified, and we are led to a much easier solution as a result of this simplification.

## 2-2 | THE PLANE WALL

First consider the plane wall where a direct application of Fourier's law [Equation (1-1)] may be made. Integration yields

$$q = -\frac{kA}{\Delta x}(T_2 - T_1) \qquad \textbf{[2-1]}$$

when the thermal conductivity is considered constant. The wall thickness is $\Delta x$, and $T_1$ and $T_2$ are the wall-face temperatures. If the thermal conductivity varies with temperature according to some linear relation $k = k_0(1 + \beta T)$, the resultant equation for the heat flow is

$$q = -\frac{k_0 A}{\Delta x}\left[(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2)\right] \qquad \textbf{[2-2]}$$

If more than one material is present, as in the multilayer wall shown in Figure 2-1, the analysis would proceed as follows: The temperature gradients in the three materials are shown, and the heat flow may be written

$$q = -k_A A\frac{T_2 - T_1}{\Delta x_A} = -k_B A\frac{T_3 - T_2}{\Delta x_B} = -k_C A\frac{T_4 - T_3}{\Delta x_C}$$

Note that the heat flow must be the same through all sections.

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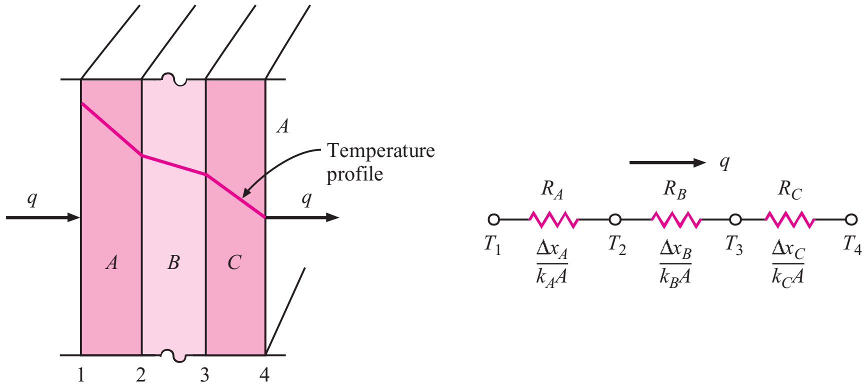
**Figure 2-1** | One-dimensional heat transfer through a composite wall and electrical analog.




Solving these three equations simultaneously, the heat flow is written

$$q = \frac{T_1 - T_4}{\Delta x_A/k_A A + \Delta x_B/k_B A + \Delta x_C/k_C A} \qquad \textbf{[2-3]}$$

At this point we retrace our development slightly to introduce a different conceptual viewpoint for Fourier's law. The heat-transfer rate may be considered as a flow, and the combination of thermal conductivity, thickness of material, and area as a resistance to this flow. The temperature is the potential, or driving, function for the heat flow, and the Fourier equation may be written

$$\text{Heat flow} = \frac{\text{thermal potential difference}}{\text{thermal resistance}} \qquad \textbf{[2-4]}$$

a relation quite like Ohm's law in electric-circuit theory. In Equation (2-1) the thermal resistance is $\Delta x/kA$, and in Equation (2-3) it is the sum of the three terms in the denominator. We should expect this situation in Equation (2-3) because the three walls side by side act as three thermal resistances in series. The equivalent electric circuit is shown in Figure 2-1*b*.

The electrical analogy may be used to solve more complex problems involving both series and parallel thermal resistances. A typical problem and its analogous electric circuit are shown in Figure 2-2. The one-dimensional heat-flow equation for this type of problem may be written

$$q = \frac{\Delta T_{\text{overall}}}{\sum R_{\text{th}}} \qquad \textbf{[2-5]}$$

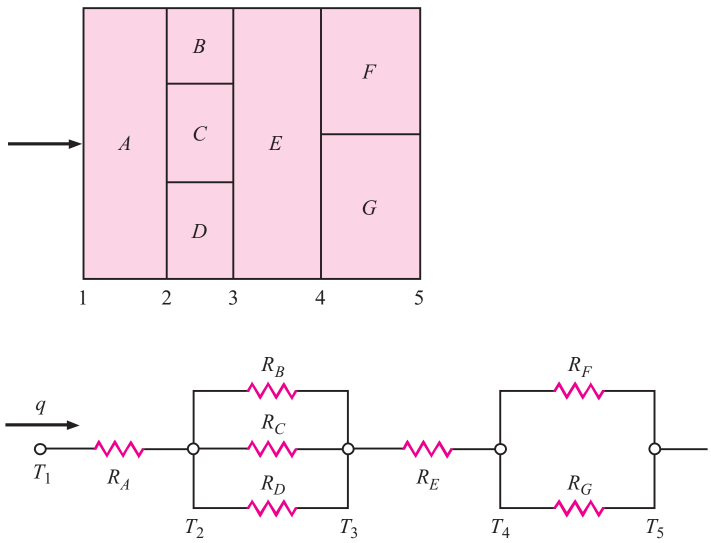
where the $R_{\text{th}}$ are the thermal resistances of the various materials. The units for the thermal resistance are °C/W or °F · h/Btu.

It is well to mention that in some systems, like that in Figure 2-2, two-dimensional heat flow may result if the thermal conductivities of materials $B$, $C$, and $D$ differ by an appreciable amount. In these cases other techniques must be employed to effect a solution.

## 2-3 | INSULATION AND *R* VALUES

In Chapter 1 we noted that the thermal conductivities for a number of insulating materials are given in Appendix A. In classifying the performance of insulation, it is a common practice

**Figure 2-2** | Series and parallel one-dimensional heat transfer through a composite wall and electrical analog.




in the building industry to use a term called the *R value,* which is defined as

$$R = \frac{\Delta T}{q/A} \tag{2-6}$$

The units for $R$ are °C · m$^2$/W or °F · ft$^2$ · h/Btu. Note that this differs from the thermal-resistance concept discussed above in that a heat flow *per unit area* is used.

At this point it is worthwhile to classify insulation materials in terms of their application and allowable temperature ranges. Table 2-1 furnishes such information and may be used as a guide for the selection of insulating materials.

## 2-4 | RADIAL SYSTEMS

### Cylinders

Consider a long cylinder of inside radius $r_i$, outside radius $r_o$, and length $L$, such as the one shown in Figure 2-3. We expose this cylinder to a temperature differential $T_i - T_o$ and ask what the heat flow will be. For a cylinder with length very large compared to diameter, it may be assumed that the heat flows only in a radial direction, so that the only space coordinate needed to specify the system is $r$. Again, Fourier's law is used by inserting the proper area relation. The area for heat flow in the cylindrical system is

$$A_r = 2\pi r L$$

so that Fourier's law is written

$$q_r = -kA_r \frac{dT}{dr} \tag{2-7}$$

or

$$q_r = -2\pi k r L \frac{dT}{dr}$$

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**Table 2-1** | Insulation types and applications.

| | Type | Temperature range, °C | Thermal conductivity, mW/m · °C | Density, kg/m$^3$ | Application |
|---|---|---|---|---|---|
| 1 | Linde evacuated superinsulation | −240–1100 | 0.0015–0.72 | Variable | Many |
| 2 | Urethane foam | −180–150 | 16–20 | 25–48 | Hot and cold pipes |
| 3 | Urethane foam | −170–110 | 16–20 | 32 | Tanks |
| 4 | Cellular glass blocks | −200–200 | 29–108 | 110–150 | Tanks and pipes |
| 5 | Fiberglass blanket for wrapping | −80–290 | 22–78 | 10–50 | Pipe and pipe fittings |
| 6 | Fiberglass blankets | −170–230 | 25–86 | 10–50 | Tanks and equipment |
| 7 | Fiberglass preformed shapes | −50–230 | 32–55 | 10–50 | Piping |
| 8 | Elastomeric sheets | −40–100 | 36–39 | 70–100 | Tanks |
| 9 | Fiberglass mats | 60–370 | 30–55 | 10–50 | Pipe and pipe fittings |
| 10 | Elastomeric preformed shapes | −40–100 | 36–39 | 70–100 | Pipe and fittings |
| 11 | Fiberglass with vapor barrier blanket | −5–70 | 29–45 | 10–32 | Refrigeration lines |
| 12 | Fiberglass without vapor barrier jacket | to 250 | 29–45 | 24–48 | Hot piping |
| 13 | Fiberglass boards | 20–450 | 33–52 | 25–100 | Boilers, tanks, heat exchangers |
| 14 | Cellular glass blocks and boards | 20–500 | 29–108 | 110–150 | Hot piping |
| 15 | Urethane foam blocks and boards | 100–150 | 16–20 | 25–65 | Piping |
| 16 | Mineral fiber preformed shapes | to 650 | 35–91 | 125–160 | Hot piping |
| 17 | Mineral fiber blankets | to 750 | 37–81 | 125 | Hot piping |
| 18 | Mineral wool blocks | 450–1000 | 52–130 | 175–290 | Hot piping |
| 19 | Calcium silicate blocks, boards | 230–1000 | 32–85 | 100–160 | Hot piping, boilers, chimney linings |
| 20 | Mineral fiber blocks | to 1100 | 52–130 | 210 | Boilers and tanks |

**Figure 2-3** | One-dimensional heat flow through a hollow cylinder and electrical analog.

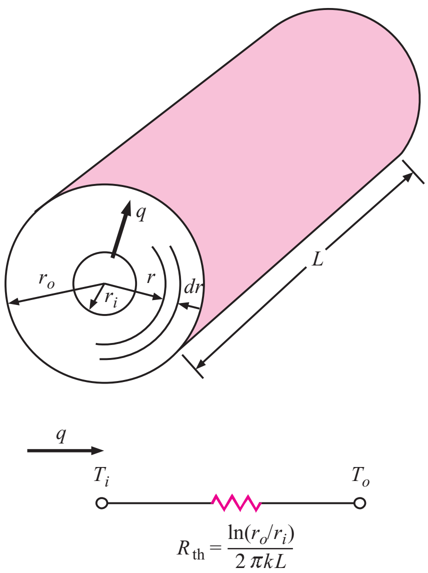
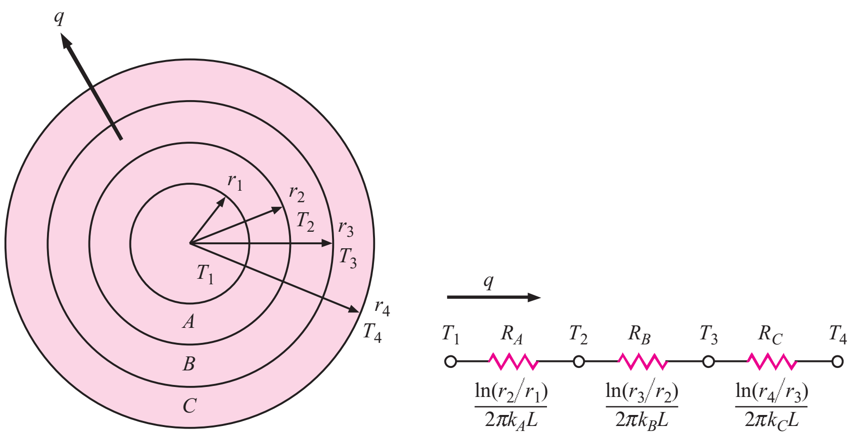


**Figure 2-4** | One-dimensional heat flow through multiple cylindrical sections and electrical analog.




with the boundary conditions

$$T = T_i \qquad \text{at } r = r_i$$
$$T = T_o \qquad \text{at } r = r_o$$

The solution to Equation (2-7) is

$$q = \frac{2\pi k L\,(T_i - T_o)}{\ln\,(r_o/r_i)} \qquad \textbf{[2-8]}$$

and the thermal resistance in this case is

$$R_{\text{th}} = \frac{\ln\,(r_o/r_i)}{2\pi k L}$$

The thermal-resistance concept may be used for multiple-layer cylindrical walls just as it was used for plane walls. For the three-layer system shown in Figure 2-4 the solution is

$$q = \frac{2\pi L\,(T_1 - T_4)}{\ln\,(r_2/r_1)/k_A + \ln\,(r_3/r_2)/k_B + \ln\,(r_4/r_3)/k_C} \qquad \textbf{[2-9]}$$

The thermal circuit is also shown in Figure 2-4.

## Spheres

Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only. The heat flow is then

$$q = \frac{4\pi k\,(T_i - T_o)}{1/r_i - 1/r_o} \qquad \textbf{[2-10]}$$

The derivation of Equation (2-10) is left as an exercise.

### Multilayer Conduction — EXAMPLE 2-1

An exterior wall of a house may be approximated by a 4-in layer of common brick [$k = 0.7$ W/m · °C] followed by a 1.5-in layer of gypsum plaster [$k = 0.48$ W/m · °C]. What thickness of loosely packed rock-wool insulation [$k = 0.065$ W/m · °C] should be added to reduce the heat loss (or gain) through the wall by 80 percent?

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### Solution

The overall heat loss will be given by

$$q = \frac{\Delta T}{\sum R_{\text{th}}}$$

Because the heat loss with the rock-wool insulation will be only 20 percent (80 percent reduction) of that before insulation

$$\frac{q \text{ with insulation}}{q \text{ without insulation}} = 0.2 = \frac{\sum R_{\text{th}} \text{ without insulation}}{\sum R_{\text{th}} \text{ with insulation}}$$

We have for the brick and plaster, for unit area,

$$R_b = \frac{\Delta x}{k} = \frac{(4)(0.0254)}{0.7} = 0.145 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

$$R_p = \frac{\Delta x}{k} = \frac{(1.5)(0.0254)}{0.48} = 0.079 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

so that the thermal resistance without insulation is

$$R = 0.145 + 0.079 = 0.224 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

Then

$$R \text{ with insulation} = \frac{0.224}{0.2} = 1.122 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

and this represents the sum of our previous value and the resistance for the rock wool

$$1.122 = 0.224 + R_{rw}$$

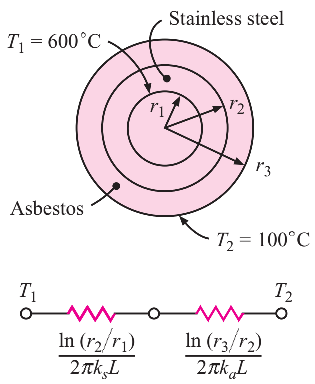
$$R_{rw} = 0.898 = \frac{\Delta x}{k} = \frac{\Delta x}{0.065}$$

so that

$$\Delta x_{rw} = 0.0584 \text{ m} = 2.30 \text{ in}$$

## EXAMPLE 2-2 Multilayer Cylindrical System

**Figure Example 2-2**




A thick-walled tube of stainless steel [18% Cr, 8% Ni, $k = 19$ W/m · °C] with 2-cm inner diameter (ID) and 4-cm outer diameter (OD) is covered with a 3-cm layer of asbestos insulation [$k = 0.2$ W/m · °C]. If the inside wall temperature of the pipe is maintained at 600°C, calculate the heat loss per meter of length. Also calculate the tube–insulation interface temperature.

### Solution

Figure Example 2-2 shows the thermal network for this problem. The heat flow is given by

$$\frac{q}{L} = \frac{2\pi(T_1 - T_2)}{\ln(r_2/r_1)/k_s + \ln(r_3/r_2)/k_a} = \frac{2\pi(600 - 100)}{(\ln 2)/19 + (\ln \frac{5}{2})/0.2} = 680 \text{ W/m}$$

This heat flow may be used to calculate the interface temperature between the outside tube wall and the insulation. We have

$$\frac{q}{L} = \frac{T_a - T_2}{\ln(r_3/r_2)/2\pi k_a} = 680 \text{ W/m}$$

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where $T_a$ is the interface temperature, which may be obtained as

$$T_a = 595.8°\text{C}$$

The largest thermal resistance clearly results from the insulation, and thus the major portion of the temperature drop is through that material.

### Convection Boundary Conditions

We have already seen in Chapter 1 that convection heat transfer can be calculated from

$$q_{\text{conv}} = hA\,(T_w - T_\infty)$$

An electric-resistance analogy can also be drawn for the convection process by rewriting the equation as

$$q_{\text{conv}} = \frac{T_w - T_\infty}{1/hA} \qquad \textbf{[2-11]}$$

where the $1/hA$ term now becomes the convection resistance.

## 2-5 | THE OVERALL HEAT-TRANSFER COEFFICIENT

Consider the plane wall shown in Figure 2-5 exposed to a hot fluid $A$ on one side and a cooler fluid $B$ on the other side. The heat transfer is expressed by

$$q = h_1 A\,(T_A - T_1) = \frac{kA}{\Delta x}\,(T_1 - T_2) = h_2 A\,(T_2 - T_B)$$

The heat-transfer process may be represented by the resistance network in Figure 2-5, and the overall heat transfer is calculated as the ratio of the overall temperature difference to the sum of the thermal resistances:

$$q = \frac{T_A - T_B}{1/h_1 A + \Delta x/kA + 1/h_2 A} \qquad \textbf{[2-12]}$$

**Figure 2-5** | Overall heat transfer through a plane wall.

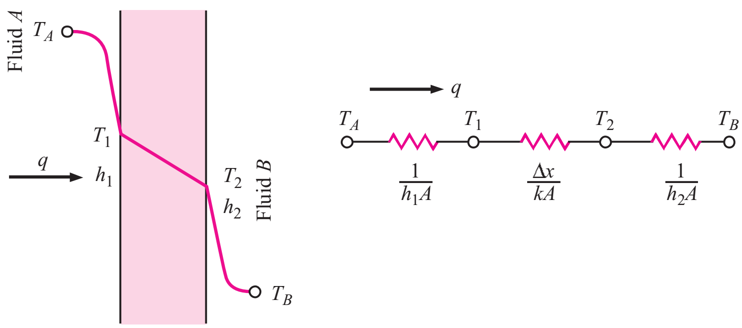

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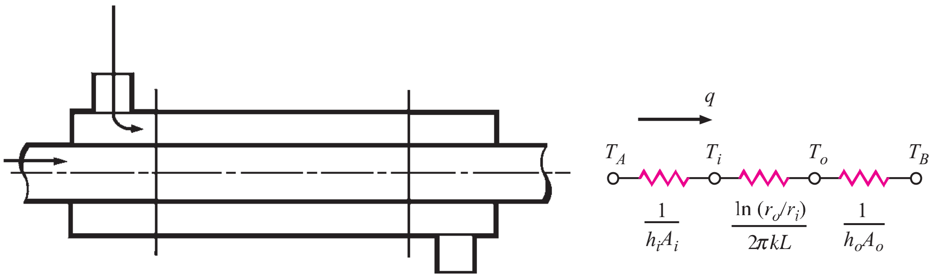
**Figure 2-6** | Resistance analogy for hollow cylinder with convection boundaries.




Observe that the value $1/hA$ is used to represent the convection resistance. The overall heat transfer by combined conduction and convection is frequently expressed in terms of an overall heat-transfer coefficient $U$, defined by the relation

$$q = UA\,\Delta T_{\text{overall}} \tag{2-13}$$

where $A$ is some suitable area for the heat flow. In accordance with Equation (2-12), the overall heat-transfer coefficient would be

$$U = \frac{1}{1/h_1 + \Delta x/k + 1/h_2}$$

The overall heat-transfer coefficient is also related to the $R$ value of Equation (2-6) through

$$U = \frac{1}{R \text{ value}}$$

For a hollow cylinder exposed to a convection environment on its inner and outer surfaces, the electric-resistance analogy would appear as in Figure 2-6 where, again, $T_A$ and $T_B$ are the two fluid temperatures. Note that the area for convection is not the same for both fluids in this case, these areas depending on the inside tube diameter and wall thickness. The overall heat transfer would be expressed by

$$q = \frac{T_A - T_B}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h_o A_o}} \tag{2-14}$$

in accordance with the thermal network shown in Figure 2-6. The terms $A_i$ and $A_o$ represent the inside and outside surface areas of the inner tube. The overall heat-transfer coefficient may be based on either the inside or the outside area of the tube. Accordingly,

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{A_i \ln(r_o/r_i)}{2\pi k L} + \dfrac{A_i}{A_o}\dfrac{1}{h_o}} \tag{2-15}$$

$$U_o = \frac{1}{\dfrac{A_o}{A_i}\dfrac{1}{h_i} + \dfrac{A_o \ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h_o}} \tag{2-16}$$

The general notion, for either the plane wall or cylindrical coordinate system, is that

$$UA = 1/\Sigma R_{\text{th}} = 1/R_{\text{th,overall}}$$

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Calculations of the convection heat-transfer coefficients for use in the overall heat-transfer coefficient are made in accordance with the methods described in later chapters. Some typical values of the overall heat-transfer coefficient for heat exchangers are given in Table 10-1. Some values of $U$ for common types of building construction system are given in Table 2-2 and may be employed for calculations involving the heating and cooling of buildings.

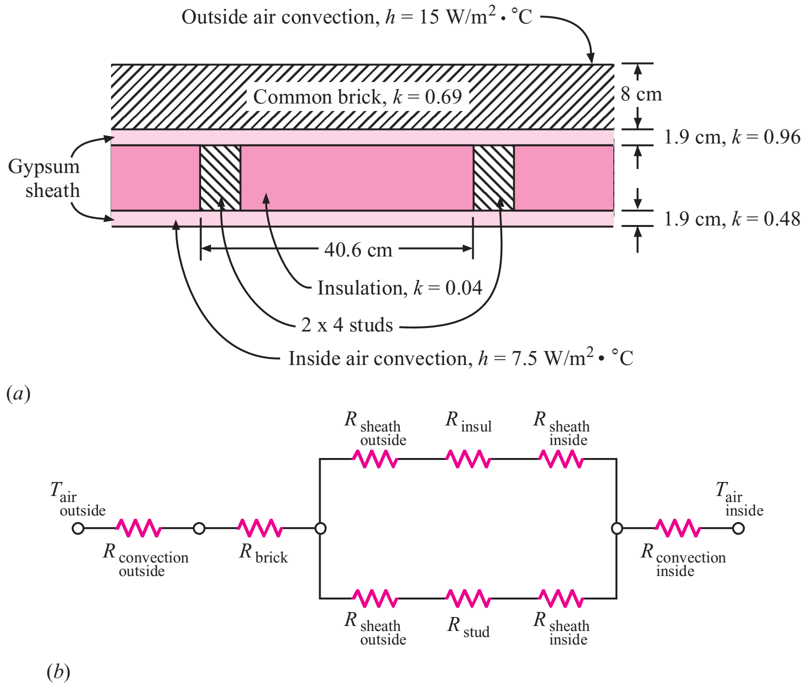
**Table 2-2** | Overall heat transfer coefficients for common construction systems according to James and Goss [12].

| | Description of construction system | $U$, Btu/hr · ft$^2$ · °F | $U$, W/m$^2$ · °C |
|---|---|---|---|
| 1 | 2 × 3 in double-wood stud wall, 406 mm OC, polyisocyanurate (0.08-mm vapor retarder, 19-mm insulation), fiberglass batts in cavity, 12.7-mm plywood | 0.027 | 0.153 |
| 2 | 2 × 4 in wood stud wall, 406 mm OC, polyisocyanurate foil-faced, fiberglass batts in cavity, 15-mm plywood | 0.060 | 0.359 |
| 3 | 2 × 4 in wood stud wall, 406 mm OC, 38-mm polyisocyanurate, foil-faced, cellular polyurethane in cavity, 19-mm plywood | 0.039 | 0.221 |
| 4 | 2 × 4 in wood stud wall, 406 mm OC, 15-mm exterior sheathing, 0.05-mm polyethylene vapor barrier, no fill in cavity | 0.326 | 1.85 |
| 5 | Nominal 4-in concrete-block wall with brick facade and extruded polystyrene insulation | 0.080 | 0.456 |
| 6 | 2 × 4 in wood stud wall, 406 mm OC, fiberglass batt insulation in cavity, 16-mm plywood | 0.084 | 0.477 |
| 7 | 2 × 4 in wood stud wall, 406 mm OC, fiberglass batt insulation in cavity, 16-mm plywood, clay brick veneer | 0.060 | 0.341 |
| 8 | 2 × 4 in wood stud wall, 406 mm OC, fiberglass batt in cavity, 13-mm plywood, aluminum or vinyl siding | 0.074 | 0.417 |
| 9 | 2 × 4 in wood stud wall, 406 mm OC, polyurethane foam in cavity, extruded polystyrene sheathing, aluminum siding | 0.040 | 0.228 |
| 10 | 2 × 4 in steel stud wall, 406 mm OC, fiberglass batts in cavity, 41-mm air space, 13-mm plaster board | 0.122 | 0.691 |
| 11 | Aluminum motor home roof with fiberglass insulation in cavity (32 mm) | 0.072 | 0.41 |
| 12 | 2 × 6 in wood stud ceiling, 406 mm OC, fiberglass foil-faced insulation in cavity, reflective airspace ($\varepsilon \approx 0.05$) | 0.065 | 0.369 |
| 13 | 8-in (203-mm) normal-weight structural concrete ($\rho = 2270$ kg/m$^3$) wall, 18-mm board insulation, painted off-white | 0.144 | 0.817 |
| 14 | 10-in (254-mm) concrete-block-brick cavity wall, no insulation in cavities | 0.322 | 1.83 |
| 15 | 8-in (203-mm) medium-weight concrete block wall, perlite insulation in cores | 0.229 | 1.3 |
| 16 | 8-in (203-mm) normal-weight structural concrete, ($\rho = 2270$ kg/m$^3$) including steel reinforcement bars (Note: actual thickness of concrete is 211 mm.) | 0.764 | 4.34 |
| 17 | 8-in (203-mm) lightweight structural concrete ($\rho = 1570$ kg/m$^3$) including steel reinforcement bars (Note: Actual thickness of concrete is 210 mm.) | 0.483 | 2.75 |
| 18 | 8-in (203-mm) low-density concrete wall ($\rho = 670$ kg/m$^3$) including steel reinforcement bars (Note: Actual thickness of concrete is 216 mm.) | 0.216 | 1.23 |
| 19 | Corrugated sheet steel wall with 10.2-in (260-mm.) fiberglass batt in cavity | 0.030 | 0.17 |
| 20 | Corrugated sheet steel wall with (159-mm) fiberglass batt in cavity | 0.054 | 0.31 |
| 21 | Metal building roof deck, 25 mm polyisocyanurate, foil-faced ($\varepsilon \approx 0.03$), 203-mm reflective air space | 0.094 | 0.535 |
| 22 | Metal building roof deck, 25-mm foil-faced polyisocyanurate, 38-mm fiberglass batts in cavity | 0.065 | 0.366 |

## EXAMPLE 2-3 Heat Transfer Through a Composite Wall

"Two-by-four" wood studs have actual dimensions of $4.13 \times 9.21$ cm and a thermal conductivity of 0.1 W/m · °C. A typical wall for a house is constructed as shown Figure Example 2-3. Calculate the overall heat-transfer coefficient and $R$ value of the wall.

**Figure Example 2-3** | (*a*) Construction of a dwelling wall; (*b*) thermal resistance model.




### ■ Solution

The wall section may be considered as having two parallel heat-flow paths: (1) through the studs, and (2) through the insulation. We will compute the thermal resistance for each, and then combine the values to obtain the overall heat-transfer coefficient.

**1.** *Heat transfer through studs* ($A = 0.0413$ m$^2$ for unit depth). This heat flow occurs through six thermal resistances:

a. Convection resistance outside of brick

$$R = \frac{1}{hA} = \frac{1}{(15)(0.0413)} = 1.614\ °\text{C/W}$$

b. Conduction resistance in brick

$$R = \Delta x / kA = \frac{0.08}{(0.69)(0.0413)} = 2.807\ °\text{C/W}$$

c. Conduction resistance through outer sheet

$$R = \frac{\Delta x}{kA} = \frac{0.019}{(0.96)(0.0413)} = 0.48\ °\text{C/W}$$

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d. Conduction resistance through wood stud

$$R = \frac{\Delta x}{kA} = \frac{0.0921}{(0.1)(0.0413)} = 22.3\ ^\circ\mathrm{C/W}$$

e. Conduction resistance through inner sheet

$$R = \frac{\Delta x}{kA} = \frac{0.019}{(0.48)(0.0413)} = 0.96\ ^\circ\mathrm{C/W}$$

f. Convection resistance on inside

$$R = \frac{1}{hA} = \frac{1}{(7.5)(0.0413)} = 3.23\ ^\circ\mathrm{C/W}$$

The total thermal resistance through the wood stud section is

$$R_{\text{total}} = 1.614 + 2.807 + 0.48 + 22.3 + 0.96 + 3.23 = 31.39\ ^\circ\mathrm{C/W} \qquad [a]$$

**2.** *Insulation section* ($A = 0.406 - 0.0413$ m$^2$ for unit depth). Through the insulation section, five of the materials are the same, but the resistances involve different area terms, i.e., $40.6 - 4.13$ cm instead of 4.13 cm, so that each of the previous resistances must be multiplied by a factor of $4.13/(40.6 - 4.13) = 0.113$. The resistance through the insulation is

$$R = \frac{\Delta x}{kA} = \frac{0.0921}{(0.04)(0.406 - 0.0413)} = 6.31$$

and the total resistance through the insulation section is

$$R_{\text{total}} = (1.614 + 2.807 + 0.48 + 0.96 + 3.23)(0.113) + 6.31 = 7.337\ ^\circ\mathrm{C/W} \qquad [b]$$

The overall resistance for the section is now obtained by combining the parallel resistances in Equations (*a*) and (*b*) to give

$$R_{\text{overall}} = \frac{1}{(1/31.39) + (1/7.337)} = 5.947\ ^\circ\mathrm{C/W} \qquad [c]$$

This value is related to the overall heat-transfer coefficient by

$$q = UA\Delta T = \frac{\Delta T}{R_{\text{overall}}} \qquad [d]$$

where $A$ is the area of the total section = 0.406 m$^2$. Thus,

$$U = \frac{1}{RA} = \frac{1}{(5.947)(0.406)} = 0.414\ \mathrm{W/m^2 \cdot {}^\circ C}$$

As we have seen, the $R$ value is somewhat different from thermal resistance and is given by

$$R \text{ value} = \frac{1}{U} = \frac{1}{0.414} = 2.414^\circ\mathrm{C \cdot m^2/W}$$

## ■ Comment

This example illustrates the relationships between the concepts of thermal resistance, the overall heat-transfer coefficient, and the $R$ value. Note that the $R$ value involves a unit area concept, while the thermal resistance does not.

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## EXAMPLE 2-4 Cooling Cost Savings with Extra Insulation

A small metal building is to be constructed of corrugated steel sheet walls with a total wall surface area of about 300 m$^2$. The air conditioner consumes about 1 kW of electricity for every 4 kW of cooling supplied.[1] Two wall constructions are to be compared on the basis of cooling costs. Assume that electricity costs \$0.15/kWh. Determine the electrical energy savings of using 260 mm of fiberglass batt insulation instead of 159 mm of fiberglass insulation in the wall. Assume an overall temperature difference across the wall of 20°C on a hot summer day in Texas.

**Solution**

Consulting Table 2-2 (Numbers 19 and 20) we find that overall heat transfer coefficients for the two selected wall constructions are

$$U(\text{260-mm fiberglass}) = 0.17\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

$$U\ (\text{159-mm fiberglass}) = 0.31\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

The heat gain is calculated from $q = UA\Delta T$, so for the two constructions

$$q\ (\text{260-mm fiberglass}) = (0.17)(300)(20) = 1020\ \text{W}$$

$$q\ (\text{159-mm fiberglass}) = (0.31)(300)(20) = 1860\ \text{W}$$

$$\text{Savings due to extra insulation} = 840\ \text{W}$$

The energy consumed to supply this extra cooling is therefore

$$\text{Extra electric power required} = (840)(1/4) = 210\ \text{W}$$

and the cost is

$$\text{Cost} = (0.210\text{kW})(0.15\$/\text{kWh}) = 0.0315\ \$/\text{hr}$$

Assuming 10-h/day operation for 23 days/month this cost becomes

$$(0.0315)(10)(23) = \$7.25/\text{month}$$

Both of these cases are rather well insulated. If one makes a comparison to a 2 × 4 wood stud wall with no insulation (Number 4 in Table 2-2) fill in the cavity ($U = 1.85\ \text{W/m}^2 \cdot {}^\circ\text{C}$), the heating load would be

$$q = (1.85)(300)(20) = 11{,}100\ \text{W}$$

and the savings compared with the 260-mm fiberglass insulation would be

$$11{,}100 - 1020 = 10{,}080\ \text{W}$$

producing a corresponding electric power saving of \$0.378/h or \$86.94/month. Clearly the insulated wall will pay for itself. It is a matter of conjecture whether the 260-mm of insulation will pay for itself in comparison to the 159-mm insulation.

[1]This is not getting something for nothing. Consult any standard thermodynamics text for the reason for this behavior.

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### Overall Heat-Transfer Coefficient for a Tube

**EXAMPLE 2-5**

Water flows at 50°C inside a 2.5-cm-inside-diameter tube such that $h_i = 3500\ \mathrm{W/m^2 \cdot °C}$. The tube has a wall thickness of 0.8 mm with a thermal conductivity of 16 W/m · °C. The outside of the tube loses heat by free convection with $h_o = 7.6\ \mathrm{W/m^2 \cdot °C}$. Calculate the overall heat-transfer coefficient and heat loss per unit length to surrounding air at 20°C.

**■ Solution**

There are three resistances in series for this problem, as illustrated in Equation (2-14). With $L = 1.0$ m, $d_i = 0.025$ m, and $d_o = 0.025 + (2)(0.0008) = 0.0266$ m, the resistances may be calculated as

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3500)\pi(0.025)(1.0)} = 0.00364\ °\mathrm{C/W}$$

$$R_t = \frac{\ln(d_o/d_i)}{2\pi k L} = \frac{\ln(0.0266/0.025)}{2\pi(16)(1.0)} = 0.00062\ °\mathrm{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(7.6)\pi(0.0266)(1.0)} = 1.575\ °\mathrm{C/W}$$

Clearly, the outside convection resistance is the largest, and *overwhelmingly so*. This means that it is the controlling resistance for the total heat transfer because the other resistances (in series) are negligible in comparison. We shall base the overall heat-transfer coefficient on the outside tube area and write

$$q = \frac{\Delta T}{\sum R} = U A_o \Delta T \qquad [a]$$

$$U_o = \frac{1}{A_o \sum R} = \frac{1}{[\pi(0.0266)(1.0)](0.00364 + 0.00062 + 1.575)} = 7.577\ \mathrm{W/m^2 \cdot °C}$$

or a value very close to the value of $h_o = 7.6$ for the outside convection coefficient. The heat transfer is obtained from Equation (*a*), with

$$q = UA_o\,\Delta T = (7.577)\pi(0.0266)(1.0)(50 - 20) = 19\ \mathrm{W}\ \text{(for 1.0 m length)}$$

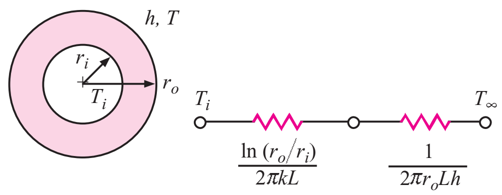
**■ Comment**

This example illustrates the important point that many practical heat-transfer problems involve multiple modes of heat transfer acting in combination; in this case, as a series of thermal resistances. It is not unusual for one mode of heat transfer to dominate the overall problem. In this example, the total heat transfer could have been computed very nearly by just calculating the free convection heat loss from the outside of the tube maintained at a temperature of 50°C. Because the inside convection and tube wall resistances are so small, there are correspondingly small temperature drops, and the outside temperature of the tube will be very nearly that of the liquid inside, or 50°C.

## 2-6 | CRITICAL THICKNESS OF INSULATION

Let us consider a layer of insulation which might be installed around a circular pipe, as shown in Figure 2-7. The inner temperature of the insulation is fixed at $T_i$, and the outer

**Figure 2-7** | Critical insulation thickness.




surface is exposed to a convection environment at $T_\infty$. From the thermal network the heat transfer is

$$q = \frac{2\pi L\,(T_i - T_\infty)}{\dfrac{\ln(r_o/r_i)}{k} + \dfrac{1}{r_o h}} \tag{2-17}$$

Now let us manipulate this expression to determine the outer radius of insulation $r_o$, which will maximize the heat transfer. The maximization condition is

$$\frac{dq}{dr_o} = 0 = \frac{-2\pi L\,(T_i - T_\infty)\left(\dfrac{1}{kr_o} - \dfrac{1}{hr_o^2}\right)}{\left[\dfrac{\ln(r_o/r_i)}{k} + \dfrac{1}{r_o h}\right]^2}$$

which gives the result

$$r_o = \frac{k}{h} \tag{2-18}$$

Equation (2-18) expresses the critical-radius-of-insulation concept. If the outer radius is less than the value given by this equation, then the heat transfer will be *increased* by adding more insulation. For outer radii greater than the critical value an increase in insulation thickness will cause a decrease in heat transfer. The central concept is that for sufficiently small values of $h$ the convection heat loss may actually increase with the addition of insulation because of increased surface area.

## EXAMPLE 2-6 Critical Insulation Thickness

Calculate the critical radius of insulation for asbestos [$k = 0.17$ W/m · °C] surrounding a pipe and exposed to room air at 20°C with $h = 3.0$ W/m$^2$ · °C. Calculate the heat loss from a 200°C, 5.0-cm-diameter pipe when covered with the critical radius of insulation and without insulation.

### ■ Solution

From Equation (2-18) we calculate $r_o$ as

$$r_o = \frac{k}{h} = \frac{0.17}{3.0} = 0.0567 \text{ m} = 5.67 \text{ cm}$$

The inside radius of the insulation is $5.0/2 = 2.5$ cm, so the heat transfer is calculated from Equation (2-17) as

$$\frac{q}{L} = \frac{2\pi\,(200 - 20)}{\dfrac{\ln(5.67/2.5)}{0.17} + \dfrac{1}{(0.0567)(3.0)}} = 105.7 \text{ W/m}$$

Without insulation the convection from the outer surface of the pipe is

$$\frac{q}{L} = h(2\pi r)(T_i - T_o) = (3.0)(2\pi)(0.025)(200 - 20) = 84.8 \text{ W/m}$$

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So, the addition of 3.17 cm (5.67 − 2.5) of insulation actually *increases* the heat transfer by 25 percent.

As an alternative, fiberglass having a thermal conductivity of 0.04 W/m · °C might be employed as the insulation material. Then, the critical radius would be

$$r_o = \frac{k}{h} = \frac{0.04}{3.0} = 0.0133 \text{ m} = 1.33 \text{ cm}$$

Now, the value of the critical radius is less than the outside radius of the pipe (2.5 cm), so addition of *any* fiberglass insulation would cause a *decrease* in the heat transfer. In a practical pipe insulation problem, the total heat loss will also be influenced by radiation as well as convection from the outer surface of the insulation.

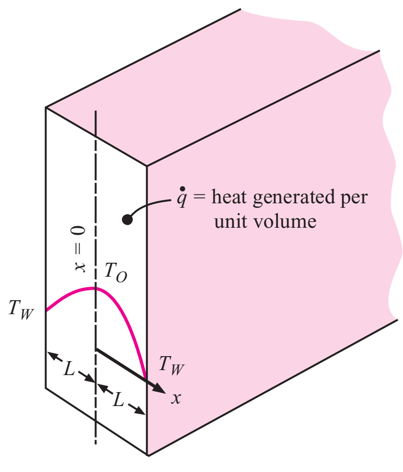
## 2-7 | HEAT-SOURCE SYSTEMS

A number of interesting applications of the principles of heat transfer are concerned with systems in which heat may be generated internally. Nuclear reactors are one example; electrical conductors and chemically reacting systems are others. At this point we shall confine our discussion to one-dimensional systems, or, more specifically, systems where the temperature is a function of only one space coordinate.

### Plane Wall with Heat Sources

Consider the plane wall with uniformly distributed heat sources shown in Figure 2-8. The thickness of the wall in the $x$ direction is $2L$, and it is assumed that the dimensions in the other directions are sufficiently large that the heat flow may be considered as one-dimensional. The heat generated per unit volume is $\dot{q}$, and we assume that the thermal conductivity does not vary with temperature. This situation might be produced in a practical situation by passing a current through an electrically conducting material. From Chapter 1,

**Figure 2-8** | Sketch illustrating one-dimensional conduction problem with heat generation.



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the differential equation that governs the heat flow is

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \qquad \textbf{[2-19]}$$

For the boundary conditions we specify the temperatures on either side of the wall, i.e.,

$$T = T_w \qquad \text{at } x = \pm L \qquad \textbf{[2-20]}$$

The general solution to Equation (2-19) is

$$T = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2 \qquad \textbf{[2-21]}$$

Because the temperature must be the same on each side of the wall, $C_1$ must be zero. The temperature at the midplane $(x = 0)$ is denoted by $T_0$ and from Equation (2-21)

$$T_0 = C_2$$

The temperature distribution is therefore

$$T - T_0 = -\frac{\dot{q}}{2k}x^2 \qquad \textbf{[2-22}\boldsymbol{a}\textbf{]}$$

or

$$\frac{T - T_0}{T_w - T_0} = \left(\frac{x}{L}\right)^2 \qquad \textbf{[2-22}\boldsymbol{b}\textbf{]}$$

a parabolic distribution. An expression for the midplane temperature $T_0$ may be obtained through an energy balance. At steady-state conditions the total heat generated must equal the heat lost at the faces. Thus

$$2\left(-kA\left.\frac{dT}{dx}\right]_{x=L}\right) = \dot{q}A\,2L$$

where $A$ is the cross-sectional area of the plate. The temperature gradient at the wall is obtained by differentiating Equation (2-22$b$):

$$\left.\frac{dT}{dx}\right]_{x=L} = (T_w - T_0)\left.\left(\frac{2x}{L^2}\right)\right]_{x=L} = (T_w - T_0)\,\frac{2}{L}$$

Then

$$-k(T_w - T_0)\,\frac{2}{L} = \dot{q}L$$

and

$$T_0 = \frac{\dot{q}L^2}{2k} + T_w \qquad \textbf{[2-23]}$$

This same result could be obtained by substituting $T = T_w$ at $x = L$ into Equation (2-22$a$).

The equation for the temperature distribution could also be written in the alternative form

$$\frac{T - T_w}{T_0 - T_w} = 1 - \frac{x^2}{L^2} \qquad \textbf{[2-22}\boldsymbol{c}\textbf{]}$$

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## 2-8 | CYLINDER WITH HEAT SOURCES

Consider a cylinder of radius $R$ with uniformly distributed heat sources and constant thermal conductivity. If the cylinder is sufficiently long that the temperature may be considered a function of radius only, the appropriate differential equation may be obtained by neglecting the axial, azimuth, and time-dependent terms in Equation (1-3$b$),

$$\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}+\frac{\dot{q}}{k}=0 \qquad \textbf{[2-24]}$$

The boundary conditions are

$$T=T_w \qquad \text{at } r=R$$

and heat generated equals heat lost at the surface:

$$\dot{q}\pi R^2 L=-k2\pi RL\left.\frac{dT}{dr}\right]_{r=R}$$

Since the temperature function must be continuous at the center of the cylinder, we could specify that

$$\frac{dT}{dr}=0 \qquad \text{at } r=0$$

However, it will not be necessary to use this condition since it will be satisfied automatically when the two boundary conditions are satisfied.

We rewrite Equation (2-24)

$$r\frac{d^2T}{dr^2}+\frac{dT}{dr}=\frac{-\dot{q}r}{k}$$

and note that

$$r\frac{d^2T}{dr^2}+\frac{dT}{dr}=\frac{d}{dr}\left(r\frac{dT}{dr}\right)$$

Then integration yields

$$r\frac{dT}{dr}=\frac{-\dot{q}r^2}{2k}+C_1$$

and

$$T=\frac{-\dot{q}r^2}{4k}+C_1\ln r+C_2$$

From the second boundary condition above,

$$\left.\frac{dT}{dr}\right]_{r=R}=\frac{-\dot{q}R}{2k}=\frac{-\dot{q}R}{2k}+\frac{C_1}{R}$$

Thus

$$C_1=0$$

We could also note that $C_1$ must be zero because at $r=0$ the logarithm function becomes infinite.

From the first boundary condition,

$$T=T_w=\frac{-\dot{q}R^2}{4k}+C_2 \qquad \text{at } r=R$$

so that

$$C_2 = T_w + \frac{\dot{q}R^2}{4k}$$

The final solution for the temperature distribution is then

$$T - T_w = \frac{\dot{q}}{4k}(R^2 - r^2) \qquad \textbf{[2-25a]}$$

or, in dimensionless form,

$$\frac{T - T_w}{T_0 - T_w} = 1 - \left(\frac{r}{R}\right)^2 \qquad \textbf{[2-25b]}$$

where $T_0$ is the temperature at $r = 0$ and is given by

$$T_0 = \frac{\dot{q}R^2}{4k} + T_w \qquad \textbf{[2-26]}$$

It is left as an exercise to show that the temperature gradient at $r = 0$ is zero.

For a hollow cylinder with uniformly distributed heat sources the appropriate boundary conditions would be

$$T = T_i \quad \text{at } r = r_i \text{ (inside surface)}$$
$$T = T_o \quad \text{at } r = r_o \text{ (outside surface)}$$

The general solution is still

$$T = -\frac{\dot{q}r^2}{4k} + C_1 \ln r + C_2$$

Application of the new boundary conditions yields

$$T - T_o = \frac{\dot{q}}{4k}(r_o^2 - r^2) + C_1 \ln \frac{r}{r_o} \qquad \textbf{[2-27]}$$

where the constant $C_1$ is given by

$$C_1 = \frac{T_i - T_o + \dot{q}(r_i^2 - r_o^2)/4k}{\ln(r_i/r_o)} \qquad \textbf{[2-28]}$$

**EXAMPLE 2-7** Heat Source with Convection

A current of 200 A is passed through a stainless-steel wire [$k = 19$ W/m · °C] 3 mm in diameter. The resistivity of the steel may be taken as 70 $\mu\Omega \cdot$ cm, and the length of the wire is 1 m. The wire is submerged in a liquid at 110 °C and experiences a convection heat-transfer coefficient of 4 k W/m$^2$ · °C. Calculate the center temperature of the wire.

**Solution**

All the power generated in the wire must be dissipated by convection to the liquid:

$$P = I^2R = q = hA(T_w - T_\infty) \qquad \textbf{[a]}$$

The resistance of the wire is calculated from

$$R = \rho \frac{L}{A} = \frac{(70 \times 10^{-6})(100)}{\pi(0.15)^2} = 0.099\ \Omega$$

where $\rho$ is the resistivity of the wire. The surface area of the wire is $\pi dL$, so from Equation ($a$),

$$(200)^2(0.099) = 4000\pi(3 \times 10^{-3})(1)(T_w - 110) = 3960 \text{ W}$$

and

$$T_w = 215°\text{C} \quad [419°\text{F}]$$

The heat generated per unit volume $\dot{q}$ is calculated from

$$P = \dot{q}V = \dot{q}\pi r^2 L$$

so that

$$\dot{q} = \frac{3960}{\pi\,(1.5\times10^{-3})^2(1)} = 560.2\ \text{MW/m}^3 \quad [5.41\times10^7\ \text{Btu/h}\cdot\text{ft}^3]$$

Finally, the center temperature of the wire is calculated from Equation (2-26):

$$T_0 = \frac{\dot{q}r_o^2}{4k} + T_w = \frac{(5.602\times10^8)(1.5\times10^{-3})^2}{(4)(19)} + 215 = 231.6°\text{C} \quad [449°\text{F}]$$

## 2-9 | CONDUCTION-CONVECTION SYSTEMS

The heat that is conducted through a body must frequently be removed (or delivered) by some convection process. For example, the heat lost by conduction through a furnace wall must be dissipated to the surroundings through convection. In heat-exchanger applications a finned-tube arrangement might be used to remove heat from a hot liquid. The heat transfer from the liquid to the finned tube is by convection. The heat is conducted through the material and finally dissipated to the surroundings by convection. Obviously, an analysis of combined conduction-convection systems is very important from a practical standpoint.

We shall defer part of our analysis of conduction-convection systems to Chapter 10 on heat exchangers. For the present we wish to examine some simple extended-surface problems. Consider the one-dimensional fin exposed to a surrounding fluid at a temperature $T_\infty$ as shown in Figure 2-9. The temperature of the base of the fin is $T_0$. We approach the problem by making an energy balance on an element of the fin of thickness $dx$ as shown in the figure. Thus

$$\text{Energy in left face} = \text{energy out right face} + \text{energy lost by convection}$$

The defining equation for the convection heat-transfer coefficient is recalled as

$$q = hA\,(T_w - T_\infty) \qquad \textbf{[2-29]}$$

where the area in this equation is the surface area for convection. Let the cross-sectional area of the fin be $A$ and the perimeter be $P$. Then the energy quantities are

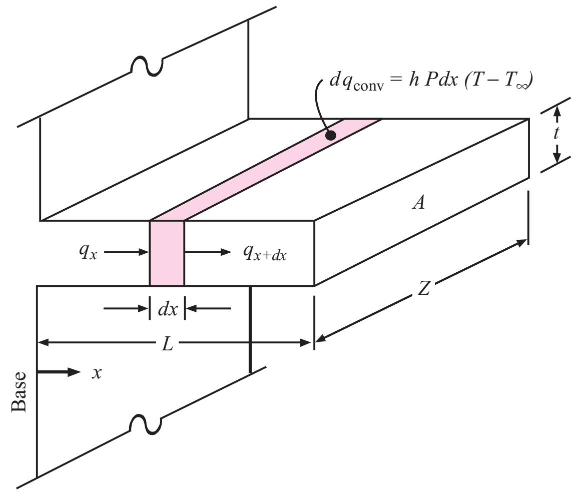
$$\text{Energy in left face} = q_x = -kA\frac{dT}{dx}$$

$$\text{Energy out right face} = q_{x+dx} = -kA\left.\frac{dT}{dx}\right]_{x+dx}$$

$$= -kA\left(\frac{dT}{dx} + \frac{d^2T}{dx^2}\,dx\right)$$

$$\text{Energy lost by convection} = hP\,dx\,(T - T_\infty)$$

**Figure 2-9** | Sketch illustrating one-dimensional conduction and convection through a rectangular fin.




Here it is noted that the differential surface area for convection is the product of the perimeter of the fin and the differential length $dx$. When we combine the quantities, the energy balance yields

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0 \qquad \textbf{[2-30}\boldsymbol{a}\textbf{]}$$

Let $\theta = T - T_\infty$. Then Equation (2-30$a$) becomes

$$\frac{d^2\theta}{dx^2} - \frac{hP}{kA}\theta = 0 \qquad \textbf{[2-30}\boldsymbol{b}\textbf{]}$$

One boundary condition is

$$\theta = \theta_0 = T_0 - T_\infty \qquad \text{at } x = 0$$

The other boundary condition depends on the physical situation. Several cases may be considered:

**CASE 1** The fin is very long, and the temperature at the end of the fin is essentially that of the surrounding fluid.

**CASE 2** The fin is of finite length and loses heat by convection from its end.

**CASE 3** The end of the fin is insulated so that $dT/dx = 0$ at $x = L$.

If we let $m^2 = hP/kA$, the general solution for Equation (2-30$b$) may be written

$$\theta = C_1 e^{-mx} + C_2 e^{mx} \qquad \textbf{[2-31]}$$

For case 1 the boundary conditions are

$$\theta = \theta_0 \qquad \text{at } x = 0$$
$$\theta = 0 \qquad \text{at } x = \infty$$

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and the solution becomes

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx} \qquad \textbf{[2-32]}$$

For case 3 the boundary conditions are

$$\theta = \theta_0 \quad \text{at } x = 0$$

$$\frac{d\theta}{dx} = 0 \quad \text{at } x = L$$

Thus

$$\theta_0 = C_1 + C_2$$

$$0 = m(-C_1 e^{-mL} + C_2 e^{mL})$$

Solving for the constants $C_1$ and $C_2$, we obtain

$$\frac{\theta}{\theta_0} = \frac{e^{-mx}}{1 + e^{-2mL}} + \frac{e^{mx}}{1 + e^{2mL}} \qquad \textbf{[2-33}\boldsymbol{a}\textbf{]}$$

$$= \frac{\cosh[m(L - x)]}{\cosh mL} \qquad \textbf{[2-33}\boldsymbol{b}\textbf{]}$$

The hyperbolic functions are defined as

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

The solution for case 2 is more involved algebraically, and the result is

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh m(L - x) + (h/mk)\sinh m(L - x)}{\cosh mL + (h/mk)\sinh mL} \qquad \textbf{[2-34]}$$

All of the heat lost by the fin must be conducted into the base at $x = 0$. Using the equations for the temperature distribution, we can compute the heat loss from

$$q = -kA \left.\frac{dT}{dx}\right]_{x=0}$$

An alternative method of integrating the convection heat loss could be used:

$$q = \int_0^L hP(T - T_\infty)\,dx = \int_0^L hP\,\theta\,dx$$

In most cases, however, the first equation is easier to apply. For case 1,

$$q = -kA(-m\theta_0 e^{-m(0)}) = \sqrt{hPkA}\,\theta_0 \qquad \textbf{[2-35]}$$

For case 3,

$$q = -kA\theta_0 m\left(\frac{1}{1 + e^{-2mL}} - \frac{1}{1 + e^{+2mL}}\right) \qquad \textbf{[2-36]}$$

$$= \sqrt{hPkA}\,\theta_0 \tanh mL$$

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The heat flow for case 2 is

$$q = \sqrt{hPkA}\,(T_0 - T_\infty)\,\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} \qquad \textbf{[2-37]}$$

In this development it has been assumed that the substantial temperature gradients occur only in the $x$ direction. This assumption will be satisfied if the fin is sufficiently thin. For most fins of practical interest the error introduced by this assumption is less than 1 percent. The overall accuracy of practical fin calculations will usually be limited by uncertainties in values of the convection coefficient $h$. It is worthwhile to note that the convection coefficient is seldom uniform over the entire surface, as has been assumed above. If severe nonuniform behavior is encountered, numerical finite-difference techniques must be employed to solve the problem. Such techniques are discussed in Chapter 3.

## 2-10 | FINS

In the foregoing development we derived relations for the heat transfer from a rod or fin of uniform cross-sectional area protruding from a flat wall. In practical applications, fins may have varying cross-sectional areas and may be attached to circular surfaces. In either case the area must be considered as a variable in the derivation, and solution of the basic differential equation and the mathematical techniques become more tedious. We present only the results for these more complex situations. The reader is referred to References 1 and 8 for details on the mathematical methods used to obtain the solutions.

To indicate the effectiveness of a fin in transferring a given quantity of heat, a new parameter called *fin efficiency* is defined by

$$\text{Fin efficiency} = \frac{\text{actual heat transferred}}{\begin{array}{c}\text{heat that would be transferred}\\ \text{if entire fin area were}\\ \text{at base temperature}\end{array}} = \eta_f$$

For case 3, the fin efficiency becomes

$$\eta_f = \frac{\sqrt{hPkA}\,\theta_0 \tanh mL}{hPL\theta_0} = \frac{\tanh mL}{mL} \qquad \textbf{[2-38]}$$

The fins discussed were assumed to be sufficiently deep that the heat flow could be considered one-dimensional. The expression for $mL$ may be written

$$mL = \sqrt{\frac{hP}{kA}}\,L = \sqrt{\frac{h(2z+2t)}{kzt}}\,L$$

where $z$ is the depth of the fin, and $t$ is the thickness. Now, if the fin is sufficiently deep, the term $2z$ will be large compared with $2t$, and

$$mL = \sqrt{\frac{2hz}{ktz}}\,L = \sqrt{\frac{2h}{kt}}\,L$$

Multiplying numerator and denominator by $L^{1/2}$ gives

$$mL = \sqrt{\frac{2h}{kLt}}\,L^{3/2}$$

$Lt$ is called the profile area of the fin, which we define as

$$A_m = Lt$$

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so that

$$mL = \sqrt{\frac{2h}{kA_m}}L^{3/2} \tag{2-39}$$

We may therefore use the expression in Equation (2-39) to compute the efficiency of a fin with insulated tip as given by Equation (2-38).

Harper and Brown [2] have shown that the solution in case 2 may be expressed in the same form as Equation (2-38) when the length of the fin is extended by one-half the thickness of the fin. In effect, lengthening of the fin by $t/2$ is assumed to represent the same convection heat transfer as half the fin tip area placed on top and bottom of the fin. A corrected length $L_c$ is then used in all the equations that apply for the case of the fin with an insulated tip. Thus

$$L_c = L + \frac{t}{2} \tag{2-40}$$

The error that results from this approximation will be less than 8 percent when

$$\left(\frac{ht}{2k}\right)^{1/2} \le \frac{1}{2} \tag{2-41}$$

If a straight cylindrical rod extends from a wall, the corrected fin length is calculated from

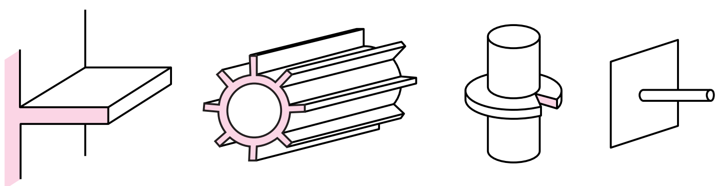
$$L_c = L + \frac{\pi d^2/4}{\pi d} = L + d/4 \tag{2-42}$$

Again, the real fin is extended a sufficient length to produce a circumferential area equal to that of the tip area.

Examples of other types of fins are shown in Figure 2-10. Figure 2-11 presents a comparison of the efficiencies of a triangular fin and a straight rectangular fin corresponding to case 2. Figure 2-12 shows the efficiencies of circumferential fins of rectangular cross-sectional area. Notice that the corrected fin lengths $L_c$ and profile area $A_m$ have been used in Figures 2-11 and 2-12. We may note that as $r_{2c}/r_1 \to 1.0$, the efficiency of the circumferential fin becomes identical to that of the straight fin of rectangular profile.

It is interesting to note that the fin efficiency reaches its maximum value for the trivial case of $L = 0$, or no fin at all. Therefore, we should not expect to be able to maximize fin performance with respect to fin length. It is possible, however, to maximize the efficiency with respect to the quantity of fin material (mass, volume, or cost), and such a maximization process has rather obvious economic significance. We have not discussed the subject of radiation heat transfer from fins. The radiant transfer is an important consideration in a

**Figure 2-10** | Different types of finned surfaces. (*a*) Straight fin of rectangular profile on plane wall, (*b*) straight fin of rectangular profile on circular tube, (*c*) cylindrical tube with radial fin of rectangular profile, (*d*) cylindrical-spine or circular-rod fin.



(*a*) (*b*) (*c*) (*d*)

**Figure 2-11** | Efficiencies of straight rectangular and triangular fins.

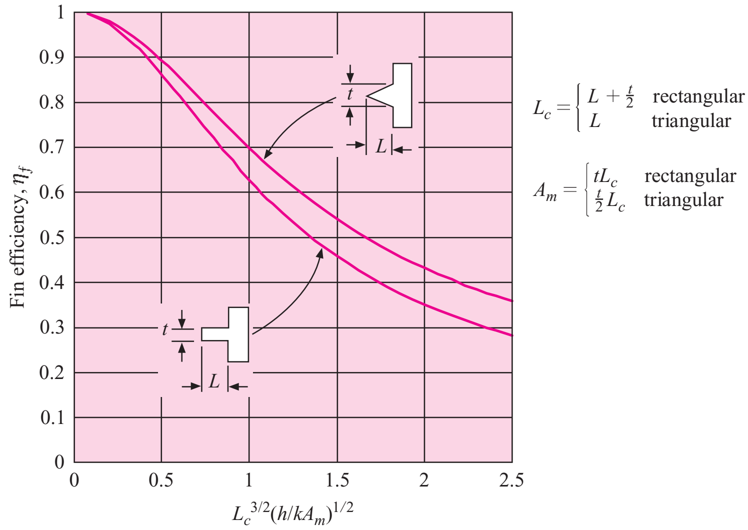
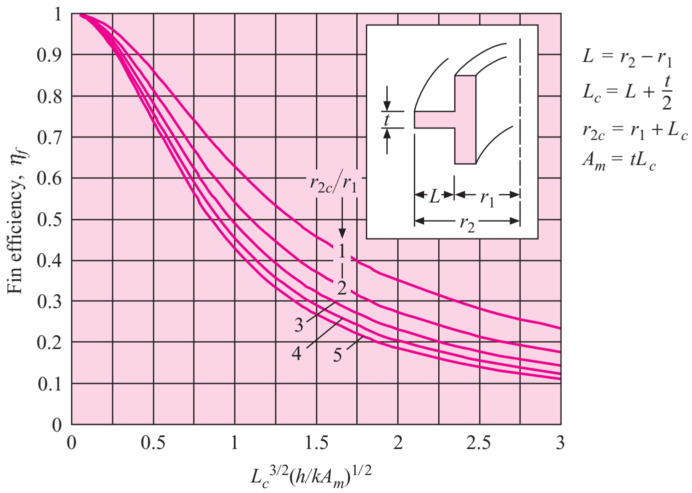



**Figure 2-12** | Efficiencies of circumferential fins of rectangular profile, according to Reference 3.




number of applications, and the interested reader should consult Siegel and Howell [9] for information on this subject.

In some cases a valid method of evaluating fin performance is to compare the heat transfer with the fin to that which would be obtained without the fin. The ratio of these quantities is

$$\frac{q \text{ with fin}}{q \text{ without fin}} = \frac{\eta_f A_f h \theta_0}{h A_b \theta_0}$$

where $A_f$ is the total surface area of the fin and $A_b$ is the base area. For the insulated-tip fin described by Equation (2-36),

$$A_f = PL$$
$$A_b = A$$

and the heat ratio would become

$$\frac{q \text{ with fin}}{q \text{ without fin}} = \frac{\tanh mL}{\sqrt{hA/kP}}$$

This term is sometimes called the *fin effectiveness.*

## Thermal Resistance for Fin-Wall Combinations

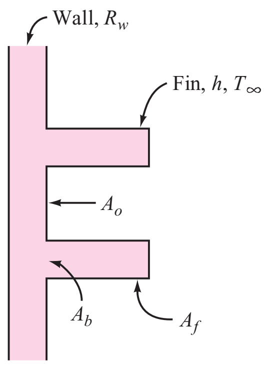
Consider a fin attached to a wall as illustrated in either Figure 2-11 or Figure 2-12. We may calculate a thermal resistance for the wall using either $R_w = \Delta x/kA$ for a plane wall, or $R_w = \ln(r_o/r_i)/2\pi kL$ for a cylindrical wall. In the absence of the fin the convection resistance at the surface would be $1/hA$. The *combined* conduction and convection resistance $R_f$ for the fin is related to the heat lost by the fin through

$$q_f = \eta_f A_f h\theta_o = \frac{\theta_o}{R_f} \tag{2-43}$$

or, the fin resistance may be expressed as

$$R_f = \frac{1}{\eta_f A_f h} \tag{2-44}$$

**Figure 2-13** | Heat loss from fin-wall combination.




The overall heat transfer through the fin-wall combination is then

$$q_f = \frac{T_i - T_\infty}{R_{wf} + R_f} \tag{2-45}$$

where $T_i$ is the inside wall temperature and $R_{wf}$ is the wall resistance at the fin position. This heat transfer is only for the fin portion of the wall. Now consider the wall section shown in Figure 2-13, having a wall area $A_b$ for the fin and area $A_o$ for the open section of the wall exposed directly to the convection environment. The open wall heat transfer is

$$q_o = \frac{T_i - T_\infty}{R_{wo} + R_o} \tag{2-46}$$

where now

$$R_o = \frac{1}{hA_o} \tag{2-47}$$

and $R_{wo}$ is the wall resistance for the open wall section. This value is $R_{wo} = \Delta x/k_w A_o$ for a plane wall, where $\Delta x$ is the wall thickness. A logarithmic form would be employed for a cylindrical wall, as noted above. The total heat lost by the wall is therefore

$$q_{\text{total}} = q_f + q_o \tag{2-48}$$

which may be expressed in terms of the thermal resistances by

$$q_{\text{total}} = (T_i - T_\infty)\left[\frac{1}{R_{wf}+R_f} + \frac{1}{R_{wo}+R_o}\right]$$
$$= (T_i - T_\infty)\frac{R_{wo}+R_o+R_{wf}+R_f}{(R_{wf}+R_f)(R_{wo}+R_o)} \qquad \textbf{[2-49]}$$

## Conditions When Fins Do Not Help

At this point we should remark that the installation of fins on a heat-transfer surface will not necessarily increase the heat-transfer rate. If the value of $h$, the convection coefficient, is large, as it is with high-velocity fluids or boiling liquids, the fin may produce a *reduction* in heat transfer because the conduction resistance then represents a larger impediment to the heat flow than the convection resistance. To illustrate the point, consider a stainless-steel pin fin that has $k = 16\ \text{W/m}\cdot{}^\circ\text{C}$, $L = 10$ cm, $d = 1$ cm and that is exposed to a boiling-water convection situation with $h = 5000\ \text{W/m}^2\cdot{}^\circ\text{C}$. From Equation (2-36) we can compute

$$\frac{q \text{ with fin}}{q \text{ without fin}} = \frac{\tanh mL}{\sqrt{hA/kp}}$$
$$= \frac{\tanh\left\{\left[\dfrac{5000\pi(1\times10^{-2})(4)}{16\pi(1\times10^{-2})^2}\right]^{1/2}(10\times10^{-2})\right\}}{\left[\dfrac{5000\pi(1\times10^{-2})^2}{(4)(16)\pi(1\times10^{-2})}\right]^{1/2}}$$
$$= 1.13$$

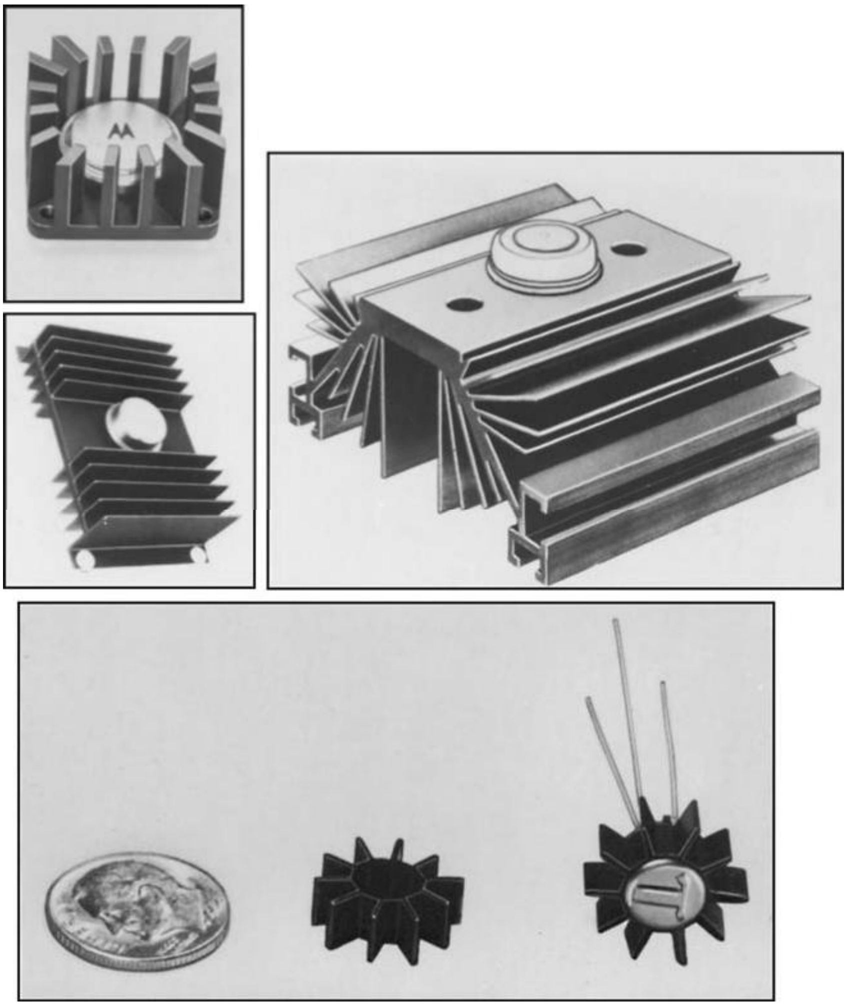
Thus, this rather large pin produces an increase of only 13 percent in the heat transfer.

Still another method of evaluating fin performance is discussed in Problem 2-68. Kern and Kraus [8] give a very complete discussion of extended-surface heat transfer. Some photographs of different fin shapes used in electronic cooling applications are shown in Figure 2-14. These fins are obviously not one-dimensional, i.e., they cannot be characterized with a single space coordinate.

## Cautionary Remarks Concerning Convection Coefficients for Fins

We have already noted that the convection coefficient may vary with type of fluid, flow velocity, geometry, etc. As we shall see in Chapters 5, 6, and 7, empirical correlations for $h$ frequently have uncertainties of the order of $\pm25$ percent. Moreover, the correlations are based on controlled laboratory experiments that are infrequently matched in practice. What this means is that the assumption of constant $h$ used in the derivation of fin performance may be in considerable error and the value of $h$ may vary over the fin surface. For the heat-transfer practitioner, complex geometries like those shown in Figure 2-14 must be treated with particular care. These configurations usually must be tested under near or actual operating conditions in order to determine their performance with acceptable reliability. These remarks are not meant to discourage the reader, but rather to urge prudence when estimating the performance of complex finned surfaces for critical applications.

**Figure 2-14** | Some fin arrangements used in electronic cooling applications.



Source: Courtesy Wakefield Engineering Inc., Wakefield, Mass.

## Influence of Thermal Conductivity on Fin Temperature Profiles

**EXAMPLE 2-8**

Compare the temperature distributions in a straight cylindrical rod having a diameter of 2 cm and a length of 10 cm and exposed to a convection environment with $h = 25\ \text{W/m}^2 \cdot {}^\circ\text{C}$, for three fin materials: copper [$k = 385\ \text{W/m} \cdot {}^\circ\text{C}$], stainless steel [$k = 17\ \text{W/m} \cdot {}^\circ\text{C}$], and glass [$k = 0.8\ \text{W/m} \cdot {}^\circ\text{C}$]. Also compare the relative heat flows and fin efficiencies.

### Solution

We have

$$\frac{hP}{kA} = \frac{(25)\pi(0.02)}{k\pi(0.01)^2} = \frac{5000}{k}$$

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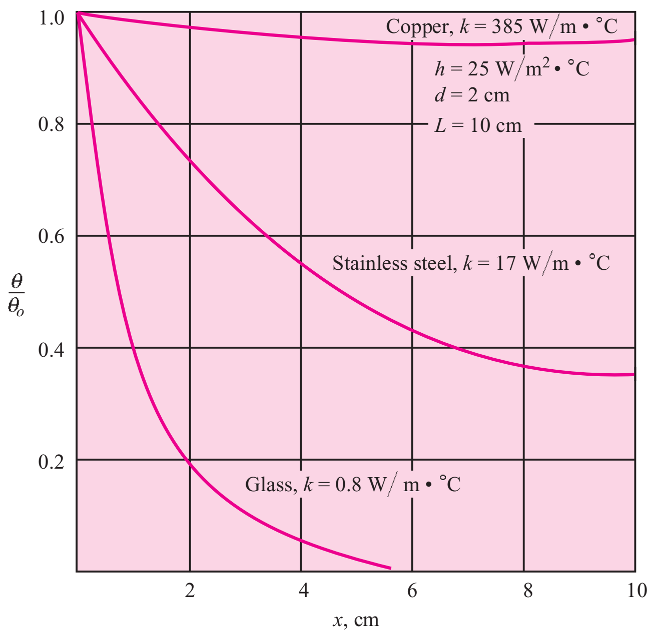
The terms of interest are therefore

| Material | $\frac{hP}{kA}$ | $m$ | $mL$ |
|---|---|---|---|
| Copper | 12.99 | 3.604 | 0.3604 |
| Stainless steel | 294.1 | 17.15 | 1.715 |
| Glass | 6250 | 79.06 | 7.906 |

These values may be inserted into Equation (2-33*a*) to calculate the temperatures at different $x$ locations along the rod, and the results are shown in Figure Example 2-8. We notice that the glass behaves as a "very long" fin, and its behavior could be calculated from Equation (2-32). The fin efficiencies are calculated from Equation (2-38) by using the corrected length approximation of Equation (2-42). We have

$$L_c = L + \frac{d}{4} = 10 + \frac{2}{4} = 10.5 \text{ cm}$$

**Figure Example 2-8**




The parameters of interest for the heat-flow and efficiency comparisons are now tabulated as

| Material | $hPkA$ | $mL_c$ |
|---|---|---|
| Copper | 0.190 | 0.3784 |
| Stainless steel | 0.0084 | 1.8008 |
| Glass | $3.9 \times 10^{-4}$ | 8.302 |

To compare the heat flows we could either calculate the values from Equation (2-36) for a unit value of $\theta_0$ or observe that the fin efficiency gives a relative heat-flow comparison because the maximum heat transfer is the same for all three cases; i.e., we are dealing with the same fin size, shape, and value of $h$. We thus calculate the values of $\eta_f$ from Equation (2-38) and the above values of $mL_c$.

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| Material | $\eta_f$ | $q$ relative to copper, % |
|---|---|---|
| Copper | 0.955 | 100 |
| Stainless steel | 0.526 | 53.1 |
| Glass | 0.124 | 12.6 |

The temperature profiles in the accompanying figure can be somewhat misleading. The glass has the steepest temperature gradient at the base, but its much lower value of $k$ produces a lower heat-transfer rate.

## Straight Aluminum Fin — EXAMPLE 2-9

An aluminum fin [$k = 200$ W/m · °C] 3.0 mm thick and 7.5 cm long protrudes from a wall, as in Figure 2-9. The base is maintained at 300°C, and the ambient temperature is 50°C with $h = 10$ W/m$^2$ · °C. Calculate the heat loss from the fin per unit depth of material.

### Solution

We may use the approximate method of solution by extending the fin a fictitious length $t/2$ and then computing the heat transfer from a fin with insulated tip as given by Equation (2-36). We have

$$L_c = L + t/2 = 7.5 + 0.15 = 7.65 \text{ cm [3.01 in]}$$

$$m = \sqrt{\frac{hP}{kA}} = \left[\frac{h(2z + 2t)}{ktz}\right]^{1/2} \approx \sqrt{\frac{2h}{kt}}$$

when the fin depth $z \gg t$. So,

$$m = \left[\frac{(2)(10)}{(200)(3 \times 10^{-3})}\right]^{1/2} = 5.774$$

From Equation (2-36), for an insulated-tip fin

$$q = (\tanh mL_c)\sqrt{hPkA}\,\theta_0$$

For a 1 m depth

$$A = (1)(3 \times 10^{-3}) = 3 \times 10^{-3} \text{ m}^2 \text{ [4.65 in}^2\text{]}$$

and

$$q = (5.774)(200)(3 \times 10^{-3})(300 - 50) \tanh [(5.774)(0.0765)]$$
$$= 359 \text{ W/m [373.5 Btu/h} \cdot \text{ft]}$$

## Circumferential Aluminum Fin — EXAMPLE 2-10

Aluminum fins 1.5 cm wide and 1.0 mm thick are placed on a 2.5-cm-diameter tube to dissipate the heat. The tube surface temperature is 170°, and the ambient-fluid temperature is 25°C. Calculate the heat loss per fin for $h = 130$ W/m$^2$ · °C. Assume $k = 200$ W/m · °C for aluminum.

### Solution

For this example we can compute the heat transfer by using the fin-efficiency curves in Figure 2-12. The parameters needed are

$$L_c = L + t/2 = 1.5 + 0.05 = 1.55 \text{ cm}$$
$$r_1 = 2.5/2 = 1.25 \text{ cm}$$
$$r_{2c} = r_1 + L_c = 1.25 + 1.55 = 2.80 \text{ cm}$$

$$r_{2c}/r_1 = 2.80/1.25 = 2.24$$

$$A_m = t(r_{2c} - r_1) = (0.001)(2.8 - 1.25)(10^{-2}) = 1.55 \times 10^{-5}\ \text{m}^2$$

$$L_c^{3/2}\left(\frac{h}{kA_m}\right)^{1/2} = (0.0155)^{3/2}\left[\frac{130}{(200)(1.55 \times 10^{-5})}\right]^{1/2} = 0.396$$

From Figure 2-12, $\eta_f = 82$ percent. The heat that would be transferred if the entire fin were at the base temperature is (both sides of fin exchanging heat)

$$\begin{aligned} q_{\text{max}} &= 2\pi(r_{2c}^2 - r_1^2)h(T_0 - T_\infty) \\ &= 2\pi(2.8^2 - 1.25^2)(10^{-4})(130)(170 - 25) \\ &= 74.35\ \text{W}\ [253.7\ \text{Btu/h}] \end{aligned}$$

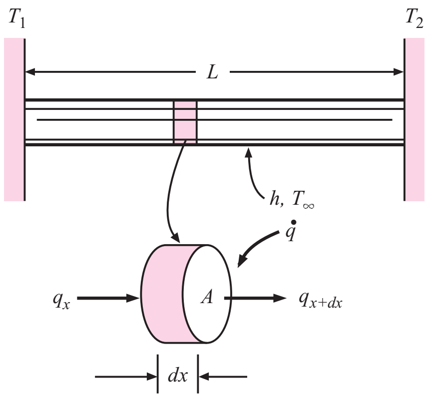
The actual heat transfer is then the product of the heat flow and the fin efficiency:

$$q_{\text{act}} = (0.82)(74.35) = 60.97\ \text{W}\ [208\ \text{Btu/h}]$$

## EXAMPLE 2-11 Rod with Heat Sources

A rod containing uniform heat sources per unit volume $\dot{q}$ is connected to two temperatures as shown in Figure Example 2-11. The rod is also exposed to an environment with convection coefficient $h$ and temperature $T_\infty$. Obtain an expression for the temperature distribution in the rod.

**Figure Example 2-11**




### Solution

We first must make an energy balance on the element of the rod shown, similar to that used to derive Equation (2-30). We have

Energy in left face + heat generated in element
= energy out right face + energy lost by convection

or

$$-kA\frac{dT}{dx} + \dot{q}A\,dx = -kA\left(\frac{dT}{dx} + \frac{d^2T}{dx^2}dx\right) + hP\,dx\,(T - T_\infty)$$

Simplifying, we have

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) + \frac{\dot{q}}{k} = 0 \qquad [a]$$

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or, with $\theta = T - T_\infty$ and $m^2 = hP/kA$

$$\frac{d^2\theta}{dx} - m^2\theta + \frac{\dot{q}}{k} = 0 \qquad [b]$$

We can make a further variable substitution as

$$\theta' = \theta - \dot{q}/km^2$$

so that our differential equation becomes

$$\frac{d^2\theta'}{dx^2} - m^2\theta' = 0 \qquad [c]$$

which has the general solution

$$\theta' = C_1 e^{-mx} + C_2 e^{mx} \qquad [d]$$

The two end temperatures are used to establish the boundary conditions:

$$\theta' = \theta'_1 = T_1 - T_\infty - \dot{q}/km^2 = C_1 + C_2$$
$$\theta' = \theta'_2 = T_2 - T_\infty - \dot{q}/km^2 = C_1 e^{-mL} + C_2 e^{mL}$$

Solving for the constants $C_1$ and $C_2$ gives

$$\theta' = \frac{(\theta'_1 e^{2mL} - \theta'_2 e^{mL})e^{-mx} + (\theta'_2 e^{mL} - \theta'_1)e^{mx}}{e^{2mL} - 1} \qquad [e]$$

For an infinitely long heat-generating fin with the left end maintained at $T_1$, the temperature distribution becomes

$$\theta'/\theta'_1 = e^{-mx} \qquad [f]$$

a relation similar to Equation (2-32) for a non-heat-generating fin.

**■ Comment**

Note that the above relationships assume one-dimensional behavior, i.e., temperature dependence only on the $x$-coordinate and temperature uniformity across the area $A$. For sufficiently large heat generation rates and/or cross-section areas, the assumption may no longer be valid. In these cases, the problem must be treated as multidimensional using the techniques described in Chapter 3.

## 2-11 | THERMAL CONTACT RESISTANCE

Imagine two solid bars brought into contact as indicated in Figure 2-15, with the sides of the bars insulated so that heat flows only in the axial direction. The materials may have different thermal conductivities, but if the sides are insulated, the heat flux must be the same through both materials under steady-state conditions. Experience shows that the actual temperature profile through the two materials varies approximately as shown in Figure 2-15*b*. The temperature drop at plane 2, the contact plane between the two materials, is said to be the result of a *thermal contact resistance*. Performing an energy balance on the two materials, we obtain

$$q = k_A A \frac{T_1 - T_{2A}}{\Delta x_A} = \frac{T_{2A} - T_{2B}}{1/h_c A} = k_B A \frac{T_{2B} - T_3}{\Delta x_B}$$

or

$$q = \frac{T_1 - T_3}{\Delta x_A / k_A A + 1/h_c A + \Delta x_B / k_B A} \qquad [2\text{-}50]$$

where the quantity $1/h_c A$ is called the thermal contact resistance and $h_c$ is called the contact coefficient. This factor can be extremely important in a number of applications because of the many heat-transfer situations that involve mechanical joining of two materials.

The physical mechanism of contact resistance may be better understood by examining a joint in more detail, as shown in Figure 2-16. The actual surface roughness is exaggerated to implement the discussion. No real surface is perfectly smooth, and the actual surface roughness is believed to play a central role in determining the contact resistance. There are two principal contributions to the heat transfer at the joint:

1. The solid-to-solid conduction at the spots of contact
2. The conduction through entrapped gases in the void spaces created by the contact

The second factor is believed to represent the major resistance to heat flow, because the thermal conductivity of the gas is quite small in comparison to that of the solids.

**Figure 2-15** | Illustrations of thermal-contact-resistance effect: (*a*) physical situation; (*b*) temperature profile.

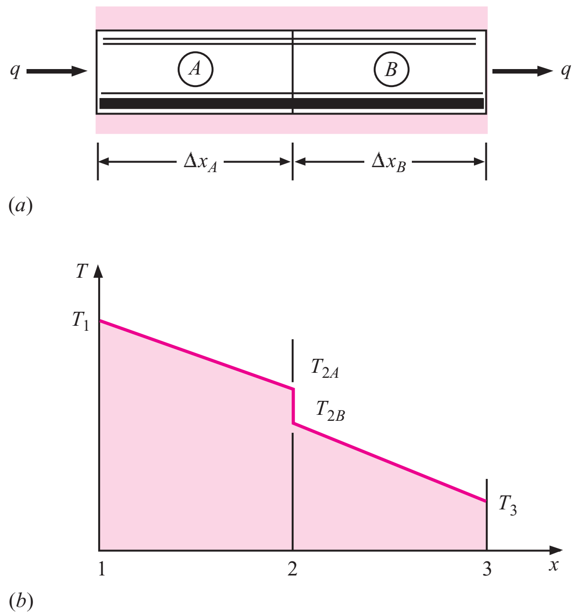


**Figure 2-16** | Joint-roughness model for analysis of thermal contact resistance.

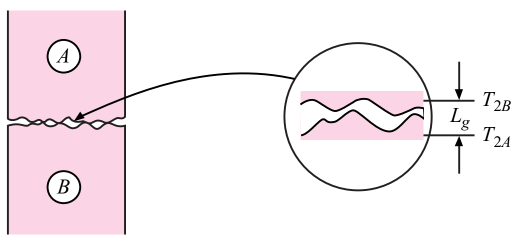

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**Table 2-3** | Contact conductance of typical surfaces.

| | Roughness | | | | $1/h_c$ | |
|---|---|---|---|---|---|---|
| **Surface type** | **$\mu$ in** | **$\mu$m** | **Temperature, °C** | **Pressure, atm** | **h · ft$^2$ · °F/Btu** | **m$^2$ · °C/W × 10$^4$** |
| 416 Stainless, ground, air | 100 | 2.54 | 90–200 | 3–25 | 0.0015 | 2.64 |
| 304 Stainless, ground, air | 45 | 1.14 | 20 | 40–70 | 0.003 | 5.28 |
| 416 Stainless, ground, with 0.001-in brass shim, air | 100 | 2.54 | 30–200 | 7 | 0.002 | 3.52 |
| Aluminum, ground, air | 100 | 2.54 | 150 | 12–25 | 0.0005 | 0.88 |
| | 10 | 0.25 | 150 | 12–25 | 0.0001 | 0.18 |
| Aluminum, ground, with 0.001-in brass shim, air | 100 | 2.54 | 150 | 12–200 | 0.0007 | 1.23 |
| Copper, ground, air | 50 | 1.27 | 20 | 12–200 | 0.00004 | 0.07 |
| Copper, milled, air | 150 | 3.81 | 20 | 10–50 | 0.0001 | 0.18 |
| Copper, milled, vacuum | 10 | 0.25 | 30 | 7–70 | 0.0005 | 0.88 |

Designating the contact area by $A_c$ and the void area by $A_v$, we may write for the heat flow across the joint

$$q = \frac{T_{2A} - T_{2B}}{L_g/2k_A A_c + L_g/2k_B A_c} + k_f A_v \frac{T_{2A} - T_{2B}}{L_g} = \frac{T_{2A} - T_{2B}}{1/h_c A}$$

where $L_g$ is the thickness of the void space and $k_f$ is the thermal conductivity of the fluid which fills the void space. The *total* cross-sectional area of the bars is $A$. Solving for $h_c$, the contact coefficient, we obtain

$$h_c = \frac{1}{L_g}\left(\frac{A_c}{A}\frac{2k_A k_B}{k_A + k_B} + \frac{A_v}{A}k_f\right) \qquad \textbf{[2-51]}$$

In most instances, air is the fluid filling the void space and $k_f$ is small compared with $k_A$ and $k_B$. If the contact area is small, the major thermal resistance results from the void space. The main problem with this simple theory is that it is extremely difficult to determine effective values of $A_c$, $A_v$, and $L_g$ for surfaces in contact.

From the physical model, we may tentatively conclude:

1. The contact resistance should increase with a decrease in the ambient gas pressure when the pressure is decreased below the value where the mean free path of the molecules is large compared with a characteristic dimension of the void space, since the effective thermal conductance of the entrapped gas will be decreased for this condition.
2. The contact resistance should be decreased for an increase in the joint pressure since this results in a deformation of the high spots of the contact surfaces, thereby creating a greater contact area between the solids.

A very complete survey of the contact-resistance problem is presented in References 4, 6, 7, 10, 11. Unfortunately, there is no satisfactory theory that will predict thermal contact resistance for all types of engineering materials, nor have experimental studies yielded completely reliable empirical correlations. This is understandable because of the many complex surface conditions that may be encountered in practice.

Radiation heat transfer across the joint can also be important when high temperatures are encountered. This energy transfer may be calculated by the methods discussed in Chapter 8.

For design purposes the contact conductance values given in Table 2-3 may be used in the absence of more specific information. Thermal contact resistance can be reduced markedly, perhaps as much as 75 percent, by the use of a "thermal grease" like Dow 340.

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**EXAMPLE 2-12** Influence of Contact Conductance on Heat Transfer

Two 3.0-cm-diameter 304 stainless-steel bars, 10 cm long, have ground surfaces and are exposed to air with a surface roughness of about 1 $\mu$m. If the surfaces are pressed together with a pressure of 50 atm and the two-bar combination is exposed to an overall temperature difference of 100°C, calculate the axial heat flow and temperature drop across the contact surface.

**Solution**

The overall heat flow is subject to three thermal resistances, one conduction resistance for each bar, and the contact resistance. For the bars

$$R_{\text{th}} = \frac{\Delta x}{kA} = \frac{(0.1)(4)}{(16.3)\pi(3 \times 10^{-2})^2} = 8.679°\text{C/W}$$

From Table 2-2 the contact resistance is

$$R_c = \frac{1}{h_c A} = \frac{(5.28 \times 10^{-4})(4)}{\pi(3 \times 10^{-2})^2} = 0.747°\text{C/W}$$

The total thermal resistance is therefore

$$\sum R_{\text{th}} = (2)(8.679) + 0.747 = 18.105$$

and the overall heat flow is

$$q = \frac{\Delta T}{\sum R_{\text{th}}} = \frac{100}{18.105} = 5.52 \text{ W} \quad [18.83 \text{ Btu/h}]$$

The temperature drop across the contact is found by taking the ratio of the contact resistance to the total thermal resistance:

$$\Delta T_c = \frac{R_c}{\sum R_{\text{th}}} \Delta T = \frac{(0.747)(100)}{18.105} = 4.13°\text{C} \quad [39.43°\text{F}]$$

In this problem the contact resistance represents about 4 percent of the total resistance.

## REVIEW QUESTIONS

1. What is meant by the term *one-dimensional* when applied to conduction problems?
2. What is meant by thermal resistance?
3. Why is the one-dimensional heat-flow assumption important in the analysis of fins?
4. Define fin efficiency.
5. Why is the insulated-tip solution important for the fin problems?
6. What is meant by thermal contact resistance? Upon what parameters does this resistance depend?

## LIST OF WORKED EXAMPLES

**2-1** Multilayer conduction
**2-2** Multilayer cylindrical system
**2-3** Heat transfer through a composite wall
**2-4** Cooling cost savings with extra insulation
**2-5** Overall heat-transfer coefficient for a tube

**2-6** Critical insulation thickness
**2-7** Heat source with convection
**2-8** Influence of thermal conductivity on fin temperature profiles
**2-9** Straight aluminum fin
**2-10** Circumferential aluminum fin
**2-11** Rod with heat sources
**2-12** Influence of contact conductance on heat transfer

# PROBLEMS

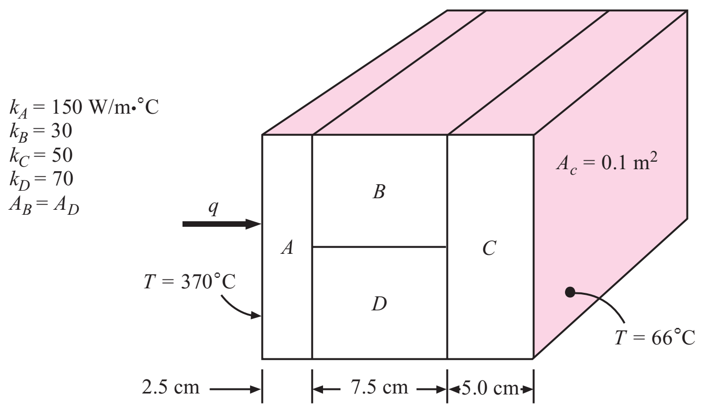
**2-1** A wall 2 cm thick is to be constructed from material that has an average thermal conductivity of 1.3 W/m · °C. The wall is to be insulated with material having an average thermal conductivity of 0.35 W/m · °C, so that the heat loss per square meter will not exceed 1830 W. Assuming that the inner and outer surface temperatures of the insulated wall are 1300 and 30°C, calculate the thickness of insulation required.

**2-2** A certain material 2.5 cm thick, with a cross-sectional area of 0.1 $m^2$, has one side maintained at 35°C and the other at 95°C. The temperature at the center plane of the material is 62°C, and the heat flow through the material is 1 kW. Obtain an expression for the thermal conductivity of the material as a function of temperature.

**2-3** A composite wall is formed of a 2.5-cm copper plate, a 3.2-mm layer of asbestos, and a 5-cm layer of fiberglass. The wall is subjected to an overall temperature difference of 560°C. Calculate the heat flow per unit area through the composite structure.

**2-4** Find the heat transfer per unit area through the composite wall in Figure P2-4. Assume one-dimensional heat flow.

**Figure P2-4**




**2-5** One side of a copper block 5 cm thick is maintained at 250°C. The other side is covered with a layer of fiberglass 2.5 cm thick. The outside of the fiberglass is maintained at 35°C, and the total heat flow through the copper-fiberglass combination is 52 kW. What is the area of the slab?

**2-6** An outside wall for a building consists of a 10-cm layer of common brick and a 2.5-cm layer of fiberglass [$k = 0.05$ W/m · °C]. Calculate the heat flow through the wall for a 25°C temperature differential.

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**2-7** One side of a copper block 4 cm thick is maintained at 175°C. The other side is covered with a layer of fiberglass 1.5 cm thick. The outside of the fiberglass is maintained at 80°C, and the total heat flow through the composite slab is 300 W. What is the area of the slab?

**2-8** A plane wall is constructed of a material having a thermal conductivity that varies as the square of the temperature according to the relation $k = k_0(1 + \beta T^2)$. Derive an expression for the heat transfer in such a wall.

**2-9** A steel tube having $k = 46$ W/m · °C has an inside diameter of 3.0 cm and a tube wall thickness of 2 mm. A fluid flows on the inside of the tube producing a convection coefficient of 1500 W/m$^2$ · °C on the inside surface, while a second fluid flows across the outside of the tube producing a convection coefficient of 197 W/m$^2$ · °C on the outside tube surface. The inside fluid temperature is 223°C while the outside fluid temperature is 57°C. Calculate the heat lost by the tube per meter of length.

**2-10** A certain material has a thickness of 30 cm and a thermal conductivity of 0.04 W/m · °C. At a particular instant in time, the temperature distribution with $x$, the distance from the left face, is $T = 150x^2 - 30x$, where $x$ is in meters. Calculate the heat-flow rates at $x = 0$ and $x = 30$ cm. Is the solid heating up or cooling down?

**2-11** A 0.025-mm-diameter stainless steel wire having $k = 16$ W/m · °C is connected to two electrodes. The length of the wire is 80 cm and it is exposed to a convection environment at 20°C with $h = 500$ W/m$^2$ · °C. A voltage is impressed on the wire that produces temperatures at each electrode of 200°C. Determine the total heat lost by the wire.

**2-12** A wall is constructed of 2.0 cm of copper, 3.0 mm of asbestos sheet [$k =$ 0.166 W/m · °C], and 6.0 cm of fiberglass. Calculate the heat flow per unit area for an overall temperature difference of 500°C.

**2-13** A certain building wall consists of 6.0 in of concrete [$k = 1.2$ W/m · °C], 2.0 in of fiberglass insulation, and $\frac{3}{8}$ in of gypsum board [$k = 0.05$ W/m · °C]. The inside and outside convection coefficients are 2.0 and 7.0 Btu/h · ft$^2$ · °F, respectively. The outside air temperature is 20°F, and the inside temperature is 72°F. Calculate the overall heat-transfer coefficient for the wall, the $R$ value, and the heat loss per unit area.

**2-14** A wall is constructed of a section of stainless steel [$k = 16$ W/m · °C] 4.0 mm thick with identical layers of plastic on both sides of the steel. The overall heat-transfer coefficient, considering convection on both sides of the plastic, is 120 W/m$^2$ · °C. If the overall temperature difference across the arrangement is 60°C, calculate the temperature difference across the stainless steel.

**2-15** An ice chest is constructed of Styrofoam [$k = 0.033$ W/m · °C] with inside dimensions of 25 by 40 by 100 cm. The wall thickness is 5.0 cm. The outside of the chest is exposed to air at 25°C with $h = 10$ W/m$^2$ · °C. If the chest is completely filled with ice, calculate the time for the ice to completely melt. State your assumptions. The enthalpy of fusion for water is 330 kJ/kg.

**2-16** A spherical tank, 1 m in diameter, is maintained at a temperature of 120°C and exposed to a convection environment. With $h = 25$ W/m$^2$ · °C and $T_\infty = 15$°C, what thickness of urethane foam should be added to ensure that the outer temperature of the insulation does not exceed 40°C? What percentage reduction in heat loss results from installing this insulation?

**2-17** A hollow sphere is constructed of aluminum with an inner diameter of 4 cm and an outer diameter of 8 cm. The inside temperature is 100°C and the outer temperature is 50°C. Calculate the heat transfer.

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**2-18** Suppose the sphere in Problem 2-16 is covered with a 1-cm layer of an insulating material having $k = 50$ m W/m · °C and the outside of the insulation is exposed to an environment with $h = 20$ W/m$^2$ · °C and $T_\infty = 10$°C. The inside of the sphere remains at 100°C. Calculate the heat transfer under these conditions.

**2-19** In Appendix A, dimensions of standard steel pipe are given. Suppose a 3-in schedule 80 pipe is covered with 1 in of an insulation having $k = 60$ m W/m · °C and the outside of the insulation is exposed to an environment having $h = 10$ W/m$^2$ · °C and $T_\infty = 20$°C. The temperature of the inside of the pipe is 250°C. For unit length of the pipe calculate (*a*) overall thermal resistance and (*b*) heat loss.

**2-20** A steel pipe with 5-cm OD is covered with a 6.4-mm asbestos insulation [$k = 0.096$ Btu/h · ft · °F] followed by a 2.5-cm layer of fiberglass insulation [$k = 0.028$ Btu/h · ft · °F]. The pipe-wall temperature is 315°C, and the outside insulation temperature is 38°C. Calculate the interface temperature between the asbestos and fiberglass.

**2-21** Derive an expression for the thermal resistance through a hollow spherical shell of inside radius $r_i$ and outside radius $r_o$ having a thermal conductivity $k$. (See Equation 2–10.)

**2-22** A 1.0-mm-diameter wire is maintained at a temperature of 400°C and exposed to a convection environment at 40°C with $h = 120$ W/m$^2$ · °C. Calculate the thermal conductivity that will just cause an insulation thickness of 0.2 mm to produce a "critical radius." How much of this insulation must be added to reduce the heat transfer by 75 percent from that which would be experienced by the bare wire?

**2-23** A 2.0-in schedule 40 steel pipe (see Appendix A) has $k = 27$ Btu/h · ft · °F. The fluid inside the pipe has $h = 30$ Btu/h · ft$^2$ · °F, and the outer surface of the pipe is covered with 0.5-in fiberglass insulation with $k = 0.023$ Btu/h · ft · °F. The convection coefficient on the outer insulation surface is 2.0 Btu/h · ft$^2$ · °F. The inner fluid temperature is 320°F and the ambient temperature is 70°F. Calculate the heat loss per foot of length.

**2-24** Derive a relation for the critical radius of insulation for a sphere.

**2-25** A cylindrical tank 80 cm in diameter and 2.0 m high contains water at 80°C. The tank is 90 percent full, and insulation is to be added so that the water temperature will not drop more than 2°C per hour. Using the information given in this chapter, specify an insulating material and calculate the thickness required for the specified cooling rate.

**2-26** A hot steam pipe having an inside surface temperature of 250°C has an inside diameter of 8 cm and a wall thickness of 5.5 mm. It is covered with a 9-cm layer of insulation having $k = 0.5$ W/m · °C, followed by a 4-cm layer of insulation having $k = 0.25$ W/m · °C. The outside temperature of the insulation is 20°C. Calculate the heat lost per meter of length. Assume $k = 47$ W/m · °C for the pipe.

**2-27** A house wall may be approximated as two 1.2-cm layers of fiber insulating board, an 8.0-cm layer of loosely packed asbestos, and a 10-cm layer of common brick. Assuming convection heat-transfer coefficients of 12 W/m$^2$ · °C on both sides of the wall, calculate the overall heat-transfer coefficient for this arrangement.

**2-28** Calculate the $R$ value for the following insulations: (*a*) urethane foam, (*b*) fiberglass mats, (*c*) mineral wool blocks, (*d*) calcium silicate blocks.

**2-29** An insulation system is to be selected for a furnace wall at 1000°C using first a layer of mineral wool blocks followed by fiberglass boards. The outside of the insulation is exposed to an environment with $h = 15$ W/m$^2$ · °C and $T_\infty = 40$°C. Using the data of Table 2-1, calculate the thickness of each insulating material such that the

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interface temperature is not greater than 400°C and the outside temperature is not greater than 55°C. Use mean values for the thermal conductivities. What is the heat loss in this wall in watts per square meter?

**2-30** Derive an expression for the temperature distribution in a plane wall having uniformly distributed heat sources and one face maintained at a temperature $T_1$ while the other face is maintained at a temperature $T_2$. The thickness of the wall may be taken as $2L$.

**2-31** A 5-cm-diameter steel pipe is covered with a 1-cm layer of insulating material having $k = 0.22$ W/m · °C followed by a 3-cm-thick layer of another insulating material having $k = 0.06$ W/m · °C. The entire assembly is exposed to a convection surrounding condition of $h = 60$ W/m$^2$ · °C and $T_\infty = 15$°C. The outside surface temperature of the steel pipe is 400°C. Calculate the heat lost by the pipe-insulation assembly for a pipe length of 20 m. Express in Watts.

**2-32** Derive an expression for the temperature distribution in a plane wall in which distributed heat sources vary according to the linear relation

$$\dot{q} = \dot{q}_w[1 + \beta(T - T_w)]$$

where $\dot{q}_w$ is a constant and equal to the heat generated per unit volume at the wall temperature $T_w$. Both sides of the plate are maintained at $T_w$, and the plate thickness is $2L$.

**2-33** A circumferential fin of rectangular profile is constructed of stainless steel with $k = 43$ W/m · °C and a thickness of 1.0 mm. The fin is installed on a tube having a diameter of 3.0 cm and the outer radius of the fin is 4.0 cm. The inner tube is maintained at 250°C and the assembly is exposed to a convection environment having $T_\infty = 35$°C and $h = 45$ W/m$^2$ · °C. Calculate the heat lost by the fin.

**2-34** A plane wall 6.0 cm thick generates heat internally at the rate of 0.3 MW/m$^3$. One side of the wall is insulated, and the other side is exposed to an environment at 93°C. The convection heat-transfer coefficient between the wall and the environment is 570 W/m$^2$ · °C. The thermal conductivity of the wall is 21 W/m · °C. Calculate the maximum temperature in the wall.

**2-35** Consider a shielding wall for a nuclear reactor. The wall receives a gamma-ray flux such that heat is generated within the wall according to the relation

$$\dot{q} = \dot{q}_0 e^{-ax}$$

where $\dot{q}_0$ is the heat generation at the inner face of the wall exposed to the gamma-ray flux and $a$ is a constant. Using this relation for heat generation, derive an expression for the temperature distribution in a wall of thickness $L$, where the inside and outside temperatures are maintained at $T_i$ and $T_0$, respectively. Also obtain an expression for the maximum temperature in the wall.

**2-36** Repeat Problem 2-35, assuming that the outer surface is adiabatic while the inner surface temperature is maintained at $T_i$.

**2-37** Rework Problem 2-32 assuming that the plate is subjected to a convection environment on both sides of temperature $T_\infty$ with a heat-transfer coefficient $h$. $T_w$ is now some reference temperature not necessarily the same as the surface temperature.

**2-38** Heat is generated in a 2.5-cm-square copper rod at the rate of 35.3 MW/m$^3$. The rod is exposed to a convection environment at 20°C, and the heat-transfer coefficient is 4000 W/m$^2$ · °C. Calculate the surface temperature of the rod.

**2-39** A plane wall of thickness $2L$ has an internal heat generation that varies according to $\dot{q} = \dot{q}_0 \cos ax$, where $\dot{q}_0$ is the heat generated per unit volume at the center of the wall ($x = 0$) and $a$ is a constant. If both sides of the wall are maintained at a constant temperature of $T_w$, derive an expression for the total heat loss from the wall per unit surface area.

**2-40** A certain semiconductor material has a conductivity of 0.0124 W/cm · °C. A rectangular bar of the material has a cross-sectional area of 1 cm$^2$ and a length of 3 cm. One end is maintained at 300°C and the other end at 100°C, and the bar carries a current of 50 A. Assuming the longitudinal surface is insulated, calculate the midpoint temperature in the bar. Take the resistivity as $1.5 \times 10^{-3}\,\Omega \cdot$ cm.

**2-41** The temperature distribution in a certain plane wall is

$$\frac{T - T_1}{T_2 - T_1} = C_1 + C_2 x^2 + C_3 x^3$$

where $T_1$ and $T_2$ are the temperatures on each side of the wall. If the thermal conductivity of the wall is constant and the wall thickness is $L$, derive an expression for the heat generation per unit volume as a function of $x$, the distance from the plane where $T = T_1$. Let the heat-generation rate be $\dot{q}_0$ at $x = 0$.

**2-42** Electric heater wires are installed in a solid wall having a thickness of 8 cm and $k = 2.5$ W/m · °C. The right face is exposed to an environment with $h = 50$ W/m$^2$ · °C and $T_\infty = 30$°C, while the left face is exposed to $h = 75$ W/m$^2$ · °C and $T_\infty = 50$°C. What is the maximum allowable heat-generation rate such that the maximum temperature in the solid does not exceed 300°C?

**2-43** Two 5.0-cm-diameter aluminum bars, 2 cm long, have ground surfaces and are joined in compression with a 0.025-mm brass shim at a pressure exceeding 20 atm. The combination is subjected to an overall temperature difference of 200°C. Calculate the temperature drop across the contact join.

**2-44** A 3.0-cm-thick plate has heat generated uniformly at the rate of $5 \times 10^5$ W/m$^3$. One side of the plate is maintained at 200°C and the other side at 45°C. Calculate the temperature at the center of the plate for $k = 16$ W/m · °C.

**2-45** Heat is generated uniformly in a stainless steel plate having $k = 20$ W/m · °C. The thickness of the plate is 1.0 cm and the heat-generation rate is 500 MW/m$^3$. If the two sides of the plate are maintained at 100 and 200°C, respectively, calculate the temperature at the center of the plate.

**2-46** A plate having a thickness of 4.0 mm has an internal heat generation of 200 MW/m$^3$ and a thermal conductivity of 25 W/m · °C. One side of the plate is insulated and the other side is maintained at 100°C. Calculate the maximum temperature in the plate.

**2-47** A 3.2-mm-diameter stainless-steel wire 30 cm long has a voltage of 10 V impressed on it. The outer surface temperature of the wire is maintained at 93°C. Calculate the center temperature of the wire. Take the resistivity of the wire as 70 $\mu\Omega \cdot$ cm and the thermal conductivity as 22.5 W/m · °C.

**2-48** The heater wire of Example 2-7 is submerged in a fluid maintained at 93°C. The convection heat-transfer coefficient is 5.7 kW/m$^2$ · °C. Calculate the center temperature of the wire.

**2-49** An electric current is used to heat a tube through which a suitable cooling fluid flows. The outside of the tube is covered with insulation to minimize heat loss to the surroundings, and thermocouples are attached to the outer surface of the tube to measure the temperature. Assuming uniform heat generation in the tube, derive an expression for the convection heat-transfer coefficient on the inside of the tube in

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terms of the measured variables: voltage $E$, current $I$, outside tube wall temperature $T_0$, inside and outside radii $r_i$ and $r_o$, tube length $L$, and fluid temperature $T_f$.

**2-50** Derive an expression for the temperature distribution in a sphere of radius $r$ with uniform heat generation $\dot{q}$ and constant surface temperature $T_w$.

**2-51** A stainless-steel sphere [$k = 16$ W/m · °C] having a diameter of 4 cm is exposed to a convection environment at 20°C, $h = 15$ W/m$^2$ · °C. Heat is generated uniformly in the sphere at the rate of 1.0 MW/m$^3$. Calculate the steady-state temperature for the center of the sphere.

**2-52** An aluminum-alloy electrical cable has $k = 190$ W/m · °C, a diameter of 30 mm, and carries an electric current of 230 A. The resistivity of the cable is 2.9 $\mu\Omega \cdot$ cm, and the outside surface temperature of the cable is 180°C. Calculate the maximum temperature in the cable if the surrounding air temperature is 15°C.

**2-53** Derive an expression for the temperature distribution in a hollow cylinder with heat sources that vary according to the linear relation

$$\dot{q} = a + br$$

with $\dot{q}_i$ the generation rate per unit volume at $r = r_i$. The inside and outside temperatures are $T = T_i$ at $r = r_i$ and $T = T_o$ at $r = r_o$.

**2-54** The outside of a copper wire having a diameter of 2 mm is exposed to a convection environment with $h = 5000$ W/m$^2$ · °C and $T_\infty = 100$°C. What current must be passed through the wire to produce a center temperature of 150°C? Repeat for an aluminum wire of the same diameter. The resistivity of copper is 1.67 $\mu\Omega \cdot$ cm.

**2-55** A hollow tube having an inside diameter of 2.5 cm and a wall thickness of 0.4 mm is exposed to an environment at $h = 100$ W/m$^2$ · °C and $T_\infty = 40$°C. What heat-generation rate in the tube will produce a maximum tube temperature of 250°C for $k = 24$ W/m · °C?

**2-56** Water flows on the inside of a steel pipe with an ID of 2.5 cm. The wall thickness is 2 mm, and the convection coefficient on the inside is 500 W/m$^2$ · °C. The convection coefficient on the outside is 12 W/m$^2$ · °C. Calculate the overall heat-transfer coefficient. What is the main determining factor for $U$?

**2-57** The pipe in Problem 2-56 is covered with a layer of asbestos [$k = 0.18$ W/m · °C] while still surrounded by a convection environment with $h = 12$ W/m$^2$ · °C. Calculate the critical insulation radius. Will the heat transfer be increased or decreased by adding an insulation thickness of ($a$) 0.5 mm, ($b$) 10 mm?

**2-58** Calculate the overall heat-transfer coefficient for Problem 2-4.

**2-59** Calculate the overall heat-transfer coefficient for Problem 2-5.

**2-60** Air flows at 120°C in a thin-wall stainless-steel tube with $h = 65$ W/m$^2$ · °C. The inside diameter of the tube is 2.5 cm and the wall thickness is 0.4 mm. $k = 18$ W/m · °C for the steel. The tube is exposed to an environment with $h = 6.5$ W/m$^2$ · °C and $T_\infty = 15$°C. Calculate the overall heat-transfer coefficient and the heat loss per meter of length. What thickness of an insulation having $k = 40$ mW/m · °C should be added to reduce the heat loss by 90 percent?

**2-61** An insulating glass window is constructed of two 5-mm glass plates separated by an air layer having a thickness of 4 mm. The air layer may be considered stagnant so that pure conduction is involved. The convection coefficients for the inner and outer surfaces are 12 and 50 W/m$^2$ · °C, respectively. Calculate the overall heat-transfer coefficient for this arrangement, and the $R$ value. Repeat the calculation for a single glass plate 5 mm thick.

**2-62** A wall consists of a 1-mm layer of copper, a 4-mm layer of 1 percent carbon steel, a 1-cm layer of asbestos sheet, and 10 cm of fiberglass blanket. Calculate the overall heat-transfer coefficient for this arrangement. If the two outside surfaces are at 10 and 150°C, calculate each of the interface temperatures.

**2-63** A circumferential fin of rectangular profile has a thickness of 0.7 mm and is installed on a tube having a diameter of 3 cm that is maintained at a temperature of 200°C. The length of the fin is 2 cm and the fin material is copper. Calculate the heat lost by the fin to a surrounding convection environment at 100°C with a convection heat-transfer coefficient of 524 W/m$^2 \cdot$ °C.

**2-64** A thin rod of length $L$ has its two ends connected to two walls which are maintained at temperatures $T_1$ and $T_2$, respectively. The rod loses heat to the environment at $T_\infty$ by convection. Derive an expression ($a$) for the temperature distribution in the rod and ($b$) for the total heat lost by the rod.

**2-65** A rod of length $L$ has one end maintained at temperature $T_0$ and is exposed to an environment of temperature $T_\infty$. An electrical heating element is placed in the rod so that heat is generated uniformly along the length at a rate $\dot{q}$. Derive an expression ($a$) for the temperature distribution in the rod and ($b$) for the total heat transferred to the environment. Obtain an expression for the value of $\dot{q}$ that will make the heat transfer zero at the end that is maintained at $T_0$.

**2-66** One end of a copper rod 30 cm long is firmly connected to a wall that is maintained at 200°C. The other end is firmly connected to a wall that is maintained at 93°C. Air is blown across the rod so that a heat-transfer coefficient of 17 W/m$^2 \cdot$ °C is maintained. The diameter of the rod is 12.5 mm. The temperature of the air is 38°C. What is the net heat lost to the air in watts?

**2-67** Verify the temperature distribution for case 2 in Section 2-9, i.e., that

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

Subsequently show that the heat transfer is

$$q = \sqrt{hPkA}\,(T_0 - T_\infty)\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

**2-68** An aluminum rod 2.0 cm in diameter and 12 cm long protrudes from a wall that is maintained at 250°C. The rod is exposed to an environment at 15°C. The convection heat-transfer coefficient is 12 W/m$^2 \cdot$ °C. Calculate the heat lost by the rod.

**2-69** Derive Equation (2-35) by integrating the convection heat loss from the rod of case 1 in Section 2-9.

**2-70** Derive Equation (2-36) by integrating the convection heat loss from the rod of case 3 in Section 2-9.

**2-71** A long, thin copper rod 5 mm in diameter is exposed to an environment at 20°C. The base temperature of the rod is 120°C. The heat-transfer coefficient between the rod and the environment is 20 W/m$^2 \cdot$ °C. Calculate the heat given up by the rod.

**2-72** A very long copper rod [$k = 372$ W/m $\cdot$ °C] 2.5 cm in diameter has one end maintained at 90°C. The rod is exposed to a fluid whose temperature is 40°C. The heat-transfer coefficient is 3.5 W/m$^2 \cdot$ °C. How much heat is lost by the rod?

**2-73** An aluminum fin 1.5 mm thick is placed on a circular tube with 2.7-cm OD. The fin is 6 mm long. The tube wall is maintained at 150°C, the environment temperature is 15°C, and the convection heat-transfer coefficient is 20 W/m$^2$ · °C. Calculate the heat lost by the fin.

**2-74** A straight fin of rectangular profile has a thermal conductivity of 14 W/m · °C, thickness of 2.0 mm, and length of 23 mm. The base of the fin is maintained at a temperature of 220°C while the fin is exposed to a convection environment at 23°C with $h = 25$ W/m$^2$ · °C. Calculate the heat lost per meter of fin depth.

**2-75** A circumferential fin of rectangular profile is constructed of a material having $k = 55$ W/m · °C and is installed on a tube having a diameter of 3 cm. The length of fin is 3 cm and the thickness is 2 mm. If the fin is exposed to a convection environment at 20°C with a convection coefficient of 68 W/m$^2$ · °C and the tube wall temperature is 100°C, calculate the heat lost by the fin.

**2-76** The total efficiency for a finned surface may be defined as the ratio of the total heat transfer of the combined area of the surface and fins to the heat that would be transferred if this total area were maintained at the base temperature $T_0$. Show that this efficiency can be calculated from

$$\eta_t = 1 - \frac{A_f}{A}(1 - \eta_f)$$

where

$\eta_t$ = total efficiency
$A_f$ = surface area of all fins
$A$ = total heat-transfer area, including fins and exposed tube or other surface
$\eta_f$ = fin efficiency

**2-77** A triangular fin of stainless steel (18% Cr, 8% Ni) is attached to a plane wall maintained at 460°C. The fin thickness is 6.4 mm, and the length is 2.5 cm. The environment is at 93°C, and the convection heat-transfer coefficient is 28 W/m$^2$ · °C. Calculate the heat lost from the fin.

**2-78** A 2.5-cm-diameter tube has circumferential fins of rectangular profile spaced at 9.5-mm increments along its length. The fins are constructed of aluminum and are 0.8 mm thick and 12.5 mm long. The tube wall temperature is maintained at 200°C, and the environment temperature is 93°C. The heat-transfer coefficient is 110 W/m$^2$ · °C. Calculate the heat loss from the tube per meter of length.

**2-79** A circumferential fin of rectangular profile surrounds a 2-cm-diameter tube. The length of the fin is 5 mm, and the thickness is 2.5 mm. The fin is constructed of mild steel. If air blows over the fin so that a heat-transfer coefficient of 25 W/m$^2$ · °C is experienced and the temperatures of the base and air are 260 and 93°C, respectively, calculate the heat transfer from the fin.

**2-80** A straight rectangular fin 2.0 cm thick and 14 cm long is constructed of steel and placed on the outside of a wall maintained at 200°C. The environment temperature is 15°C, and the heat-transfer coefficient for convection is 20 W/m$^2$ · °C. Calculate the heat lost from the fin per unit depth.

**2-81** An aluminum fin 1.6 mm thick surrounds a tube 2.5 cm in diameter. The length of the fin is 12.5 mm. The tube-wall temperature is 200°C, and the environment temperature is 20°C. The heat-transfer coefficient is 60 W/m$^2$ · °C. What is the heat lost by the fin?

**2-82** Obtain an expression for the optimum thickness of a straight rectangular fin for a given profile area. Use the simplified insulated-tip solution.

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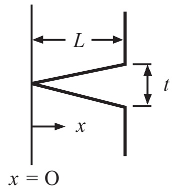
**2-83** Derive a differential equation (do not solve) for the temperature distribution in a straight triangular fin. For convenience, take the coordinate axis as shown in Figure P2-83 and assume one-dimensional heat flow.

**Figure P2-83**




**2-84** A circumferential fin of rectangular profile is installed on a 10-cm-diameter tube maintained at 120°C. The fin has a length of 15 cm and thickness of 2 mm. The fin is exposed to a convection environment at 23°C with $h = 60$ W/m$^2 \cdot$ °C and the fin conductivity is 120 W/m $\cdot$ °C. Calculate the heat lost by the fin expressed in watts.

**2-85** A long stainless-steel rod [$k = 16$ W/m $\cdot$ °C] has a square cross section 12.5 by 12.5 mm and has one end maintained at 250°C. The heat-transfer coefficient is 40 W/m$^2 \cdot$ °C, and the environment temperature is 90°C. Calculate the heat lost by the rod.

**2-86** A straight fin of rectangular profile is constructed of duralumin (94% Al, 3% Cu) with a thickness of 2.1 mm. The fin is 17 mm long, and it is subjected to a convection environment with $h = 75$ W/m$^2 \cdot$ °C. If the base temperature is 100°C and the environment is at 30°C, calculate the heat transfer per unit length of fin.

**2-87** A certain internal-combustion engine is air-cooled and has a cylinder constructed of cast iron [$k = 35$ Btu/h $\cdot$ ft $\cdot$ °F]. The fins on the cylinder have a length of $\frac{5}{8}$ in and thickness of $\frac{1}{8}$ in. The convection coefficient is 12 Btu/h $\cdot$ ft$^2 \cdot$ °F. The cylinder diameter is 4 in. Calculate the heat loss per fin for a base temperature of 450°F and environment temperature of 100°F.

**2-88** A 1.5-mm-diameter stainless-steel rod [$k = 19$ W/m $\cdot$ °C] protrudes from a wall maintained at 45°C. The rod is 12 mm long, and the convection coefficient is 500 W/m$^2 \cdot$ °C. The environment temperature is 20°C. Calculate the temperature of the tip of the rod. Repeat the calculation for $h = 200$ and 1500 W/m$^2 \cdot$ °C.

**2-89** An aluminum block is cast with an array of pin fins protruding like that shown in Figure 2-10*d* and subjected to room air at 20°C. The convection coefficient between the pins and the surrounding air may be assumed to be $h = 13.2$ W/m$^2 \cdot$ °C. The pin diameters are 2 mm and their length is 25 mm. The base of the aluminum block may be assumed constant at 70°C. Calculate the total heat lost by an array of 15 by 15, that is, 225 fins.

**2-90** A finned tube is constructed as shown in Figure 2-10*b*. Eight fins are installed as shown and the construction material is aluminum. The base temperature of the fins may be assumed to be 100°C and they are subjected to a convection environment at 30°C with $h = 15$ W/m$^2 \cdot$ °C. The longitudinal length of the fins is 15 cm and the peripheral length is 2 cm. The fin thickness is 2 mm. Calculate the total heat dissipated by the finned tube. Consider only the surface area of the fins.

**2-91** Circumferential fins of rectangular profile are constructed of aluminum and attached to a copper tube having a diameter of 25 mm and maintained at 100°C. The length of the fins is 2 cm and thickness is 2 mm. The arrangement is exposed to a convection environment at 30°C with $h = 15$ W/m$^2 \cdot$ °C. Assume that a number of fins is installed such that the total fin surface area equals that of the total surface fine area in Problem 2-90. Calculate the total heat lost by the fins.

**2-92** A 2-cm-diameter glass rod 6 cm long [$k = 0.8$ W/m $\cdot$ °C] has a base temperature of 100°C and is exposed to an air convection environment at 20°C. The temperature at the tip of the rod is measured as 35°C. What is the convection heat-transfer coefficient? How much heat is lost by the rod?

**2-93** A straight rectangular fin has a length of 2.5 cm and a thickness of 1.5 mm. The thermal conductivity is 55 W/m $\cdot$ °C, and it is exposed to a convection environment

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at 20°C and $h = 500$ W/m$^2 \cdot$ °C. Calculate the maximum possible heat loss for a base temperature of 200°C. What is the actual heat loss?

**2-94** A straight rectangular fin has a length of 3.5 cm and a thickness of 1.4 mm. The thermal conductivity is 55 W/m · °C. The fin is exposed to a convection environment at 20°C and $h = 500$ W/m$^2 \cdot$ °C. Calculate the maximum possible heat loss for a base temperature of 150°C. What is the actual heat loss for this base temperature?

**2-95** A circumferential fin of rectangular profile is constructed of 1 percent carbon steel and attached to a circular tube maintained at 150°C. The diameter of the tube is 5 cm, and the length is also 5 cm with a thickness of 2 mm. The surrounding air is maintained at 20°C and the convection heat-transfer coefficient may be taken as 100 W/m$^2 \cdot$ °C. Calculate the heat lost from the fin.

**2-96** A circumferential fin of rectangular profile is constructed of aluminum and surrounds a 3-cm-diameter tube. The fin is 2 cm long and 1 mm thick. The tube wall temperature is 200°C, and the fin is exposed to a fluid at 20°C with a convection heat-transfer coefficient of 80 W/m$^2 \cdot$ °C. Calculate the heat loss from the fin.

**2-97** A 1.0-cm-diameter steel rod [$k = 20$ W/m · °C] is 20 cm long. It has one end maintained at 50°C and the other at 100°C. It is exposed to a convection environment at 20°C with $h = 50$ W/m$^2 \cdot$ °C. Calculate the temperature at the center of the rod.

**2-98** A circumferential fin of rectangular profile is constructed of copper and surrounds a tube having a diameter of 1.25 cm. The fin length is 6 mm and its thickness is 0.3 mm. The fin is exposed to a convection environment at 20°C with $h = 55$ W/m$^2 \cdot$ °C and the fin base temperature is 100°C. Calculate the heat lost by the fin.

**2-99** A straight rectangular fin of steel (1% C) is 2 cm thick and 17 cm long. It is placed on the outside of a wall which is maintained at 230°C. The surrounding air temperature is 25°C, and the convection heat-transfer coefficient is 23 W/m$^2 \cdot$ °C. Calculate the heat lost from the fin per unit depth and the fin efficiency.

**2-100** A straight fin having a triangular profile has a length of 5 cm and a thickness of 4 mm and is constructed of a material having $k = 23$ W/m · °C. The fin is exposed to surroundings with a convection coefficient of 20 W/m$^2 \cdot$ °C and a temperature of 40°C. The base of the fin is maintained at 200°C. Calculate the heat lost per unit depth of fin.

**2-101** A circumferential aluminum fin is installed on a 25.4-mm-diameter tube. The length of the fin is 12.7 mm and the thickness is 1.0 mm. It is exposed to a convection environment at 30°C with a convection coefficient of 56 W/m$^2 \cdot$ °C. The base temperature is 125°C. Calculate the heat lost by the fin.

**2-102** A circumferential fin of rectangular profile is constructed of stainless steel (18% Cr, 8% Ni). The thickness of the fin is 2.0 mm, the inside radius is 2.0 cm, and the length is 8.0 cm. The base temperature is maintained at 135°C and the fin is exposed to a convection environment at 15°C with $h = 20$ W/m$^2 \cdot$ °C. Calculate the heat lost by the fin.

**2-103** A rectangular fin has a length of 2.5 cm and thickness of 1.1 mm. The thermal conductivity is 55 W/m · °C. The fin is exposed to a convection environment at 20°C and $h = 500$ W/m$^2 \cdot$ °C. Calculate the heat loss for a base temperature of 125°C.

**2-104** A 1.0-mm-thick aluminum fin surrounds a 2.5-cm-diameter tube. The length of the fin is 1.25 cm. The fin is exposed to a convection environment at 30°C with $h = 75$ W/m$^2 \cdot$ °C. The tube surface is maintained at 100°C. Calculate the heat lost by the fin.

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**2-105** A glass rod having a diameter of 1 cm and length of 5 cm is exposed to a convection environment at a temperature of 20°C. One end of the rod is maintained at a temperature of 180°C. Calculate the heat lost by the rod if the convection heat-transfer coefficient is 20 W/m$^2$ · °C.

**2-106** A stainless steel rod has a square cross section measuring 1 by 1 cm. The rod length is 8 cm, and $k = 18$ W/m · °C. The base temperature of the rod is 300°C. The rod is exposed to a convection environment at 50°C with $h = 45$ W/m$^2$ · °C. Calculate the heat lost by the rod and the fin efficiency.

**2-107** Copper fins with a thickness of 1.0 mm are installed on a 2.5-cm-diameter tube. The length of each fin is 12 mm. The tube temperature is 275°C and the fins are exposed to air at 35°C with a convection heat-transfer coefficient of 120 W/m$^2$ · °C. Calculate the heat lost by each fin.

**2-108** A straight fin of rectangular profile is constructed of stainless steel (18% Cr, 8% Ni) and has a length of 5 cm and a thickness of 2.5 cm. The base temperature is maintained at 100°C and the fin is exposed to a convection environment at 20°C with $h = 47$ W/m$^2$ · °C. Calculate the heat lost by the fin per meter of depth, and the fin efficiency.

**2-109** A circumferential fin of rectangular profile is constructed of duralumin and surrounds a 3-cm-diameter tube. The fin is 3 cm long and 1 mm thick. The tube wall temperature is 200°C, and the fin is exposed to a fluid at 20°C with a convection heat-transfer coefficient of 80 W/m$^2$ · °C. Calculate the heat loss from the fin.

**2-110** A circular fin of rectangular profile is attached to a 3.0-cm-diameter tube maintained at 100°C. The outside diameter of the fin is 9.0 cm and the fin thickness is 1.0 mm. The environment has a convection coefficient of 50 W/m$^2$ · °C and a temperature of 30°C. Calculate the thermal conductivity of the material for a fin efficiency of 60 percent.

**2-111** A circumferential fin of rectangular profile having a thickness of 1.0 mm and a length of 2.0 cm is placed on a 2.0-cm-diameter tube. The tube temperature is 150°C, the environment temperature is 20°C, and $h = 150$ W/m$^2$ · °C. The fin is aluminum. Calculate the heat lost by the fin.

**2-112** Two 1-in-diameter bars of stainless steel [$k = 17$ W/m · °C] are brought into end-to-end contact so that only 0.1 percent of the cross-sectional area is in contact at the joint. The bars are 7.5 cm long and subjected to an axial temperature difference of 300°C. The roughness depth in each bar ($L_g/2$) is estimated to be 1.3 $\mu$m. The surrounding fluid is air, whose thermal conductivity may be taken as 0.035 W/m · °C for this problem. Estimate the value of the contact resistance and the axial heat flow. What would the heat flow be for a continuous 15-cm stainless-steel bar?

**2-113** When the *joint pressure* for two surfaces in contact is increased, the high spots of the surfaces are deformed so that the contact area $A_c$ is increased and the roughness depth $L_g$ is decreased. Discuss this effect in the light of the presentation of Section 2-11. (Experimental work shows that joint conductance varies almost directly with pressure.)

**2-114** Two aluminum plates 5 mm thick with a ground roughness of 100 $\mu$in are bolted together with a contact pressure of 20 atm. The overall temperature difference across the plates is 80°C. Calculate the temperature drop across the contact joint.

**2-115** Fins are frequently installed on tubes by a press-fit process. Consider a circumferential aluminum fin having a thickness of 1.0 mm to be installed on a 2.5-cm-diameter aluminum tube. The fin length is 1.25 cm, and the contact conductance may be

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taken from Table 2-2 for a 100-$\mu$in ground surface. The convection environment is at 20°C, and $h = 125$ W/m$^2 \cdot$ °C. Calculate the heat transfer for each fin for a tube wall temperature of 200°C. What percentage reduction in heat transfer is caused by the contact conductance?

**2-116** An aluminum fin is attached to a transistor that generates heat at the rate of 300 mW. The fin has a total surface area of 9.0 cm$^2$ and is exposed to surrounding air at 27°C. The contact conductance between transistor and fin is $0.9 \times 10^{-4}$ m$^2 \cdot$ °C/W, and the contact area is 0.5 cm$^2$. Estimate the temperature of the transistor, assuming the fin is uniform in temperature.

**2-117** A plane wall 20 cm thick with uniform internal heat generation of 200 kW/m$^3$ is exposed to a convection environment on both sides at 50°C with $h = 400$ W/m$^2 \cdot$ °C. Calculate the center temperature of the wall for $k = 20$ W/m $\cdot$ °C.

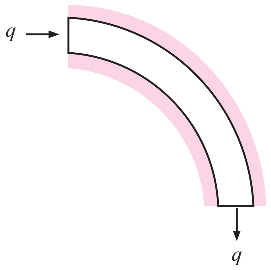
**2-118** Suppose the wall of Problem 2-117 is only 10 cm thick and has one face insulated. Calculate the maximum temperature in the wall assuming all the other conditions are the same. Comment on the results.

**2-119** A circumferential fin of rectangular profile is constructed of aluminum and placed on a 6-cm-diameter tube maintained at 120°C. The length of the fin is 3 cm and its thickness is 2 mm. The fin is exposed to a convection environment at 20°C with $h = 220$ W/m$^2 \cdot$ °C. Calculate the heat lost by the fin expressed in Watts.

**2-120** A straight aluminum fin of triangular profile has a base maintained at 200°C and is exposed to a convection environment at 25°C with $h = 45$ W/m$^2 \cdot$ °C. The fin has a length of 8 mm and a thickness of 2.0 mm. Calculate the heat lost per unit depth of fin.

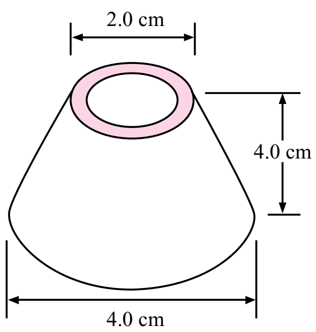
**2-121** One hundred circumferential aluminum fins of rectangular profile are mounted on a 1.0-m tube having a diameter of 2.5 cm. The fins are 1 cm long and 2.0 mm thick. The base temperature is 180°C, and the convection environment is at 20°C with $h = 50$ W/m$^2 \cdot$ °C. Calculate the total heat lost from the finned-tube arrangement over the 1.0-m length.

**Figure P2-122**




**2-122** The cylindrical segment shown in Figure P2-122 has a thermal conductivity of 100 W/m $\cdot$ °C. The inner and outer radii are 1.5 and 1.7 cm, respectively, and the surfaces are insulated. Calculate the circumferential heat transfer per unit depth for an imposed temperature difference of 50°C. What is the thermal resistance?

**Figure P2-123**




**2-123** The truncated hollow cone shown in Figure P2-123 is used in laser-cooling applications and is constructed of copper with a thickness of 0.5 mm. Calculate the thermal resistance for one-dimensional heat flow. What would be the heat transfer for a temperature difference of 300°C?

**2-124** A tube assembly is constructed of copper with an inside diameter of 1.25 cm, wall thickness of 0.8 mm, and circumferential fins around the periphery. The fins have a thickness of 0.3 mm and length of 3 mm, and are spaced 6 mm apart. If the convection heat transfer coefficient from the tube and fins to the surrounding air is 50 W/m$^2 \cdot$ °C, calculate the thermal resistance for a 30-cm length of the tube-fin combination. What is the fin efficiency for this arrangement? If the inside tube temperature is 100°C and the surrounding air temperature is 20°C, what is the heat loss per meter of tube length? What fraction of the loss is by the fins?

**2-125** Calculate the $R$ value for the fin-tube combination in Problem 2-116.

**2-126** Repeat Problem 2-124 for aluminum fins installed on a copper tube.

**2-127** Repeat Problem 2-125 for aluminum fins installed on a copper tube.

**2-128** A stainless-steel rod having a length of 10 cm and diameter of 2 mm has a resistivity of 70 $\mu\Omega \cdot$ cm and thermal conductivity of 16 W/m $\cdot$ °C. The rod is exposed to a convection environment with $h = 100$ W/m$^2 \cdot$ °C and $T = 20$°C. Both ends of the rod are maintained at $T = 100$°C. What voltage must be impressed on the rod to dissipate twice as much heat to the surroundings as in a zero-voltage condition?

**2-129** Suppose the rod in Problem 2-128 is very long. What would the zero-voltage heat transfer be in this case?

**2-130** Suppose the cylindrical segment of Problem 2-122 has a periphery exposed to a convection environment with $h = 75$ W/m$^2 \cdot$ °C and $T_\infty = 30$°C instead of to the insulated surface. For this case, one end is at 50°C while the other end is at 100°C. What is the heat lost by the segment to the surroundings in this circumstance? What is the heat transfer at each end of the segment?

## Design-Oriented Problems

**2-131** Suppose you have a choice between a straight triangular or rectangular fin constructed of aluminum with a base thickness of 3.0 mm. The convection coefficient is 50 W/m$^2 \cdot$ °C. Select the fin with the least weight for a given heat flow.

**2-132** Consider aluminum circumferential fins with $r_1 = 1.0$ cm, $r_2 = 2.0$ cm, and thicknesses of 1.0, 2.0, and 3.0 mm. The convection coefficient is 160 W/m$^2 \cdot$ °C. Compare the heat transfers for six 1.0-mm fins, three 2.0-mm fins, and two 3.0-mm fins. What do you conclude? Repeat for $h = 320$ W/m$^2 \cdot$ °C.

**2-133** "Pin fins" of aluminum are to be compared in terms of their relative performance as a function of diameter. Three "pins" having diameters of 2, 5, and 10 mm with a length of 5 cm are exposed to a convection environment with $T_\infty = 20$°C, and $h = 40$ W/m$^2 \cdot$ °C. The base temperature is 200°C. Calculate the heat transfer for each pin. How does it vary with pin diameter?

**2-134** Calculate the heat transfer per unit mass for the pin fins in Problem 2-133. How does it vary with diameter?

**2-135** A straight rectangular fin has a length of 1.5 cm and a thickness of 1.0 mm. The convection coefficient is 20 W/m$^2 \cdot$ °C. Compare the heat-transfer rates for aluminum and magnesium fins.

**2-136** Suppose both fins in Problem 2-129 are to dissipate the same heat. Which would be lower in weight? Assume that the thickness is the same for both fins but adjust the lengths until the heat transfers are equal.

**2-137** Insulating materials are frequently installed with a reflective coating to reduce the radiation heat transfer between the surface and the surroundings. An insulating material is installed on a furnace oven wall that is maintained at 200°C. The energy cost of the fuel firing the oven is $8.25/GJ and the insulation installation must be justified by the savings in energy costs over a three-year period. Select an appropriate insulation from Table 2-1 and/or Table A-3 and determine a suitable quantity of insulation that will pay for itself over a three-year period. For this computation assume that the outer surface of the insulation radiates like a blackbody and that the heat loss can be determined from Equation (1-12). For the calculation use Table 1-2 as a guide for selecting the convection heat-transfer coefficient. Next, consider the same type of insulating material but with a reflective coating having $\epsilon = 0.1$. The radiation transfer may still be calculated with Equation (1-12). Determine the quantity of the

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reflective insulating material required to be economical. How much higher cost per unit thickness or volume could be justified for the reflective material over that of the nonreflective? Comment on uncertainties which may exist in your analysis.

**2-138** A thin-wall stainless-steel tube is to be used as an electric heating element that will deliver a convection coefficient of 5000 $W/m^2 \cdot °C$ to water at 100°C. Devise several configurations to accomplish a total heat transfer of 10 kW. Specify the length, outside diameter, wall thickness, maximum tube temperature, and necessary voltage that must be imposed on the tube. Take the resistivity of stainless steel as 70 $\mu\Omega \cdot cm$.

**2-139** Thin cylindrical or spherical shells may be treated as a plane wall for sufficiently large diameters in relation to the thickness of the shell. Devise a scheme for quantifying the error that would result from such a treatment.

**2-140** A 2.5-cm-diameter steel pipe is maintained at 100°C by condensing steam on the inside. The pipe is to be used for dissipating heat to a surrounding room at 20°C by placing circular steel fins around the outside surface of the pipe. The convection loss from the pipe and fins occurs by free convection, with $h = 8.0$ $W/m^2 \cdot °C$. Examine several cases of fin thickness, fin spacing, and fin outside diameters to determine the overall heat loss per meter of pipe length. Take $k = 43$ $W/m \cdot °C$ for the steel fins and assume $h$ is uniform over all surfaces. Make appropriate conclusions about the results of your study.

**2-141** A pipe having a diameter of 5.3 cm is maintained at 200°C by steam flowing inside. The pipe passes through a large factory area and loses heat by free convection from the outside with $h = 7.2$ $W/m^2 \cdot °C$. Using information from Table 2-1 and/or Table A-3, select two alternative insulating materials that could be installed to lower the outside surface temperature of the insulation to 30°C when the pipe is exposed to room air at 20°C. If the energy loss from the steam costs $8.00/$10^9$ J, what are the allowable costs of the insulation materials per unit volume to achieve a payback period of three years where

$$\begin{aligned}&(\text{energy cost saved per year}) \times 3\\ &\quad = (\text{cost of installed insulation/unit volume}) \times \text{volume}\end{aligned}$$

**2-142** It is frequently represented that the energy savings resulting from installation of extra ceiling insulation in a home will pay for the insulation cost within a three-year period. You are asked to evaluate this claim. For the evaluation it may be assumed that 1 kW of electrical input to an air-conditioning unit will produce about $1.26 \times 10^4$ kJ/h of cooling and that electricity is priced at $0.085/kWh. Assume that an existing home has ceiling insulation with an $R$ value of 7.0°F $\cdot$ $ft^2 \cdot h/Btu$ and is to be upgraded to an $R$ value of either 15 or 30. Choose two alternative insulation materials from Table 2-1 and/or Table A-3 and calculate the allowable costs per unit volume of insulating material to accomplish the three-year payback with the two specified $R$ values. For this calculation, (energy cost saved/year) × 3 = (insulation cost per unit volume) × volume. Make your own assumptions regarding (1) temperature difference between the interior of the house and the attic area and (2) the hours of operation for the air-conditioning system during an annual period. Comment on the results and assumptions.

**2-143** A finned wall like that shown in Figure 2-10*a* is constructed of aluminum alloy with $k = 160$ $W/m \cdot °C$. The wall thickness is 2.0 mm and the fins are straight with rectangular profile. The inside of the wall is maintained at a constant temperature of 70°C and the fins are exposed to a convection environment at 25°C with $h = 8$ $W/m^2 \cdot °C$ (free convection). The assembly will be cast from the aluminum material and must

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dissipate 30 W of heat under the conditions noted. Assuming a square array, determine suitable combinations of numbers of fins, fin spacing, dimension of the square, and fin thickness to accomplish this cooling objective. Assume a uniform value of $h$ for both the fin and wall surfaces.

**2-144** Repeat Problem 2-143 for cooling with forced convection, which produces a convection coefficient of $h = 20 \text{ W/m}^2 \cdot °\text{C}$.

**2-145** Consider a pin fin as shown in Figure 2-10*d*. Assume that the fin is exposed to an evacuated space such that convection is negligible and that the radiation loss per unit surface area is given by

$$q_{\text{rad}}/A = \epsilon\sigma(T^4 - T_s^4)$$

where $\epsilon$ is a surface emissivity constant, $\sigma$ is the Stefan-Boltzmann constant, and the temperatures are expressed in degrees Kelvin. Derive a differential equation for the temperature in the pin fin as a function of $x$, the distance from the base. Let $T_0$ be the base temperature, and write the appropriate boundary conditions for the differential equation.

**2-146** Consider two special cases for the fin in Problem 2-145: (*a*) an insulated-tip fin losing heat by radiation and (*b*) a very long fin losing heat by radiation. Write the appropriate boundary conditions for these two cases.

**2-147** Consider another special case for the fin of Problem 2-145; where the surrounding radiation boundary temperature is negligible, that is,

$$T_s^4 \ll T^4$$

Write the resulting simplified differential equation under this condition.

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CHAPTER

# 3

# Steady-State Conduction—Multiple Dimensions

## 3-1 | INTRODUCTION

In Chapter 2 steady-state heat transfer was calculated in systems in which the temperature gradient and area could be expressed in terms of one space coordinate. We now wish to analyze the more general case of two-dimensional heat flow. For steady state with no heat generation, the Laplace equation applies.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad \textbf{[3-1]}$$

assuming constant thermal conductivity. The solution to this equation may be obtained by analytical, numerical, or graphical techniques.

The objective of any heat-transfer analysis is usually to predict heat flow or the temperature that results from a certain heat flow. The solution to Equation (3-1) will give the temperature in a two-dimensional body as a function of the two independent space coordinates $x$ and $y$. Then the heat flow in the $x$ and $y$ directions may be calculated from the Fourier equations

$$q_x = -kA_x \frac{\partial T}{\partial x} \qquad \textbf{[3-2]}$$

$$q_y = -kA_y \frac{\partial T}{\partial y} \qquad \textbf{[3-3]}$$

These heat-flow quantities are directed either in the $x$ direction or in the $y$ direction. The total heat flow at any point in the material is the resultant of the $q_x$ and $q_y$ at that point. Thus the total heat-flow vector is directed so that it is perpendicular to the lines of constant temperature in the material, as shown in Figure 3-1. So if the temperature distribution in the material is known, we may easily establish the heat flow.

## 3-2 | MATHEMATICAL ANALYSIS OF TWO-DIMENSIONAL HEAT CONDUCTION

We first consider an analytical approach to a two-dimensional problem and then indicate the numerical and graphical methods that may be used to advantage in many other problems.