2.4.2 Methods of Inverse Z-T [find f(kT)]

1- (P.F.M)

The z-transform – The partial fraction expansion method

Note that 1. If (Z) is not exit in num., multiply Num, den by (Z)

Example.

$$X(z) = \frac{10z+5}{(z-1)(z-1/5)}$$

$$\Downarrow$$

$$\frac{X(z)}{z} = \frac{10z+5}{z(z-1)(z-1/5)} = 25\frac{1}{z} + \frac{75}{4}\frac{1}{z-1} - \frac{175}{4}\frac{1}{z-1/5}$$

$$\Downarrow$$

$$X(z) = 25 + \frac{75}{4}\frac{z}{z-1} - \frac{175}{4}\frac{z}{z-1/5} = 25 + \frac{75}{4}\frac{1}{1-z^{-1}} - \frac{175}{4}\frac{1}{1-1/5z^{-1}}$$

$$\Downarrow$$

$$x(k) = 25\delta(k) + \frac{75}{4}1^k - \frac{175}{4}(1/5)^k \qquad k \geq 0$$

Example.

$$X(z) = X(z) = \frac{3z+1}{z^2-z+1/2}$$

$$\Downarrow$$

$$\frac{X(z)}{z} = \frac{3z+1}{z(z-(1/2+1/2j))(z-(1/2-1/2j))} = \frac{10}{z} - \frac{5+8j}{z-(1/2+1/2j)} - \frac{5-8j}{z-(1/2-1/2j)}$$

$$\Downarrow$$

$$X(z) = 10 - \frac{(5+8j)z}{z-(1/2+1/2j)} - \frac{(5-8j)z}{z-(1/2-1/2j)} = 10 - \frac{5+8j}{1-(1/2+1/2j)z^{-1}} - \frac{5-8j}{1-(1/2-1/2j)z^{-1}}$$

$$\Downarrow$$

$$x(k) = 10\delta(k) - 10\left(\frac{1}{\sqrt{2}}\right)^k \cos\frac{k\pi}{4} + 16\left(\frac{1}{\sqrt{2}}\right)^k \sin\frac{k\pi}{4} \qquad k \geq 0$$

Example:

Find F(kT), where

$$f(z)=\frac{10z}{(z-1)(z-2)}$$
$$\frac{f(z)}{z}=\frac{10}{(z-1)(z-2)}$$
$$\frac{f(z)}{z}=\frac{A}{z-1}+\frac{B}{z-2}$$
$$A=-10$$
$$B=10$$
$$f(z)=\frac{-10z}{z-1}+\frac{10z}{z-2}$$
$$f(kT)=-10+10(2)^k$$

There are some of formulas

$$\frac{z}{z-1}=(1)^k=1$$
$$\frac{z}{z-2}=(2)^k$$
$$\frac{z}{z-a}=(a)^k$$

$$\frac{az}{(z-a)^2}=k(a)^k$$

Note that

1-Must be the operator (z) in the numerator of F(z) or G(z). If not exist , we must added {multiplied the num. and den. by z}.

2- apply the same procedures for repeated poles as in (P.F.M).

2- long division method

Find F(kT) for

$$f(z) = \frac{10z}{(z-1)(z-2)}$$

$$f(z) = \frac{10z}{z^2 - 3z + 2}$$

$$f(z) = \frac{10z^{-1}}{1 - 3z^{-1} + 2z^{-2}}$$

longdivision

$$f(kT) = 10z^{-1} + 30z^{-2} + 70z^{-3} + - - - - -$$

$$f(0) = 0$$

$$f(1T) = 10$$

$$f(2T) = 30$$

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Stopped as required of counter (k).

3- Residual method

The law is:-

$$f(kT) = \sum \mathrm{Re}\, of\, f(z) Z^{k-1} at\, poles\, of\, f(z)$$

Find F(kT) for

$$f(z) = \frac{10z}{(z-3)(z-4)}$$

$$f(kT) = (z-3)\frac{10z}{(z-3)(z-4)} z^{k-1} \downarrow_{at z=3} + \frac{(z-4)10z}{(z-3)(z-4)} z^{k-1} \downarrow_{at z=4}$$

$$f(kT) = -10z^k + 10z^k$$

$$f(kT) = -10(3)^k + 10(4)^k$$

apply the same procedures for repeated poles as in (residual method of Z-T).

Example:

Find F(kT) for

$$Z^{-1}\left[\frac{z^2}{(z-1)(z-0.5)}\right]=\frac{z^2\cdot z^{k-1}}{(z-1)(z-0.5)}(z-1)\bigg|_{z=1}+\frac{z^2\cdot z^{k-1}}{(z-1)(z-0.5)}(z-0.5)\bigg|_{z=0.5}$$

$$=2-(0.5)^{\frac{kT}{T}}$$

4-Difference Equation Method

This method can be used, when G(z) is given as (num/den)

Note that for <u>step input</u>

R(k)=1 , for all values of k=0,1,2,3,4,----------

And for <u>impulse input</u>

R(0)=1, only at k=0 and R(k)=0 for all values of k=1,2,3,4-----------

Example:

Find C(kT) or time-response of :-

$$\frac{C(z)}{R(z)}=\frac{10z}{(z-1)(z-2)}$$

$$=\frac{10z^{-1}}{1-3z^{-1}+2z^{-2}}$$

$$C(z)-3z^{-1}C(z)+2z^{-2}C(z)=10z^{-1}R(z)$$

$$C(kT)-3C(kT-1)+2C(kT-2)=10R(k-1)$$

$$C(kT)=3C(k-1)-2C(k-2)+10R(k-1)$$

K=0, C(0)=0

K=1, C(1)=10

K=2, C(2)=30 and so on

Note that

If the given system is open-loop, convert to closed-loop and apply the method to find the transient response.

2.4.3 Modified of Z-T

For systems containing time delays, Z-T of time-delay system is called modified Z-T.

The laws are:-

$$G(z,m) = z^{-1}(1-z^{-1})\sum \operatorname{Re} s of G(s)\frac{ze^{mTs}}{(z-e^{Ts})} at poles of G(s)$$

TM=T-TΔ, Where Δ is the system delay and (T) is sampling time

Or

M=1- Δ

Example:

Find the pluse-Transfer –Function(P.T.F) or G(z,m) for:- let T=1sec.

$$G(s) = \frac{e^{-0.4s}}{(s+1)}$$

$$withZOH$$

$$G(z,m) = z^{-1}(1-z^{-1})[\frac{1}{s(s+1)} s \frac{ze^{mTs}}{(z-e^{Ts})} \downarrow_{s=0} + \frac{1}{s(s+1)}(s+1)\frac{ze^{mTs}}{(z-e^{Ts})} \downarrow_{s=-1}]$$

$$mT = T - \Delta T = 1 - 0.4*1 = 0.6$$

$$G(z,m) = \frac{z-1}{z}[\frac{(1-e^{-mT})z + e^{-mT} - e^{-T}}{(z-1)(z-e^{-T})}]$$

$$G(z,m) = \frac{z-1}{z}[\frac{(1-e^{-0.6})z + e^{-0.6} - e^{-1}}{(z-1)(z-e^{-1})}]$$

$$G(z,m) = \frac{C(z)}{R(z)}$$

$$or$$

$$C(z,m) = R(z) * G(z,m)$$

$$= \frac{(1-e^{-0.6})z + e^{-0.6} - e^{-1}}{(z-1)(z-e^{-1)}}$$

This is for step input. So that the output can be calculated by using long-division:-

$$C(z,m) = 0 + (1-e^{-0.6})z^{-1} + (1-e^{-1.6})z^{-2} + -------$$

$$C(0) = 0$$

$$C(0.6) = 1 - e^{-0.6}$$

$$C(1+0.6) = 1 - e^{-1.6}$$

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Example:

$$G(s)=\frac{e^{-2.5s}}{(s+1)}$$

withZOH

$$G(z,m)=z^{-3}(1-z^{-1})[\frac{1}{s(s+1)}s\frac{ze^{mTs}}{(z-e^{Ts})}\downarrow_{s=0}+\frac{1}{s(s+1)}(s+1)\frac{ze^{mTs}}{(z-e^{Ts})}\downarrow_{s=-1}]$$

$$\Delta T=2.5$$

$$\Delta=\frac{2.5}{1}$$

$z^{-1} \times z^{-2}$

let

$$e^{-2.5}=e^{-2}*e^{-0.5}$$

Thenew

$$\Delta=0.5$$

$$mT=T-\Delta T=1-0.5*1=0.5$$

$$G(z,m)=\frac{z-1}{z^3}[\frac{(1-e^{-mT})z+e^{-mT}-e^{-T}}{(z-1)(z-e^{-T})}]$$

$$G(z,m)=[\frac{(1-e^{-0.5})z+e^{-0.5}-e^{-1}}{z^3(z-e^{-1})}]$$

$$G(z,m)=\frac{C(z)}{R(z)}$$

or

$$C(z,m)=R(z)*G(z,m)$$

$$=\frac{0.3934z+0.2386}{z^4-1.3678z^3+0.3678z^2}$$

$$C(z,m)=0.3934z^{-3}+0.399z^{-4}+0.2650z^{-5}+-----$$

$$C(0)=0$$

$$C(0.5)=0$$

$$C(1.5)=0$$

$$C(2.5)=0.3934$$

$$C(3.5)=0.399$$

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Example:

Find F(z,m), for the system is delayed Δ=0.27 and T=0.1sec

$$F(s) = \frac{(s+3)}{(s+1)(s+2)}$$

Since the F(z,m) is function and not control system, don't use ZOH device.

ΔT=0.27

$$\Delta = \frac{0.27}{0.1} = 2.7$$

$$e^{2.7} = e^{2} * e^{-0.7}$$

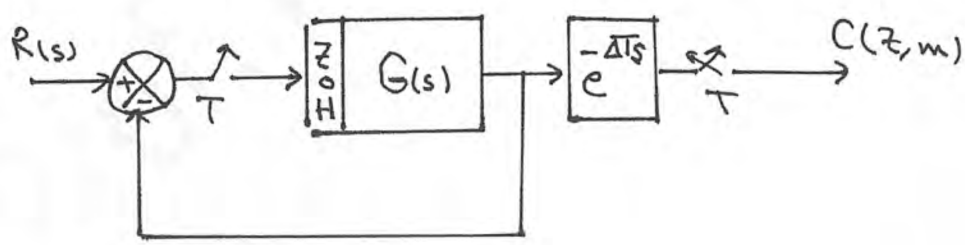
$$\therefore mT = T - \Delta T = 0.1 - 0.7 * 0.1 = 0.03$$

$$F(z,m) = z^{-1} z^{-2} [\frac{2ze^{-0.03}}{z - e^{-0.1}} - \frac{ze^{-0.06}}{z - e^{-0.2}}]$$

$$F(z,m) = \frac{z - 0.7358}{z^4 - 1.724z^3 + 0.741z^2}$$

Note that

For this block-diagram:-



The closed-loop T.F is:-

$$\frac{C(z,m)}{R(z)} = \frac{G(z,m)}{1 + G(z)}$$

And complete the solution in the same manner.