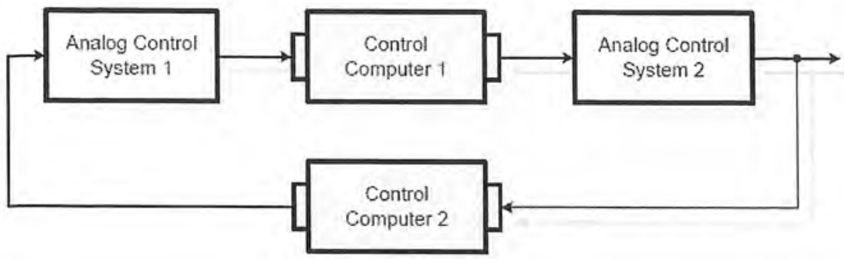
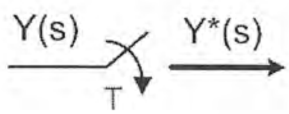
Hybrid Control Systems



- In practice, several analog controllers and a number of micro-controllers / control computers (which are synchronized by the *same* clock) may be utilized as a part of a complex control scheme.
- From the stand-point of control engineering, it is highly desirable to develop equivalent (discrete-time) transfer functions representing the *essential* features of such control systems.

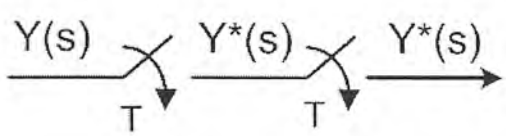
2.7 Properties of Sampled Data Systems

Property 1: *(sampled signal)*



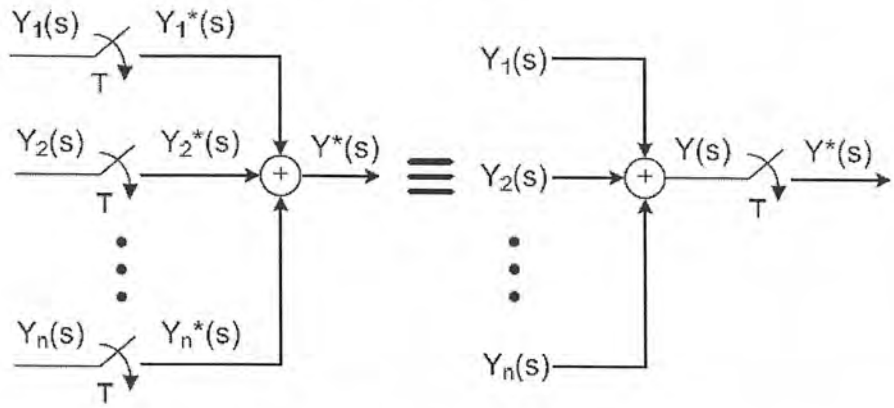
$$Y(z) = Z\{Y^*(s)\}$$

Property 2: *(re-sampled signal)*



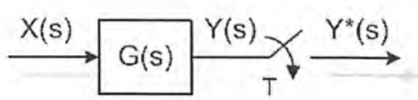
$$Y^*(s) = \left[Y^*(s)\right]^*$$
$$Y(z) = Z\{Y^*(s)\}$$

Property 3: *(summation of sampled signals)*



$$Y^*(s) = Y_1^*(s) \pm Y_2^*(s) \pm \cdots \pm Y_n^*(s)$$
$$Y(z) = Z\{Y_1^*(s)\} \pm Z\{Y_2^*(s)\} \pm \cdots \pm Z\{Y_n^*(s)\}$$
$$Y(z) = Y_1(z) \pm Y_2(z) \pm \cdots \pm Y_n(z)$$

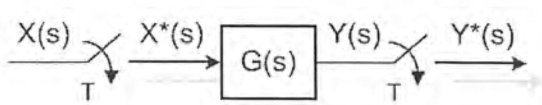
Property 4: *(sampled response)*



$$Y(s) = G(s)X(s)$$
$$Y^*(s) = [G(s)X(s)]^*$$
$$Y(z) \doteq Z\{G(s)X(s)\}$$

Notice that $Y^*(s) \neq G^*(s)X^*(s)$

Property 5: *(sampled impulse response)*

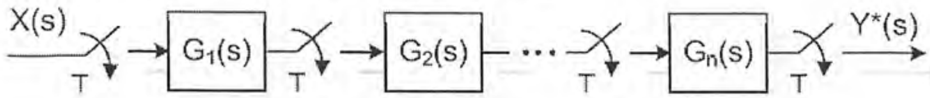


$$Y(s) = G(s)X^*(s)$$
$$Y^*(s) = [G(s)X^*(s)]^*$$
$$Y^*(s) = [G(s)]^* X^*(s) = G^*(s)X^*(s)$$

$X(z) \doteq Z\{X^*(s)\}$ $G(z) \doteq Z\{G^*(s)\}$

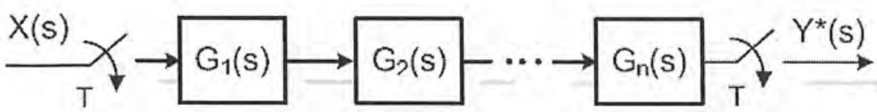
$\therefore Y(z) = G(z)X(z)$

Property 6: *(cascaded impulse responses)*



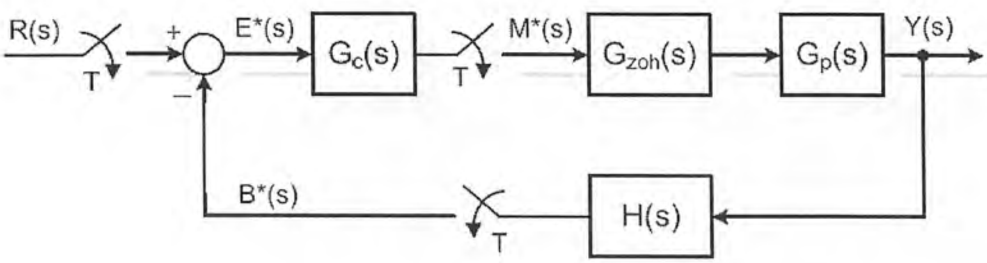
$$Y^*(s) = G_n^*(s) \ldots G_2^*(s)\, G_1^*(s) X^*(s)$$
$$Y(z) \triangleq G_n(z) \ldots G_2(z)\, G_1(z) X(z)$$

Property 7: *(sampled impulse response of a cascaded system)*



$$Y^*(s) = [G_n(s) \ldots G_2(s) G_1(s) X^*(s)]^*$$
$$Y^*(s) = [G_n(s) \ldots G_2(s) G_1(s)]^* X^*(s)$$
$$Y(z) \triangleq Z\{G_n(s) \ldots G_2(s) G_1(s)\} X(z)$$

Example 1



- Consider the block diagram of the illustrated system. Obtain the following transfer functions:
  a) B(z)/R(z)
  b) Y(z)/R(z)

Solution – Part (a)

Using the Property 3:

$E^*(s) = R^*(s) - B^*(s) \Rightarrow$ $\boxed{E(z) = R(z) - B(z)}$ *(1)*

From Property 5:

$M^*(s) = [G_c(s)E^*(s)]^* = G_c^*(s)E^*(s) \Rightarrow$ $\boxed{M(z) = G_c(z)E(z)}$ *(2)*

where $G_c(z) \triangleq Z\{G_c^*(s)\} = Z\{G_c(s)\}$

Sub. ① in ② and the Result in ③

Similarly, the Property 7 yields

$B^*(s) = [H(s)G_p(s)G_{zoh}(s)M^*(s)]^*$

$B^*(s) = [H(s)G_p(s)G_{zoh}(s)]^* M^*(s) \Rightarrow$ $\boxed{B(z) = G(z)M(z)}$ *(3)*

where $G(z) \triangleq Z\{H(s)G_p(s)G_{zoh}(s)\}$

Combining Eqns. (1), (2), and (3) gives

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$$[1 + G(z)G_c(z)]B(z) = G(z)G_c(z)R(z)$$

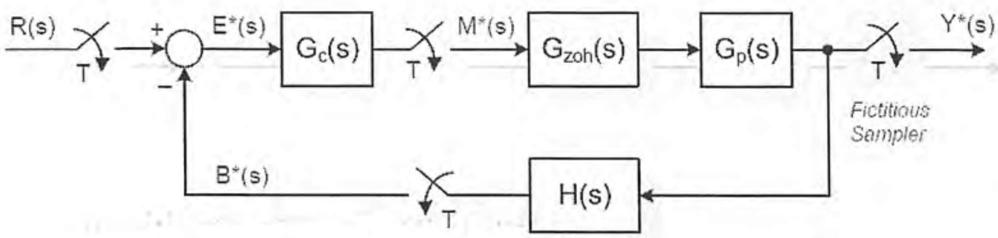
Thus,

$$\boxed{\frac{B(z)}{R(z)} = \frac{G(z)G_c(z)}{1 + G(z)G_c(z)}}$$

where

$$G_c(z) \triangleq Z\{G_c(s)\}$$
$$G(z) \triangleq Z\{H(s)G_p(s)G_{zoh}(s)\}$$

Solution – Part (b)



To obtain the desired transfer function Y(z)/R(z), a *fictitious* sampler must be added to the system.

Using the Property 3:

$$E^*(s) = R^*(s) - B^*(s) \Rightarrow \qquad \boxed{E(z) = R(z) - B(z)} \quad (4)$$

From Property 5:

$$M^*(s) = [G_c(s)E^*(s)]^* = G_c^*(s)E^*(s) \Rightarrow \qquad \boxed{M(z) = G_c(z)E(z)} \quad (5)$$

where $G_c(z) \triangleq Z\{G_c^*(s)\}$

Similarly, the Property 7 yields

$$Y^*(s) = [G_p(s)G_{zoh}(s)M^*(s)]^*$$

$$Y^*(s) = [G_p(s)G_{zoh}(s)]^* M^*(s) \Rightarrow \qquad \boxed{Y(z) = G_1(z)M(z)} \quad (6)$$

where $G_1(z) \triangleq Z\{G_p(s)G_{zoh}(s)\}$

Finally, the Property 7 gives

$$B^*(s) = [H(s)G_p(s)G_{zoh}(s)M^*(s)]^*$$

$$B^*(s) = [H(s)G_p(s)G_{zoh}(s)]^* M^*(s) \Rightarrow \qquad \boxed{B(z) = G(z)M(z)} \quad (7)$$

where $G(z) \triangleq Z\{H(s)G_p(s)G_{zoh}(s)\}$

Dividing Eqn. (7) by (6) leads to

$$\boxed{\frac{B(z)}{Y(z)} = \frac{G(z)}{G_1(z)} \triangleq G_2(z)} \quad (8)$$

Combining Eqns. (4), (5), (6), and (8) gives

$$Y(z) = G_1(z)G_c(z)[R(z) - G_2(z)Y(z)]$$

Thus, $Y(z)[1 + \underbrace{G_1(z)G_2(z)}_{G(z)}G_c(z)] = G_1(z)G_c(z)R(z)$

$$\boxed{\frac{Y(z)}{R(z)} = \frac{G_1(z)G_c(z)}{1 + G(z)G_c(z)}}$$

where
$G_c(z) \triangleq Z\{G_c(s)\}$

$G_1(z) \triangleq Z\{G_p(s)G_{zoh}(s)\}$

$G(z) \triangleq Z\{H(s)G_p(s)G_{zoh}(s)\}$