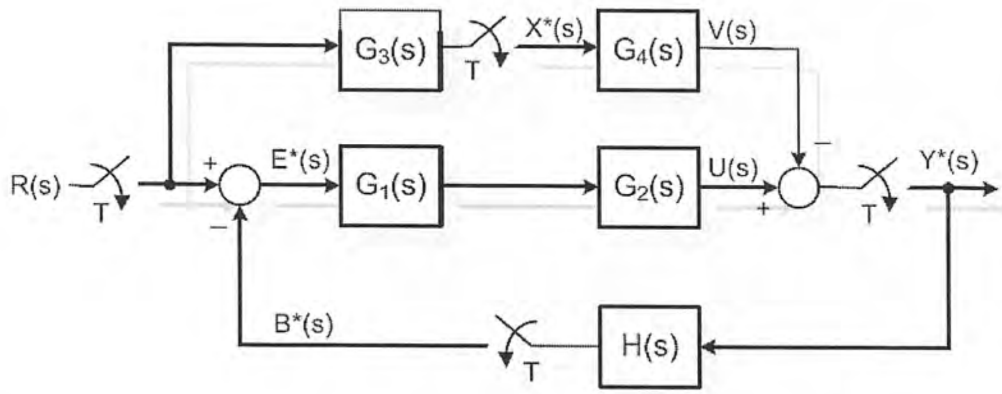
Example 2



- Obtain the transfer function Y(z)/R(z) for the system shown.

Solution

$E^*(s) = R^*(s) - B^*(s) \Rightarrow$ $\boxed{E(z) = R(z) - B(z)}$ *(1)*

$X^*(s) = [G_3(s)R^*(s)]^* = G_3^*(s)R^*(s) \Rightarrow$ $\boxed{X(z) = G_3(z)R(z)}$ *(2)*

$B^*(s) = [H(s)Y^*(s)]^* = H^*(s)Y^*(s) \Rightarrow$ $\boxed{B(z) = H(z)Y(z)}$ *(3)*

$$Y^*(s) = [U(s) - V(s)]^* = U^*(s) - V^*(s)$$

$$Y^*(s) = \underbrace{[G_1(s)G_2(s)E^*(s)]^*}_{U^*(s)} - \underbrace{[G_4(s)X^*(s)]^*}_{V^*(s)}$$

$$Y^*(s) = [G_1(s)G_2(s)]^* E^*(s) - G_4^*(s)X^*(s) \Rightarrow$$

$\boxed{Y(z) = G_{12}(z)E(z) - G_4(z)X(z)}$ *(4)*

where $G_{12}(z) \triangleq Z\{G_1(s)G_2(s)\}$

Combining Eqns. (1), (2), (3), and (4) yields

$$Y(z) = G_{12}(z)[R(z) - H(z)Y(z)] - G_4(z)G_3(z)R(z)$$

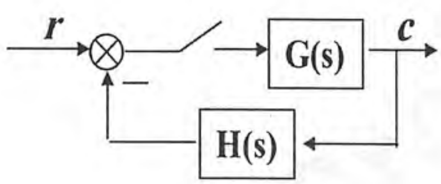
Hence,

$$Y(z)[1 + G_{12}(z)H(z)] = [G_{12}(z) - G_3(z)G_4(z)]R(z)$$

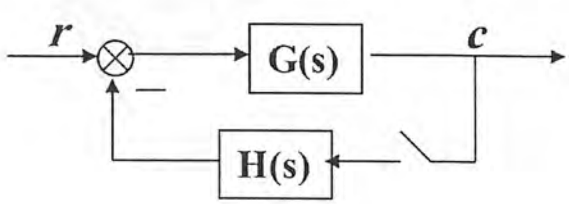
$$\boxed{\frac{Y(z)}{R(z)} = \frac{G_{12}(z) - G_3(z)G_4(z)}{1 + G_{12}(z)H(z)}}$$

where $G_{12}(z) \triangleq Z\{G_1(s)G_2(s)\}$

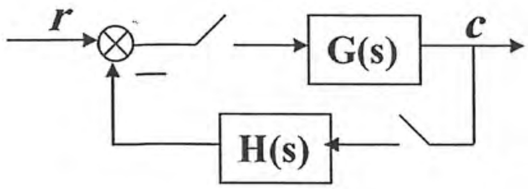
2.8 Z-transfer (pulse) function(P.T.F)



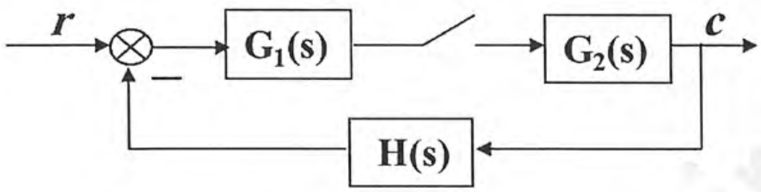
$$C(z) = \frac{R(z)G(z)}{1 + GH(z)} \Rightarrow GH(z) = Z[G(s)H(s)]$$



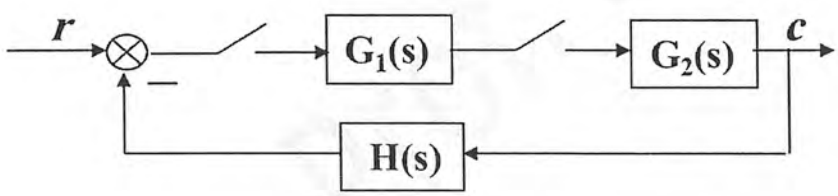
$$C(z) = \frac{RG(z)}{1 + GH(z)} \Rightarrow RG(z) = Z[R(s)G(s)]$$



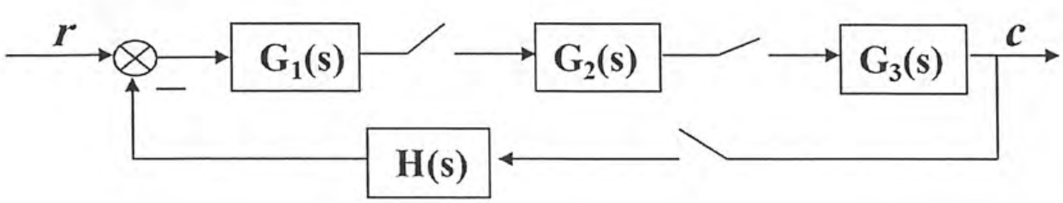
$$C(z) = \frac{R(z)G(z)}{1 + G(z)H(z)}$$



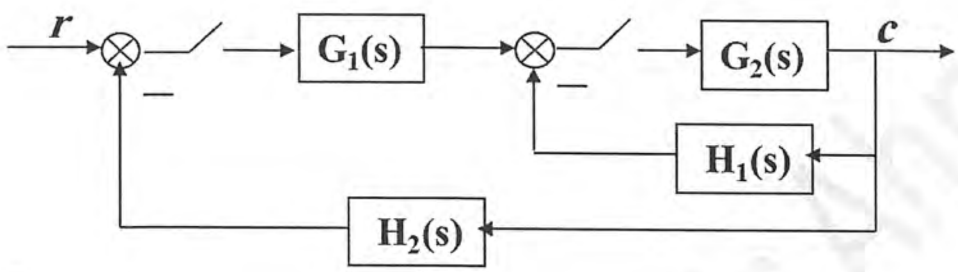
$$C(z) = \frac{RG_1(z)G_2(z)}{1 + G_1G_2H(z)}$$



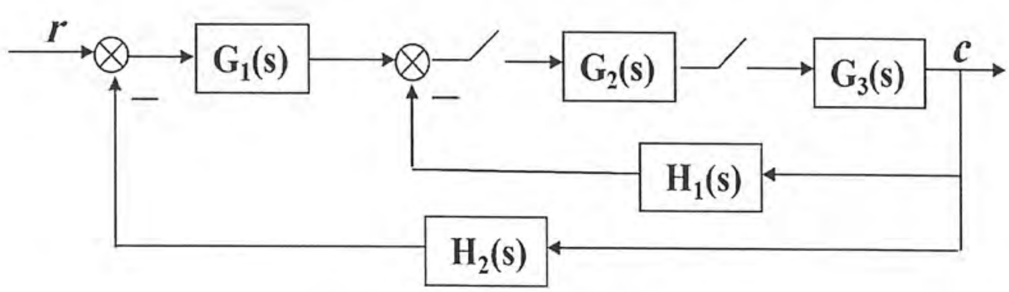
$$C(z) = \frac{R(z)G_1(z)G_2(z)}{1 + G_1(z)G_2H(z)}$$



$$C(z)=\frac{RG_1(z)G_2(z)G_3(z)}{1+G_2(z)G_3(z)G_1(z)H(z)}$$

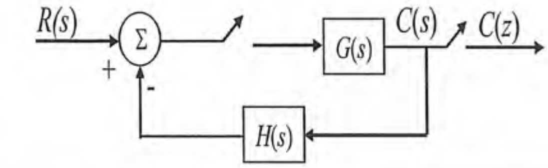
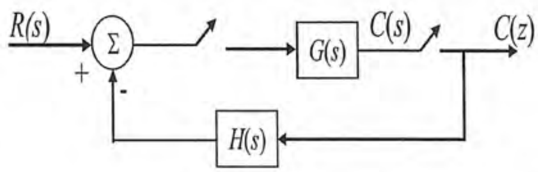
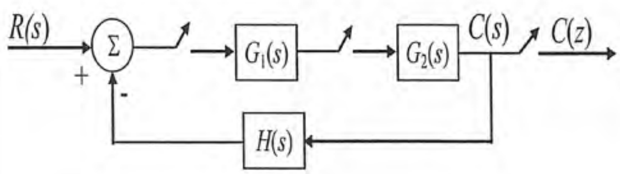
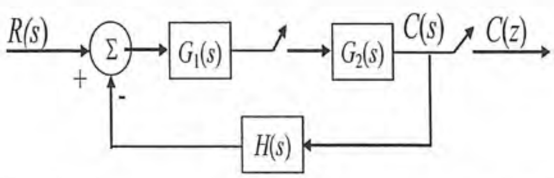
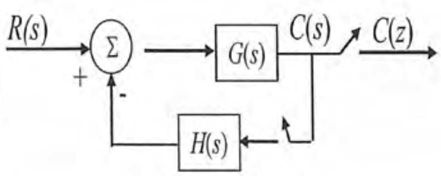


$$C(z)=\frac{R(z)G_1(z)G_2(z)}{1+G_2H_1(z)+G_1(z)G_2H_2(z)}$$



$$C(z)=\frac{RG_1(z)G_2(z)G_3(z)}{1+G_2(z)G_3H_1(z)+G_2(z)G_1G_3H_2(z)}$$

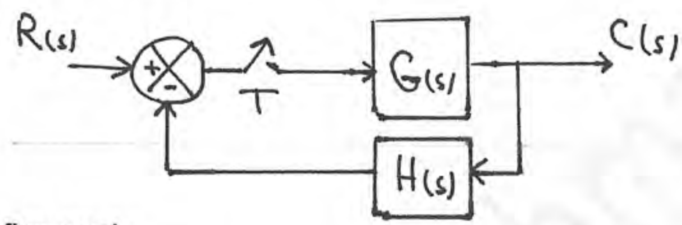
Table 3.2 five typical configurations for closed-loop discrete-time system

	$C(z)=\frac{G(z)R(z)}{1+GH(z)}$
	$C(z)=\frac{G(z)R(z)}{1+G(z)H(z)}$
	$C(z)=\frac{G_1(z)G_2(z)R(z)}{1+G_1(z)G_2H(z)}$
	$C(z)=\frac{G_2(z)G_1R(z)}{1+G_1G_2H(z)}$
	$C(z)=\frac{GR(z)}{1+GH(z)}$

2.9 Signal Flow Graphs(S.F.G)

1- The determination of (T.F) for sampled-data-system is difficult because a (T.F) for ideal sampler does not exit. A procedures using (S.F.G) can be used to find (T.F) of discrete system .

2- Let us find the (T.F) of the system shown below:-



Procedures

1-constrict the original signal flow graph

2- Assign a variable to each sampler i/p. Then the sampler o/p is thus variables starred.

3- Considering each sampler o/ps to be a (source), express the sampler i/ps and the systems o/p in terms of each sampler o/p

$$E = R - GHE^* \text{...........(1)}$$
$$C = GE^* \text{..................(2)}$$

4-Take the starred transform of these equations(1,2) and solve by any convenient method.

$$E^* = R^* - \overline{GHE^*}^* \text{............(3)}$$
$$C^* = G^*E^* \text{...................(4)}$$

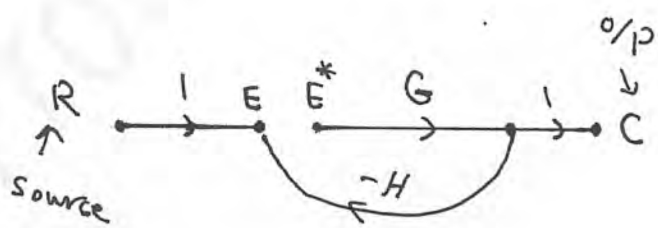
or

$$E^* = \frac{R^*}{1+\overline{GH}^*}$$

and

$$C^* = \frac{G^*}{1+\overline{GH}^*}R^*$$

$$\frac{C(z)}{R(z)} = \frac{G(z)}{1+GH(z)}$$



where $C^* \Rightarrow C(z)$
$R^* \Rightarrow R(z)$

Note that

The systems equations can also be solved by constructing a (S.F.G) from these equations(3,4) and applying ***Masons formula***

(using the rules for control subject in 3rd year)

Examples

Return to all examples in P56 and P60 (solved by using S.F.G method)

2.10 Discretization Methods

- The design of control system consists of two main steps:
  - The mathematical model of the plant is obtained to analyze its behaviour.
  - Using the model, an appropriate controller is designed to get the desired response from the controlled system.
- In continuous-time domain, the dynamics of the plant are represented by a set of *differential equations*.

- In control literature, there exist various approximation techniques to convert such equations conveniently into discrete-time forms without utilizing z-transforms.
- The methods discussed here approximate the time derivative (d/dt) in ODE using a corresponding difference equation:
  1. Forward difference (Euler's rule)
  2. Backward difference
  3. Trapezoidal (integration) rule
     - Also known as *Tustin* or *bilinear* transformation
  4. Pole-Zero matching method.

1) Forward Difference

Utilizing the definition of derivative, we have

$$\frac{dx}{dt} \cong \frac{x((k+1)T) - x(kT)}{(k+1)T - kT} = \frac{x(k+1) - x(k)}{T}$$

In terms of forward time-shift operator q:

$$\frac{dx}{dt} \cong \frac{q-1}{T} x(k) \Rightarrow \quad \frac{d}{dt} \cong \frac{q-1}{T}$$

$x(k+1) = q^1 = z^1$
$x(k+2) = q^2 = z^2$
⋮

Since d/dt corresponds to s variable while q operator is equivalent to z variable, one can write

$$s \cong \frac{z-1}{T}$$

The s-variable in the transfer function is conveniently replaced by this expression leading to a new transfer function of z:

$$G'(z) \cong G(s = \tfrac{z-1}{T})$$

2) Backward Difference

Utilizing the definition of derivative, we get

$$\frac{dx}{dt} \cong \frac{x(kT) - x((k-1)T)}{kT - (k-1)T} = \frac{x(k) - x(k-1)}{T}$$

In terms of backward time-shift operator q^{-1}:

$$\frac{dx}{dt} \cong \frac{1-q^{-1}}{T} x(k) \Rightarrow \quad \frac{d}{dt} \cong \frac{1-q^{-1}}{T}$$

$x(k-1) = q^{-1} = z^{-1}$
$x(k-2) = q^{-2} = z^{-2}$
⋮

As d/dt corresponds to s variable while q^{-1} operator is equivalent to z^{-1} variable, we have

$$s \cong \frac{z-1}{Tz}$$

The s-variable in the transfer function is to be replaced by this expression leading to a new transfer function of z:

$$G'(z) \cong G(s = \tfrac{z-1}{Tz})$$