# Sugeno-Type Fuzzy Inference

This section discusses the so-called Sugeno, or Takagi-Sugeno-Kang, method of fuzzy inference. Introduced in 1985 [16], it is similar to the Mamdani method in many respects. The first two parts of the fuzzy inference process, fuzzifying the inputs and applying the fuzzy operator, are exactly the same. The main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant.

A typical rule in a Sugeno fuzzy model has the form:

**If Input 1 = *x* and Input 2 = *y*,then Output is *z=ax+by+c***

For a zero-order Sugeno model, the output level $z$ is a constant $(a=b=0)$.

The output level $z_i$ of each rule is weighted by the firing strength w of the rule.

For example, for an AND rule with Input 1 = $x$ and Input 2 = $y$, the firing strength is

$$w_i = AndMethod\,(F_1(x), F_2(y))$$

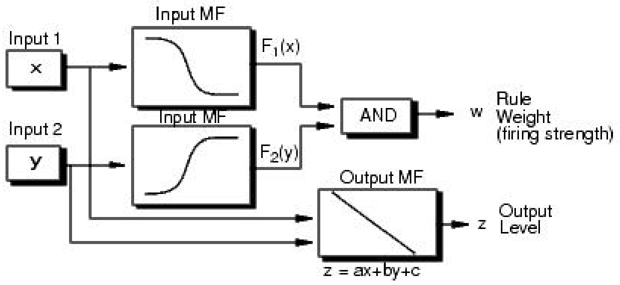
where $F_{1,2}(.)$ are the membership functions for Inputs 1 and 2.

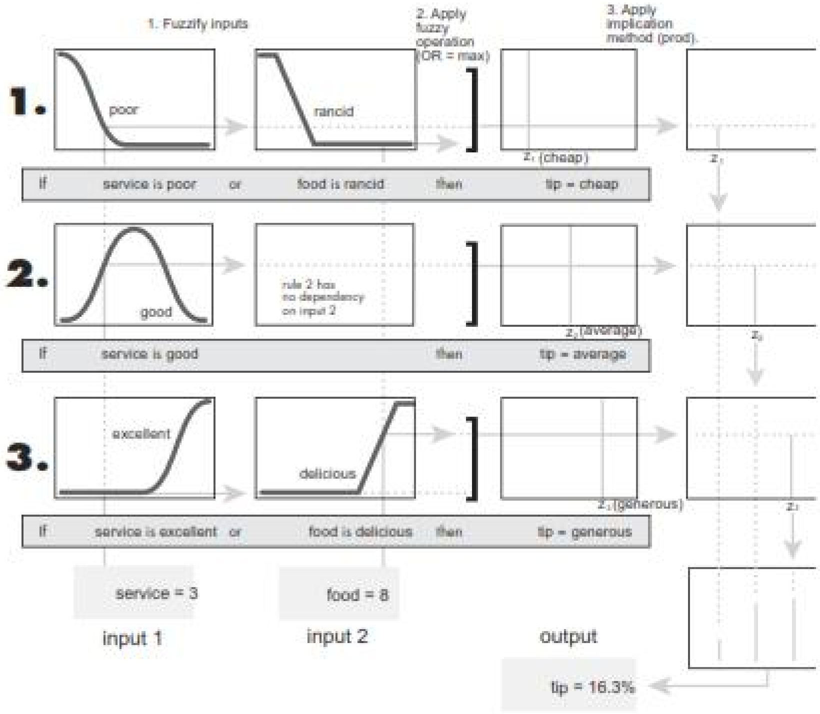
The final output of the system is the weighted average of all rule outputs, computed as

$$\text{Final Output} = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i}$$

where $N$ is the number of rules.

A Sugeno rule operates as shown in the following diagram.

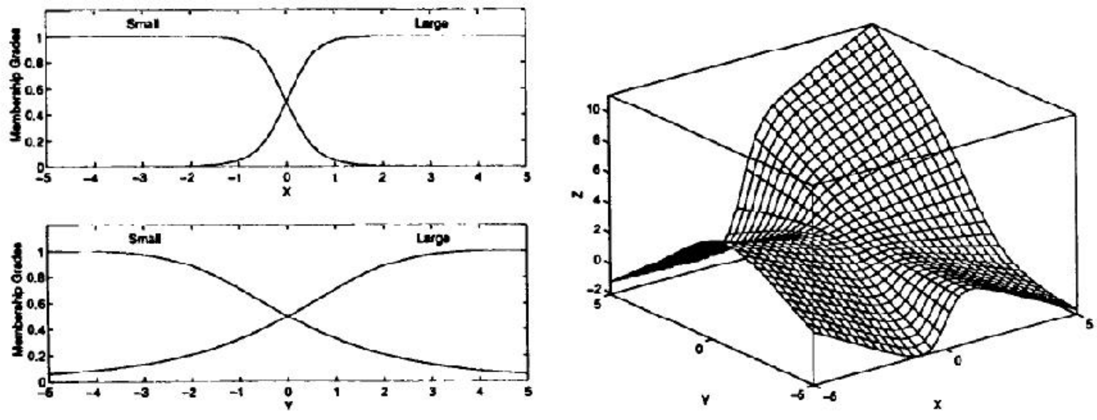



- Higher-order Sugeno fuzzy models are possible, but they introduce significant complexity with little obvious merit.
- Because of the linear dependence of each rule on the input variables, the Sugeno method is ideal for acting as an interpolating supervisor of multiple linear controllers that are to be applied, respectively, to different operating conditions of a dynamic nonlinear system.
- A Sugeno fuzzy inference system is extremely well suited to the task of smoothly interpolating the linear gains that would be applied across the input space; it is a natural and efficient gain scheduler. Similarly, a Sugeno system is suited for modeling nonlinear systems by interpolating between multiple linear models.

## Two-input, single-output Sugeno fuzzy model

$$\begin{cases} \text{If } X \text{ is small and } Y \text{ is small then } z = -x + y + 1. \\ \text{If } X \text{ is small and } Y \text{ is large then } z = -y + 3. \\ \text{If } X \text{ is large and } Y \text{ is small then } z = -x + 3. \\ \text{If } X \text{ is large and } Y \text{ is large then } z = x + y + 2. \end{cases}$$



(a) (b)

**Example: Two Lines**

To see a specific example of a system with linear output membership functions, consider one input one output system. The output variable has two linear membership functions with parameters Line1 [-1 -1] and Line2 [1 -1] respectively.

Further, these membership functions are linear functions of the input variable. The membership function `line1` is defined by the equation

*Output=(-1)*input+(-1)*

and the membership function `line2` is defined by the equation

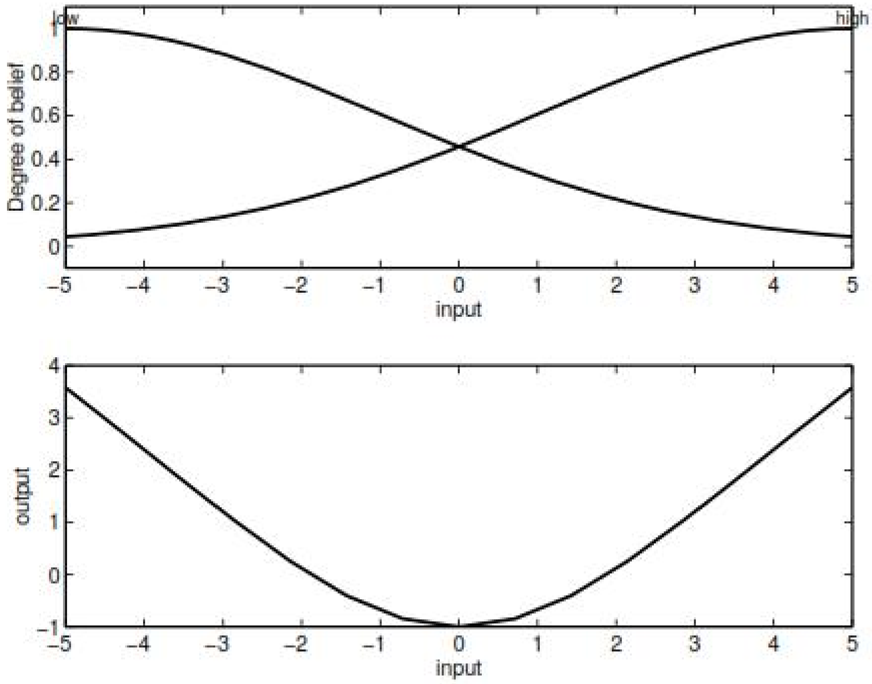
*Output=(1)*input+(-1)*

The input membership functions and rules define which of these output functions are expressed by:

```
1. If (input is low) then (output is line1)
2. If (input is high) then (output is line2)
```

The input membership function `low` generally refers to input values less than zero, while `high` refers to values greater than zero.

The overall fuzzy system output switches smoothly from the line called `line1` to the line called `line2`.




As this example shows, Sugeno-type system gives you the freedom to incorporate linear systems into your fuzzy systems. By extension, you could build a fuzzy system that switches between several optimal linear controllers as a highly nonlinear system moves around in its operating space.

**Comparison of Sugeno and Mamdani Methods**

Because it is a more compact and computationally efficient representation than a Mamdani system, the Sugeno system lends itself to the use of adaptive techniques for constructing fuzzy models. These adaptive techniques can be used to customize the membership functions so that the fuzzy system best models the data.

## Advantages of the Sugeno Method

- It is computationally efficient.
- It works well with linear techniques (e.g., PID control).
- It works well with optimization and adaptive techniques.
- It has guaranteed continuity of the output surface.
- It is well suited to mathematical analysis.

## Advantages of the Mamdani Method

- It is intuitive.
- It has widespread acceptance.
- It is well suited to human input.

Reference:

*Fuzzy Logic Toolbox™ User's Guide,* © COPYRIGHT 1995–2012 The MathWorks, Inc.