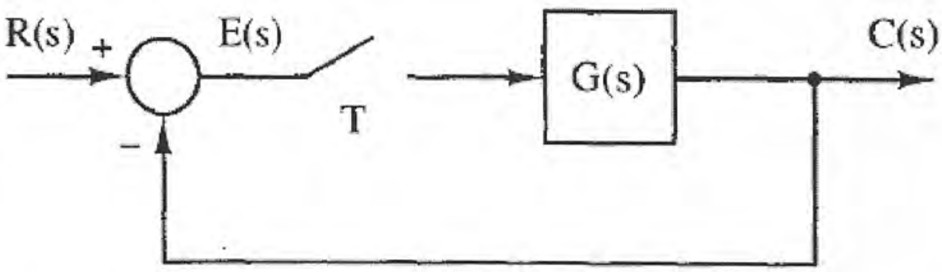
Chapter 4

4.1 STEADY-STATE ACCURACY

An important characteristic of a control system is its ability to follow, or track, certain inputs with a minimum of error. The control system designer attempts to minimize the system error to certain anticipated inputs. In this section the effects of the system transfer characteristics on the steady-state system errors are considered.

Consider the system of Figure



$$\frac{C(z)}{R(z)} = \frac{G(z)}{1 + G(z)}$$

the system error, $e(t)$, is defined as

$$E(z) = \mathcal{Z}[e(t)] = R(z) - C(z)$$

$$E(z) = R(z) - \frac{G(z)}{1 + G(z)}R(z) = \frac{R(z)}{1 + G(z)}$$

The steady-state errors will now be derived for two common inputs—a position (step) input and a velocity (ramp) input. First, for the unit-step input,

$$R(z) = \frac{z}{z - 1}$$

provided that $e_{ss}(kT)$ has a final value. The steady-state error is then

$$e_{ss}(kT) = \lim_{z \to 1} \frac{z}{1 + G(z)} = \frac{1}{1 + \lim_{z \to 1} G(z)}$$

We now define the position error constant as

$$K_p = \lim_{z \to 1} G(z)$$

$$e_{ss}(kT) = \frac{1}{1 + K_p}$$

(system type greater than or equal to one), $K_p = \infty$ and the steady-state error is zero.

Consider next the unit-ramp input. In this case $r(t) = t$

$$R(z) = \frac{Tz}{(z - 1)^2}$$

$$e_{ss}(kT) = \lim_{z \to 1} \frac{Tz}{(z - 1) + (z - 1)G(z)} = \frac{T}{\lim_{z \to 1} (z - 1)G(z)}$$

We now define the velocity error constant as

$$K_v = \lim_{z \to 1} \frac{1}{T}(z - 1)G(z)$$

$$e_{ss}(kT) = \frac{1}{K_v}$$

(system type greater than or equal to 2), $K_v = \infty$ and $e_{ss}(kT)$ is zero.

The development above illustrates that, in general, increased system gain and/or the addition of poles at $z = 1$ to the open-loop forward-path transfer function tend to decrease steady-state errors.

both large gains and poles of $G(z)$ at $z = 1$ have destabilizing effects on the system. Generally, trade-offs exist between small steady-state errors and adequate system stability (or acceptable system transient response).

Example

The steady-state errors will be calculated for the system of Figure open-loop function is given as

P94

$$G(s) = \frac{1 - \epsilon^{-Ts}}{s}\left[\frac{K}{s(s + 1)}\right]$$

Thus

$$G(z) = K\mathfrak{z}\left[\frac{1 - \epsilon^{-Ts}}{s^2(s + 1)}\right] = \frac{K(z - 1)}{z}\mathfrak{z}\left[\frac{1}{s^2(s + 1)}\right]$$

$$= \frac{K(z - 1)}{z}\frac{z[(\epsilon^{-T} + T - 1)z + (1 - \epsilon^{-T} - T\epsilon^{-T})]}{(z - 1)^2(z - \epsilon^{-T})}$$

$$= \frac{K[(\epsilon^{-T} + T - 1)z + (1 - \epsilon^{-T} - T\epsilon^{-T})]}{(z - 1)(z - \epsilon^{-T})}$$

the system is type 1 and

Kv= K

Since $G(z)$ has one pole at $z = 1$, the steady-state error to a step input is zero, and to

a ramp input is,

$$e_{ss}(kT) = \frac{1}{K_v} = \frac{1}{K}$$

<u>Note that</u>

(z-1)is represent a pole at z=1 or (Type one system)

$(z-1)^2$ *is represent a 2-pole at z=1 or (Type two system)*

Example:

The open-loop T.F is:-

$$G(z) = \frac{0.98z + 0.66}{(z-1)(z-0.368)}$$

Compute [illegible] constants and Ess .

Sol

$$Kp = \lim_{z \to 1} \frac{0.98z + 0.66}{(z-1)(z-0.368)}$$

$$Kp = \infty$$

$$Ess = \frac{1}{1+Kp}$$

$$Ess = 0$$

$$Kv = \frac{1}{T} \lim_{z \to 1} [(z-1) \frac{0.98z + 0.66}{(z-1)(z-0.368)}$$

$$Kv = \frac{2.59}{T}$$

$$Ess = \frac{1}{Kv} = \frac{T}{2.59}$$

Note that

In sometimes the (Ess) can be used as one of the design requirements in control system.

Example

Let the O.L.T.F is:-

$$G(s) = \frac{K}{s(s+1)}$$

Calculate the gain(K) for Ess=1%. Suppose the i/p signal ramp and T=0.1sec

Sol

$$G(z) = (1 - z^{-1}) Z\left[\frac{G(s)}{s}\right]$$

$$G(z) = \frac{(0.005z + 0.00347)K}{(z-1)(z-0.9048)}$$

$$Kv = \frac{1}{T} \lim_{z \to 1} [(z-1)G(z)]$$

$$Kv = 0.918K$$

$$Ess = \frac{1}{Kv} = \frac{1}{0.918K} = 0.01$$

$$K = 108.93$$

NOTES

1-May be gives in question Ess large than or small a tolerance band.

And calculate (K) for this band.

2- May be there is a controller D(z) in feed forward path with a system G(z). so that the O.L.T.F is become G(z)D(z) instead of G(z) alone.

In general

Steady State Error and System Type

System	Steady-state errors in response to		
	Step input $r(t) = 1$	Ramp input $r(t) = t$	Acceleration input $r(t) = \frac{1}{2}t^2$
Type 0 system	$\frac{1}{1 + K_p}$	∞	∞
Type 1 system	0	$\frac{1}{K_v}$	∞
Type 2 system	0	0	$\frac{1}{K_a}$

4.2 How to calculate ζ, Wn for discrete control system

Suppose , you have a 2nd order continuous control system

if $\frac{C(s)}{R(s)} = \frac{Wn^2}{s^2 + 2\zeta Wns + Wn^2}$

Which has the poles:-

$$S_{1,2} = -\zeta wn \pm jwn\sqrt{1-\zeta^2}$$
$$Z_{1,2} = e^{-\zeta WnT} e^{\pm WdTj}$$
$$Z_{1,2} = R\angle -\theta rad$$
$$R = e^{-\zeta WnT} \text{..........(1)}$$
$$\theta = WdT \text{...............(2)}$$
$$-\ln R = \zeta WnT \text{.....(3)}$$
$$\theta = WnT\sqrt{1-\zeta^2} \text{............(4)}$$
$$Div(3) to (4)$$
$$\zeta = \frac{-\ln R}{\sqrt{(\ln R)^2 + \theta^2}} \text{.................(5)}$$
$$Wn = \frac{1}{T}\sqrt{(\ln R)^2 + \theta^2} \text{..............(6)}$$
$$\tau = \frac{-T}{\ln R} \text{.................................(7)}$$

Thus given the complex pole locations in the Z-domain, we find (ζ, Wn, τ) by using equations (5,6,7).

Example:

$$\frac{C(z)}{R(z)} = \frac{0.368z + 0.264}{z^2 - z + 0.632}$$

Sol:- T = 1 sec

$$z^2 - z + 0.632 = z^2 - 2a + a^2 + b^2$$
$$a = 0.5$$
$$b = 0.618$$
$$poles Z_{1,2} = 0.5 \mp 0.618j$$
$$R = \sqrt{a^2 + b^2} \text{.........(*)}$$
$$\theta = \tan^{-1}\frac{b}{a} \text{............(**)}$$
$$R = 0.795$$
$$\theta = 51^{\circ} = 0.89 rad$$
$$using (5,6,7)$$, we calculate :-
$$\zeta = 0.25$$
$$Wn = 0.919 rad/\sec$$
$$\tau = 4.36 \sec$$

To analyze the same example in continuous system

The original system is

$$G(s) = \frac{1}{s^2 + s + 1}$$
$$\zeta = 0.5$$
$$Wn = 1 rad/\sec$$
$$\tau = 2 \sec$$

Note that

The effects of the sampling (T=1 sec) is seem to destabilizing, however, if (T=0.1sec) instead of (T=1sec in above example) , there is little effect from sampling.

Example:

If the c/cs equation is

$$Q(z) = z^2 - 1.9z + 0.91$$

Find ζ, Wn, τ

Using eq(5,6,7), we get:-

ζ=0.46

Wn=1.0245rad/sec

$\tau = 2.12\sec$

4.3 How to calculate G(s) from the discrete control system G(z)

Example:

Let , G(z)

$$G(z) = \frac{C(z)}{R(z)} = \frac{0.2909z + 0.1693}{z^2 + 0.7417z + 0.2019}$$

$$T = 0.2\sec$$

Find G(s)

Sol:-

You know

$$\frac{C(s)}{R(s)} = \frac{Wn^2}{s^2 + 2\zeta Wns + Wn^2} = \frac{wn^2}{(s + \sigma + Wdj)(s + \sigma - Wdj)}$$

$$S_{1,2} = -\zeta wn \pm jwn\sqrt{1 - \zeta^2}$$

$$S1,2 = -\sigma \pm jWd$$

$\theta / T = Wd$..............(1) → From equ. 2 P.112

$-\sigma = \ln R / T$...........(2) → From equ. 3 P.112

Calculate ϕ and R using equations(*) and(**) respectively

R=0.4494

Φ=34.372=2.5417rad

Use above equ. (1)and (2)

To calculate (σ) and (Wd) and sub. In the standard 2nd order equation G(s).

Note that

In sometimes, you have ζ and Wn for digital control system, how to calculate the c/cs equation Q(z)

Use equ.(5) and (6) to calculate (R) and (ϕ).Where:-

$$R = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

From these equations, we get (a) and (b)

$$Q(z) = (z - a + bj)(z - a - bj)$$

Example:

If ζ=0.25 and Wn=0.919rad/sec and T=1 sec.

R=0.795

ϕ=0.89

a=0.5 and b=0.6177

$$Q(z) = z^2 - z + 0.632$$

Chapter 5

Design of digital control system

- The Research Procedure in Control Science

- Plant
- Sensor
- Actuator
- Computer
- Communication
- Noise
- Disturbance

- Differential eqn
- Laplace transform
- Transfer function
- State space form

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- Difference eqn
- z transform
- Transfer function
- State space form

- Root locus
- Bode diagram
- Nyquist plot

- Stability
- Robustness
- Sensitivity
- Controllability
- Observability

- Estimator
- Identification
- Regulation
- Tracking
- PID
- Pole placement

- Optimal Control LQR/LQG
- Adaptive control
- Robust control

5.1 Design of Discrete-Time Controllers

Two approaches are possible to the design of digital control laws:

1. **"Direct" method**
   - Discretization of the plant model
   - Design of the controller in the discrete-time domain

2. **"Indirect" Method**
   - Simplest approach, it does not requires specific knowledge of design techniques in the discrete-time domain
   - Some limitations are given by the choice of the sampling time