Recall from ( * ) the closed-loop transfer function of the cruise control system with the PI control law, i.e.,

$$H(s) = \frac{Y(s)}{R(s)} = \frac{G(s)D(s)}{1+G(s)D(s)} = \frac{0.001k_p s + 0.001k_i}{s^2 + (0.1 + 0.001k_p)s + 0.001k_i}$$

and the desired transfer function that produces desired performance

$$H_{\text{desired}}(s) = \frac{0.67}{s^2 + 1.15s + 0.67}$$

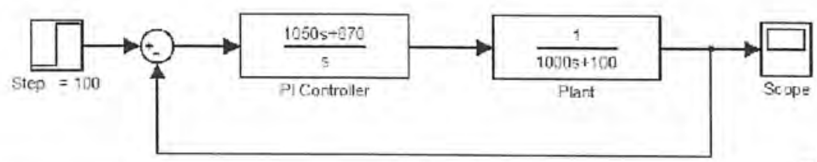
Comparing the coefficients on the denominators, we have

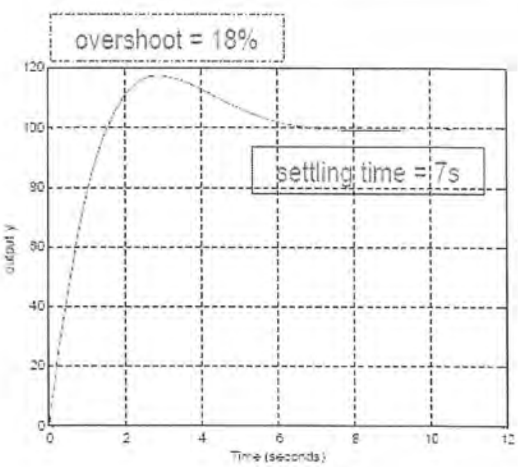
$$0.1 + 0.001k_p = 1.15 \quad \Rightarrow \quad k_p = 1050$$

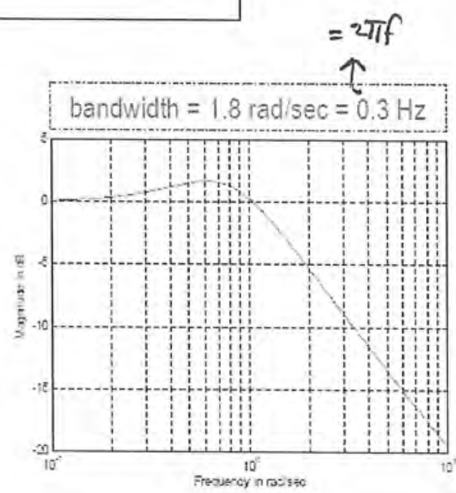
$$0.001k_i = 0.67 \quad \Rightarrow \quad k_i = 670$$

and the resulting closed-loop transfer function $H(s) = \dfrac{1.05s + 0.67}{s^2 + 1.15s + 0.67}$

## *Verification through SIMULINK*



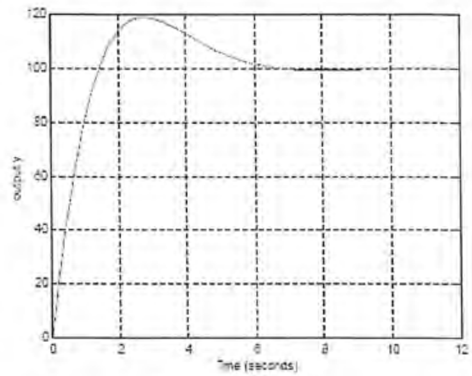
## *Digital controller with a sampling rate 30 times the bandwidth*

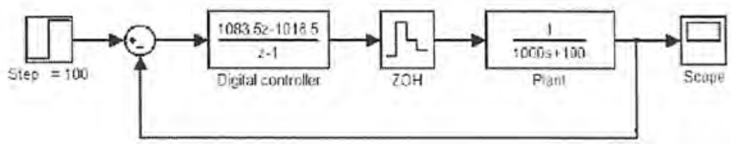
We first discretize the continuous-time PI control law with $T = 1/(30 \times 0.3) \approx 0.1$ seconds using a bilinear transformation method, i.e.,

$$D(z) = D(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} = \frac{1050s+670}{s}\Bigg|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

$$= \frac{1050\,\frac{2}{T}\left(\frac{z-1}{z+1}\right)+670}{\frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

$$= \frac{1083.5z-1016.5}{z-1}$$




Performance is about the same as the continuous-time case.

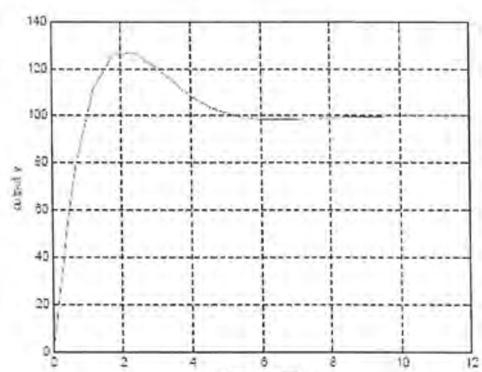
## *Digital controller with a sampling rate 6 times the bandwidth*

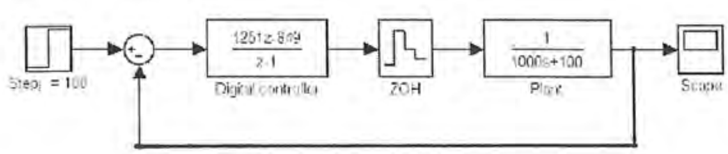
We now discretize the continuous-time PI control law with $T = 1/(6 \times 0.3) \approx 0.6$ seconds using the bilinear transformation method, i.e.,

$$D(z) = D(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} = \frac{1050s+670}{s}\Bigg|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

$$= \frac{1050\,\frac{2}{0.6}\left(\frac{z-1}{z+1}\right)+670}{\frac{2}{0.6}\left(\frac{z-1}{z+1}\right)}$$

$$= \frac{1251z-849}{z-1}$$




Performance is not as good as the continuous-time case.

## Example

For $G(s) = \dfrac{1}{s(s+0.8)}$

Design (PD) controller [use bilinear Transformation] such that design requirements are:-

$\zeta$ =0.7 and Wn=4rad/sec . let T=0.02 sec.

**sol**

**D(s)=Kp+Kds**

$$\frac{C(s)}{R(s)} = \frac{D(s)G(s)}{1+D(s)G(s)} = \frac{?}{s^2 + (Kd+0.8)s + Kp}$$

$$\frac{C(s)}{R(s)} = \frac{?}{s^2 + 2*0.7*4s + 4^2}$$

Kp=16, Kd=4.8

$$D(z) = D(s)_{\downarrow s=\frac{2}{T}\frac{z-1}{z+1}} = 480\frac{z-1}{z+1} + 16$$

## 5.2.1.1 Design of a PID controller using Pole-Zero cancellation

In this method, we design (PI) or (PD) by choosing (selecting) the Zero of (PI) or )PD) in such way to cancel one pole of the plant G(s)[canceling the pole which is close to the imaginary axis].

## Example

If $G(s) = \dfrac{1}{(s+1)(s+10)}$

Design (PI) controller use (bilinear Transform) to obtain:-

Ess=0, ξ=0.7 and Ts $\langle$ 1 sec . Let T=0.02sec.

**Sol**

$$D(s)=\frac{K(s+a)}{s}=Kp+\frac{Ki}{s}=\frac{Kps+Ki}{s}=\frac{Kp[s+\frac{Ki}{Kp}]}{s}$$

$$G(s)D(s)=\frac{K(s+a)}{s}\frac{1}{(s+1)(s+10)}=\frac{K}{s(s+10)}$$

$$a=1$$

$$\frac{C(s)}{R(s)}=\frac{G(s)D(s)}{1+G(s)D(s)}=\frac{?}{s^2+10s+K}=\frac{?}{s^2+2\zeta Wns+Wn^2}$$

$$10=2\zeta Wn$$

$$Wn=\frac{10}{2*0.7}=7.142rad/\sec$$

$$K=Wn^2=51.02$$

$$D(s)=51.02\frac{s+1}{s}\downarrow_{s=\frac{2}{T}\frac{z-1}{z+1}}=51.02\frac{z-0.9802}{z-1}$$

To check (Ts) less than 1sec

$$Ts=\frac{4}{\zeta Wn}=\frac{4}{5}=0.8\sec$$

Less than 1 sec(satisfied the requirement).

$$G(z)=1.86*10^{-4}\frac{z+0.9293}{(z-0.8187)(z-0.9802)}$$

$$G(z)D(z)=51.02*1.86*10^{-4}\frac{(z+0.9293)(z-0.9802)}{(z-1)(z-0.8187)(z-0.9802)}$$

The whole system became Type one because (z-1), so that:-

$$Ess=\lim_{z\to 1}\frac{1}{1+G(z)D(z)}=\frac{1}{\infty}=0$$

## Example

$$G(s)=\frac{1}{s(s+1)(s+10)}$$

Design (PD) controller use (bilinear Transform) to obtain:-

ξ=0.7 and Ts $\langle 1$ sec . Let T=0.02sec. P128

The solution is same procedure as example **above** ,where

**D(s)=K(s+a)**

$$D(z)=D(s)_{\downarrow s=\frac{2}{T}\frac{z-1}{z+1}}=5153\frac{(z-0.98)}{(z+1)}$$

There is a problem in this controller the term (z+1) causes instability , so that sub. (z+1) by (z).

$$D(z)=5153\frac{(z-0.98)}{z}$$

## <u>Note that</u>

**A**-In general , there are a number of ways to implement integration and derivative digitally.

[1]- **Conventional version**

## Derivative Control

The continuous derivative control is

$$u(t) = k_d \dot{e}(t) \Rightarrow D(s) = k_d s$$

The discrete derivative control is

$$u(k) = k_d \frac{e(k) - e(k-1)}{T} \Rightarrow D(z) = k_d \frac{1 - z^{-1}}{T} = k_d \frac{z-1}{Tz}$$

## Integral Control

The continuous integral control is $u(t) = k_i \int_{t_0}^{t} e(t) dt \Rightarrow D(s) = k_i \frac{1}{s}$

The discrete integral control is

$$u(k) = u(k-1) + k_i T e(k) \Rightarrow D(z) = \frac{k_i T}{1 - z^{-1}} = \frac{k_i T z}{z-1}$$

## Digital PID Control (conventional version)

$$D(z) = k_P + k_I \frac{z}{z-1} + k_D \frac{z-1}{z} = \frac{(k_P + k_I + k_D) z^2 - (k_P + 2k_D) z + k_D}{z(z-1)}$$

## Digital PI Control (conventional version)

Digital PI control consists of only P and I actions and is given by

$$D(z) = k_P + k_I \frac{z}{z-1} = \frac{(k_P + k_I) z - k_P}{z-1}$$

## Digital PD Control (conventional version)

Digital PD control consists of only P and D actions and is given by

$$D(z) = k_P + k_D \frac{z-1}{z} = \frac{(k_P + k_D) z - k_D}{z}$$

## [2]- Bilinear version

Digital PID Control (via bilinear transformation)

$$D(z)=\left(k_p+\frac{k_i}{s}+k_d s\right)_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}=k_p+\frac{k_i T(z+1)}{2(z-1)}+\frac{2k_d(z-1)}{T(z+1)}=\frac{\alpha_2 z^2+\alpha_1 z+\alpha_0}{(z-1)(z+1)}$$

where $\alpha_0$, $\alpha_1$ and $\alpha_2$ are design parameters.

Digital PI Control (via bilinear transformation) – the same as the previous version

$$D(z)=\left(k_p+\frac{k_i}{s}\right)_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}=k_p+\frac{k_i T(z+1)}{2(z-1)}=\frac{\alpha_1 z+\alpha_0}{z-1}$$

Digital PD Control (via bilinear transformation)

$$D(z)=\left(k_p+k_d s\right)_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}=k_p+\frac{2k_d(z-1)}{T(z+1)}=\frac{\alpha_1 z+\alpha_0}{z+1}$$

## [3]- Mixing version(Backward and Tustin)

$$(I)=\frac{Ki}{s}=\frac{KiT(z+1)}{2(z-1)}$$

$$P+I=\frac{(Ki\frac{T}{2}+Kp)[z+\frac{Ki\frac{T}{2}-Kp}{\frac{KiT}{2}+Kp}]}{z-1}$$

$$(D)=Kd\frac{(z-1)}{Tz}$$

$$P+D=\frac{(Kp+\frac{Kd}{T})[z-\frac{Kd}{Kd+KpT}]}{z}$$

**B-** To design (PID) , there are three terms must be calculated(Kp,Ki,Kd).

So that the c/cs equation for comparison must be 3rd order

$$\frac{C(s)}{R(s)}=\frac{?}{(s^2+2\zeta Wns+Wn^2)(s+\alpha)}$$

Where ( $\alpha$ ) is a parameter which values between (10-30), which related to very decay poles(non-dominant pole).

## 5.2.1.2 Design Lag-Lead compensators

General T.F is:-

$$D(s) = \frac{K(s+a)}{(s+b)}$$

If 1- a $\rangle b$ lag-compensator, like (P+I) controller[ Improve Ess]. { Lag }

2- b $\rangle a$ lead-compensator, like(P+D) controller [ Improve Dynamic response (Mp),(Ts)]. { Lead }

## Example

$$G(s) = \frac{12}{s(s+2.5)}$$

Design a digital lead-compensator will satisfy:-

$$\zeta = 0.8$$
$$Wn = 5rad/\sec$$
$$T = 0.1\sec$$

Use bilinear Transform.

**Sol**

after design

$$C.L.T.F = \frac{25}{s^2+8s+25} = Q(s) = \frac{D(s)G(s)}{1+D(s)G(s)}$$

$$or \quad = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$D(s) = \frac{Q(s)}{G(s)[1-Q(s)]} = \frac{\dfrac{25}{s^2+8s+25}}{\dfrac{12}{s(s+2.5)}[1-\dfrac{25}{s^2+8s+25}]}$$

$$D(s) = \frac{2.083(s+2.5)}{(s+8)}$$

Since b=8 more than a=2.5, the design is correct.