$$D(z) = D(s)_{\downarrow s=\frac{2}{T}\frac{z-1}{z+1}} = \frac{46.86z - 39.1}{28z - 12}$$

Note that

We can solve the example by using P-Z cancellation

$$G(s)D(s) = \frac{12}{s(s+2.5)} \frac{K(s+a)}{(s+b)}$$
$$a = 2.5$$
$$= \frac{12K}{s(s+b)}$$
$$C.L.T.F = \frac{C(s)}{R(s)} = \frac{12K}{s^2 + bs + 12K} = \frac{25}{s^2 + 8s + 25}$$
$$b = 8$$
$$K = 2.08$$
$$D(s) = \frac{2.08(s+2.5)}{(s+8)}$$

And complete the solution to find D(z).

Example

$$G(s) = \frac{1}{s(10s+1)}$$

Design a digital lead-compensator will satisfy:-

$$\zeta = 0.5$$
$$Ts = 10\sec$$
$$T = 0.2\sec$$

Use P-Z matching Transform. And P-Z cancellation method

Sol

In the beginning convert the Den.(10s+1) to (s+a).

$$G(s)D(s) = \frac{1}{10s(s+0.1)}\frac{K(s+a)}{(s+b)}$$

$$a = 0.1$$

$$\frac{C(s)}{R(s)} = \frac{0.1K}{s^2 + bs + 0.1K} = \frac{}{s^2 + 0.8s + .64}$$

$$Ts = \frac{4}{\zeta * Wn} = 10$$

$$Wn = 0.8 rad/\sec$$

K=6.4 and b=0.8

$$D(s) = 6.4\frac{(s+0.1)}{(s+0.8)}$$

$$D(z) = K_z\frac{(z - e^{-0.1T})}{(z - e^{-0.8T})} = K_z\frac{(z-0.98)}{(z-0.852)}\downarrow_{z=1} = D(s)\downarrow_{s=0}$$

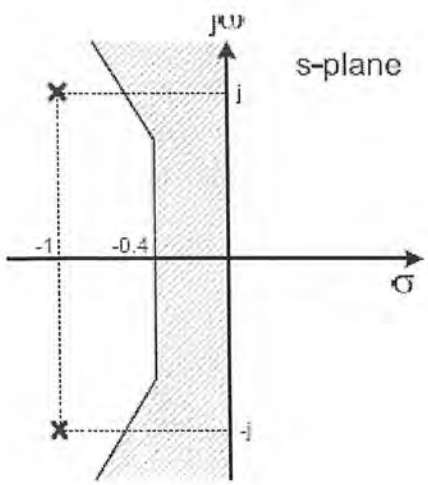
$$K_z = 5.92$$

$$D(z) = \frac{5.92(z-0.98)}{(z-0.852)}$$

We can complete the solution to find the closed-loop transient response C(kT) , and calculate Mp, Ts ,Tr and all design specifications.

Example

Repeat the above ex. For the following design specifications:- like this C.L.poles



Use : P-Z matching Transform

Let T=0.1 sec

Let the controller be a lead-lag compensator:

$$G_c(s) = K\frac{s+b}{s+a}$$

Hence, the open-loop TF becomes

$$G_{OL}(s) = G_c(s)G_p(s)$$

$$G_{OL}(s) = K\frac{s+b}{s+a}\cdot\frac{0.1}{s(s+0.1)}$$

If b is selected as 0.1, the zero-pole cancellation takes place:

$$G_{OL}(s) = K\frac{\cancel{s+0.1}}{s+a}\cdot\frac{0.1}{s\cancel{(s+0.1)}}$$

Then, the characteristic poly. becomes

$$A(s) = num\{1+G_{OL}(s)\}$$

$$A(s) = s^2 + as + K0.1 \quad \text{-----} (1)$$

Let us select the location of dominant poles in the design region as

$$p_{1,2} = -1 \pm j$$

which corresponds to a choice of $\omega_d = 1$ [rad/s] and $\zeta\omega_n = 1$ [rad/s].

Desired characteristic polynomial becomes

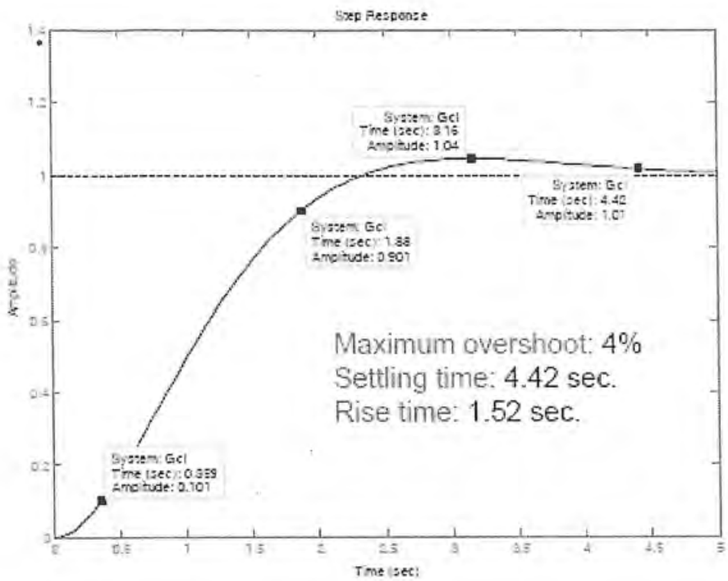
$$A_d(s) = (s+1+j)(s+1-j)$$

$$A_d(s) = s^2 + 2s + 2 \quad \text{-------} (2)$$

Matching the coefficients of A(s) (1) and A_d(s) (2) yields the controller parameters as $a = 2$ and $K = 20$. Therefore,

$$G_c(s) = 20\frac{s+0.1}{s+2}$$

Let us check the transient response of the system using Matlab.



Fig(1)

Consequently, the discrete TF will have the following form:

$$G_c(z) = K_0 \frac{z-\beta}{z-\alpha}$$

Using $z = e^{sT}$ yields

$$\alpha = e^{pole \times T} = e^{-2\times 0.1} = 0.8187$$

$$\beta = e^{zero \times T} = e^{-0.1\times 0.1} = 0.99$$

Finally, the DC gains of both transfer functions should be matched:

$$DC\ Gain = \lim_{z\to 1} G_c(z) = K_0 \frac{1-0.99}{1-0.8187} = 0.0552K_0$$

$$= \lim_{s\to 0} G_c(s) = 20\frac{0.1}{2} = 1 \Rightarrow \quad K_0 = 18.13$$

Use P-Z matching method

Discrete-time TF becomes

$$G_c(z) = 18.13 \frac{z-0.99}{z-0.8187}$$

One can verify this by Matlab:

```
>> c2d(Gc,T,'matched')
Transfer function:
18.22 z - 18.04
---------------
  z - 0.8187

Sampling time: 0.1
```

Matlab Script

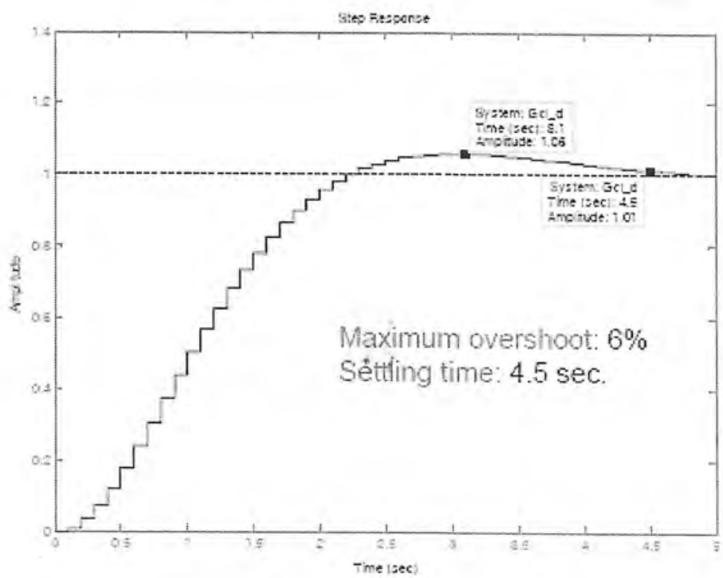
```
%
% Antenna Servo-control Example
%
Gp = tf([0.1],[1 0.1 0]);      % Plant's TF (continuous-time)
Gc = tf([20 2],[1 2]);         % TF of the designed controller
Gcl = feedback(Gp*Gc,1);       % Closed-loop TF

figure(1)                      % Simulate the CT control system
  step(Gcl,5);                 % (for a unit step input)

tr = 1.88 - 0.359;             % Read the rise time off the figure 5%-10%
T = 0.1;                       % Sampling period T << tr/10
Gcd = c2d(Gc,T,'matched');     % Obtain the DT controller
Gpd = c2d(Gp,T,'zoh');         % Discrete TF of the plant w/ ZOH
Gcl_d = feedback(Gpd*Gcd,1);   % Closed-loop TF of DTS

figure(2)                      % Simulate the DTS
  step(Gcl_d,5);               % (for a unit step input)
```

From Fig(1)
tr ≈ 1.5

$\therefore \frac{1.5}{10} > T$

$0.15 > T$

$\therefore T = 0.1$ sec.



Fig(2)

Also there is another solution by using the **formula**

$$D(s) = \frac{Q(s)}{G(s)[1-Q(s)]} ------(*)$$

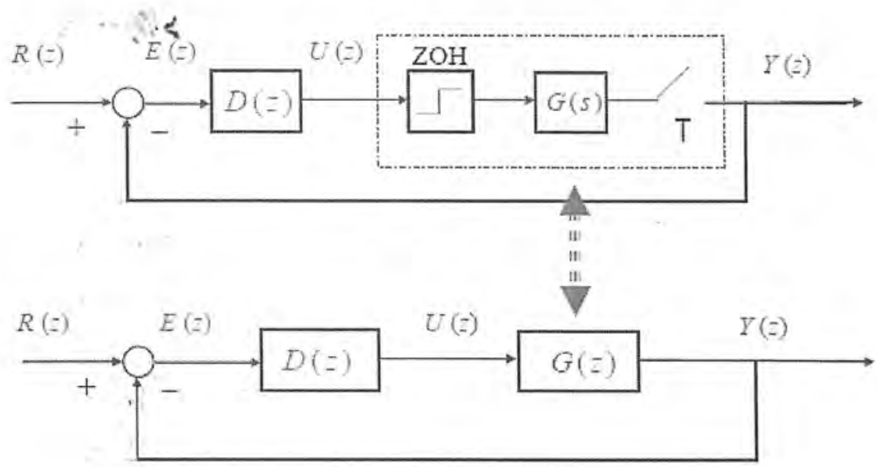
→ P133
P134

H.W:- solve these two examples by using this formula.

5.2.2 Digital control system design by discrete method

(direct method)

In this approach, we discretize the continuous-time plant first or directly work on a discrete time plant to design a digital controller using the well-known controllers.



PI
PD

where $G(z) = (1 - z^{-1})Z\left\{\frac{G(s)}{s}\right\}$ and $D(z)$ is taken to be a PID controller.

One could discretize the plant G(s) to obtain a sampled-data system or discrete-time system, and apply digital control design methods to design a digital controller.

Digital PID control system design via pole placement technique

PID Control

PID control is widely used in process control and most of industrial control systems. Unknown source reports that more than 90% of industrial processes are actually controlled by PID type of controllers. PID control consists of three essential components, namely, P (proportional control), I (integral control) and D (derivative control).

Proportional Control

A discrete implementation of proportional control is identical to continuous. The continuous is

$$u(t) = k_p e(t) \quad \Rightarrow \quad D(s) = k_p$$

The discrete is

$$u(k) = k_p e(k) \quad \Rightarrow \quad D(z) = k_p$$

where $e(t)$ or $e(k)$ is the error signal as given in the feedback block diagram.

While the **integral** and **derivative** as mentioned early [see p 130].

Design example:

Note that: In this example ,use the conventional (PI) controller.

Consider a car (BMW), which has a weight m = 1000 kg. Assuming the average friction coefficient b = 100, design a cruise control system such that the car can reach 100 km/h from 0 km/h in 8 s with an overshoot less 20%. Let the step input = 100

Assuming the sampling period T = 0.6 seconds, design a digital PI controller that achieve the above specifications.

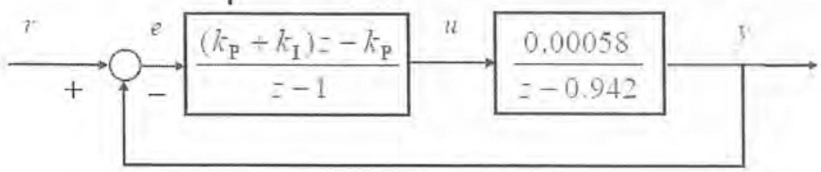
$$G(s) = \frac{1}{ms+b} = \frac{1}{1000s+100} \quad \Rightarrow$$

$$G(z) = (1-z^{-1})Z\left\{\frac{G(s)}{s}\right\} = (1-z^{-1})Z\left\{\frac{0.001}{s(s+0.1)}\right\} = \frac{z-1}{z}Z\left\{\frac{0.01}{s} - \frac{0.01}{s+0.1}\right\}$$

$$= 0.01 \cdot \frac{z-1}{z} \cdot \left[\frac{z}{z-1} - \frac{z}{z-e^{-0.1\times 0.6}}\right] = \boxed{\frac{0.00058}{z-0.942}} \leftarrow \text{discretized plant with T = 0.6}$$

Discretized plant with digital PI controller:

→ See P130



The resulting closed-loop transfer function from r to y is given by

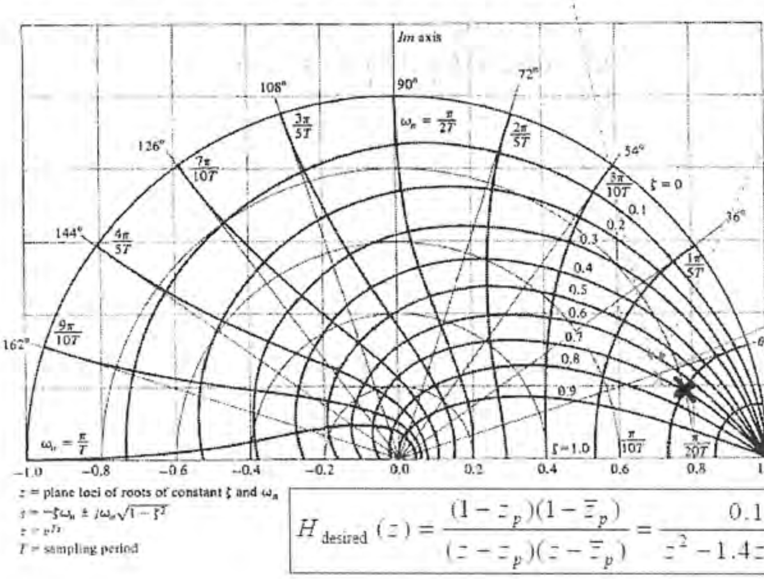
$$H(z) = \frac{G(z)D(z)}{1+G(z)D(z)} = \frac{\frac{0.00058}{z-0.942}\cdot\frac{(k_P+k_I)z-k_P}{z-1}}{1+\frac{0.00058}{z-0.942}\cdot\frac{(k_P+k_I)z-k_P}{z-1}}$$

$$= \frac{0.00058\,(k_P+k_I)z - 0.00058\,k_P}{z^2 + [0.00058\,(k_P+k_I) - 1.942]z + (0.942 - 0.00058\,k_P)}$$

Desired closed-loop transfer function:

→ P 124

From the design in , we obtain the desired $\zeta = 0.7$ and $\omega_n = 0.82$ in continuous setting, which would achieve the design specifications. Using the following chart with $T = 0.6$



$$\omega_n = 0.82 \approx 0.16\frac{\pi}{T}$$

The desired poles:

$$z_p = 0.7 + j0.2$$

$$H_{desired}(z) = \frac{(1-z_p)(1-\bar{z}_p)}{(z-z_p)(z-\bar{z}_p)} = \frac{0.13}{z^2 - 1.4z + 0.53}$$