Determination of the PI controller parameters

Comparing the denominator of the actual closed-loop transfer function

$$H(z) = \frac{0.00058\,(k_P + k_I)z - 0.00058\,k_P}{z^2 + [0.00058\,(k_P + k_I) - 1.942\,]z + (0.942 - 0.00058\,k_P)}$$

with that of the desired one

$$\boxed{H_{\text{desired}}(z) = \frac{0.13}{z^2 - 1.4z + 0.53}}$$

we obtain

$$\begin{cases} 0.00058\,(k_P + k_I) - 1.942 = -1.4 \\ 0.942 - 0.00058\,k_P = 0.53 \end{cases} \Rightarrow \quad \begin{matrix} k_I = 224 \\ k_P = 710 \end{matrix}$$

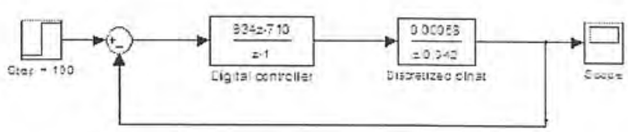
and a digital PI controller

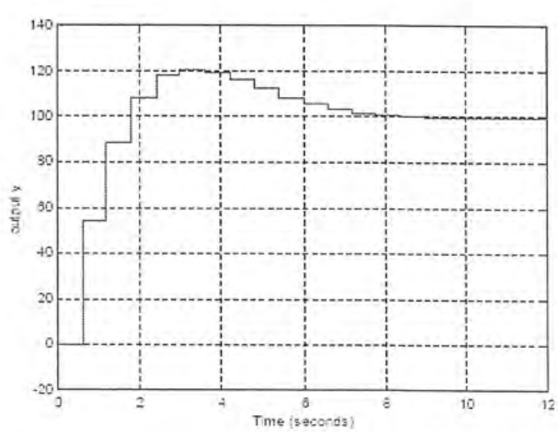
$$\boxed{D(z) = k_P + k_I\frac{z}{z-1} = \frac{(k_P + k_I)z - k_P}{z-1} = \frac{934z - 710}{z-1}}$$

Note: we cannot do much with the numerators of these transfer functions. It does affect the overall performance.

Simulation of the digital controller with discretized plant

We simulate the digital controller with the discretized plant to see whether the specifications are fulfilled in the discrete-time setting:

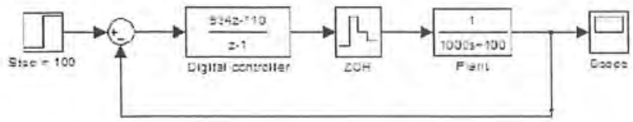


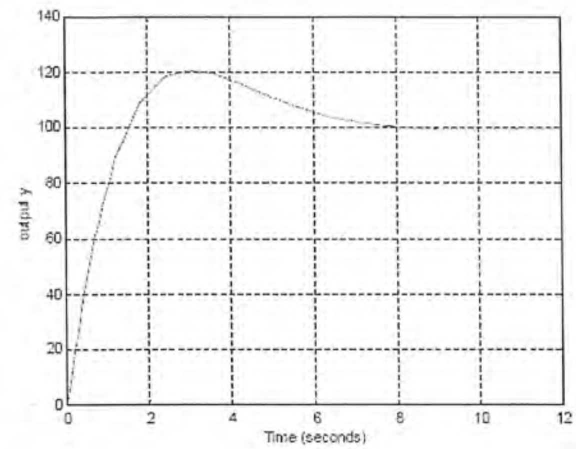


<u>Remark:</u> The overshoot is slightly larger than the design specification. But, the settling time meets the specification. The performance can be fine tuned by re-selecting the desired pole locations in z-plane.

Simulation of the digital controller with actual plant

We simulate the digital controller with the actual plant.





Remark: When the control law is implemented onto the actual continuous-time plant, the overshoot and the settling time are above the same as those obtained with the discretized system. All design specifications are met with a sampling period $T = 0.6$ seconds.

Example

Let the control system is:-

$$G(s) = \frac{10}{(s+1)(s+2)}$$

Design digital controller by using mixing (PI) controller, so that Ess=0, let T=0.1sec.[use discrete design and P-Z cancellation].

$$G(z) = (1 - z^{-1})\mathcal{Z}\{\frac{G(s)}{s}\} = \frac{0.0453(z+0.9048)}{(z-0.9048)(z-0.8187)}$$

The Ess of the uncompensated system is;-

$$Kp = \lim_{z \to 1} G(z) = 5$$

$$Ess = \frac{1}{Kp+1} = 0.166$$ { before Controller }

$$D(z)G(z) = \frac{(Ki\frac{T}{2} + Kp)(z + \frac{KiT - 2Kp}{KiT + 2Kp})}{(z-1)} * \frac{0.0453(z+0.9048)}{(z-0.9048)(z-0.8187)}$$

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We can use cancellation method, we get:-

$$\frac{KiT-2Kp}{KiT+2Kp}=-0.9048$$

$$\frac{Kp}{Ki}=1.0004$$

If we assume Kp=1, then Ki=0.9996

$$D(z)=1.05\frac{(z-0.9048)}{(z-1)}$$

$$D(z)G(z)=\frac{0.0476(z+0.9048)}{(z-1)(z-0.8187)}$$

So that the system became type one(z-1)and the Ess=0 for step input.

Remarks on higher order systems

When the given plant has a dynamic order higher than 1 and/or a general PID controller is used, the overall closed-loop transfer function from r to y will have an order larger than 2, e.g.,

$$H(z)=\frac{b_m z^m+b_{m-1}z^{m-1}+\cdots+b_1 z+b_0}{z^n+a_{n-1}z^{n-1}+\cdots+a_1 z+a_0}, \qquad m\le n,\ n>2$$

In this case, we should place the poles of the above transfer function by comparing it to the following desired transfer function

$$\boxed{H_{\text{desired}}(z)=\frac{*}{(z-\alpha_1)\cdots(z-\alpha_{n-2})\cdot(z-z_p)(z-\bar{z}_p)}}$$

i.e., by placing all the rest poles close to the origin, which is the fastest location in digital control. Eventually, dynamics associated with the poles close to the origin will die out very fast and the overall system is dominated by the pair left.

Example

Let the control system is:-

$$G(s) = \frac{6*10^7}{s^2}$$

Design digital controller by using conventional (PD) controller, so that Wn=820rad/sec and $\zeta = 0.8$, let T=0.0001sec.[use discrete design].

meaning

sol

$$G(z) = (1 - z^{-1})Z\{\frac{G(s)}{s}\} = \frac{(0.3z + 0.3)}{(z^2 - 2z + 1)}$$

$$D(z) = \frac{(Kp + Kd)z - Kd}{z} \longrightarrow P130$$

$$\frac{C(z)}{R(z)} = \frac{D(z)G(z)}{1 + D(z)G(z)} = \frac{0.3[(Kp + Kd)z^2 + Kpz - Kd)]}{z^3 + (0.4Kp + 0.3Kd - 2)z^2 + (0.3Kp + 1)z - 0.3Kd} ----(*)$$

$$S1,2 = -\zeta Wn \pm jWn\sqrt{1 - \zeta^2}$$

$$S1,2 = -656 \pm j492$$

$$Z1,2 = 0.9354 \pm j0.0461$$

$$= (z^2 - 1.8707z + 0.877)$$

$$\frac{C(z)}{R(z)} = \frac{K_{dc}}{z(z^2 - 1.8707z + 0.877} = \frac{0.0063}{z(z^2 - 1.8707z + 0.877)} ----(**)$$

select close to origin

When equating equ.(*) and (**), we get Kd=0 and this wrong. We must choose the 3rd pole of equ(**), (z+α), and the value of (α) very small. If in **EXAME** is not give , use this procedure:-

Take the c/cs(den of equ(*), and sub. one of the root in Z-domain, and solve this equ by paritation to two parts(real and imag.) and equating these to zero, after that we get Kp and Kd.

$$(0.9354 + j0.0461)^3 + (0.3Kp + 0.3Kd - 2)(0.9354 + j0.0461)^2 + (0.3Kp + 1)(0.9354 + j0.0461) - 0.3Kd = 0$$

$$5425Kp - 382Kd + 23 = 0$$

$$397Kp + 259Kd - 54 = 0$$

Kp=0.0094 and Kd=0.194

Note that:-

To know the correct design , sub. These values Kp, Kd in Den of equ (*), we get:-

$$z^3 - 1.939z^2 + 1.0028z - 0.0582 = 0$$
$$roots$$
$$Z1,2 = 0.9364 \pm j0.0448$$
$$z3 = 0.0622 \approx 0(small)$$

So that, we get approximately the same roots of the design requirement.

Note that

Also in discrete design, we can use the **formula** :-

$$D(z) = \frac{Q(z)}{G(z)[1 - Q(z)]}$$

Example:-Let the control system is:-

$$G(s) = \frac{1}{(10s+1)}$$

Find the control algorithm D(z), so that the response to a step input is:-

$$C(t) = 1 - e^{-t}$$

$$G(z) = (1 - z^{-1})\mathcal{Z}\{\frac{G(s)}{s}\} = \frac{0.18}{(z-0.82)}$$

$$C(t) = 1 - e^{-t}$$

$$C(z) = \frac{z}{z-1} - \frac{z}{z-e^{-T}}$$

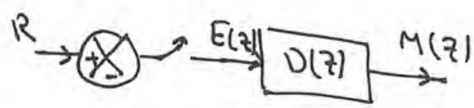
$$R(z) = step = \frac{z}{z-1}$$

$$\frac{C(z)}{R(z)} = Q(z) = \frac{0.86}{z-0.14}$$

Let T=2 sec $D(z) = \dfrac{4.8z - 3.9}{z-1}$ this calculated by using formula in P 145

$algorithm$

$$D(z) = \frac{M(z)}{E(z)} = \frac{4.8 - 3.9z^{-1}}{1 - z^{-1}}$$



$$M(z) - M(z)z^{-1} = 4.8E(z) - 3.9E(z)z^{-1}$$

$$m(k) - m(k-1) = 4.8e(k) - 3.9e(k-1)$$

$$m(k) = 4.8e(k) - 3.9e(k-1) + m(k-1)$$

This difference equ. Must be implemented in s/w of computer control system.

Note that (Important)

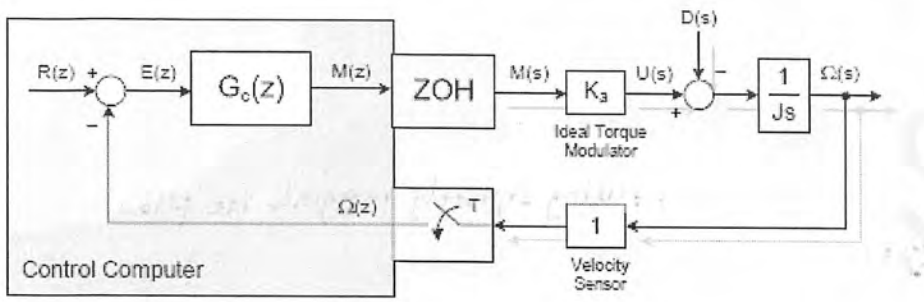
1- if the num of D(z) more than den of it , D(z) is bad(unrealizable)   X

2- if the num of D(z) equal den of it , D(z) is realizable proper.

3- if the num of D(z) less than den of it , D(z) is strictly proper.

Example

Consider the motor control system with J = 0.001 [kgm^2] and K_3 = 10 [Nm/V]. Design a PI controller which will yield a *over-damped* step-response (no overshoot) with a rise time (t_r) smaller than 0.25 [s]. use discrete method.



Let T = 0.01 sec.

Solution

Let D(s) = 0. Then, the discrete transfer function for the overall system can be expressed as

$$\frac{\Omega(z)}{M(z)} = Z\left\{(1-e^{-sT})\frac{K_a}{Js^2}\right\} = \frac{K_aT}{J}\cancel{(1-z^{-1})}\frac{z^{-1}}{(1-z^{-1})^{\cancel{2}}}$$

$$\frac{\Omega(z)}{M(z)} = G_p(z) \triangleq \frac{K_aT}{J}\cdot\frac{1}{z-1}$$

As for the PI controller, we have → using separately integral see p130

$$\frac{M(z)}{E(z)} = \overset{D(z)}{G_c(z)} = K_p + \frac{K_iT}{1-z^{-1}} = \frac{(K_p+K_iT)z-K_p}{z-1}$$

So, the open-loop TF becomes

$$G_{OL}(z) = G_c(z)G_p(z) = \frac{K(z-\beta)}{(z-1)^2} \quad \text{where} \quad K \triangleq \frac{K_aT}{J}(K_p+K_iT), \quad \beta \triangleq \frac{K_p}{K_p+K_iT}$$

Hence,

$$A(z) = num\{1+G_{OL}(z)\} = z^2 + (K-2)z + (1-K\beta)$$

$$A_d(z) = (z-a)(z-b) = z^2 - (a+b)z + ab$$

Let the design parameters are:-

```
a = 0.85; b = 1
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[These roots are given as design specifications]

Matching A(z) with $A_d(z)$ gives

$$z^0: \quad 1-K\beta = ab$$

$$z^1: \quad K-2 = -(a+b)$$

Solving these equations yields,

$$\boxed{K_p = \frac{J}{K_aT}(1-ab)} \qquad \boxed{K_i = \frac{J(1-a)(1-b)}{K_aT^2}}$$

A Matlab code is developed to check