Example5

A microcontroller is used to control a system with the following dynamics

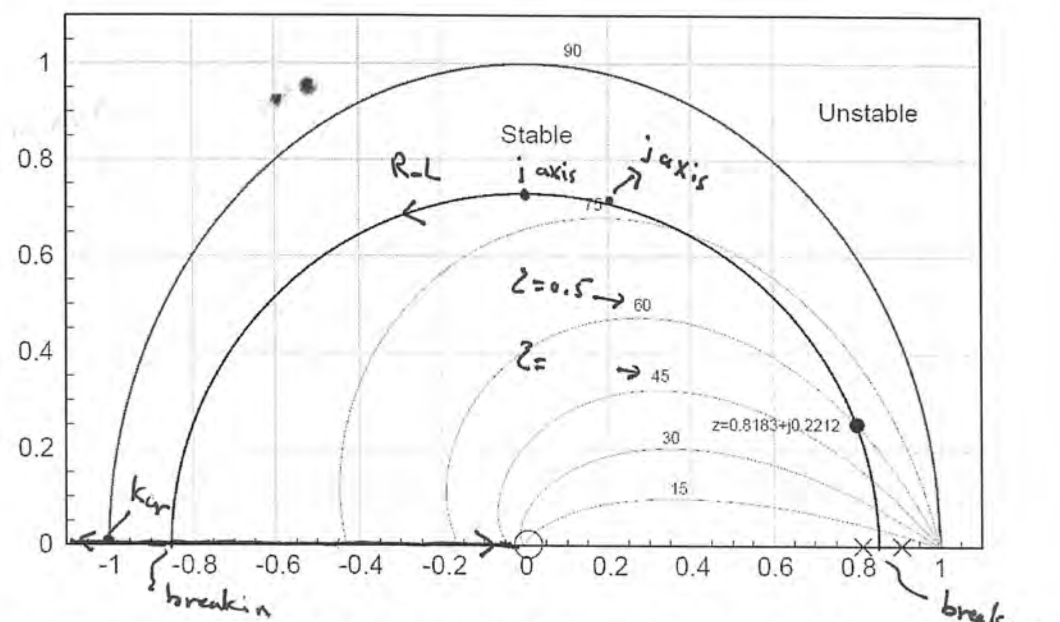
$$G(z) = \left(\frac{0.0173z}{(z-0.9048)(z-0.8187)} \right)$$

Assume a sampling rate of 10ms. Find the maximum 'k' for stability and 'k' for a damping ratio of 0.5.

Next, sketch the root locus. Include in the root-locus plot the 60 degree damping ratio line

1- 2-poles are z=0.9048 and z=0.8187
2- 2-zeros are z=0 and z= ∞
3- No. of asymp.=n-m=2-1=1
4- Angle of asymp.=180°.
5- Use equ. Of break away and in, find breakaway=0.85and break in=-0.85.
6- Use jury test to find Kcr. Or Max. gain at z=-1, Kmax=3.46 and at j-axis=0.1

7-To find (K) at zeta=0.5, intersect the root-locus with angle=60°, at this point ,find the C.L.poles, Z12=0.8138∓j0.2212 as shown in fig. To Find (K) at these C.L. poles sub. in the c/cs equ. and Find (K).



Sub. This root(C.L.Poles) in the system equ.(T.F) and find the gain(K).

Or using MATLAB

n=[0.0173 0];

d=[1 -(0.9048+0.8187) 0.9048*0.8187];

rlocus(n,d)

hold on

zgrid

rlocfind(n,d) % this intersection R-L with zeta=0.5(above plot is angle=60°)

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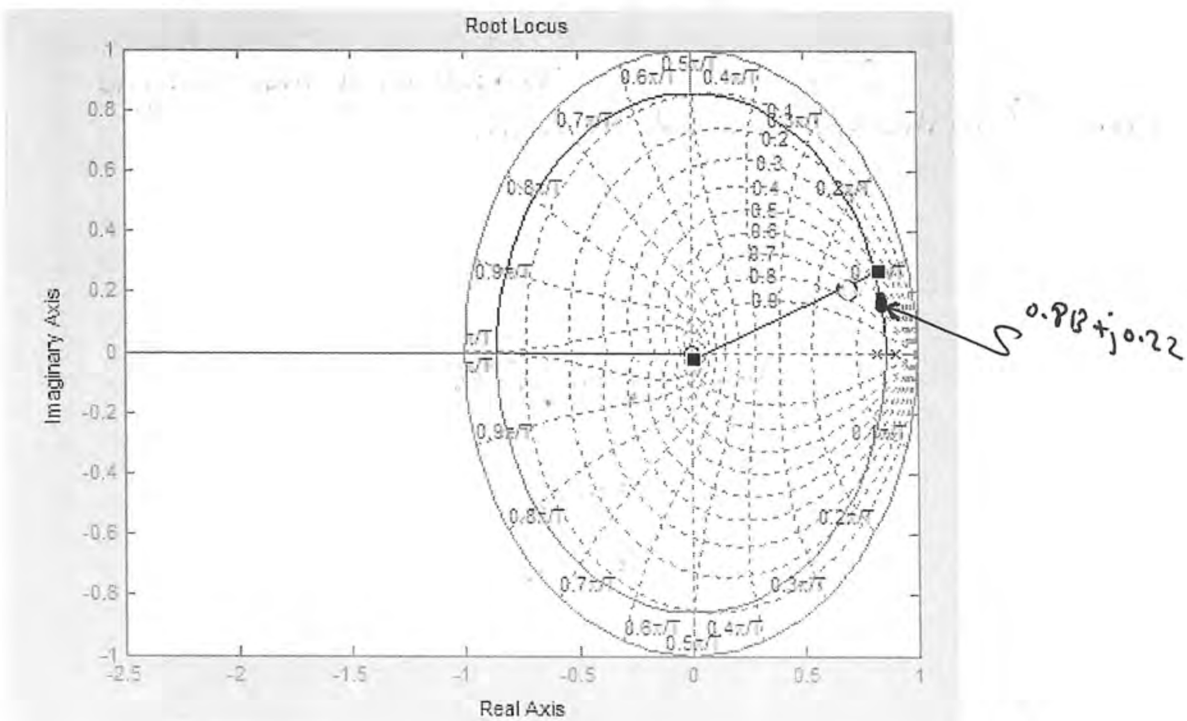
find the gain(K).may be(4.5).



Example6

Obtain the root locus plot and the critical gain for system withT.F:

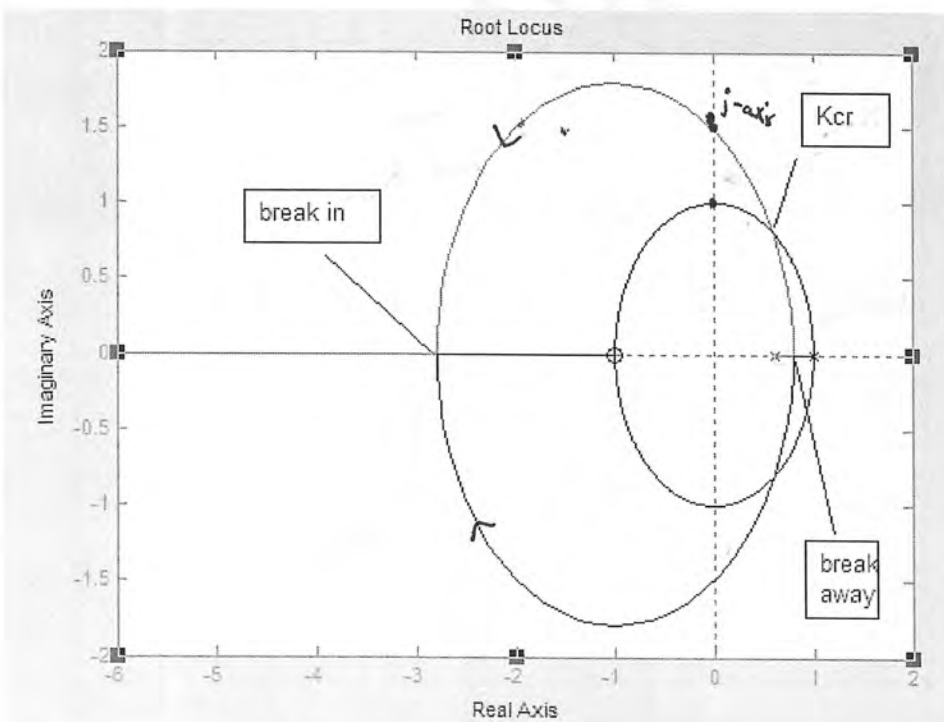
G(z)=K(z+1)/(z-1)(z-0.6065)

Sol

1- 2-poles are z=1 and z=0.6065
2- 2-zeros are z=-1 and z= ∞
3- No. of asymp.=n-m=2-1=1
4- Angle of asymp.=180.
5- Use equ. Of break away and in, find breakaway=0.792and break in=-2.792.

6-Use jury test to find Kcr. Or , Kcr=0.3935

7-Intersection with j-axis=1.5j



Example7

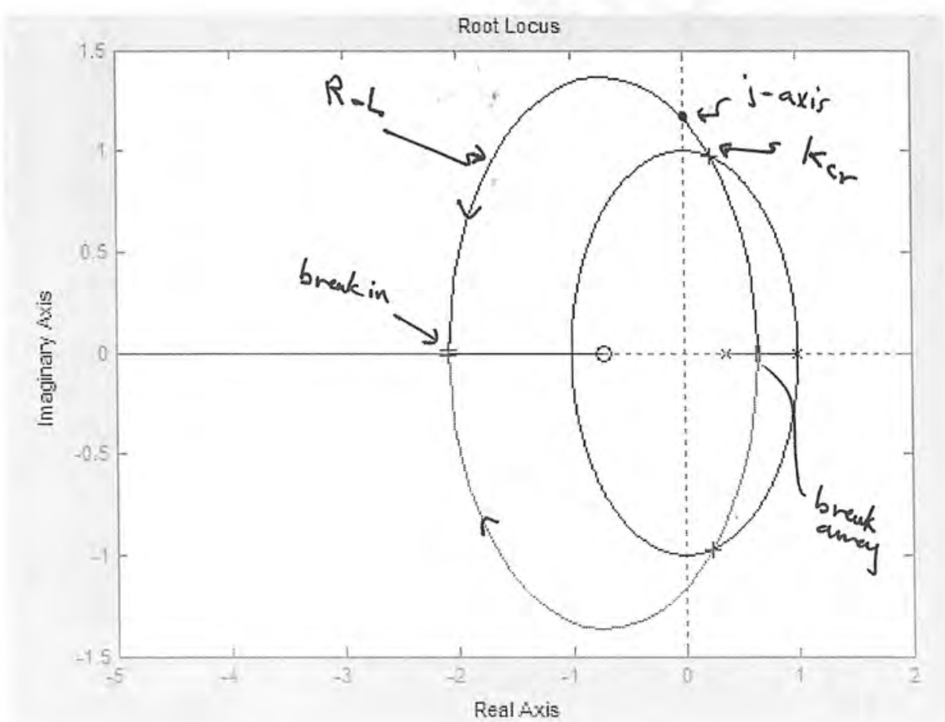
Obtain the root locus plot and the critical gain for system withT.F:

G(z)=0.368K(z+0.717)/(z-1)(z-0.368)

Sol

1- 2-poles are z=1 and z=0.368
2- 2-zeros are z=-0.717 and z= ∞
3- No. of asymp.=n-m=2-1=1
4- Angle of asymp.=180°.
5- Use equ. Of break away and in, find breakaway=0.6479 and break in=-2.0819.

6-Use jury test to find Kcr. Or , Kcr=2.39 and the intersection with j-axis=1.15j



Example8 :

If the T.F is:- Draw R-L, if T = 0.6348 sec.

$$G(s)=\frac{101}{s^2+2s+101}=\frac{101}{(s+1)^2+(10)^2}$$, and there is Controller $D(z)=\frac{(z+0.8171)}{(z-1)}$

$$G(z)=(1-z^{-1})Z\{\frac{1}{s}\frac{101}{(s+1)^2+(10)^2}\}=(1-z^{-1})Z\{\frac{A}{s}+\frac{Bs+C}{s^2+2s+101}\}$$

$$A=1, B=-1, C=-2$$

$$G(z)=(1-z^{-1})Z\{\frac{1}{s}+\frac{-s-2}{s^2+2s+101}\}=(1-z^{-1})Z\{\frac{1}{s}-\frac{s+1}{(s+1)^2+(10)^2}-\frac{1}{(s+1)^2+(10)^2}\}$$

$$G(z)=\frac{(z-1)}{z}[\frac{z}{z-1}-\frac{z^2-ze^{-T}\cos 10T}{z^2-2ze^{-T}\cos 10T+e^{-2T}}-\frac{ze^{-T}\sin 10T}{z^2-2ze^{-T}\cos 10T+e^{-2T}}]$$

$$G(z)=\frac{(z-1)}{z}[\frac{z}{z-1}-\frac{z^2-ze^{-T}}{z^2-2ze^{-T}+e^{-2T}}-\frac{0}{z^2-2ze^{-T}+e^{-2T}}]$$

since

$$\sin\frac{10*2*3.14}{10}=0$$

$$\cos\frac{10*2*3.14}{10}=1$$

$$G(z)=1-\frac{z-1}{z-e^{-T}}=\frac{0.47}{z-0.53}$$

2- with the controller D(z) $=\frac{(z+0.8171)}{(z-1)}$

$$D(z)G(z)=\frac{0.47K(z+0.8171)}{(z-1)(z-0.53)}$$

Use Root-locus Rules:-

a- 2-poles are z=1 , 0.53

b- 2-zeros are z=-0.8171 , ∞

c- No. of asymptotic. =1

d- angle of asymptotic $=180^{\circ}$

To find breakaway and break in points use

$$K=\frac{(z-1)(z-0.53)}{0.47(z-0.8171)}$$

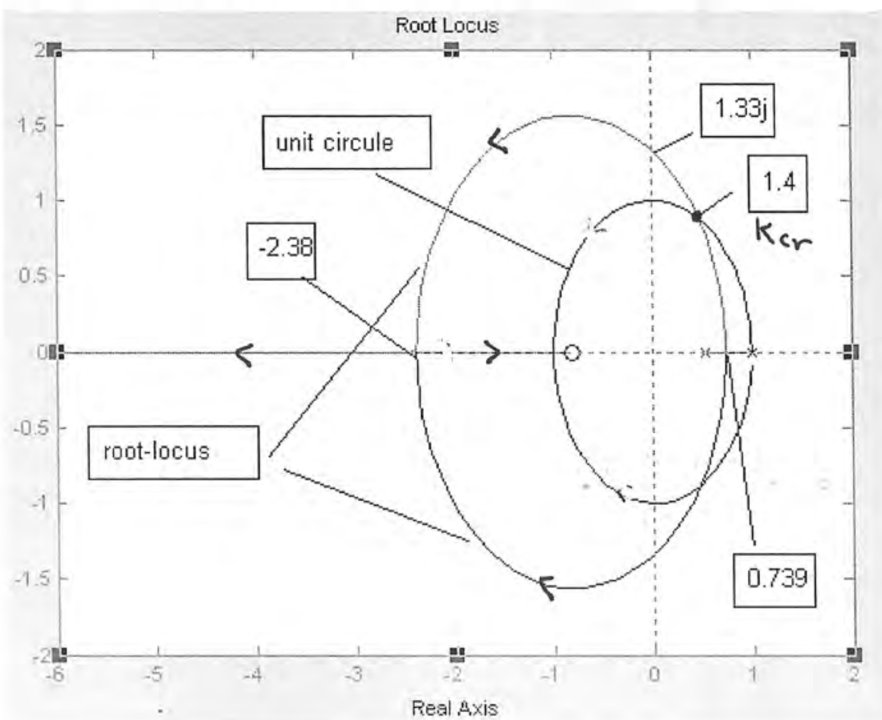
$$\frac{dK}{dz}=0$$

Break away point is = 0.739 and K at this point is =0.0746

Break in point is= -2.38 and K at this point is= 13.4

e- Intersection with j-axis is =1.33j and K at this point is = 3.26

g- Critical gain[Kcr] (use jury Test) is = 1.4



Example9

Consider the closed-loop characteristic equation

$$1 + K\frac{(z+1)(z-0.5)}{(z-1)(z-0.9)(z+0.6)} = 0$$

Draw the root-locus of it.

sol

R1: The system has $n = 3$ poles indicating three branches. They start at 1, 0.9, and -0.6 with $K = 0$.

R2: There are two finite zeros ($m = 2$). Hence two of the branches terminate on the zeros -0.9 and 0.5 at $K = \infty$.

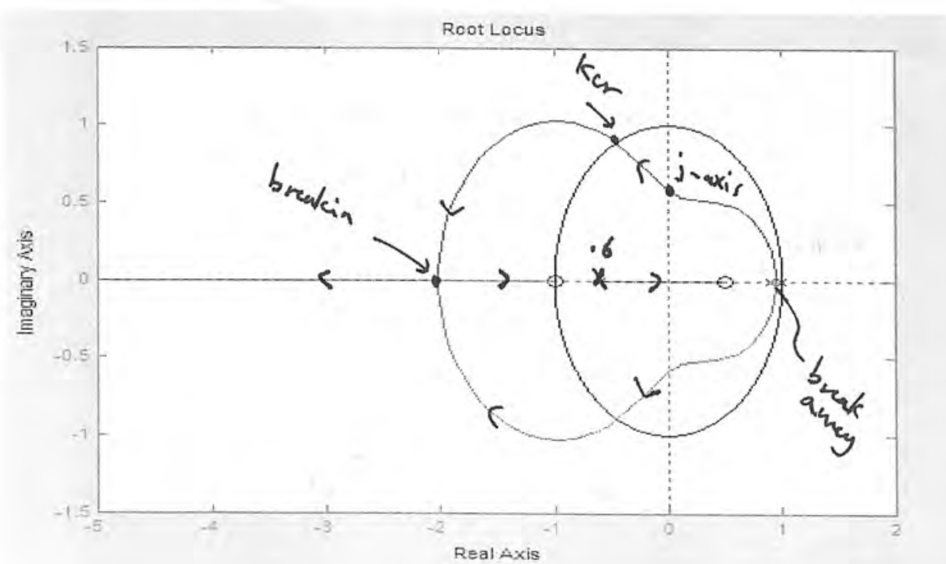
R3: One branch ($n - m = 1$) will go to ∞ along an asymptote. and the angle = 180°

R4: Sections of the root loci on the real line are between 0.9 and 1.0, -1.0 and $-\infty$, and -0.6 and 0.5; with this information the sketch shown in Fig can be easily obtained. If additional features are needed, the rest of the root-loci construction rules can be applied without change.

R5:- break away z=0.963 and break in z=-2

R6:- intersection with j-axis at z=±0.59j

R7:- use jury test, find Kcr=1.8



Example10

If $G(z) = \frac{0.01873(z+0.9356)}{(z-1)(z-0.8187)}$ **and T=0.2 sec**

Also there is in feed forward path a digital controller is:-

$$D(z) = \frac{5.4351(z-0.8187)}{(z-0.5071)}$$

A- Draw the root-locus and find (Kcr).

B- Find closed-loop poles of the whole system and (Ws), (Wd).

Sol

G(z)D(z)=0.1018K(z+0.9356)/(z-1)(z-0.5071)

A-apply all rules of R-L, we get:-

1- 2-poles are z=1 and z=0.5071
2- 2-zeros are z= -0.9356 and z=∞
3- No. of asymp.=n-m=2-1=1
4- Angle of asymp.=180°
5- Use equ. Of breakaway=0.73 and in=-2.61,
6- Use jury test to find Kcr=6.3
7- Find the intersection with j-axis=j1.38