B-

C(z)/R(z)=0.1018(z+0.9356)/(z-0.7026+j0.3296)(z-0.7026-j0.3296)

Closed-loop poles are Z12=0.7026∓0.3296j ⇐ c/cs eqn. of C(z)/R(z)

From these poles found R=0.776 and ϕ=0.4386rad

Ws=2*pi/T=31.4rad/sec

ϕ=Wd*T= or Wd=ϕ/T=2.193rad/sec

→ How To calculate c.L. poles

$$\text{Find C.L.T.F} = \frac{G(z)D(z)}{1+G(z)D(z)}$$

$$\text{c/cs equ} \quad 1+G(z)D(z) = 1 + \frac{0.1018(z+0.9356)}{(z-1)(z-0.5071)}$$

$$= z^2 - 1.4053z + 0.6023$$

↳ Roots

$$z_{1,2} = \overset{a}{0.7026} \pm j\,\overset{b}{0.3296}$$

$$R = \sqrt{a^2+b^2} = 0.7758 = 0.776$$

$$\phi = \tan^{-1}\frac{0.3296}{0.7026} = 0.4386 \text{ rad}$$

Example11

We will first investigate the effect of controller gain K and sampling time T on the relative stability of the closed loop system as shown in Figure 1.

HINT:- *USE ROOT-LOCUS RULES*

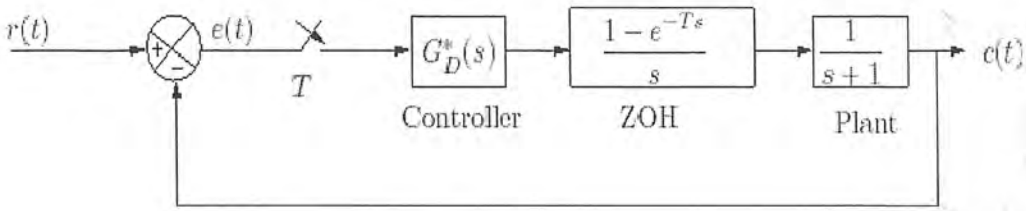


Figure1 : A discrete time control system

Let us first take **T=0.5** sec.

Sol

$$
\begin{aligned}
Z\left[G_{ho}(s)G_p(s)\right] &= Z\left[\frac{1-e^{-Ts}}{s}.\frac{1}{s+1}\right] \\
&= (1-z^{-1})Z\left[\frac{1}{s(s+1)}\right] \\
&= (1-z^{-1})Z\left[\frac{1}{s}-\frac{1}{s+1}\right] \\
&= \frac{z-1}{z}\left[\frac{z}{z-1}-\frac{z}{z-e^{-T}}\right] \\
&= \frac{1-e^{-T}}{z-e^{-T}}
\end{aligned}
$$

Let us assume that the controller is an integral controller, i.e.

$$G_D(z) = \frac{Kz}{z-1}$$

Thus,

$$
\begin{aligned}
G(z) &= G_D(z)\cdot G_hG_p(z) \\
&= \frac{Kz}{z-1}\cdot\frac{1-e^{-T}}{z-e^{-T}} = \frac{0.3935Kz}{(z-1)(z-0.6065)} \quad ---(*)
\end{aligned}
$$

The characteristic equation can be written as

$$1 + G(z) = 0$$
$$\Rightarrow \quad 1 + \frac{Kz(1-e^{-T})}{(z-1)(z-e^{-T})} = 0$$

when $T = 0.5sec.$ $\quad L(z) = \frac{0.3935Kz}{(z-1)(z-0.6065)} = G(z)$

$L(z)$
$\Downarrow$
has poles at z = 1 and z = 0.605 and zero at z = 0, ∞

Break away/ break in points are calculated by putting

$$\frac{dK}{dz} = 0$$

$$K = -\frac{(z-1)(z-0.6065)}{0.3935z}$$
$$\frac{dK}{dz} = -\frac{z^2-0.6065}{0.3935z^2} = 0$$
$$\Rightarrow z^2 = 0.6065 \Rightarrow z_1 = 0.7788 \text{ and } z_2 = -0.7788$$

Critical value of Kcr can be found out from the magnitude criterion.

$$\left|\frac{0.3935z}{(z-1)(z-0.6065)}\right| = \frac{1}{K}$$

$z = -1$
Thus

$$\left|\frac{-0.03935}{(-2)(-1.6065)}\right| = \frac{1}{K}$$
$$\text{or,} \quad K = \underline{\underline{8.165}} \; \mathbf{=Kcr}$$

Or use Jury test (apply the conditions) to find Kcr.

Figure 2 shows the root locus of the system . Two root locus branches start from two open loop poles at K = 0. If we further increase K one branch will go towards the zero and the other one will tend to infinity. The circle represents the unit circle. Thus the stable range of K is 0 < K < 8.165.

sub $z = -1$ in equ(*)

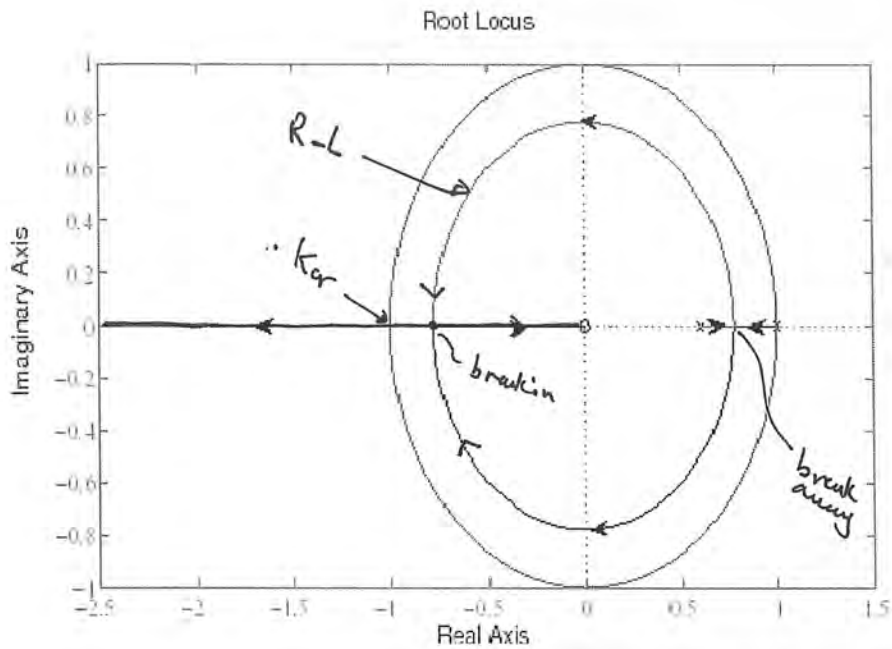


Figure 2 : Root Locus when T=0.5 sec

The same example . **If T = 1** sec,

$$G(z) = \frac{0.6321Kz}{(z-1)(z-0.3679)}$$

Break away/ break in points:

$z^2 = 0.3679 \Rightarrow z_1 = 0.6065$ and $z_2 = -0.6065$ Critical gain

$$(K_c) = \underline{\underline{4.328}}$$

Figure 3 shows the root locus . It can be seen from the figure that the radius of the inside circle reduces and the maximum value of stable K also decreases to K = 4.328.

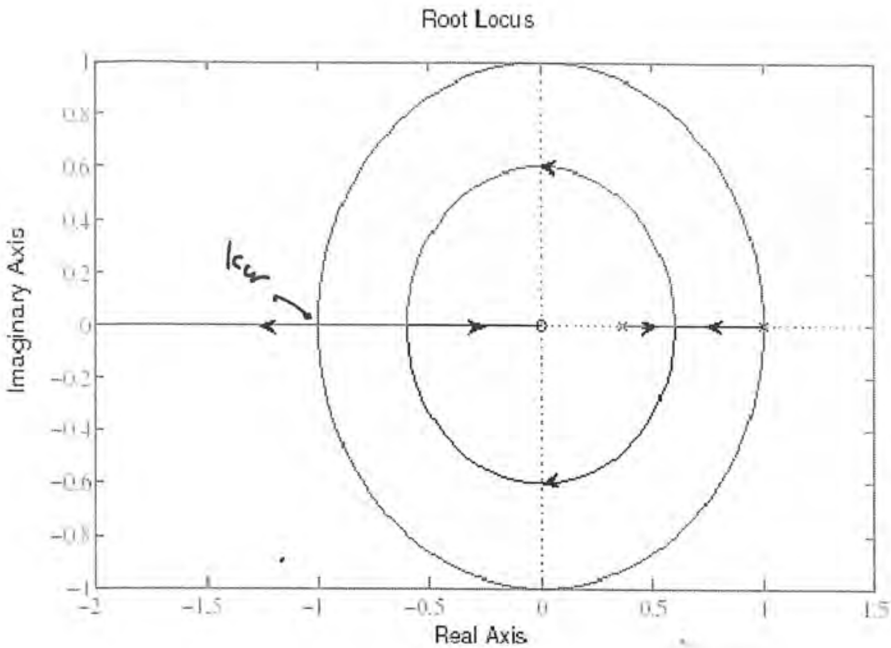


Figure 3: Root Locus when T=1 sec

Similarly if **T = 2** sec,

$$G(z) = \frac{0.8647Kz}{(z-1)(z-0.1353)}$$

One can find that the critical gain in this case further reduces to 2.626.

Effect of sampling period T

As can be seen from the previous example, large T has detrimental effect on relative stability.

As seen from the example making the sampling period smaller allows the critical gain to be larger, i.e., maximum allowable gain can be made larger by increasing sampling frequency /rate. It seems from the example that damping ratio decreases with the decrease in T. But one should take a note that damping ratio of the closed loop poles of a digital control system indicates the relative stability only if the sampling frequency is sufficiently high (8 to 10 times). If it is not the case, prediction of overshoot from the damping ratio will be erroneous and in practice the overshoot will be much higher than the predicted one.

∴ T↓ is more gain but (ζ)↓

Next, we may investigate the effect of T on the steady state error. Let us take a fixed gain K = 2.

When T = 0.5 sec. and K = 2,

$$G(z) = \frac{0.787z}{(z-1)(z-0.6065)}$$

Since this is a second order system, velocity error constant will be a non-zero finite quantity.

$$Kv = \lim_{z \to 1} \frac{(1-z^{-1})G(z)}{T} = 4$$

Thus,

$$e_{ss} = \frac{1}{4} = 0.25$$

When T = 1 sec. and K = 2

$$G(z) = \frac{1.2642z}{(z-1)(z-0.3679)}$$

$$Kv = \lim_{z \to 1} \frac{(1-z^{-1})G(z)}{T} = 2$$

$$e_{ss} = \frac{1}{2} = 0.5$$

When T = 2 sec. and K = 2

$$G(z) = \frac{1.7294z}{(z-1)(z-0.1353)}$$

$$Kv = \lim_{z \to 1} \frac{(1 - z^{-1})G(z)}{T} = 1$$

$$e_{ss} = \frac{1}{1} = 1$$

Thus, increasing sampling period (decreasing sampling frequency) has an adverse effect on the steady state error as well.

negative

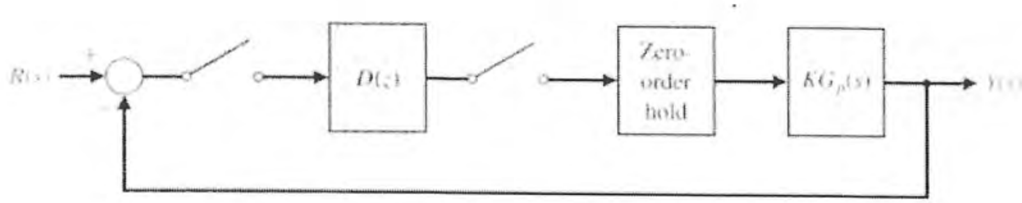
$\therefore T \uparrow \Longrightarrow E_{ss} \uparrow$

Example12

Root locus of a second-order system

Consider the system shown in Figure with $D(z) = 1$ and $G_p(s) = 1/s^2$. Then we obtain

$$\frac{Y(z)}{R(z)} = \frac{KG(z)D(z)}{1 + KG(z)D(z)}.$$



$$KG(z) = \frac{T^2 K(z+1)}{2(z-1)^2}$$

Let $T = \sqrt{2}$ and plot the root locus. We now have

$$KG(z) = \frac{K(z+1)}{(z-1)^2}$$

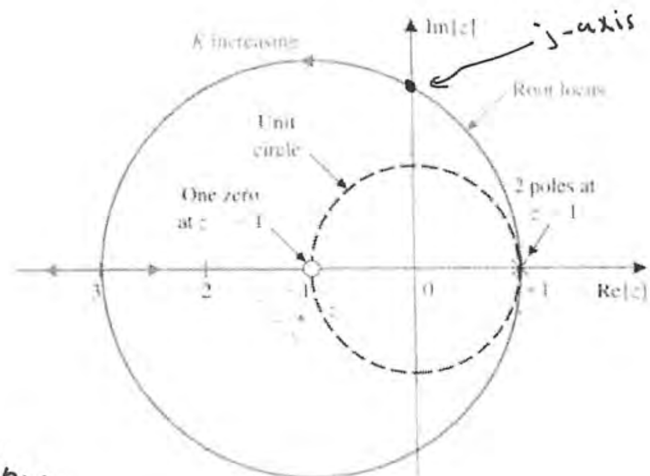
and the poles and zeros are shown on the z-plane in Figure 1[illegible]. The characteristic equation is

$$1 + KG(z) = 1 + \frac{K(z+1)}{(z-1)^2} = 0$$

Let $z = \sigma$ and solve for K to obtain

$$K = -\frac{(\sigma-1)^2}{(\sigma+1)} = F(\sigma) \Rightarrow \frac{dF(\sigma)}{dz}$$

$\therefore$ breakaway & breakin



breakaway

breakin

Then obtain the derivative $dF(\sigma)/d\sigma = 0$ and calculate the roots as $\sigma_1 = -3$ and $\sigma_2 = 1$. The locus leaves the two poles at $\sigma_2 = 1$ and reenters at $\sigma_1 = -3$, as shown in Figure [illegible]. The unit circle is also shown in Figure [illegible]. The system always has two roots outside the unit circle and is always unstable for all $K > 0$. ■