## Example13

Repeat Example12, but with adding a controller.

Let us design a compensator $D(z)$ that will result in a stable system

Let D(z):-

$$D(z) = \frac{z-a}{z-b}$$

so that

$$KG(z)D(z) = \frac{K(z+1)(z-a)}{(z-1)^2(z-b)}$$

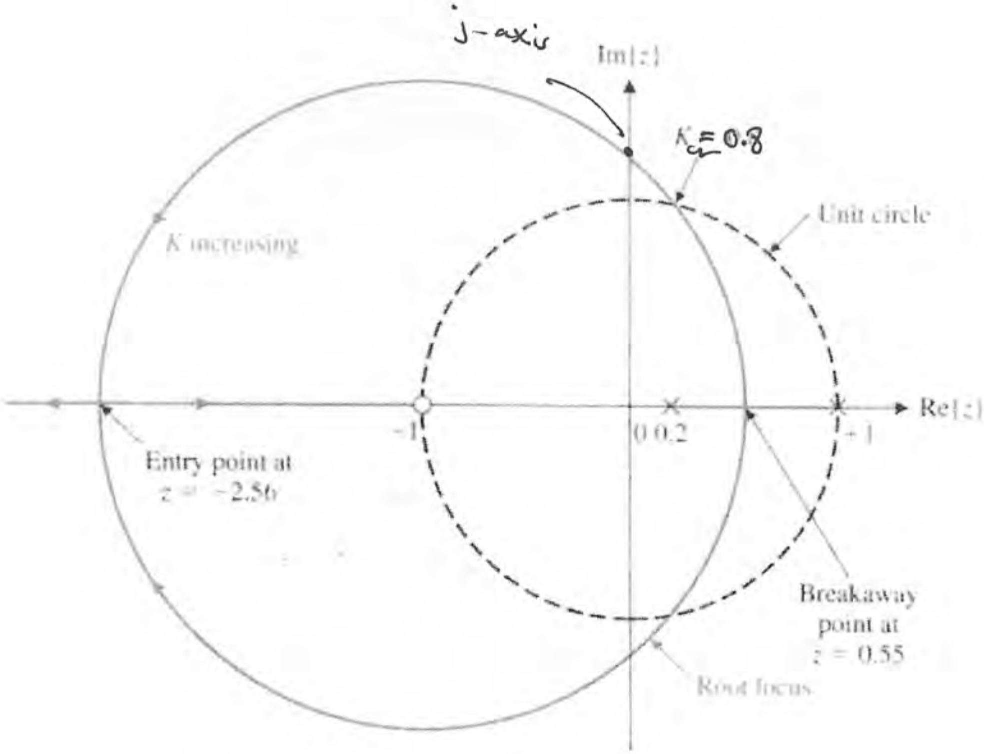
In this controller always chose the zero of it(z-a), canceling the pole of G(z), so that, a=1 and let b=0.2.

we have

$$KG(z)D(z) = \frac{K(z+1)}{(z-1)(z-0.2)}.$$

break in

Using the equation for $F(\sigma)$, we obtain the entry point as $z = -2.56$, as shown in Figure . The root locus is on the unit circle at $K_{cr} = 0.8$. Thus, the system is stable

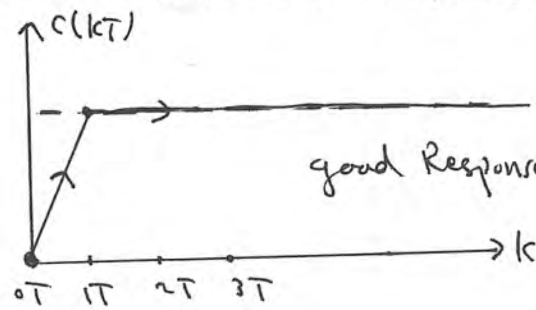



for $K < 0.8$. If we select $K = 0.25$, we find that the step response has an overshoot of 20% and a settling time (with a 2% criterion) equal to 8.5 seconds.

If the system performance were inadequate, we would improve the root locus by selecting $a = 1$ and $b = -0.98$ so that

$$KG(z)D(z) = \frac{K(z+1)}{(z-1)(z+0.98)} \approx \frac{K}{(z-1)}$$

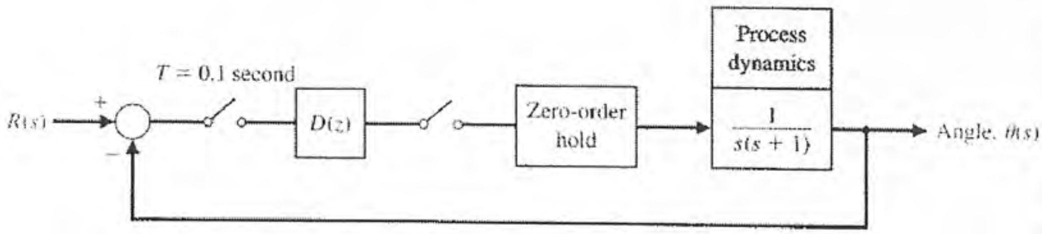
Then the root locus would lie on the real axis of the $z$-plane. When $K = 1$, the root of the characteristic equation is at the origin, and $T(z) = 1/z = z^{-1}$. Then the response of the sampled system (at the sampling instants) is the input step delayed by one sampling period. ■




good Response: $M_p = 0$
and $T_s$ is small

## Example14

Let the system as shown in fig.




Draw root-locus and find Kcr , if D(z)=proportional controller=K.

**Sol**

Combining the process and the zero-order hold in series yields

$$G(s) = G_o(s)G_p(s) = \frac{1 - e^{-sT}}{s^2(s + 1)}.$$

$$G(s) = (1 - e^{-sT})\left(\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s + 1}\right),$$

$$G(z) = Z\{G(s)\} = \frac{ze^{-T} - z + Tz + 1 - e^{-T} - Te^{-T}}{(z - 1)(z - e^{-T})},$$

$\}$ represents the $z$-transform. Choosing $T = 0.1$, we have

$$G(z) = \frac{0.004837z + 0.004679}{(z - 1)(z - 0.9048)}.$$

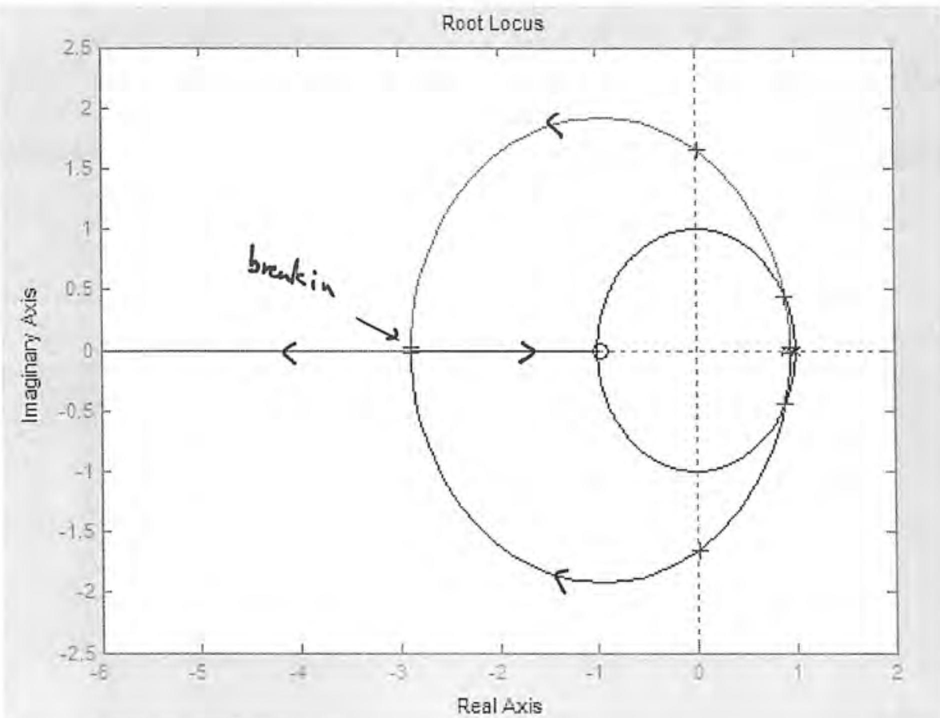
For a simple compensator, $D(z) = K$, the root locus is shown in Figure

For stability we require $K < 21$

Apply all rules of R-L, we get:-

1 - 2-poles are z=1 and z=0.9048
2 - 2-zeros are z=0.9673 and z= ∞
3 - No. of asymp.=n-m=2-1=1
4 - Angle of asymp.=180°.
5 - Use equ. Of breakaway=0.96 and in=-2.9,
6 - Use jury test to find Kcr=21

Find the intersection with j-axis=j1.65




Using an iterative approach we discover that as $K \to 21$, the step response is very oscillatory, and the percent overshoot is too large; conversely, as $K$ gets smaller, the settling time gets too long, although the percent overshoot decreases. In any case the design specifications cannot be satisfied with a simple proportional controller, $D(z) = K$. We need to utilize a more sophisticated controller.

Here we choose a compensator with the general structure

$$D(z) = K\frac{z-a}{z-b}.$$

In this controller always chose the zero of it(z-a), canceling the pole of G(z), so that, a=0.9048 and let b=0.25

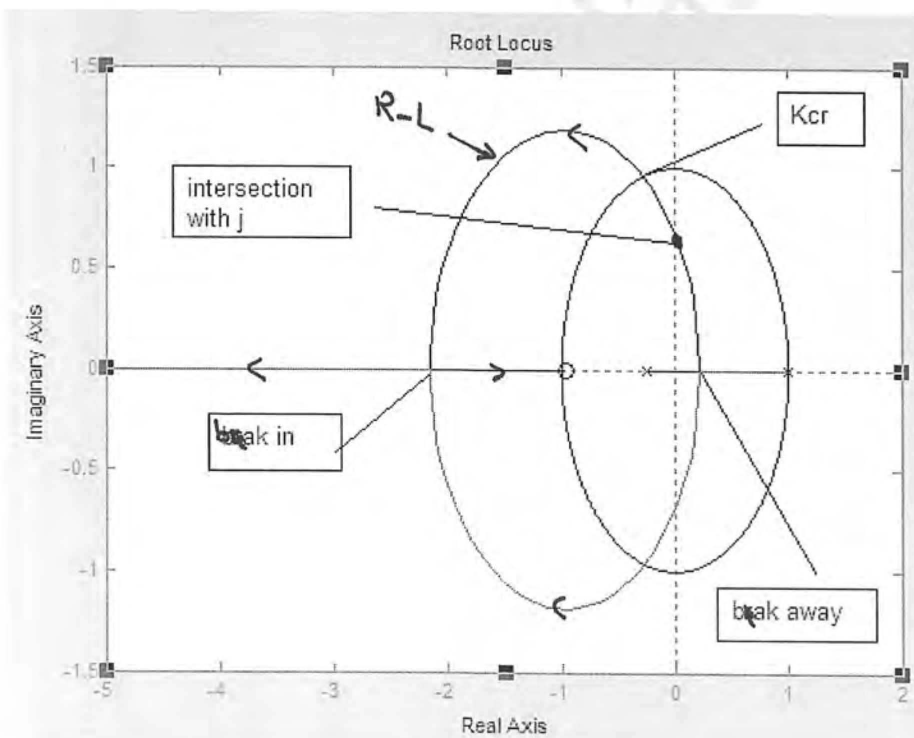
Thus the compensator is

$$D(z) = 70\,\frac{z - 0.9048}{z + 0.25}.$$

So that KG(z)D(z)=70(0.004837z+0.004673)/(z-1)(z+0.25)

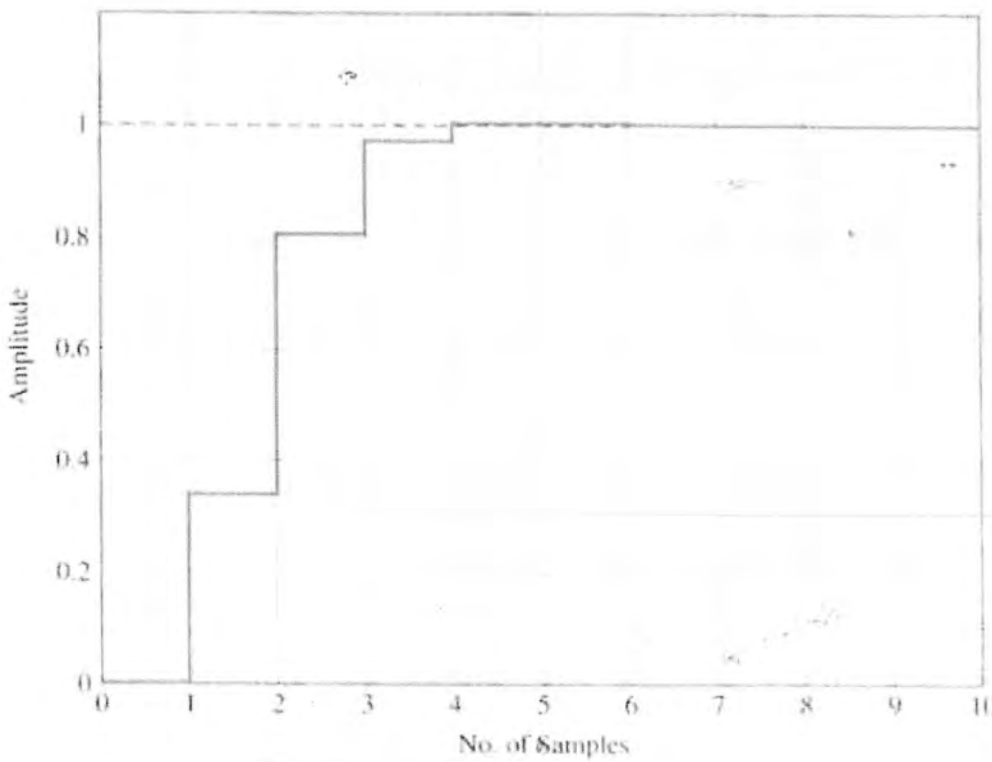
1- 2-poles are z=1 and z=-0.25
2- 2-zeros are z= 0.9674 and z=
3- No. of asymp.=n-m=2-1=1
4- Angle of asymp.=180°.
5- Use equ. Of break away=0.22 and in=-2.16
6- Use jury test to find Kcr=268.7

Find the intersection with j-axis=j0.68




The R-L after design (add compensator) as shown in fig.

The closed-loop step response is shown in Figure . Notice that the percent overshoot specification ($P.O. \leq 5\%$) is satisfied, and the system response settles in less than 10 samples (10 samples = 1 second because the sampling time is 0.1 second). ■



**Example15:-(repeated Ex6)** → P167

Let $G(z)=\dfrac{K(z+1)}{(z-1)(z-0.6065)}$

And T=0.1sec

1-Draw R-L and find Kcr

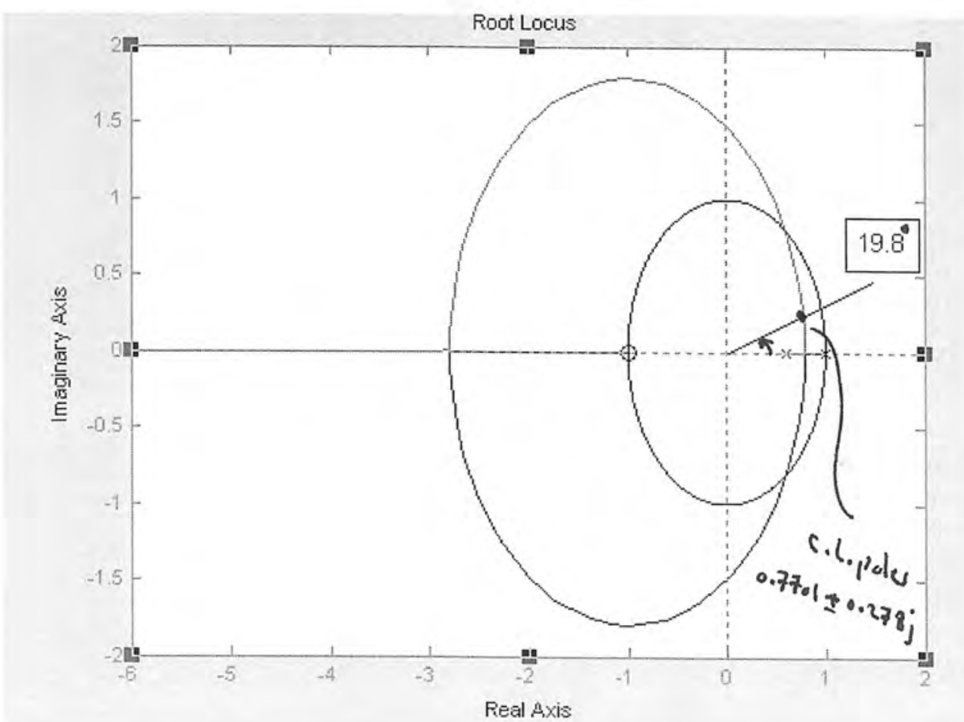
2- What is the value of (K) will yield a $\zeta=0.5, Wn=4rad/\sec$

3-With this value of (K) determine Wd and the number of samples per cycles of damped oscillation.

**Sol**

**1-R-L**

The same procedures as Ex6



## 2-

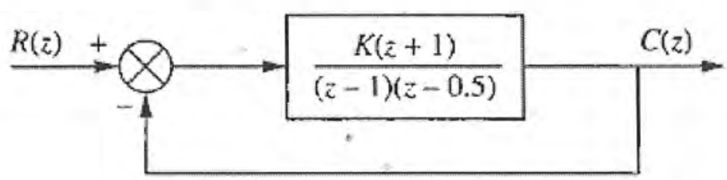
$$M = e^{-\zeta W n T} = e^{-0.1*0.5*4} = 0.8187$$
$$\phi = TWn\sqrt{1-\zeta^2} = 0.3464 rad = 19.87^\circ$$
$$Z12 = 0.8187\langle 19.87^\circ$$
$$= 0.7701 \pm j0.278$$
$$\frac{K(z+1)}{(z-1)(z-0.6065)} = 1 \downarrow z = 0.7701 + j0.278$$
$$K = 0.0649$$

## 3-

$$Wd = Wn\sqrt{1-\zeta^2} = 3.464 rad/\sec$$
$$No.of samples = \frac{2 * Pi}{\phi} = 18.14 = 18$$

## Example16:

Sketch the root locus for the system shown in Figure




1- 2-poles are z=1 and z=0.5
2- 2-zeros are z= -1 and z= $\infty$
3- No. of asymp.=n-m=2-1=1
4- Angle of asymp.=180°
5- Use equ. Of break away=0.72 and in=-2.73
6- Use jury test to find Kcr=0.5

Find the intersection with j-axis=j1.4