because there are 2- intersection with unit-circle

The figure shows that the stable range of K_p is $0.239 < K_p < 6.31$. The best achievable overshoot is 45.5% , for $K_p = 1$, which is very high for any practical system. To improve the relative stability, we need to introduce D action. Let us modify the controller to a PID controller for which the transfer function in z-domain is given as below.

$$G_D(z) = \frac{(K_pT + K_d)z^2 + (K_iT^2 - K_pT - 2K_d)z + Kd}{Tz(z-1)}$$

To satisfy velocity error constant, $K_i \geq 1$. If we assume 15% overshoot (corresponding to $\xi \cong 0.5$) and 2 sec settling time (corresponding to $\omega_n \cong 4$), the desired dominant poles can be calculated as,

$$\begin{aligned} s_{1,2} &= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \\ &= -2 \pm j3.46 \end{aligned}$$

Thus the closed loop poles in z-plane

$$\begin{aligned} z_{1,2} &= exp(T(-2 \pm j3.46)) \\ &\cong 0.77 \pm j0.28 \end{aligned}$$

T = 0.1 sec.

The pole zero map including the poles of the PID controller is shown in Figure 6 where the red cross denotes the desired poles.

big

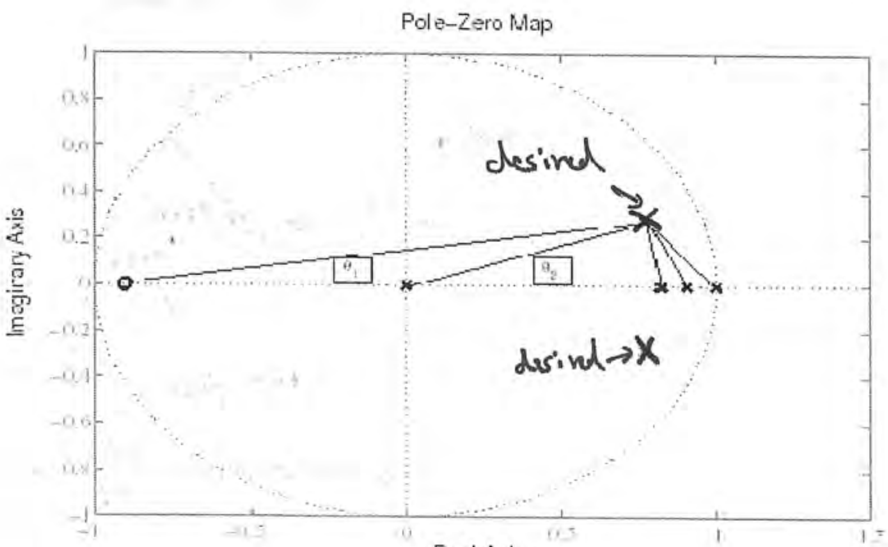


Figure 6: Pole zero map including poles of the PID controller

Let us denote the angle contribution starting from the zero to the right most pole as θ_1, θ_2, θ_3, θ_4 and θ_5 respectively. The angles can be calculated as $\theta_1 = 9.5°$, $\theta_2 = 20°$, $\theta_3 = 99.9°$, $\theta_4 = 115.7°$ and $\theta_5 = 129.4°$.

Net angle contribution is A = 9.5° - 20° - 99.9° - 115.7° - 129.4° = -355.5° . Angle deficiency is -355.5° + 180° = - 175.5°

Thus the two zeros of PID controller must provide an angle of 175.5°. Let us place the two zeros at the same location, z_{pid} .

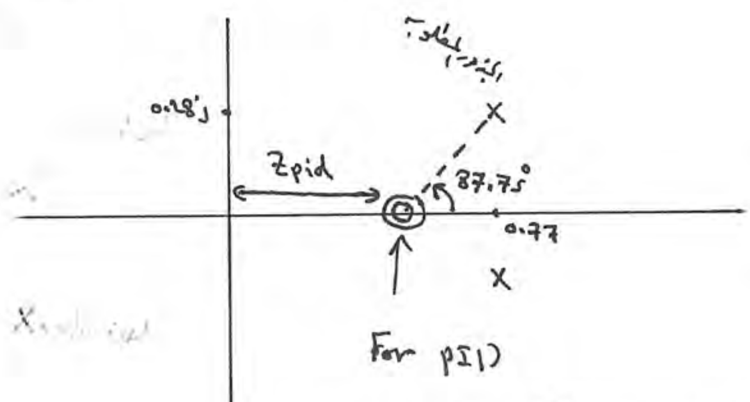
Since the required angle by individual zero is 87.75° , we can easily say that the zeros must lie on the left of the desired closed loop pole.

$$\begin{aligned} \tan^{-1}\frac{0.28}{0.77 - z_{pid}} &= 87.75^o \\ \text{or, } \frac{0.28}{0.77 - z_{pid}} &= \tan(87.75^o) = 25.45 \\ \text{or, } 0.77 - z_{pid} &= \frac{0.28}{25.45} = 0.011 \\ \text{or, } z_{pid} &= 0.77 - 0.011 = 0.759 \end{aligned}$$

The controller is then written as

$$G_D(z) = K\frac{(z - 0.759)^2}{z(z-1)}$$

. The root locus of the compensated system (with PID controller) is shown in Figure 7. This figure shows that the desired closed loop pole corresponds to K = 4.33 .



$$\therefore \tan 87.75 = \frac{\text{مقابل}}{\text{مجاور}} = \frac{0.28}{0.77 - z_{pid}}$$

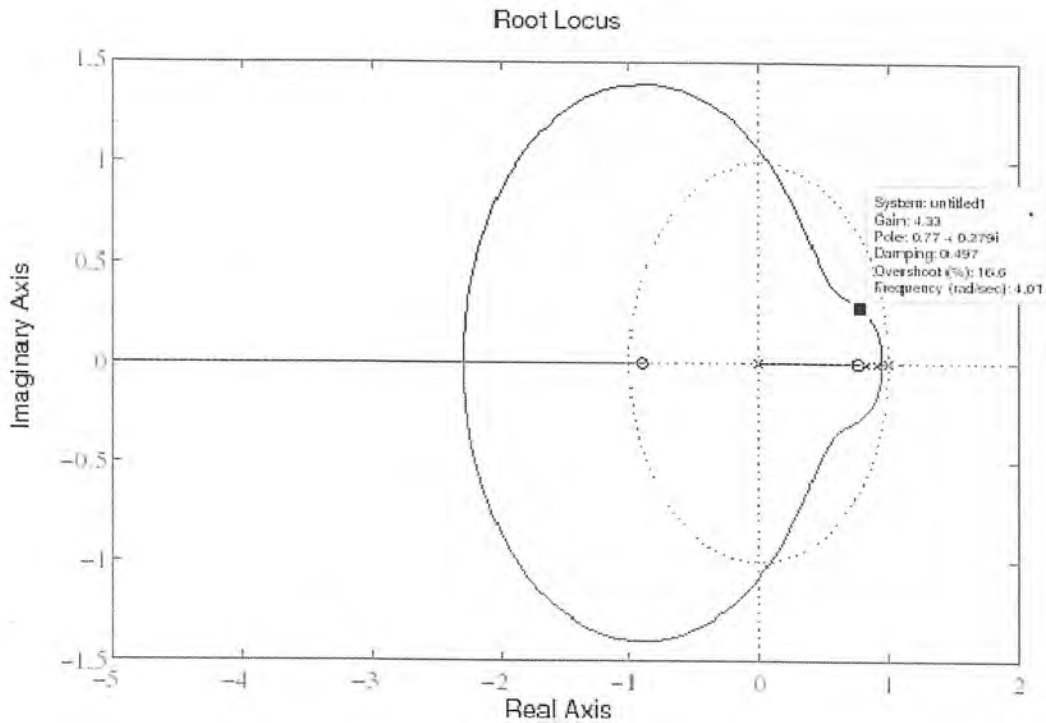


Figure 7: Root locus of compensated system

Thus the required controller is

$$G_D(z) = 4.33\frac{z^2 - 1.518z + 0.5761}{z(z-1)}$$

. If we compare the above transfer function with the general PID controller, K_p and K_d can be computed as follows.

$$\begin{aligned} K_d/T &= 0.5761 * 4.33 \;\Rightarrow\; K_d = 0.2495 \\ K_p + K_d/T &= 4.33 \;\Rightarrow\; K_p = 1.835 \\ K_iT - K_p - 2K_d/T &= -1.518 * 4.33 \;\Rightarrow\; K_i = 2.521 \end{aligned}$$

Note that the above K_i satisfies the constraint $K_i \geq 1$. One should keep in mind that the design is based on second order dominant pole pair approximation. But, in practice, there will be other poles and zeros of the closed loop system which might not be insignificant compared to the desired poles. Thus the actual overshoot of the system may differ from the designed one.

6.4 Useful MATLAB statements in design

Generally design specifications in S-domain are:-

1-Max.overshot (Mp) from this find (Zeta).

2-Settling time(Ts) from this find(sigma) or(Wn).

Example:- If T=1sec

Specifications

- $\zeta < 0.5$ (approx. 20 percent overshoot to a step input)
- $t_{settling} < 10$ sec

First calculate *s*-plane pole requirements (based on formulas for a second order continuous system)

Sigma=4/tsetting=0.4

Wn=sigma/zeta=0.8

Then transform these requirements to the *z*-plane and plot them

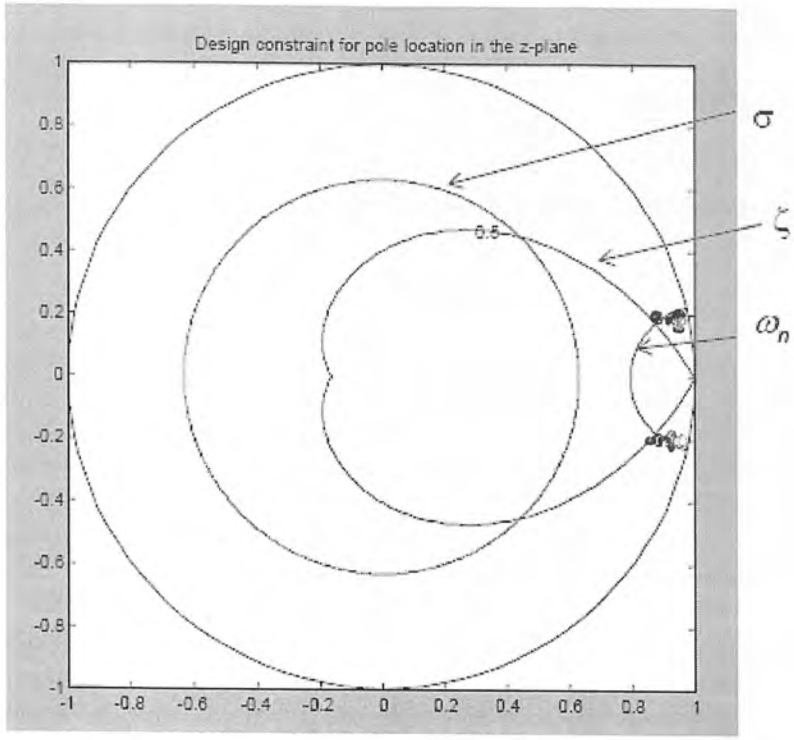
```
zeta = 0.5;

tset= 10; % settling time

T = 1;

sigma = 4/tset

wn= sigma/zeta
```



radius= exp(-sigma*T)

sigma = 0.4(Ans.) → $e^{-\sigma T} = e^{-0.4*1} = 0.67$ ←

wn = 0.8(Ans)

radius =0.67(Ans)

zgrid([zeta],[wn/T])

hold on

uc= exp(j*[0:0.01:2*pi]);

axis('equal')

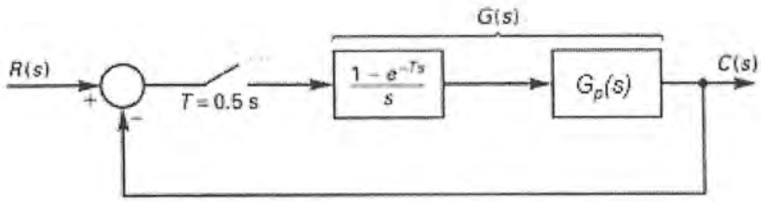
plot(radius*uc)

Example23

Root locus design

Consider a second order system with

$$G_p(s) = \frac{1}{s(10s+1)}$$



We sample with $T = 1$ sec

Specifications

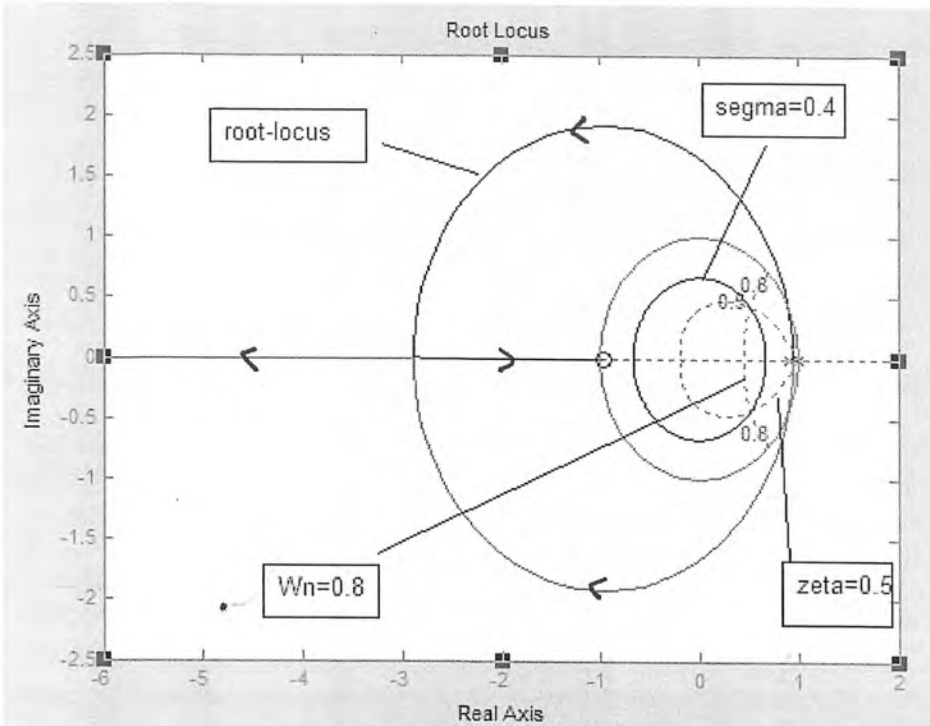
- $\zeta < 0.5$ (approx. 20 percent overshoot to a step input)
- $t_{settling} < 10$ sec

Use the same procedure to calculate (sigma) and (Wn).

```
Plant:
          0.048374 (z+0.9672)
G(z)  =  -------------------
           (z-1) (z-0.9048)
```

1- 2-poles are z=1 and z= 0.9048
2- 2-zeros are z=-0.9672 and z=∞
3- No. of asymp.=n-m=2-1=1
4- Angle of asymp.=180°.
5- Use equ. Of break away=0.948 and in=-2.86
6- Use jury test to find Kcr=2

Find the intersection with j-axis=j1.66



We never get the root locus into the target area. Compensation is needed! Let's try a lead controller

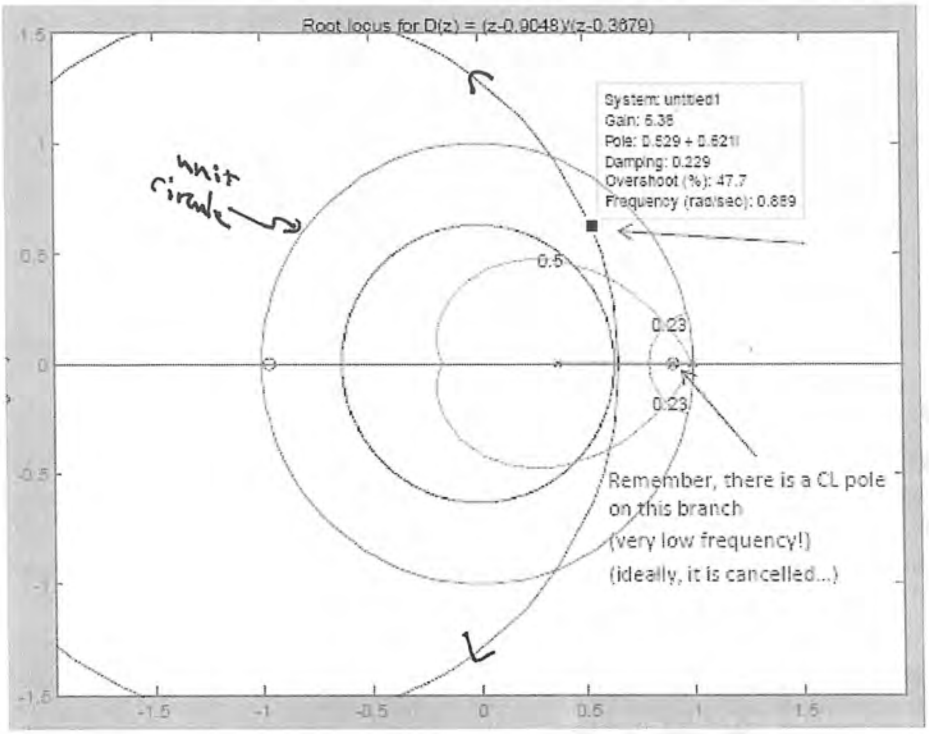
Plant:

```
         0.048374 (z+0.9672)
G(z) = -------------------
          (z-1) (z-0.9048)
```

Controller:

```
         z - 0.9048
D(z) = ----------
         z - 0.3679
```

Be careful with zero/pole cancellation!



Still the design is bad because (Mp=47.7%) and not 20%

Plant:

```
         0.048374 (z+0.9672)
G(z) = -------------------
          (z-1) (z-0.9048)
```

Controller:

```
         z - 0.80
D(z) = --------
         z - 0.05
```