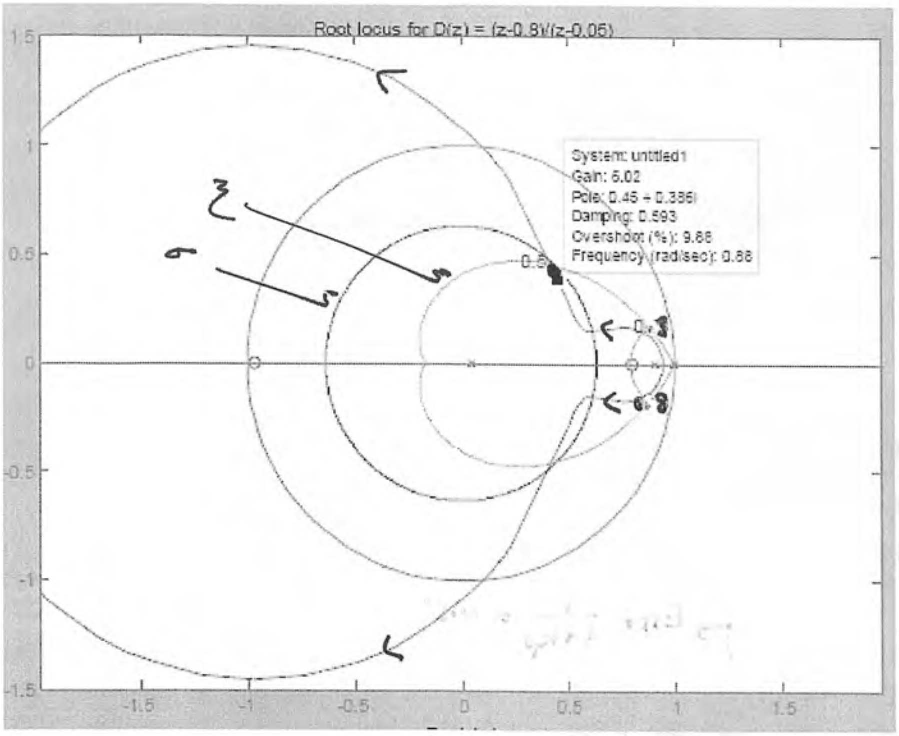


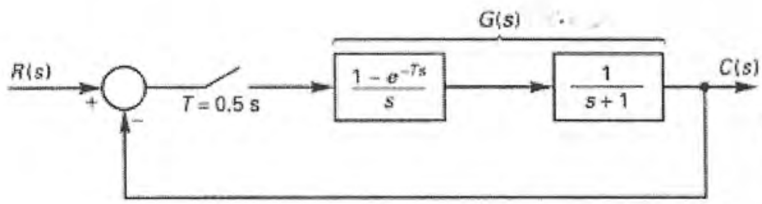
We see the R-L pass through the design specifications (zeta=0.5) and (sigma=0.4), also Mp=9.88% less than 20%.

Example24

Root locus design

Consider a first-order system with

$$G(z) = \frac{0.393}{z - 0.607} \qquad T = 0.5\text{s}$$

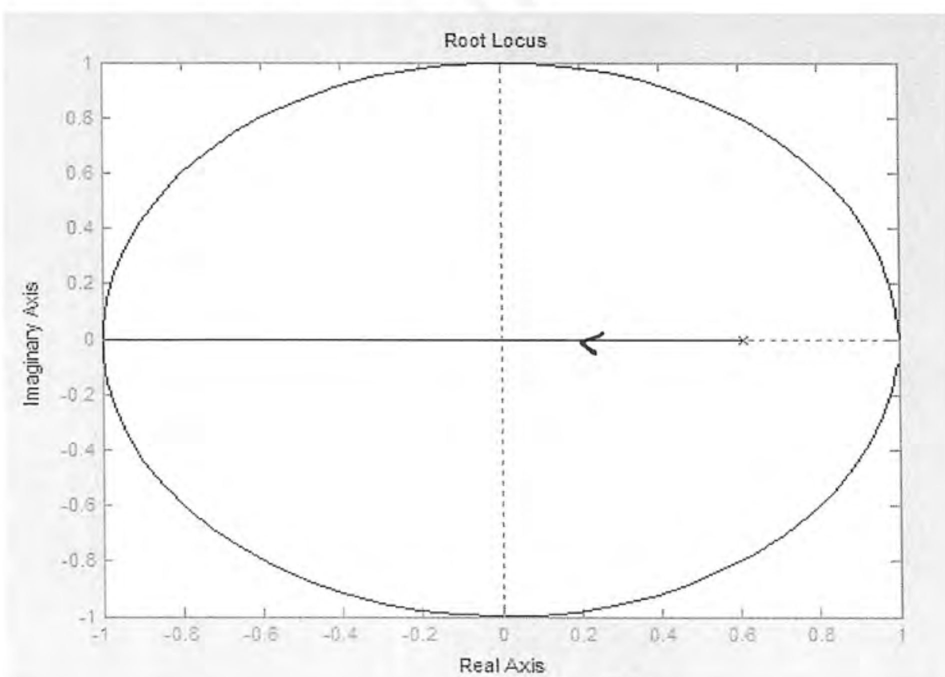


$\rightarrow E_{ss} = \frac{1}{1+K_p} = 0.5$

The steady-state error constant is $K_p = G(z)|_{z=1} = 1$, and we want to increase the system to type 1 by adding a integrator in the controller.

This can be done by a PI controller, which adds a pole at $z = 1$ (and then $K_p = \infty$):

R-L before controller design.



R-L after controller design

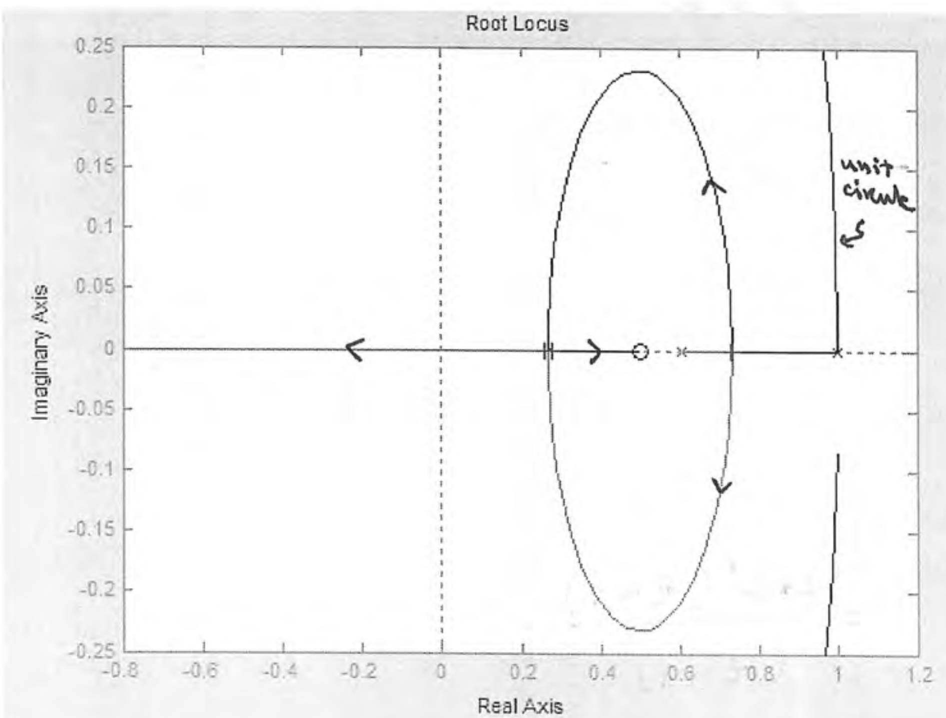
$$D(z) = K_P + \frac{K_I T z}{z-1} = \frac{(K_P + K_I T)z - K_P}{z-1} = \frac{\bar{K}(z - z_0)}{z-1}$$

$$\bar{K} = K_P + K_I T \qquad z_0 = \frac{K_P}{K_P + K_I T} = \frac{K_P}{\bar{K}}$$

The open-loop function of the compensated system is then

$$D(z)G(z) = \frac{0.393\bar{K}(z - z_0)}{(z-1)(z-0.607)}$$

```
        2 (z-0.5)
D(z) = ---------
         (z-1)
```



The system after design became type one, so that Ess=0.

Example25

Let the system T.F is:-

$$\frac{\theta(s)}{\tau(s)} = \frac{1}{Js^2} \implies \frac{1}{s^2} \text{ (unit inertia)}$$

Let

Let C.L.natutal frequency=100Hz and zeta=0.707

↳ $w_n = 2\pi f_n = 628$ rad/sec.

frequency of f_s=500 Hz (T=0.002).

This corresponds to desired s-plane and z-plane pole locations of

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \approx -444 \pm j444.$$
$$z = e^{sT} = 0.2596 \mp j0.3192 \approx 0.26 \mp j0.32$$

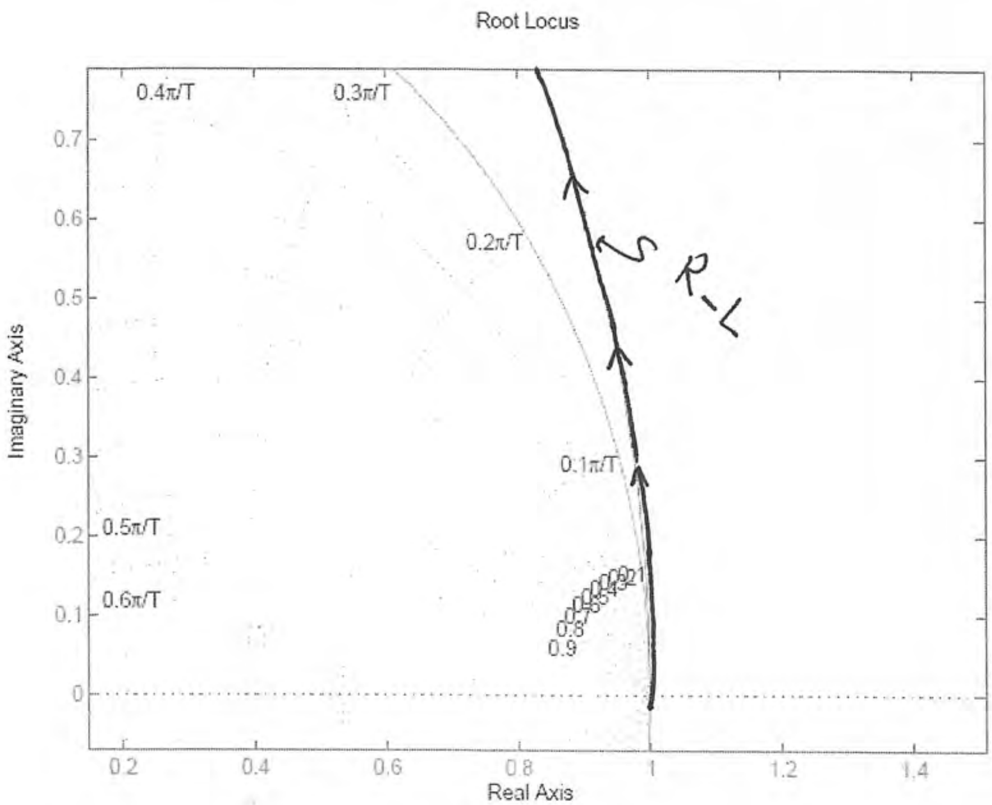
```
>> Gc = tf([0 0 1],[1 0 0])

>> Gd = c2d(Gc,T)

Transfer function:
2e-06 z + 2e-06
---------------
 z^2 - 2 z + 1
```

$= \dfrac{2*10^{-6}(z+1)}{(z-1)^2}$

Compensator Design.

The system before controller(compensator) is unstable, because double poles at z=1. With (P) controller only.

```
>> rlocus(Gd); axis equal; grid;
```



The locus (black line) starts from the double pole at $z = 1$ and goes outside the unit circle—this system is unstable.

Lead Compensator.

We need to improve the stability of this system. A lead compensator (a quasi-differentiator) will do that. This can be seen from two perspectives:

- A lead network will pull the locus to the left, thus keeping it inside the unit circle
- A lead network adds positive phase angle, thereby improving the phase margin

The structure of a lead compensator $D(z)$ is

$$D(z) = \frac{z+b}{z+a}$$

[where the zero is closer to the +1 point than the pole.] ⇐ Important Note

Compensator Pole/Zero Locations.

The philosophy on lead compensator design is to place the compensator zero in the s-plane at about one-third the distance of the desired dominant poles. Thus if the desired poles are at $-445 \pm j445$ I'd place the compensator zero at about $-445/3 \approx -150$. Now in the z-plane this zero location would be

$$z = e^{sT} = e^{(-150)(0.002)} \approx 0.74 \implies D(z) = \frac{z - 0.74}{z + a} \quad (5)$$

The location of the compensator pole is found using the root-locus *angle condition*, in which we use open-loop transfer function $D(z)G(z)$:

$$\Sigma(\text{angles of vectors from poles to desired pole}) - \Sigma(\text{angles of vectors from zeros to desired pole}) = \pm 180^\circ$$

The relevant values in this problem are:

- The open-loop poles: -1, -1, and the unknown lead compensator pole
- The open-loop zeros: 0.74, -1
- The desired closed-loop pole: 0.26+j0.32

I suggest you draw the z-plane and draw these vectors. Here are the values that I found:

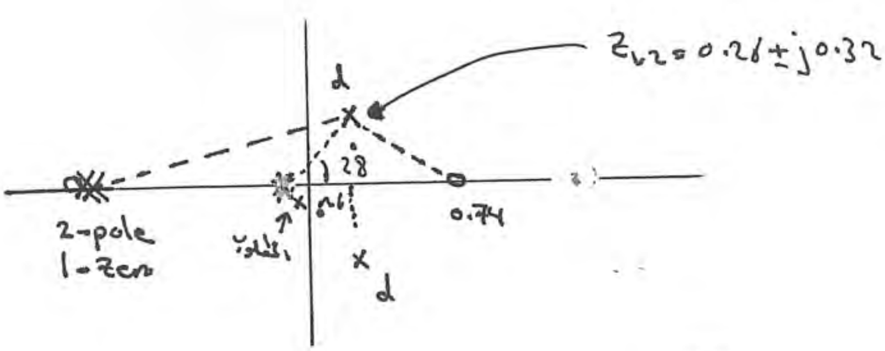
$$2(156^\circ) + \theta - 146^\circ - 14^\circ = \pm 180^\circ \quad (6)$$

Parameter θ in (6) is the angle of the vector from the lead compensator pole to the desired closed-loop pole location. From (6) this angle is

$$\theta = 28^\circ$$

This implies that the pole must be located at $z = -0.354$, thus the lead compensator is

$$D(z) = \frac{z - 0.74}{z - 0.35}$$



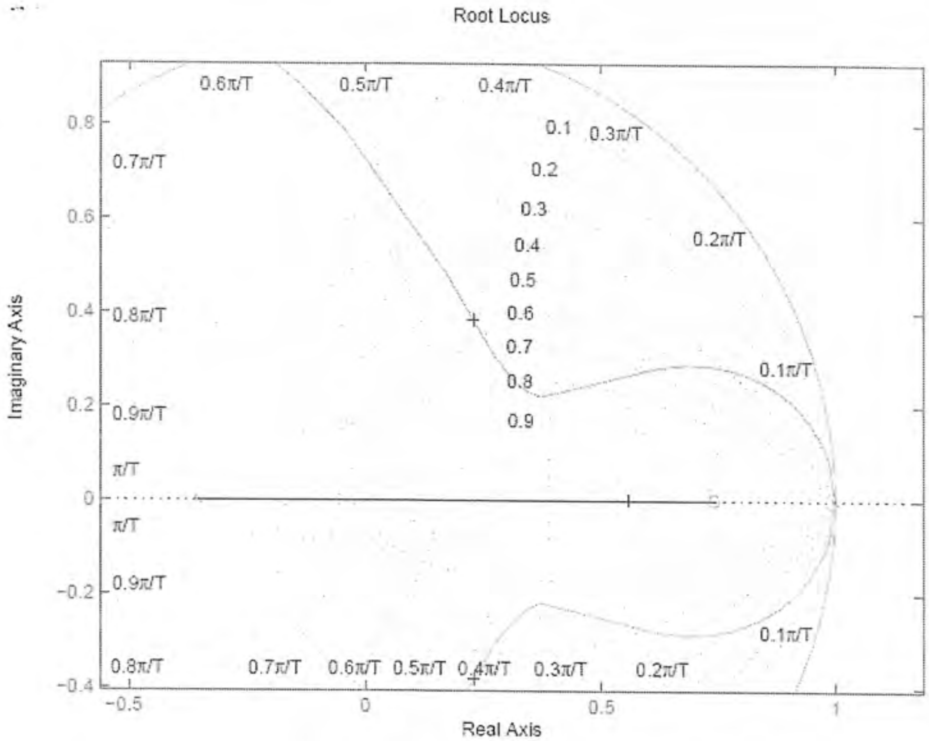
$$\tan 28^\circ = \frac{0.32}{X + 0.26}$$

$$\therefore X \approx -0.35$$

The root-locus diagram of $D(z)G(z)$ is shown on the r... ; the "cross" shows where I specified the desired pole (near $\zeta = 0.6$).

The corresponding K at this point is:

$$K = 3.14e5 = 314,000$$



$$\frac{Y(z)}{R(z)} = \frac{KD(z)G(z)}{1 + KD(z)G(z)}$$

Using MATLAB, this is

```
>> Wd = feedback(K*Dd*Gd,1)

Transfer function:
  0.6287 z^2 + 0.1635 z - 0.4652
-----------------------------------
z^3 - 1.017 z^2 + 0.4555 z - 0.1112

Sampling time: 0.002
```

Since this is a Type II system, it should definitely have a DC gain of 1.00; from MATLAB we get

```
>> dcgain(Wd)

ans = 1
```

H.W:-

Draw the R-L(by using all rules) for the followings:-

1- **G(s)=K/s(s+1) and T=0.1sec (without using ZOH).**
2- **G(s)=100K/(s^2+100) and T=0.05sec.**
3- **G(z)=(z+1)/(z-1)(z-0.5)**
4- **G(z)=K(z-0.2)/(z^2-1.5z+0.5), if zeta=0.6 and Wn=0.6,calculate C.L.poles and (K) at this pole.**
5- **G(z)=K(0.42612z+0.3608)/(z^2-1.6065z+0.6065)**
6- **G(z)=K(z+0.2)/z(z-1)**
7- **G(z)=Kz/(z-1.5)(z-0.5), and find the range of (K) for stable system.**
8- **G(s)=0.1/s(s+0.1) and T=1sec.**
9- **Repeat Ex8 , if T=2sec.**
10- **G(z)=(0.001z+0.001)/(z^2-1.9425z+0.9425)**
11- **G(s)=3086245930.9988/s(s+1.454*10^6)(s+59.23), hint you can canceling any pole or zero which is very close to =0. Let T=0.001sec.**
12- **G(z)=K(z+0.995)/(z-1)(z-0.905).**
13- **G(s)=10/s(0.2s+1), if T=0.2sec**

Example

In this example shows, how to build a control algorithm?

- The continuous system has a transfer function G(s) = 1/(s + 1).
- The sampling period is a half second, i.e. Ts = 0.5 sec.
- In this cas we will use integral control.

6.4 Control Algorithms

If we are trying to implement something like **integral control**, for example, we might use code like the following:

```
// I = Integral
I = I + Ts*Error;
Control Signal = Ki*I;
```