Example

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -0.5 & 1.5 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) \qquad T = 0.2s$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(k)$$

For the state space system above, design a full state feedback controller such that the closed loop system has equivalent s-plane poles at.

$$s_{CL} = -\sigma \pm j\omega = -2 \pm j2$$

Also draw a block diagram of the closed loop system.

Now,

$$z_{CL} = e^{s_{CL}T} = e^{-\sigma T}[\cos(\omega T) \pm j\sin(\omega T)] = 0.62 \pm j0.26$$

=> $$CE_{cl}(z) = (z-0.62)^2 + (0.26)^2 = z^2 - 1.24z + 0.45$$

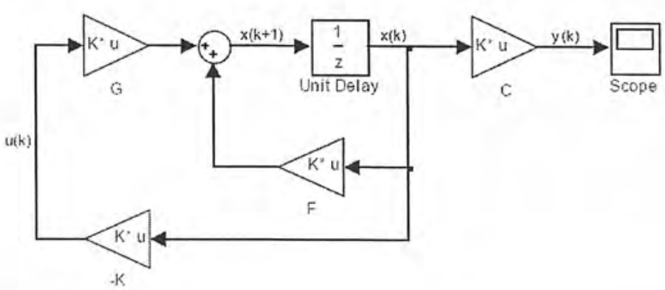
So,

$$CE_{cl}(z) = |z\mathbf{I} - \overset{A}{\mathbf{F}} + \overset{B}{\mathbf{g}}\mathbf{k}|$$

$$= \det\left(\begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -0.5 & 1.5 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \right) = \det\left(\begin{bmatrix} z & -1 \\ 0.5 + k_1 & z - 1.5 + k_2 \end{bmatrix} \right)$$

$$= z^2 + (-1.5 + k_2)z + (0.5 + k_1)$$

=> $$\mathbf{k} = \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} -0.05 & 0.26 \end{bmatrix}$$



In general for 3rd control system

If our state space system is in control canonical form then the full state feedback calculations are simplified,

$$\overset{A}{\mathbf{F}_c} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_3 & -\alpha_2 & -\alpha_1 \end{bmatrix} \qquad \overset{B}{\mathbf{g}_c} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \mathbf{k}_c = \begin{bmatrix} k_{c1} & k_{c2} & k_{c3} \end{bmatrix}$$

$A_f \qquad A - BK$

$$\Rightarrow \qquad \mathbf{F}_{cl} = \mathbf{F}_c - \mathbf{g}_c \mathbf{k}_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(\alpha_3 + k_{c1}) & -(\alpha_2 + k_{c2}) & -(\alpha_1 + k_{c3}) \end{bmatrix}$$

which corresponds to a characteristic equation of,

$$CE_{cl}(z) = z^3 + (\alpha_1 + k_{c3})z^2 + (\alpha_2 + k_{c2})z + (\alpha_3 + k_{c1}) = z^3 + a_1 z^2 + a_2 z + a_3$$

$$\Rightarrow \qquad k_{c1} = a_3 - \alpha_3, \quad k_{c2} = a_2 - \alpha_2, \quad k_{c3} = a_1 - \alpha_1$$

Notice that in the previous example the state space system was in control canonical form and thus the above shortcut could have been used. Verify the feedback gains using the shortcut for yourself (be careful with the signs!).

MATLAB: *acker.m* function works by converting any state space model to control canonical form, calculating the feedback gain matrix and then converting the gain back so that it is applicable to the original state vector. The *place.m* function is more complicated and it works for systems with multiple inputs too ! It uses the extra degrees of freedom provided by these inputs to not only place the eigenvalues of the closed loop system, but to also 'shape' the eigenvectors such that the closed loop system is 'well conditioned'.

The *acker.m* function is suitable for low order (n < 10) systems and can handle repeated pole locations. The *place.m* function is better for high order systems but it can't handle repeated poles.

```
>> k = acker(F,g,poles)
>> k = place(F,g,poles)
```

Example

Consider the continuous state space system below.

$$\underline{\dot{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}(t)$$

Design a discrete controller using state space methods such that the closed loop system has unity DC gain, an overshoot of 5% and a rise time of approximately 1.8 sec.

Use a sample period T of 0.2 sec.

Sol

3.14

Specs:

$M_p = 5\%$ => $\zeta = 0.7$ $t_r = \frac{\pi - \beta}{\omega_d}$

$t_r = 1.8s$ => $\omega_n = 1 \ \ rad/s$ $\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$

$$s_{CL} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -0.7 \pm j0.7$$ $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$z_{CL} = e^{s_{CL}T} = 0.86 \pm j0.12$$

which gives the discrete equivalent system, using the MATLAB program like p250 or p252

$$\mathbf{x}(k+1) = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 0.02 \\ 0.2 \end{bmatrix}u(k) \qquad T = 0.2s$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}(k)$$

Control:

$$CE_{cl}(z) = (z - 0.86)^2 + (0.12)^2 = z^2 - 1.72z + 0.754$$

$$CE_{cl}(z) = |\, z\mathbf{I} - \underset{A}{\mathbf{F}} + \underset{B}{\mathbf{g}}\mathbf{k} \,|$$

=> $\mathbf{k} = \begin{bmatrix} 0.85 & 1.31 \end{bmatrix}$

7.9 Some MATLAB programs

$$H(z) = \frac{2z+1}{z^2+3z+2}$$

with sampling time 0.4

```
>>num = [2 1];
>>den = [1 3 2];
>>Ts=0.4;
>>H=tf(num,den,Ts)
```

G

Matlab Output

```
Transfer function:
   2 z + 1
-------------
z^2 + 3 z + 2

Sampling time: 0.4
```

Function: Use **zpk** function to create transfer function of following form:

Example: $H(z) = 2\dfrac{z+0.5}{(z+1)(z+2)}$

with sampling time 0.4

```
>>num = [-0.5];
>>den = [-1 -2];
>>k = 2;
>>Ts=0.4;
>>H=zpk(num,den,k,Ts)
```

G

Matlab Output

```
Zero/pole/gain:
 2 (z+0.5)
-----------
(z+1) (z+2)

Sampling time: 0.4
```

Function: Use **ss** function creates state space models. For example:

$$\mathbf{x}[n]=\begin{bmatrix} x_1[n] \\ x_2[n] \end{bmatrix} \quad \mathbf{A}=\begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \quad \mathbf{B}=\begin{bmatrix} 0 \\ 3 \end{bmatrix} \quad \mathbf{C}=\begin{bmatrix} 0 & 1 \end{bmatrix} \quad \mathbf{D}=\begin{bmatrix} 0 \end{bmatrix}$$

```
>>A = [0 1;-5 -2];
>>B = [0;3];
>>C = [0 1];
>>D = [0];
>>Ts= [0.4];
>>sys=ss(A,B,C,D,Ts)
```

Matlab Output

```
a =
        x1  x2
    x1   0   1
    x2  -5  -2


b =
        u1
    x1   0
    x2   3


c =
        x1  x2
    y1   0   1

d =
        u1
    y1   0
```

Example *Given is the process*

$$\begin{cases} x_{k+1} &= \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \cdot x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot u_k \\ y_k &= \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot x + [0] \cdot u_k \end{cases}$$

The task consists in designing a controller without the integral part in order to obtain all poles at 0.5. The characteristic polynomial is

$$d(z) = |zI - \Phi| = \begin{vmatrix} z & -1 \\ k_1 & z - 1 - k_2 \end{vmatrix} = z^2 + z \cdot (-1 - k_2) + k_1$$

when $\Phi = A - BK$

which must be compared with $z^2 - z + 0.25$ yielding $k_1 = 0.25$ and $k_2 = 0$.

$\rightarrow (z - 0.5)(z - 0.5)$

Example *With the same system above, the state-feedback controller with integral gain has to be designed. Then*

$A_f = A - BK$

$T_s = 1 \sec$

$$\begin{cases} x_{k+1} &= \begin{bmatrix} 0 & 1 & 0 \\ -k_1 & 1 - k_2 & -K_e \\ -1 & 0 & T_s \end{bmatrix} \cdot x_k + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot r_k \\ y_k &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot x_k \end{cases}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & T_s \end{bmatrix}$$

The polynomial with all poles at 0.5 is $\Rightarrow (z - 0.5)(z - 0.5)(z - 0.5)$

$$z^3 - 1.5 \cdot z^2 + 0.75 \cdot z - 0.125$$

while the polynomial of the controlled system is $\Rightarrow \det|zI - A_f|$

$$z^3 + (-1 + k_2 - Ts) \cdot z^2 + (k_1 - k_2 \cdot T_s + T_s) \cdot z - K_e - k_1 \cdot T_s$$

With a coefficient comparison the state-feedback vector is

$$k_1 = 0.25, \quad k_2 = 0.5, \quad K_e = -0.125$$

and the controlled system

$$\begin{cases} x_{k+1} &= \begin{bmatrix} 0 & 1 & 0 \\ -0.25 & 0.5 & 0.125 \\ -1 & 0 & T_s \end{bmatrix} \cdot x_k + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot r_k \\ y_k &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot x_k \end{cases}$$

See p253 + 254

Example : Find out the state feedback gain matrix K for the following system using two different methods such that the closed loop poles are located at 0.5 , 0.6 and 0.7.

$$\mathbf{x}(k+1)\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\-1 & -2 & -3\end{bmatrix}\mathbf{x}(k)+\begin{bmatrix}0\\0\\1\end{bmatrix}u(k)$$

The above matrix has rank 3, so the system is controllable

Open loop characteristic equation:

$$z^3 + \underset{a_1}{3z^2} + \underset{a_2}{2z} + \underset{a_3}{1} = 0$$

Desired characteristic equation:

$$\begin{aligned}(z-0.5)(z-0.6)(z-0.7) &= 0\\ \text{or, } z^3 - \underset{\alpha_1}{1.8z^2} + \underset{\alpha_2}{1.07z} - \underset{\alpha_3}{0.21} &= 0\end{aligned}$$

Since the open loop system is already in controllable canonical form, T =1.

$$K = [\alpha_3 - a_3 \;\; \alpha_2 - a_2 \;\; \alpha_1 - a_1]$$

$$\alpha_3 = -0.21,\ \alpha_2 = 1.07,\ \alpha_1 = -1.8$$

and

$$a_3 = 1,\ a_2 = 2,\ a_1 = 3$$

Thus

$$K = [-1.21 \quad -0.93 \quad -4.8]$$

Using Ackermann's formula:

Or equ(·29)

We get the same result

Example

Consider the following discrete-time system in state-space form:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u(k).$$

Use state feedback to relocate all of the system's poles to 0.5.

Solution:

The characteristic equation of the original (open-loop) system is :

$$\det(zI - \Phi) = \det \begin{bmatrix} z & -1 \\ 0 & z+1 \end{bmatrix} = z^2 + z = 0$$

Since desired eigenvalues are both at 0.5, the desired characteristic equation is :

$$\alpha_d(z) = (z - 0.5)^2 = z^2 - z + 0.25$$

With

Control Law

$$u = -\mathrm{Kx} = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and

A B

$$\begin{aligned}\Phi_d &= \Phi - \Gamma K \\ &= \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 10 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 10k_1 & 10k_2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 \\ -10k_1 & -1-10k_2 \end{bmatrix}\end{aligned}$$

the characteristic equation of the closed-loop system is :

$$\begin{aligned}\det(z\mathbf{I} - \Phi_d) &= \det \begin{bmatrix} z & -1 \\ 10k_1 & z+1+10k_2 \end{bmatrix} \\ &= z^2 + (1+10k_2)z + 10k_1 \\ &= \alpha_c(z)\end{aligned}$$

Matching each coefficient in z in $\alpha_c(z)$ with those in $\alpha_d(z)$ yields :

$$\begin{aligned}k_1 &= 0.025 \\ k_2 &= -0.2\end{aligned}$$

Therefore, the control law is :

$$u(k) = -\begin{bmatrix} 0.025 & -0.2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

A B

Alternatively, the same answer can be obtained by using acker in Matlab. For example, K=acker(F,G,[.5;.5]) will produce the same result. It is interesting, however, that place does not work for this case of repeated roots.

Example

$$\ddot{x} = 1000x + 20i.$$

Let the sampling time be 0.01 sec.

(a) Use pole placement to design a controller for the magnetic levitator so that the closed-loop system meets the following specifications: settling time, $t_s \leq 0.25$ sec, and overshoot to an initial offset in x that is less than 20%.

(a) State-space representation of the plant :

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \; y = x, \; u = i$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1000 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 20 \end{bmatrix} u$$

$$= \mathbf{F}\mathbf{x} + \mathbf{G}u$$

A B

when $X = X_1$

$\therefore \dot{X}_1 = \dot{X} = X_2$

$\dot{X}_2 = \ddot{X} = 1000X + 20U$

$\hookrightarrow X_1$

while $y = X = X_1$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$y = x = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{H}\mathbf{x}$$

$\hookrightarrow C$

Discrete state-space equation ($T = 0.01$ sec) : $\Rightarrow$ using the MATLAB program like P250 or P252 method

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1.0504 & 0.0102 \\ 10.168 & 1.0504 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.0010 \\ 0.2034 \end{bmatrix} u(k)$$

$$y(k) = x(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

The specifications : using 1% criterion $\Rightarrow 0.25 = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\sigma}$

$$t_s < 0.25 \Longrightarrow \sigma > \frac{4.6}{0.25} = 18.4 \Longrightarrow r = |z| < e^{-18.4\times0.01} = 0.832$$

$$M_p < 20\% \Longrightarrow \zeta > 0.48$$

$s_{1,2} = 18.4 \pm j38.333 \Rightarrow z_{1,2} = e^{Ts_{1,2}} \Rightarrow R\angle\theta$

$0.832\angle 21.9^\circ$

$\therefore a \simeq 0.75$

$b \simeq 0.311$

select controller poles at $z = 0.75 \pm 0.30j$:

$$\alpha_c(z) = (z-0.75-0.30j)(z-0.75+0.30j) = 0 \Longleftrightarrow \det[z\mathbf{I}-\underbrace{\Phi}_{A}+\underbrace{\Gamma}_{B}\mathbf{K}] = 0$$

Control gain is most easily caculated using acker or place and results in:

$$\mathbf{K} = \begin{array}{cc} k_1 & k_2 \\ \end{array}$$
$$\mathbf{K} = \begin{bmatrix} 125.62 & 2.332 \end{bmatrix}$$

To ensure that the estimator roots are substantially faster then the control roots, select estimator poles at $z = 0.14 \pm 0.17j$:

$$\alpha_e(z) = (z-0.14-0.17j)(z-0.14+0.17j) = 0 \Longleftrightarrow \det[z\mathbf{I}-\Phi+\mathbf{L}\mathbf{H}] = 0$$

H.W:- find k1 and k2 in the above case.

Example

$$\frac{\theta(s)}{u(s)} = \frac{1}{s(s/a+1)} = G\ (s).$$

(a) Let $a = 0.1$ and $x_1 = \dot{\theta}$, and write the continuous-time state equations for the system.

(b) Let $T = 1$ sec, and find a state feedback gain $\mathbf{K}$ for the equivalent discrete-time system that yields closed-loop poles corresponding to the following points in the s-plane: $s = -1/2 \pm j\left(\sqrt{\frac{3}{2}}\right)$. Plot the step response of the resulting design.

(a)

$$G_2(s) = \frac{\Theta(s)}{U(s)} = \frac{0.1}{s(s+0.1)} \Rightarrow \ddot{\theta} + 0.1\dot{\theta} = 0.1U$$

$$x_1$$

$$\text{Let } \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \theta \end{bmatrix} \Rightarrow \begin{matrix} X_1 = \dot{\theta} \Rightarrow \dot{X}_1 = \ddot{\theta} = -0.1\dot{\theta} + 0.1U \\ X_2 = \theta \Rightarrow \dot{X}_2 = \dot{\theta} = X_1 \end{matrix}$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -0.1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} u$$

$$= \underset{A}{\mathbf{F}}\mathbf{x} + \underset{B}{\mathbf{G}}u$$

$$y = \theta = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underset{C}{\mathbf{H}}\mathbf{x}$$

$$y = X_2$$

(b) For $T = 1$ sec,

Using MATLAB program like P250 or P252 method

$$A \quad \Phi = \begin{bmatrix} 0.9048 & 0 \\ 0.9516 & 1 \end{bmatrix}$$

$$B \quad \Gamma = \begin{bmatrix} 0.0952 \\ 0.0484 \end{bmatrix}$$

Plant pole : $z = 1.0,\ 0.905$
Plant zero : $z = -0.967$

Controller gain design :
Desired closed-loop at,

$$s = \frac{-1}{2} \pm j\sqrt{\frac{3}{2}} \overset{z=e^{sT}}{\Longrightarrow} z = 0.206 \pm 0.571j$$

Desired characteristic equation :

$$\begin{aligned} \alpha_c(z) &= (z - 0.206 - 0.571j)(z - 0.206 + 0.571j) = 0 \\ \Longleftrightarrow &\ \det[z\mathbf{I} - \Phi + \Gamma\mathbf{K}] = 0 \end{aligned}$$

Control gain (either match coefficients or use acker or place):

$$\mathbf{K} = \begin{bmatrix} 10.58 & 10.05 \end{bmatrix}$$

Example

The transfer function of the system is thus

$$\frac{T_e(s)}{T_{ec}(s)} = \frac{e^{-\tau ds}}{s/a+1} = G_{\cdot}(s).$$

For

Td=1.5, a=1, and T=1sec.

Find G´ (z) and:-

(a) Write the discrete-time system equations in state-space form.

(b) Design a state feedback gain that yields $\alpha_c(z) = z^3$.

Sol

Use modified Z-T and $\because T = 1\,sec$

We assume ΔT=1.5, so that Δ =1.5, and this equal to

$e^{-1} * e^{-0.5} = z^{-1} * e^{-0.5}$ and m=1-0.5=0.5

we get:-

use the procedures of finding the Modified Z-T. [P33]

$$G_{\cdot}(z) = \frac{0.3935(z+0.6065)}{z^2(z-0.3679)} = \frac{}{z^3 - 0.3679z^2 + 0z + 0}$$

Find the controller canonical form:-

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0.3679 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [0.2378 \quad 0.3935 \quad 0]$$

Desired c/cs equ.has all three roots at z=0

$Q(z)= \det\left| zI-A+BK \right| = \alpha_c(z) = z^3$

And the feedback gain are:- [K1 K2 K3]

The students complete the solution.

Example

Suppose you have (zeta=0.5) and (Wn=Pi) and 3rd pole is(z=0.9) , T= 0.1 sec.

$$\begin{aligned} z_{1,2} &= e^{s_{1,2}T} \\ &= e^{(-\sigma \pm j\omega_d)T} \\ &= e^{-\sigma T} e^{\pm j\omega_d T} \\ &= e^{-\sigma T}\left(\cos(\omega_d T) + j\sin(\omega_d T)\right) \end{aligned}$$

Thus, or let :-

$$z_{1,2} = 0.62 \pm j0.38$$

Thus, the characteristic equation is,

$$CE_{cl} = (z-0.9)\left((z-0.62)^2 + (0.38)^2\right) = z^3 - 2.14z^2 + 1.645z - 0.476$$

$$\Downarrow$$

$$(z-0.62+j0.38)(z-0.62-j0.38)$$

let the system with

Now,

$$\det\left(\begin{bmatrix} z & 0 & 0 \\ 0 & z & 0 \\ 0 & 0 & z \end{bmatrix} - \overbrace{\begin{bmatrix} 1 & 0.2 & 0 \\ 0 & 1 & 0 \\ 1 & 0.2 & 1 \end{bmatrix}}^{A} + \overbrace{\begin{bmatrix} 0.2 \\ 2 \\ 0.2 \end{bmatrix}}^{B} \begin{bmatrix} \underset{K_1}{k_{p1}} & \underset{K_2}{k_{p2}} & \underset{K_3}{k_i} \end{bmatrix}\right) = z^3 - 2.14z^2 + 1.645z - 0.476$$

or using MATLAB

```
>> pcl = [0.9, 0.62 + j*0.38, 0.62 – j*0.38];
>> K = acker(F_bar, g_bar, pcl);
```

F_bar → A g_bar → B

This gives a feedback gain of,

$$\bar{\mathbf{k}} = \begin{bmatrix} 0.8037 & 0.3434 & 0.0722 \end{bmatrix}$$

$$\Rightarrow \mathbf{k}_p = \begin{bmatrix} \underset{K_1}{0.8037} & \underset{K_2}{0.3434} \end{bmatrix} \quad \text{and} \quad k_i = \underset{K_3}{0.0722}$$

Example

Consider the following system:

$$\frac{Y(z)}{U(z)} = \frac{z+1}{z^2 + 1.3z + 0.4}$$

The state-space representations in the controllable canonical form, observable canon-

CONTROLLABLE CANONICAL FORM:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.4 & -1.3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

OBSERVABLE CANONICAL FORM:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & -0.4 \\ 1 & -1.3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$