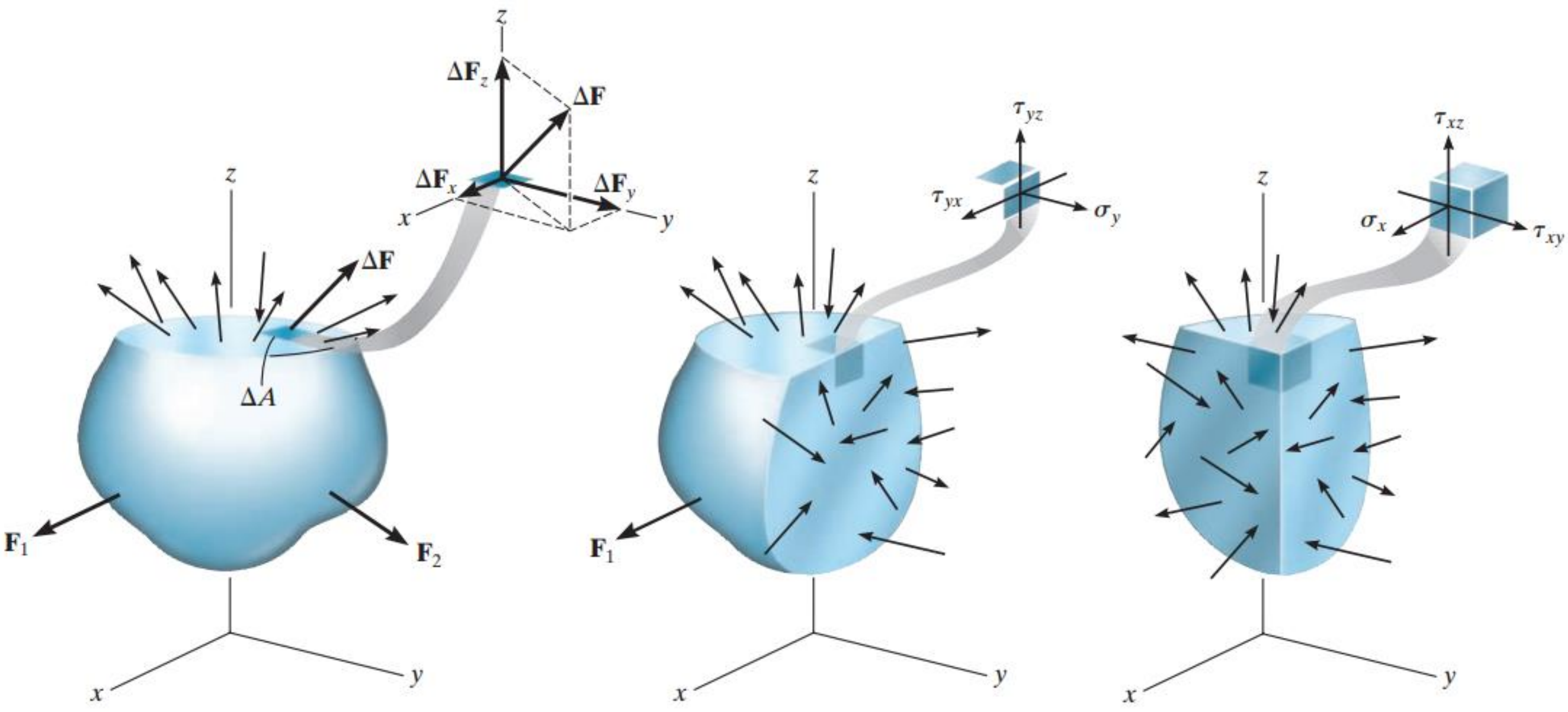
The concept of the stress

We begin by considering the sectioned area to be subdivided into small areas. A typical finite very small force Δ**F** acting on Δ**A** is shown in Figure. This force, like all the others, will have a unique direction and we will replace it by its three components, namely, **ΔFx, ΔFy, and ΔFz** and which are taken tangent, tangent, and normal to the area, respectively. As Δ**A** approaches zero, so do Δ**F** and its components; however, the quotient of the force and area will, in general, approach a finite limit. This quotient is called **stress, and as noted, it describes the intensity of the internal force acting on a specific plane (area) passing through a point.**



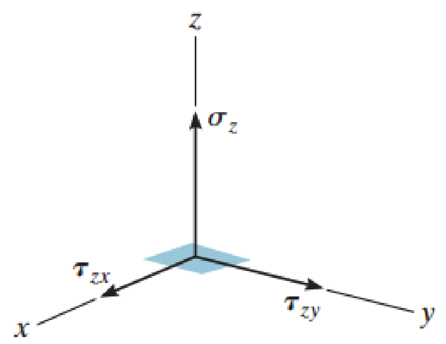
Normal Stress: The intensity of the force acting normal to is defined as the normal stress, σ(sigma). Since is normal to the area then

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

Shear Stress: The intensity of force acting tangent to is called the shear stress, τ(tau). Here we have shear stress components.

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$



Note that in this subscript notation z specifies the orientation of the area ΔA, and x and y indicate the axes along which each shear stress acts.

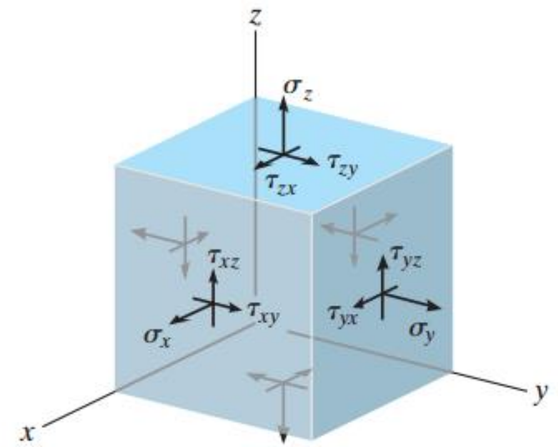
General State of Stress: If the body is further sectioned by planes parallel to the *x–z* plane and the *y–z* plane, we can then "cut out" a cubic volume element of material that represents the **state of stress acting around a chosen point in the body. This state of stress is then characterized by three components acting on each face of the element.**

SI Stress Units:

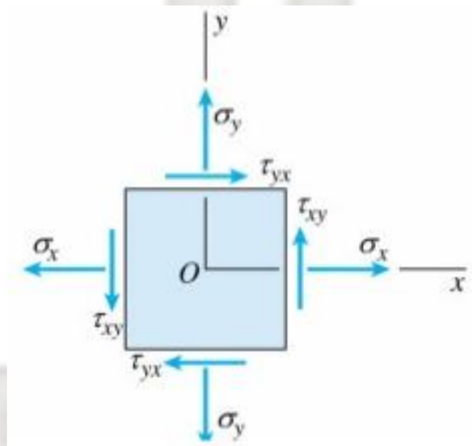
Pascal = $\dfrac{N}{m^2}$

Kilo Pascal = $1000 \times \dfrac{N}{m^2} = \dfrac{kN}{m^2}$

Mega Pascal = $1000000 \times \dfrac{N}{m^2} = \dfrac{N}{mm^2}$



In plane, there are 4 components of stresses; 2 normal stresses and 2 shear stresses, as in the plane element below:



Type of Stresses

1- Normal Stress:

a) Tensile stress

Where:

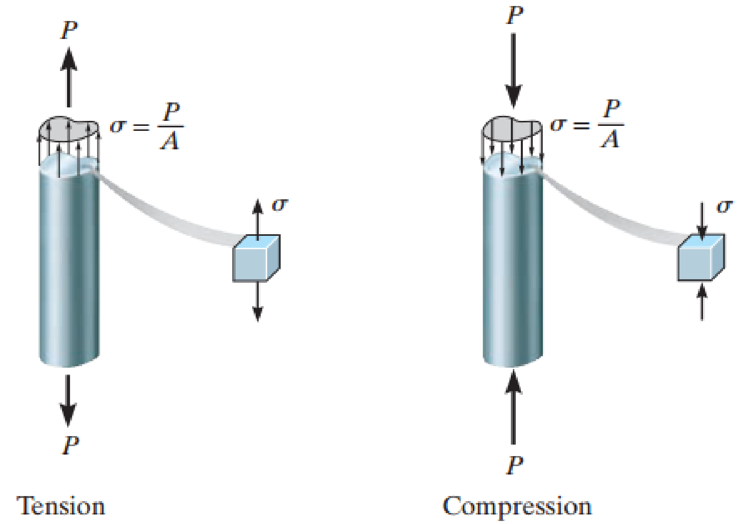
P = axial (passes through the centroid) tensile force

A = cross-sectional area

When the applied force is axial and normal, a uniform (equal) maximum normal stress can be achieved through the section.

b) Compressive stress

c) Bearing stress



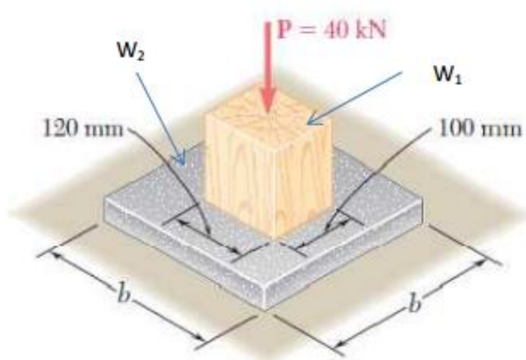
The bearing stress is a normal compressive stress happens **between two surfaces**.

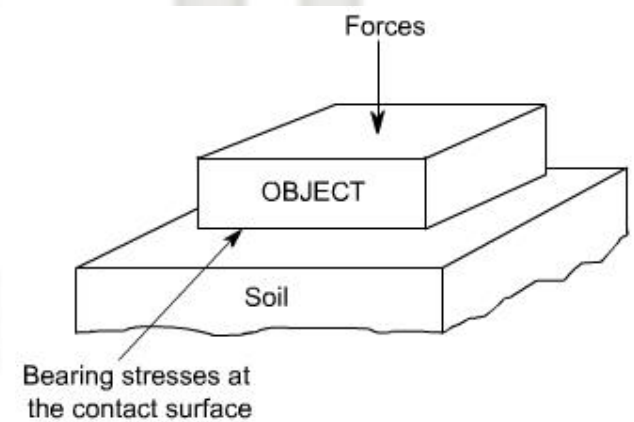
In this example, we have two bearing stresses. First, between the timber block and the steel base, this equals:

$$\sigma_b = \frac{P + W_1}{120 \times 100}$$

Second, between the steel base and the soil, this equals:

$$\sigma_b = \frac{P + W_1 + W_2}{b \times b}$$

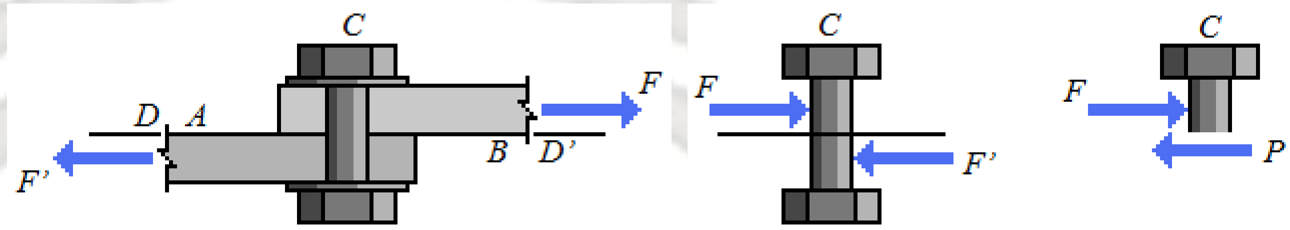




2- Shear Stress

a) Direct Shear stress:

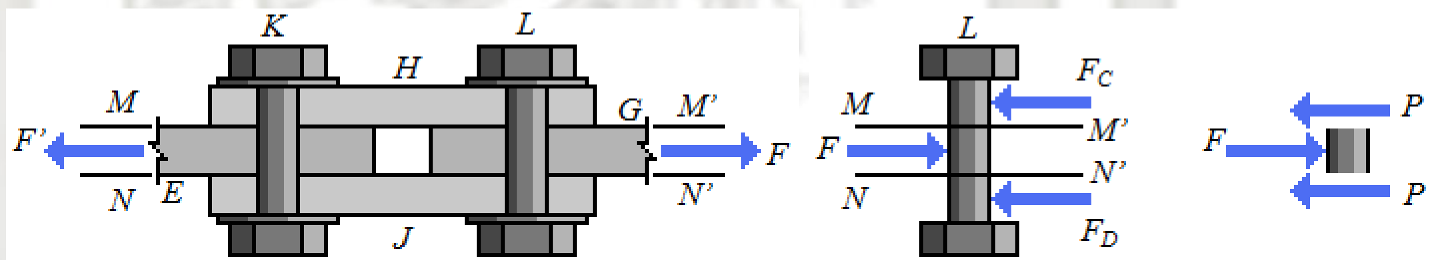
1- Single Shear



If plates A and B are connected by bolt C, shear will take place in bolt C in plane DD'. The bolt is in single shear. Observing that the shear P = F, it can be concluded that the average shearing stress is:

$$\tau_{bolt} = \frac{P}{A} = \frac{F}{A} = \frac{F}{\pi r^2}$$

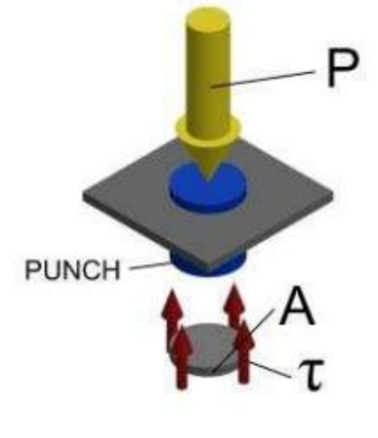
2- Double shear



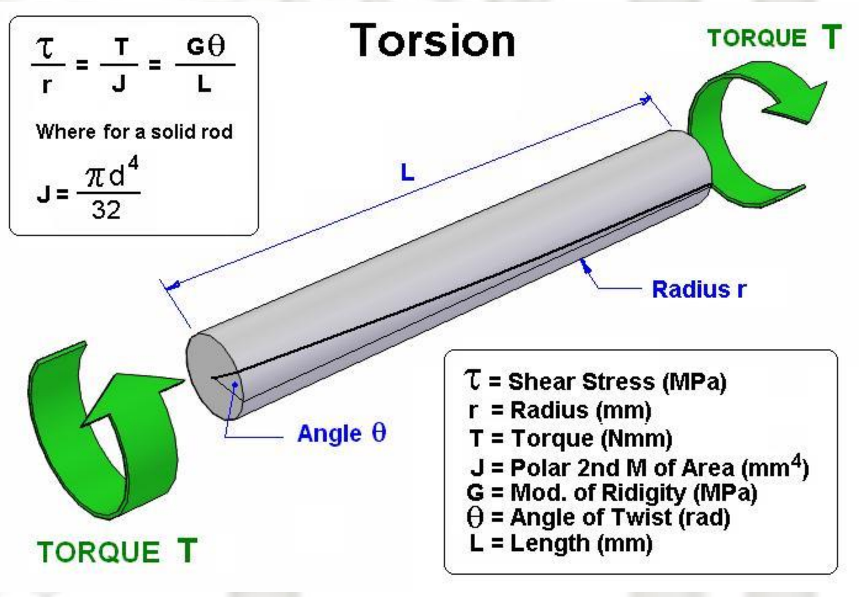
If splice plates H and J are used to connect plates E and G, shear will take place in bolts K and L in each of the two planes MM' and NN'. The bolts are in double shear. Observing that the shear P in each of the sections is P = F/2, it can be concluded that the average shearing stress is:

$$\tau_{bolt} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2\pi r^2}$$

b) Bunching Shear Stress



c) Torsional Shear Stress

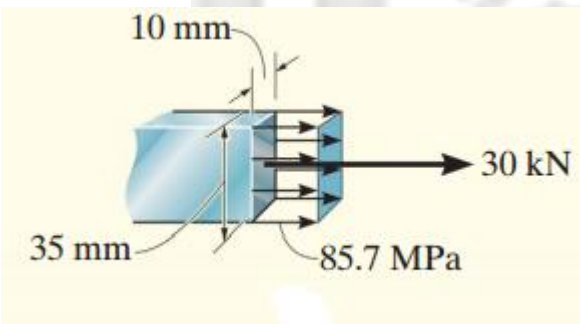


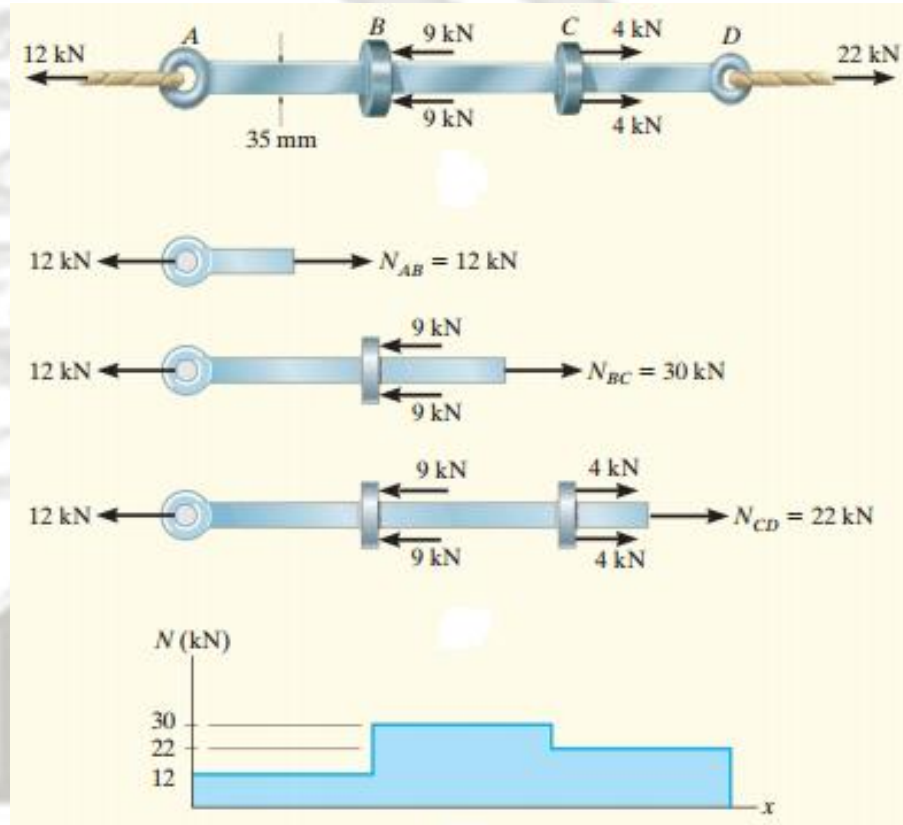
EXAMPLE 1-4

The bar in the Figure has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.

$$\sigma_{BC} = \frac{N_{BC}}{A}$$

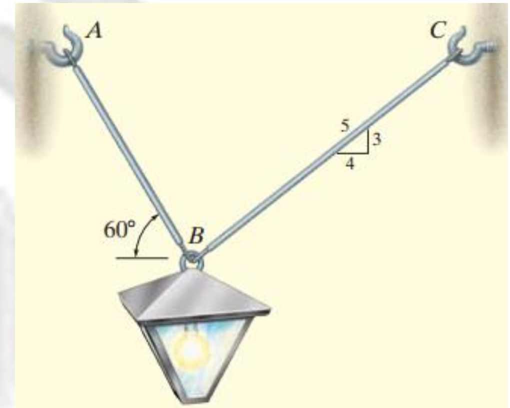
$$\sigma_{BC} = \frac{30 \times 1000}{35 \times 10} = 85.7\,MPa$$





EXAMPLE 1-5

The 80-kg lamp is supported by two rods AB and BC as shown in the Figure. If AB has a diameter of 10 mm and BC has a diameter of 8 mm, determine the average normal stress in each rod.



$$(+) \rightarrow \sum F_x = 0 \rightarrow \left[F_{BC}(\frac{4}{5}) - F_{BA}\cos 60^{\circ} = 0 \right] \times \frac{5}{4} \rightarrow (1)$$

$$(+) \uparrow \sum F_y = 0 \rightarrow \left[F_{BC}(\frac{3}{5}) - F_{BA}\sin 60^{\circ} - 784.8 = 0 \right] \times \frac{5}{3} \rightarrow (2)$$

$$F_{BC} - (\frac{5}{4})F_{BA}\cos 60^{\circ} = 0 \rightarrow (1)$$

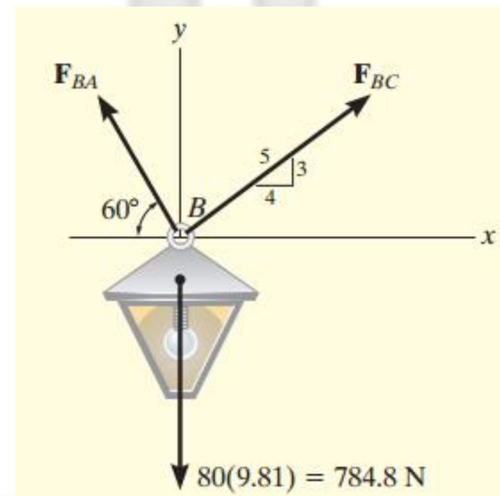
$$F_{BC} - (\frac{5}{3})F_{BA}\sin 60^{\circ} - (\frac{5}{3})784.8 = 0 \rightarrow (2)$$

$$F_{BC} = 395.2N$$

$$F_{BA} = 632.4N$$

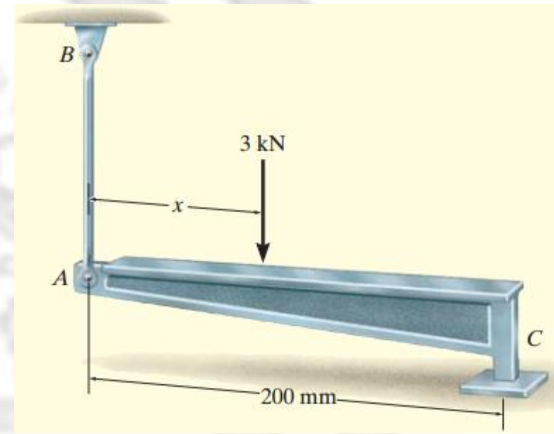
$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2}{\pi \times 4^2} = 7.86MPa$$

$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4}{\pi \times 5^2} = 8.05MPa$$



EXAMPLE 1-6

Member AC shown in the Figure. is subjected to a vertical force of 3 kN. Determine the position x of this force so that the average compressive stress at the smooth support C is equal to the average tensile stress in the tie rod AB. The rod has a cross-sectional area of 400 mm2 and the contact area at C is 650 mm2.



$$(+)\uparrow\sum F_y = 0 \rightarrow F_{AB} + F_C - 3000 = 0 \rightarrow (1)$$

$$(+)\sum M_A = 0 \rightarrow -3000\times x + F_C \times 200 = 0 \rightarrow (2)$$

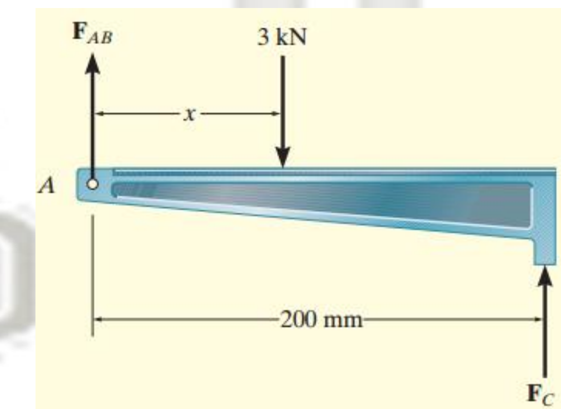
$$\sigma = \frac{F_{AB}}{400} = \frac{F_c}{650}$$

$$F_C = 1.625 F_{AB} \rightarrow (3)$$

$$F_{AB} = 1143 N$$

$$F_C = 1857 N$$

$$x = 124 mm$$



EXAMPLE 1-7

A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m2.

$$P = 400kN = 400000N$$

$$\sigma = 120MPa$$

$$A = \pi \times \frac{D^2}{4} - \pi \times \frac{100^2}{4} = \frac{\pi}{4}(D^2 - 10000)$$

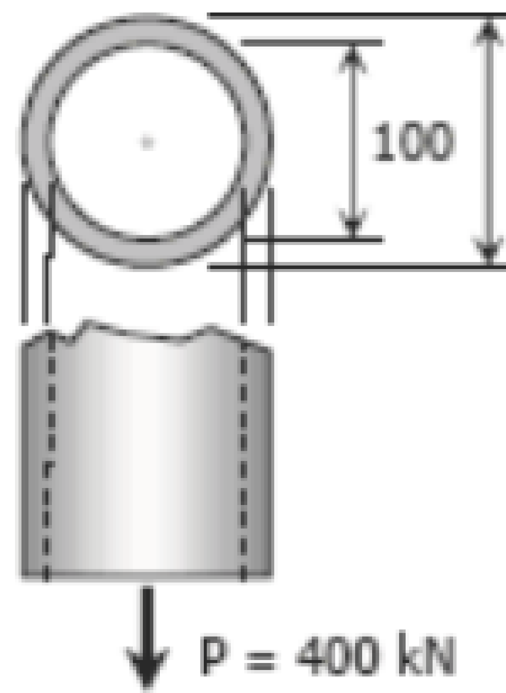
$$P = \sigma \times A$$

$$400000 = 120(\frac{\pi}{4}(D^2 - 10000)$$

$$400000 = 30\pi D^2 - 300000\pi$$

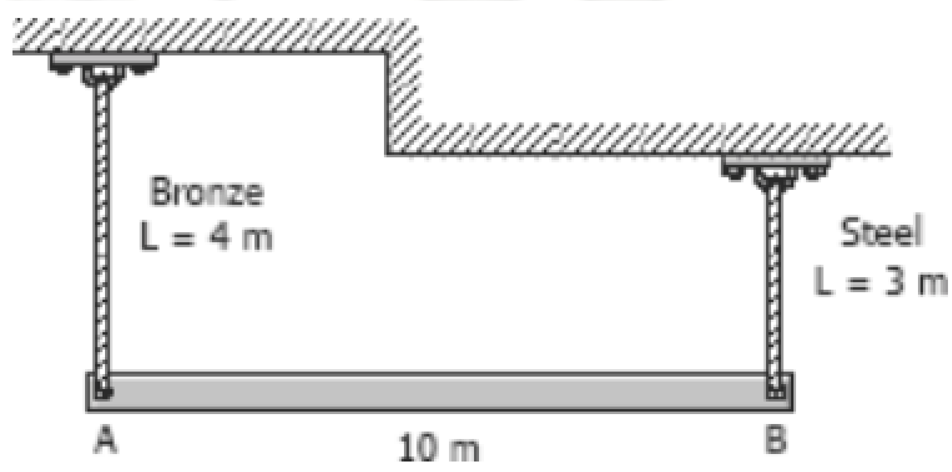
$$D^2 = \frac{400000 + 300000\pi}{30\pi}$$

$$D = 119.35mm$$



EXAMPLE 1-8

A homogeneous 800 kg bar AB is supported at either end by a cable as shown in the Figure. Calculate the smallest area of each cable if the stress is not to exceed 90 MPa in bronze and 120 MPa in steel.



$$W_{bar} = 800 \times 9.81 = 7848N$$

$$P_{bronze} = P_{steel} = \frac{7848}{2} = 3924N$$

$$P_{bronze} = \sigma_{bronze} \times A_{bronze}$$

$$3924 = 90 \times A_{bronze}$$

$$A_{bronze} = 43.6mm^2$$

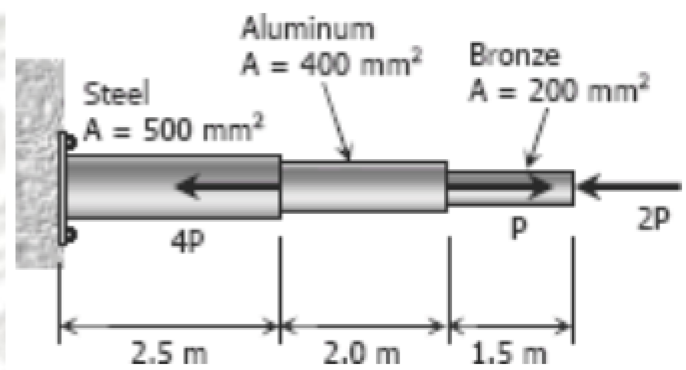
$$P_{steel} = \sigma_{steel} \times A_{steel}$$

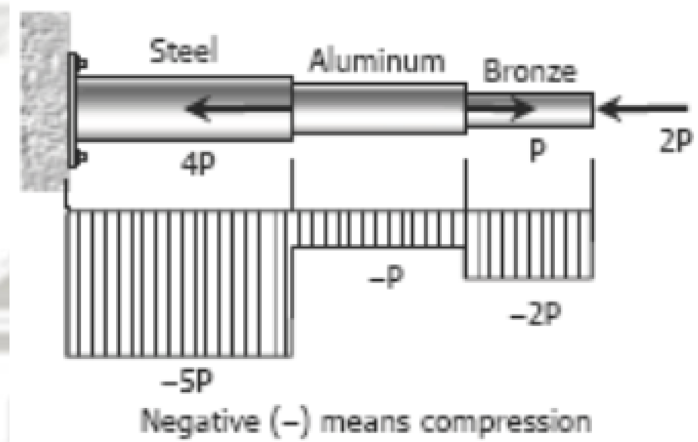
$$3924 = 120 \times A_{steel}$$

$$A_{steel} = 32.7mm^2$$

EXAMPLE 1-9

An aluminum rod is rigidly attached between a steel rod and a bronze rod as shown in the Figure. Axial loads are applied at the positions indicated. Find the maximum value of P that will not exceed a stress in steel of 140 MPa, in aluminum of 90 MPa, or in bronze of 100 MPa.





$$P_{bronze} = \sigma_{bronze} \times A_{bronze}$$

$$2P_{bronze} = 100 \times 200$$

$$P_{bronze} = 10000N$$

$$P_{alum} = \sigma_{alum} \times A_{alum}$$

$$P_{alum} = 90 \times 400$$

$$P_{alum} = 36000N$$

$$P_{steel} = \sigma_{steel} \times A_{steel}$$

$$5P_{steel} = 140 \times 500$$

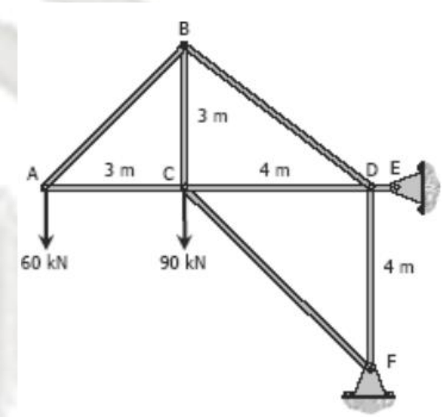
$$P_{steel} = 14000N$$

$$\therefore P_{smallest} = 10000N = 10kN$$

The safe force is P=10 kN

EXAMPLE 1-10

Find the stresses in members BC, BD, and CF for the truss shown in the Figure. Indicate the tension or compression. The cross sectional area of each member is 1600 mm^2.



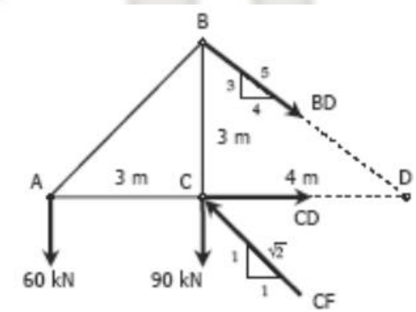
For member BD: $\sum M_c = 0 \rightarrow 3 \times \frac{4}{5} BD = 3 \times 60$

$BD = 75kN(tension)$

$BD = \sigma_{BD} \times A$

$75 \times 1000 = \sigma_{BD} \times 1600$

$\sigma_{BD} = 46.875MPa(tension)$



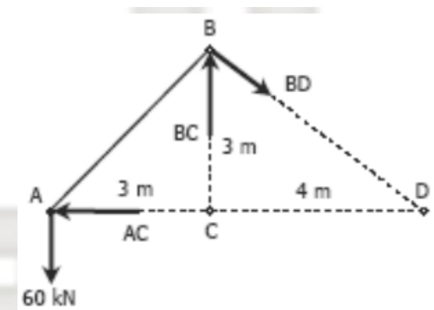
For member CF: $\sum M_D = 0 \rightarrow 4 \times \frac{1}{\sqrt{2}} CF = 4 \times 90 + 7 \times 60$

$BD = 195\sqrt{2} = 275.77kN(compression)$

$CF = \sigma_{CF} \times A$

$275.77 \times 1000 = \sigma_{CF} \times 1600$

$\sigma_{CF} = 172.357MPa(compression)$



For member BC: $\sum M_D = 0 \rightarrow 4BC = 7 \times 60$

$BC = 105kN(compression)$

$BC = \sigma_{BC} \times A$

$105 \times 1000 = \sigma_{BC} \times 1600$

$\sigma_{BC} = 65.625MPa(compression)$