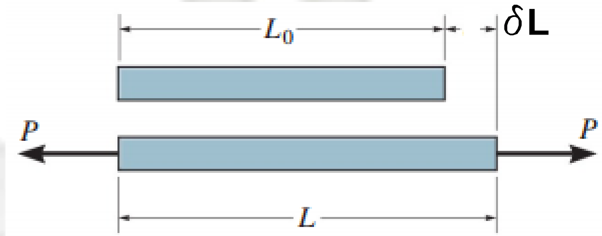
CHAPER TWO – STRAIN, HOOK'S LAW, AXIAL LOAD PROBLEMS

Introduction:

In engineering the deformation of a body is specified using the concepts of normal and shear strain. Whenever a force is applied to a body, it will tend to change the body's shape. These changes are referred to as deformation, and they may be highly visible or practically unnoticeable. Deformation of a body can also occur when the temperature of the body is changed. The figure shows the before and after positions of three different line segments on this rubber membrane which is subjected to tension. The vertical line is lengthened, the horizontal line is shortened, and the inclined line changes its length and rotates.

Normal Strain (Axial Strain):

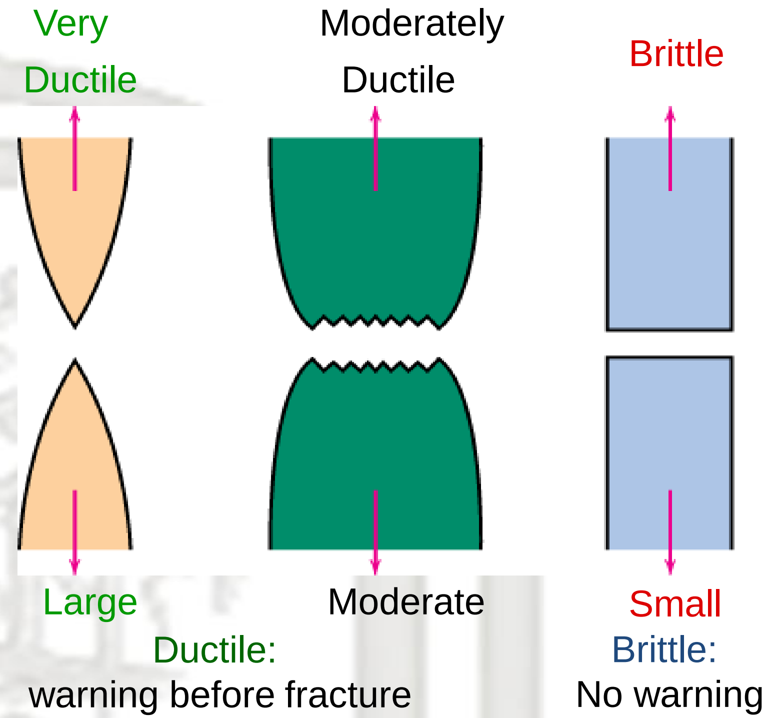
If an axial load P is applied to the bar in the Figure, it will change the bar's length Lo to a length L. We will define the average normal strain ε (epsilon) of the bar as the change in its length δ (delta) = L - Lo divided by its original length, that is:

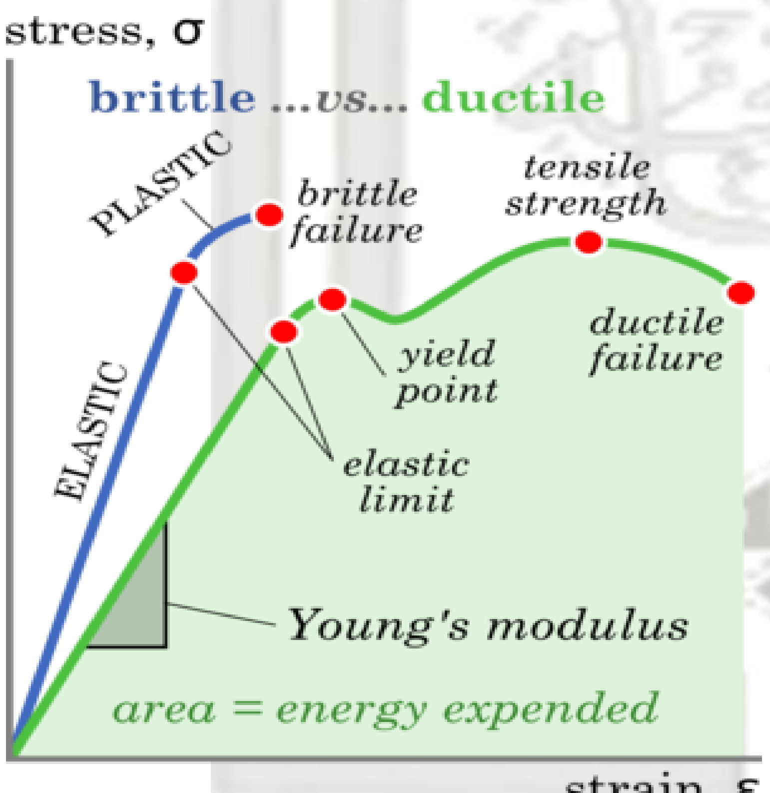


$$\varepsilon = \frac{L - L_o}{L_o} = \frac{\delta L}{L_o}$$ (dimensionless quantity)

Ductile and Brittle Materials

Engineering materials are commonly classified as either ductile or brittle materials. A ductile material lies one having a relatively large tensile strain up to the point of rupture (for example, structural **steel** or **aluminum**) whereas a brittle material has a relatively small strain up to this same point. **Cast iron** and **concrete** are examples of brittle materials.





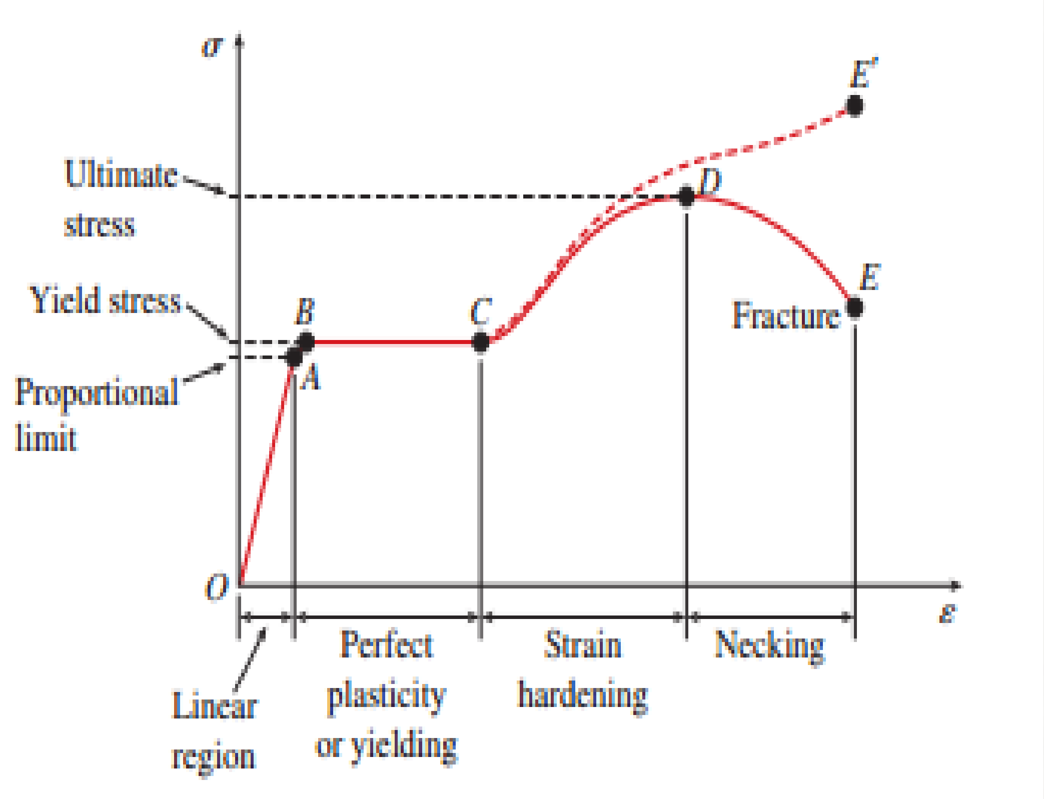
Ductile material	Brittle material
Is the ability of a Material to withstand Plastic deformation Under tensile stress Without fracture.	brittle material fractures at or near its proportional limit.
Fracture occur far Away from P.L	Fracture occur at or near P.L

Hooke’s Law

From the origin O to the point called proportional limit, the stress-strain curve is a straight line. This linear relation between elongation and the axial force causing was first noticed by Sir Robert Hooke in 1678 and is called Hooke's law that within the proportional limit, the stress is directly proportional to strain.

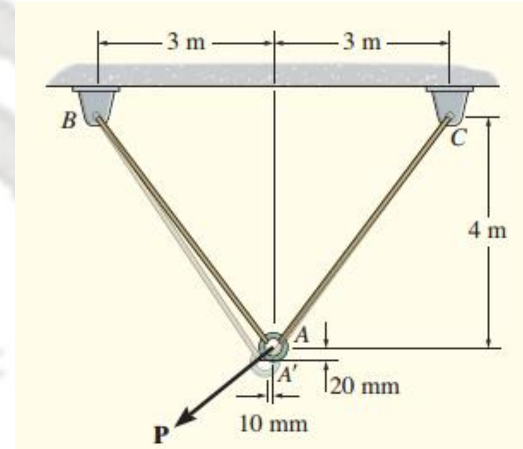
$$\sigma = E\varepsilon$$

Where: E denotes the slope of the straight-line portion OA.
The quantity E is equal to the slope of the stress-strain diagram from O to A it is called the modulus of elasticity of the material in tension, or, as it is often called, Young’s modulus of elasticity.



EXAMPLE 2-1

Determine the average normal strains in the two wires in the Figure shown if the ring at A moves to A′.



The original length of each wire is

$$L_{AB} = L_{AC} = \sqrt{3^2 + 4^2} = 5m$$

The final lengths are:

$$L_{A'B} = \sqrt{(3-0.01)^2 + (4+0.02)^2} = 5.01004m$$

$$L_{A'C} = \sqrt{(3+0.01)^2 + (4+0.02)^2} = 5.022m$$

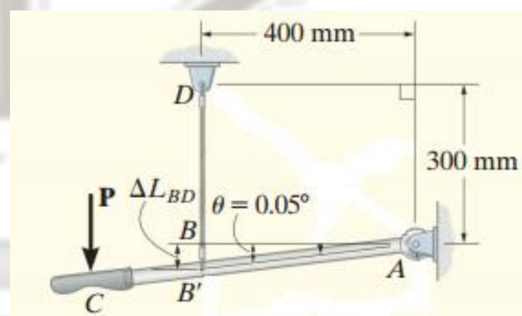
The normal strains:

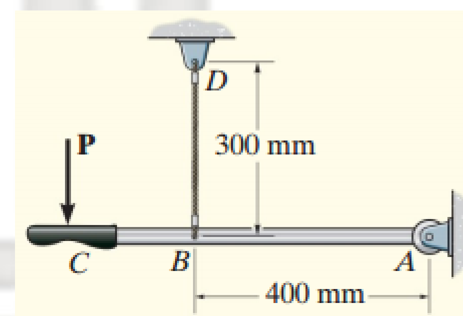
$$\varepsilon_{AB} = \frac{L_{A'B} - L_{AB}}{L_{AB}} = \frac{5.01004-5}{5} = 2.01\times10^{-3} m/m$$

$$\varepsilon_{AC} = \frac{L_{A'C} - L_{AC}}{L_{AC}} = \frac{5.022-5}{5} = 4.4\times10^{-3} m/m$$

EXAMPLE 2-2

When force **P** is applied to the rigid lever arm ABC shown in the Figure, the arm rotates counterclockwise about pin A through an angle of 0.05 °. Determine the normal strain in wire BD.





$$\tan(0.05) = \frac{BB'}{400}$$

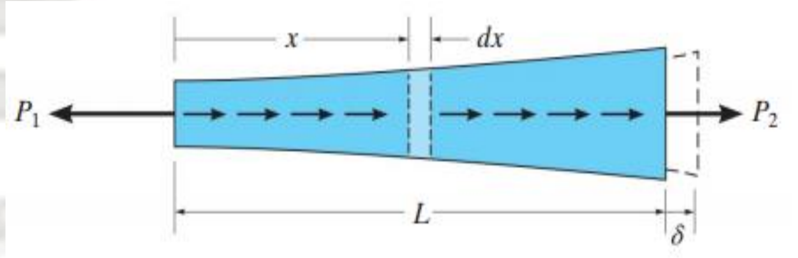
$$BB' = 400 \times \tan(0.05) = 0.349mm$$

$$L_{DB'} = 300 + 0.349 = 300.349mm$$

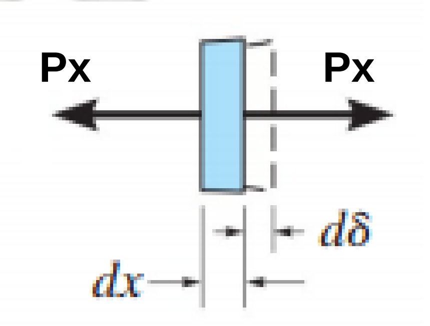
$$\varepsilon_{DB'} = \frac{L_{DB'} - L_{DB}}{L_{DB}} = \frac{300.349-300}{300} = 0.00116 m/m$$

Deflection of Axially Loaded Rods

Using Hooke's law and the definitions of stress and strain, we will now develop an equation that can be used to determine the elastic displacement of a member subjected to axial loads. The free-body diagram of the axially loaded element is shown in Figure. The resultant internal axial force will be a function of x since the external distributed loading will cause it to vary along the length of the bar.



Provided the stress does not exceed the proportional limit, we can apply Hooke's law



$$\sigma = \frac{P_x}{A_x}$$

$$\varepsilon = \frac{d\delta}{dx}$$

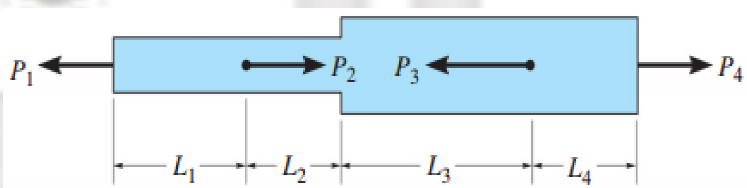
$$\frac{P_x}{A_x} = E_x\left(\frac{d\delta}{dx}\right) \rightarrow d\delta = \frac{P_x d_x}{A_x E_x}$$

$$\delta = \int_0^L \frac{P_x d_x}{A_x E_x}$$

For constant load Px, cross-sectional area Ax and Modulus of elasticity Ex, the above equation becomes:

$$\delta = \frac{PL}{AE}$$

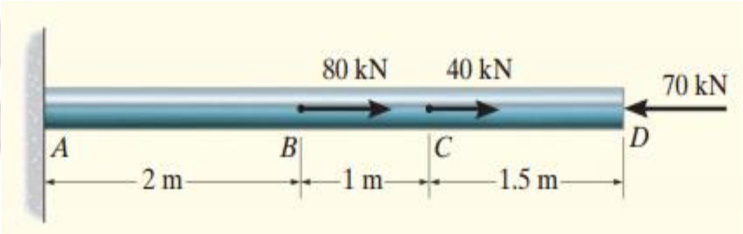
If the structural member has some segments with different load, area, and E in each segment, the total deflection equals the sum of the deflection of each segment.



$$\delta = \sum \frac{PL}{AE}$$

EXAMPLE 2-3

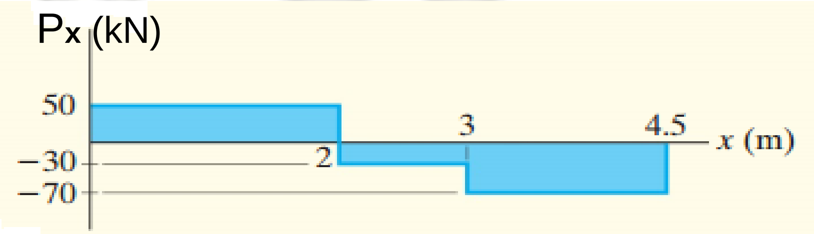
The uniform steel bar shown in the Figure has a diameter of 50 mm and is subjected to the loading shown. Determine the displacement at D. Assume E=200 GPa.



$$\delta_D = \sum \frac{PL}{AE}$$

$$\delta_D = \frac{-70\times1000\times1500}{\pi\times25^2\times200000} + \frac{-30\times1000\times1000}{\pi\times25^2\times200000} + \frac{50\times1000\times2000}{\pi\times25^2\times200000}$$

$$\delta_D = -0.0891mm$$



This negative result indicates that point D moves to the left.

EXAMPLE 2-4

A member is made of a material that has a specific weight of γ = 6 kN/m3 and modulus of elasticity of E=9 GPa. Determine how far of the cone's end is displaced due to gravity when it is suspended in the vertical position.

$$\frac{x}{y} = \frac{0.3}{3} \rightarrow x = 0.1y \rightarrow Vol. = \frac{1}{3}\pi x^2 y = \frac{\pi\times(0.1y)^2\times y}{3} = 0.01047y^3$$

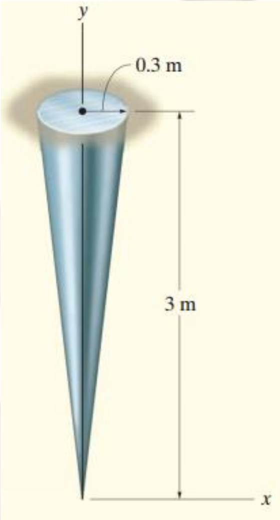
$$W = \gamma \times V$$

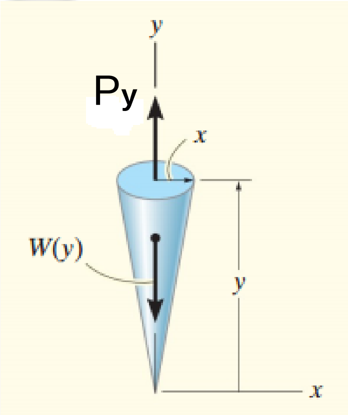
$$\sum Fy = 0 \rightarrow P_y = 6\times1000\times0.01047y^3 = 62.83y^3$$

$$A_y = \pi x^2 = 0.03142y^2$$

$$\delta = \int_0^L \frac{Pdy}{AE} = \int_0^3 \frac{62.83y^3dy}{0.03142y^2\times9\times10^9} = 222.2\times10^{-9}\int_0^3 ydy = \frac{222.2\times10^{-6}}{2}\left[y^2\right]_0^3$$

$$\delta = 1\times10^{-6}m$$

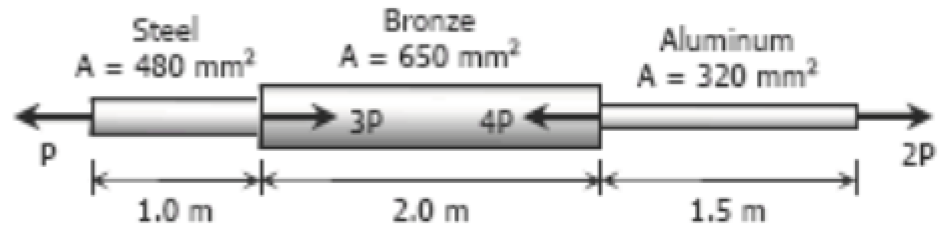


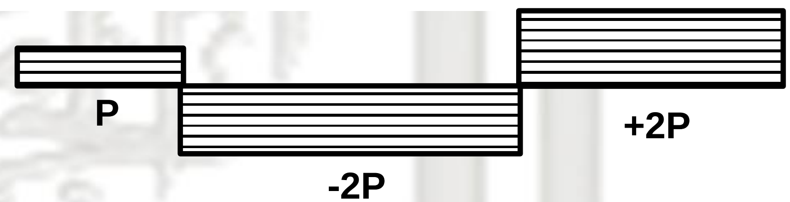


very small amount

EXAMPLE 2-5

A bronze bar is fastened between a steel bar and an aluminum bar as shown in the Figure. Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Use E_{st} = 200 GPa, E_{al} = 70 GPa, and E_{br} = 83 GPa.





$$P_{st} = \sigma_{st} \times A_{st} = 140 \times 480 = 67200N$$

$$2P_{bro} = \sigma_{bro} \times A_{bro} = 120 \times 650 \rightarrow P_{bro} = 39000N$$

$$2P_{al} = \sigma_{al} \times A_{al} = 80 \times 320 \rightarrow P_{al} = 12800N$$

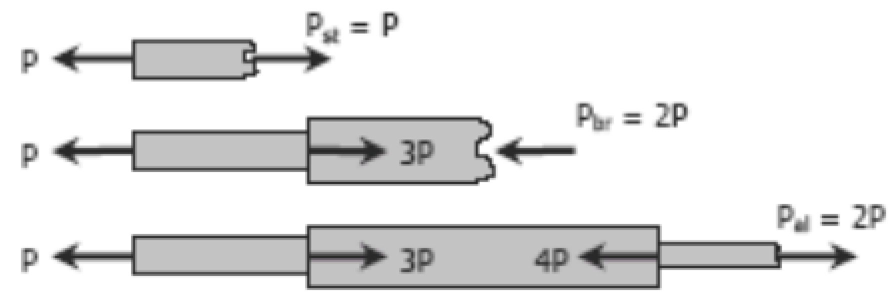
$$\delta = \delta_{st} - \delta_{bro} + \delta_{al}$$

$$3 = \frac{P \times 1000}{480 \times 200000} - \frac{2P \times 2000}{650 \times 70000} + \frac{2P \times 1500}{480 \times 83000}$$

$$3 = P\left(\frac{1}{96000} - \frac{1}{11375} + \frac{3}{26560}\right)$$

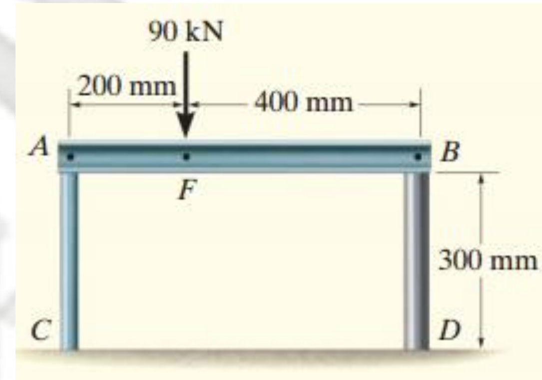
$$P = 84611N$$

Use the smallest P=12800 N



EXAMPLE 2-6

Rigid beam AB rests on the two short posts shown in Figure.. AC is made of steel and has a diameter of 20 mm, and BD is made of aluminum and has a diameter of 40 mm. Determine the displacement of point F on AB if a vertical load of 90 kN is applied over this point. Take Est = 200 GPa, Eal = 70 GPa.



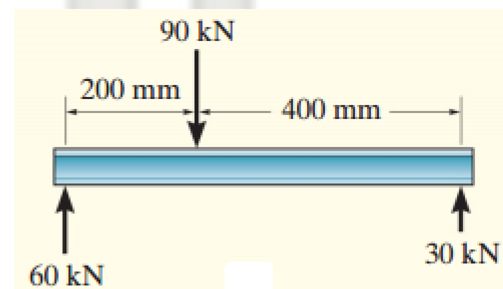
$$\delta_A = \frac{P_{AC} L_{AC}}{A_{AC} E_{st}}$$

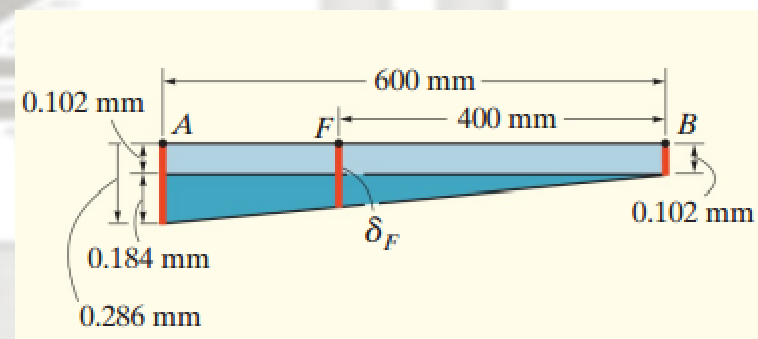
$$\delta_A = \frac{-60 \times 1000 \times 300}{\pi \times 10^2 \times 200000} = -0.286 mm = 0.286 mm \downarrow$$

$$\delta_B = \frac{P_{BD} L_{BD}}{A_{BD} E_{al}}$$

$$\delta_B = \frac{-30 \times 1000 \times 300}{\pi \times 20^2 \times 70000} = -0.102 mm = 0.102 mm \downarrow$$

$$\delta_F = 0.102 + 0.184 \times \frac{400}{600} = 0.225 mm \downarrow$$





EXAMPLE 2-7

A uniform concrete slab of total weight W is to be attached, as shown in the Figure, to two rods whose lower ends are on the same level. Determine the ratio of the areas of the rods so that the slab will remain level. Take Est = 200 GPa, Eal = 70 GPa.

$$\sum M_{al} = 0 \rightarrow 6P_{st} = 2W$$

$$P_{st} = \frac{W}{3}$$

$$\sum M_{st} = 0 \rightarrow 6P_{al} = 4W$$

$$P_{al} = \frac{2W}{3}$$

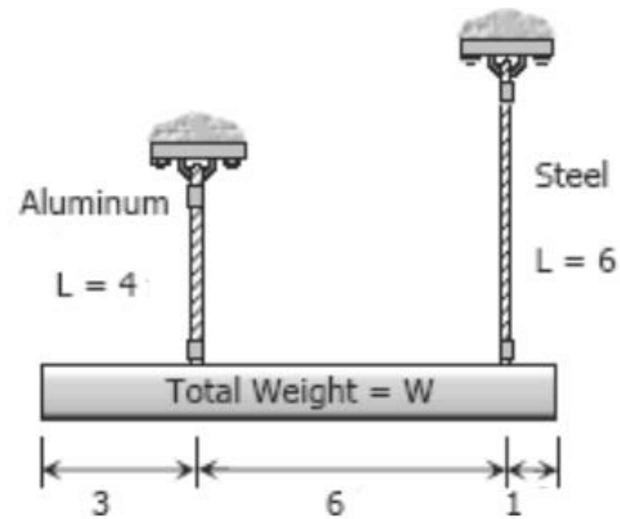
$$\delta_{st} = \delta_{al}$$

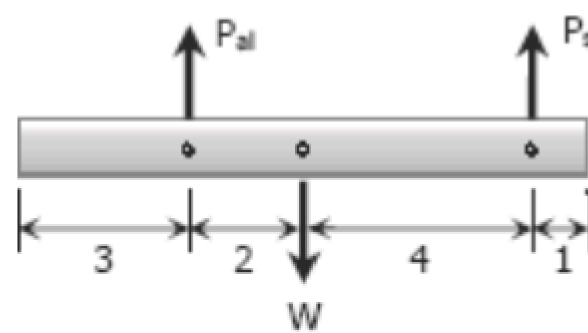
$$(\frac{PL}{AE})_{st} = (\frac{PL}{AE})_{al}$$

$$\frac{\frac{W}{3} \times 6}{A_{st} \times 200} = \frac{\frac{2W}{3} \times 4}{A_{al} \times 70}$$

$$\frac{2W}{A_{st} \times 200} = \frac{8W}{A_{al} \times 70 \times 3}$$

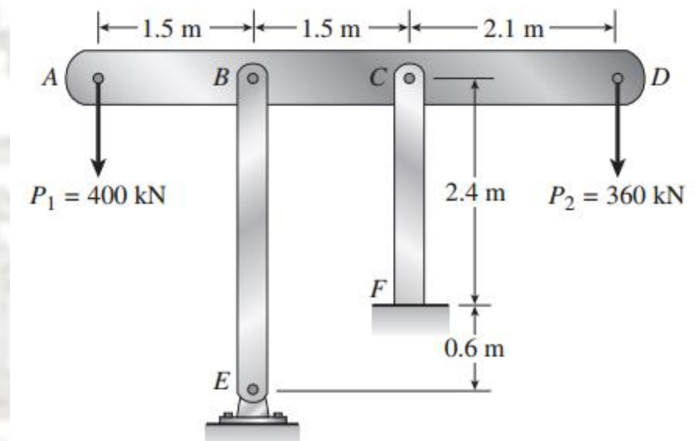
$$\frac{A_{al}}{A_{st}} = \frac{200}{70} \times \frac{8}{6} = 3.809$$





EXAMPLE 2-8

The horizontal rigid beam ABCD is supported by vertical bars BE and CF and is loaded by vertical forces P1=400 kN and P2=360 kN acting at points A and D, respectively. Bars BE and CF are made of steel (E=200 GPa) and have cross-sectional areas ABE=11,100 mm2 and ACF=9,280 mm2. The distances between various points on the bars are shown in the figure. Determine the vertical displacements of points A and D.

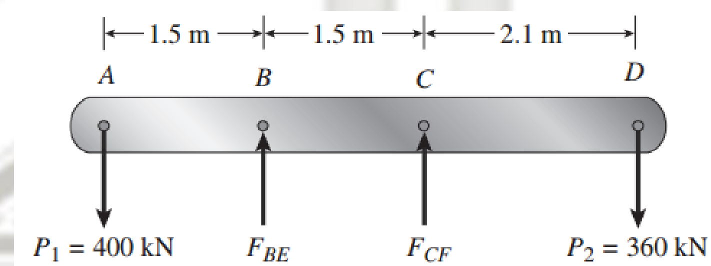


$$\sum M_B = 0 \rightarrow 400\times1.5 + F_{CF}\times1.5 = 360\times3.6$$

$$F_{CF} = 464kN$$

$$\sum M_C = 0 \rightarrow 400\times3 = F_{BE}\times1.5 + 360\times2.1$$

$$F_{BE} = 296kN$$



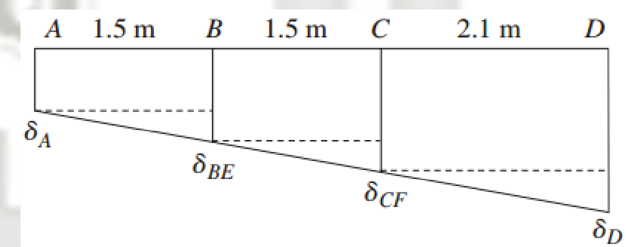
Shortening of bar BE:

$$\delta_{BE} = \frac{F_{BE}L_{BE}}{A_{BE}E} = \frac{296\times3}{11100\times200} = 0.4m$$

Shortening of bar CF:

$$\delta_{CF} = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{464\times2.4}{9280\times200} = 0.6m$$

From displacement diagram:



$$\delta_{BE} - \delta_A = \delta_{CF} - \delta_{BE}$$

$$\delta_A = 2\delta_{BE} - \delta_{CF} = 2\times0.4 - 0.6 = 0.2mm$$

$$\delta_D - \delta_{CF} = \frac{2.1}{1.5}(\delta_{CF} - \delta_{BE})$$

$$\delta_D = \frac{12}{5}\delta_{CF} - \frac{7}{5}\delta_{BE} = \frac{12}{5}\times0.6 - \frac{7}{5}\times0.4 = 0.88mm$$

EXAMPLE 2-9

For the composite column shown:

1. Determine the maximum load P if the total deflection Δ=0.25mm.
2. Draw the vertical deflection diagram.

$$\delta_{total} = (\frac{PL}{AE})_{st} + (\frac{PL}{AE})_{al}$$

$$0.25 = \frac{P \times 300}{200 \times 50 \times 50} + \frac{P \times 500}{70 \times 100 \times 100}$$

$$0.25 = 0.0006P + 0.00071P$$

$$P = 190.8kN$$

$$\delta_{al} = \frac{190.8 \times 500}{70 \times 100 \times 100} = 0.136mm$$

$$\delta_{st} = \frac{190.8 \times 300}{200 \times 50 \times 50} = 0.114mm$$