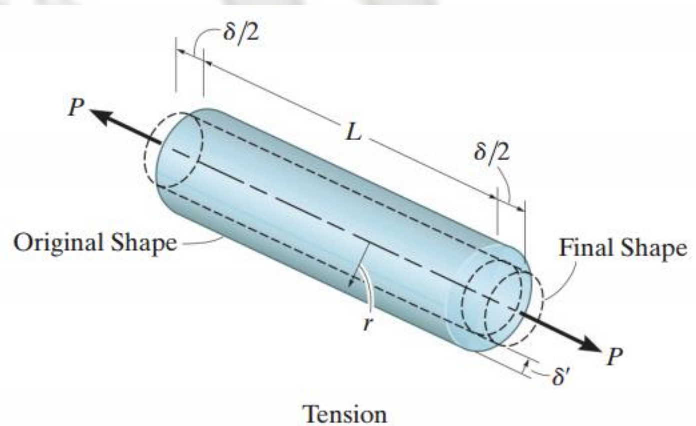
Poisson's Ratio

When a deformable body is subjected to a tensile force (or a compressive force), not only does it elongate but it also contracts laterally. In the early 1800s, the French scientist S. Poisson realized that within the elastic range the ratio of the strain in the lateral or radial direction to the strain in the longitudinal or axial direction is a constant. This ratio is referred to as Poisson's ratio.

Poisson's ratio is a dimensionless quantity with maximum possible value is 0.5, so that $0 \le \nu \le 0.5$. For most solids, it has a value that is generally between 0.25 and 0.355.

ν= 0.1- 0.16 (concrete).
ν= 0.25 (steel).
ν= 0.333 (aluminum).
ν= 0.5 (rubber).



Poisson's Ratio= Absolute (Lateral Strain / Axial Strain)

$$\nu = \left| -\frac{\varepsilon_y}{\varepsilon_x} \right| = \left| -\frac{\varepsilon_z}{\varepsilon_x} \right|$$

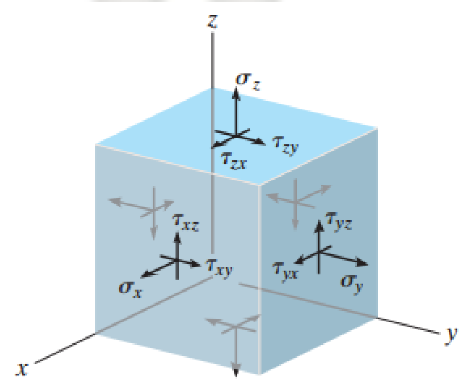
Generalized Hook's Law

Consider an element of an isotropic material in the shape of a cube subjected to a triaxial tensile stress, as shown in the figure. By using the principal of superposition, the general Hooke's law can be written as:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

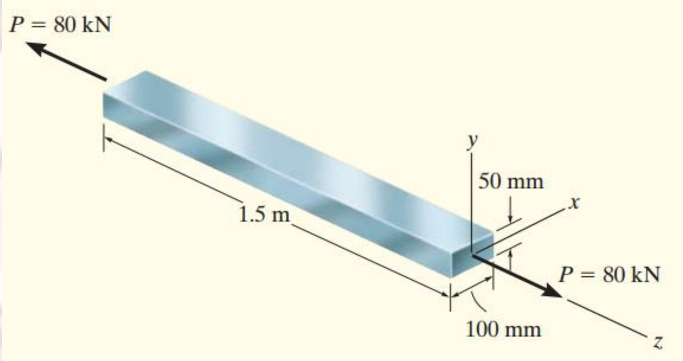
$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$



EXAMPLE 2-17

A bar made of steel has the dimensions shown in the Figure. If an axial force of P = 80 kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section. The material behaves elastically. (Assume Est=200 GPa, ν of steel=0.32).



$$\sigma = \frac{P}{A} = \frac{80000}{100 \times 50} = 16 MPa$$

$$\varepsilon_z = \frac{\sigma_z}{E} = \frac{16}{200000} = 8 \times 10^{-5}$$

$$\delta_z = \varepsilon_z \times L_z = 8 \times 10^{-5} \times 1500 = 0.12 mm$$

The lateral contraction strains in both the x and y directions are equal

$$\nu_{st} = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z}$$

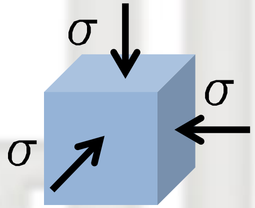
$$\varepsilon_x = \varepsilon_y = -\nu_{st} \times \varepsilon_z = -0.32 \times 8 \times 10^{-5} = -2.56 \times 10^{-5}$$

$$\delta_x = \varepsilon_x \times L_x = -2.56 \times 10^{-5} \times 100 = -2.56 \times 10^{-3} mm$$

$$\delta_y = \varepsilon_y \times L_y = -2.56 \times 10^{-5} \times 50 = -1.28 \times 10^{-3} mm$$

EXAMPLE 2-18

What material should be used in order to produce a cube has no change in its volume when it is subjected to a uniform pressure?



$$\sigma_x = \sigma_y = \sigma_z = \sigma$$

$$\varepsilon_x = 0 = -\frac{\sigma}{E} + \nu \frac{\sigma}{E} + \nu \frac{\sigma}{E} = -\frac{\sigma}{E}(1 - 2\nu)$$

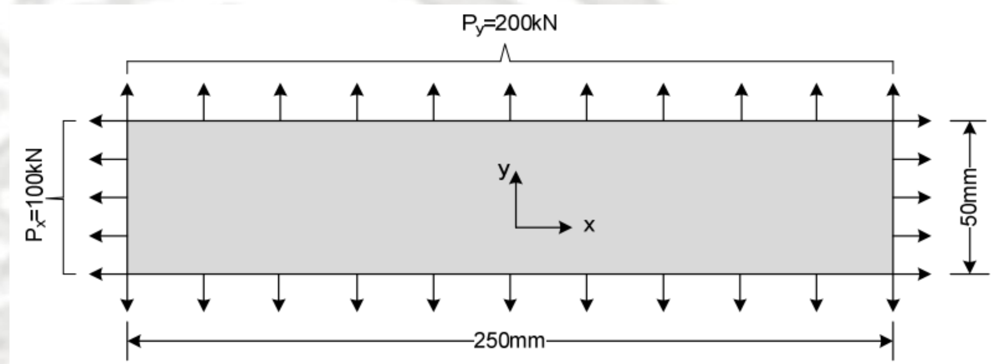
$$\because -\frac{\sigma}{E} \neq 0 \quad \therefore (1 - 2\nu) = 0 \rightarrow \nu = \frac{1}{2}$$

EXAMPLE 2-19

A piece of a steel plate of (250 x 50 x 10)mm dimensions is subjected to a biaxial force system in x and y directions, as shown in the figure. Knowing that E=200000 MPa and ν =0.25:

a) What is the change in the thickness.

b) To cause the same change in thickness as in (a) by Px alone, what must be its magnitude?



a) $$\sigma_x = \frac{P_x}{A_x} = \frac{100000}{50\times10} = 200MPa$$

$$\sigma_y = \frac{P_y}{A_y} = \frac{200000}{250\times10} = 80MPa$$

$$\sigma_z = 0$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = \frac{-\nu}{E}(\sigma_x + \sigma_y)$$

$$\varepsilon_z = \frac{-0.25}{200000}(200+80) = -0.00035$$

$$\delta_z = \varepsilon_z \times t = -0.00035\times10 = -0.0035$$

b) $$\sigma_z = \sigma_y = 0$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0 - \nu\frac{\sigma_x}{E} - 0$$

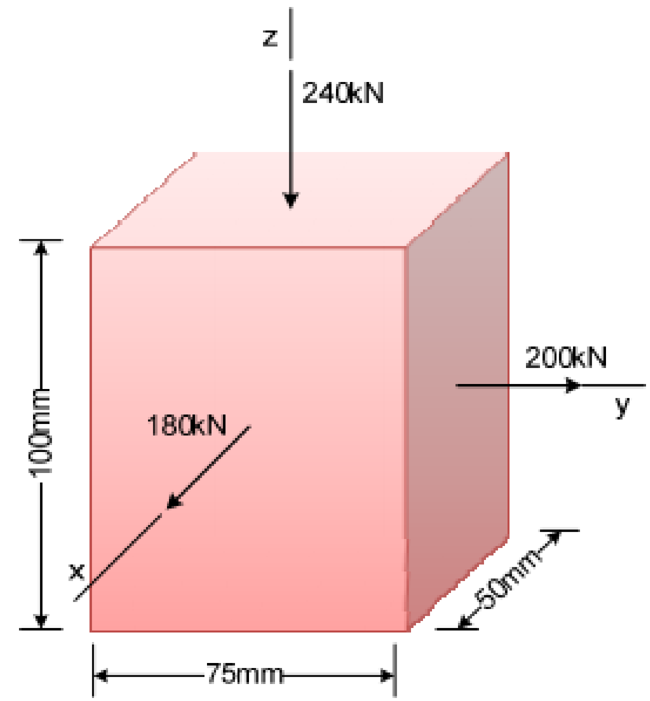
$$-0.00035 = -0.25\times\frac{\sigma_x}{200000}$$

$$\sigma_x = 280MPa$$

$$P_x = \sigma_x \times A_x = 280\times50\times10 = 140kN$$

EXAMPLE 2-20

Knowing that Poisson's ratio ν=0.25, **determine the magnitude of a single force acting only in the y-direction that would cause the same deformation in the y-direction as the initial forces.**



$$\sigma_x = \frac{P_x}{A_x} = \frac{180000}{100\times75} = 24MPa$$

$$\sigma_y = \frac{P_y}{A_y} = \frac{200000}{50\times100} = 40MPa$$

$$\sigma_z = \frac{P_z}{A_z} = \frac{-240000}{50\times75} = -64MPa$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E} = \frac{1}{E}\{40-0.25\times24-0.25\times(-64)\}$$

$$\varepsilon_y = +\frac{50}{E}$$

$$\sigma_x = \sigma_z = 0, \quad \sigma_y = ?$$

$$\varepsilon_y = +\frac{\sigma_y}{E}$$

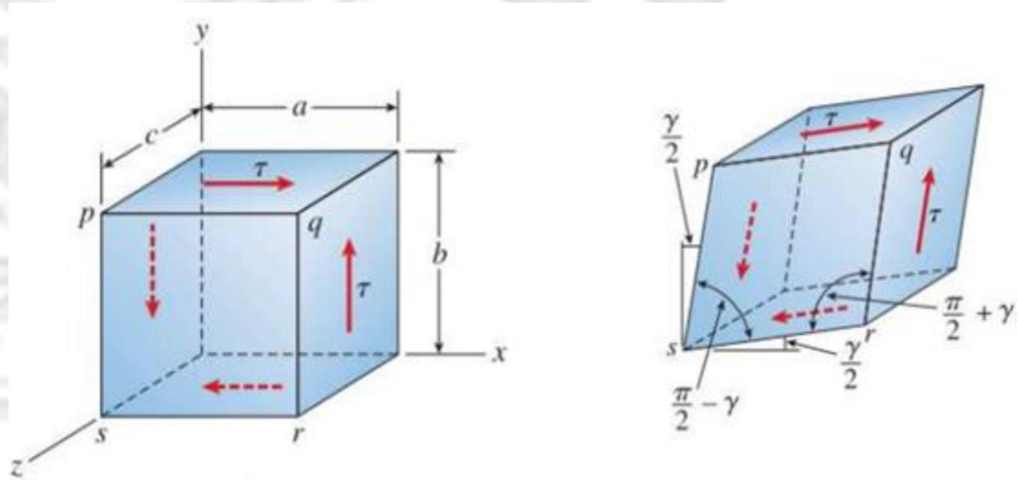
$$\frac{50}{E} = \frac{\sigma_y}{E} \rightarrow \sigma_y = 50MPa$$

$$P = \sigma A$$

$$P = 50\times10^{-3}\times50\times100 = 250kN$$

Shear Strain

Shearing forces cause shearing deformation. An element subject to shear does not change in length but undergoes a change in shape. The change in angle at the corner of an original rectangular element is called the **shear strain.** This angle is denoted by γ (gamma) and is always measured in radians (rad). For instance, the angles at the points q and s, which were π/2 before deformation, are reduced to π/2-γ. At the same time, the angles at points p and rare increased to π /2+ γ.



The shear strain is expressed as:

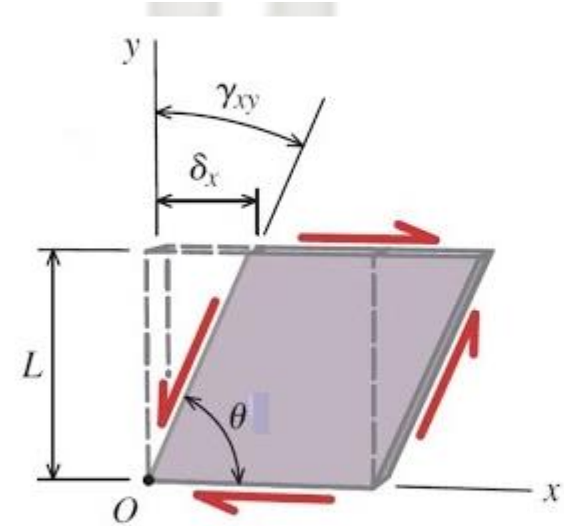
$$\gamma = \frac{\delta_{sh}}{L}$$

The ratio of the shear stress τ and the shear strain γ is called the **modulus of rigidity** or **shear modulus of elasticity** and is denoted as G, in MPa. the following equation for Hooke's law in shear.

$$\tau = \gamma \times G$$

The relationship between the shearing deformation and the applied shearing force is:

$$\delta_{sh} = \frac{VL}{A_{sh} \times G} = \frac{\tau \times L}{G}$$ where V is the shearing force acting over an area Ash.



The **modulus of elasticity** in tension and **modulus of rigidity** in shear are related by the following equation:

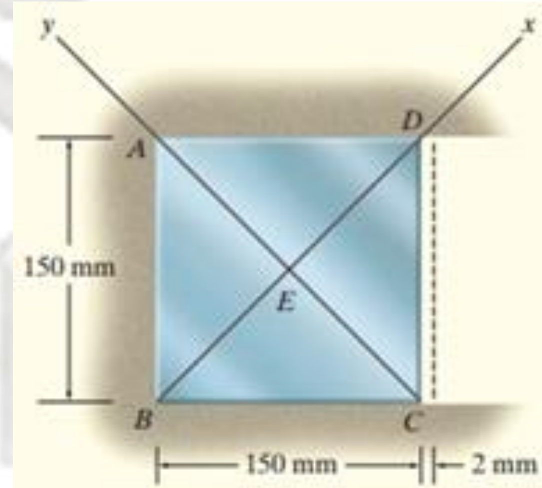
$$G = \frac{E}{2(1+\nu)}$$

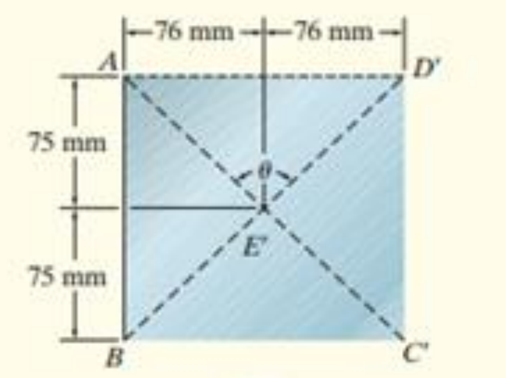
EXAMPLE 2-21

The right side CD of the plate shown in the Figure deforms by a uniform horizontal displacement of 2 mm. Determine:

(a) the average normal strain along the diagonal AC.

(b) the shear strain at E relative to the x, y axes.





(a) $$AC = \sqrt{0.15^2 + 0.15^2} = 0.21213m$$

$$AC' = \sqrt{0.15^2 + 0.152^2} = 0.21355m$$

$$\varepsilon_{AC} = \frac{AC' - AC}{AC} = \frac{0.21355 - 0.21213}{0.21213} = 0.00669$$

(b) $$\tan(\frac{\theta}{2}) = \frac{76}{75}$$

$$\theta = 90.759° = 90.759° \times \frac{\pi}{180°} = 1.58404rad$$

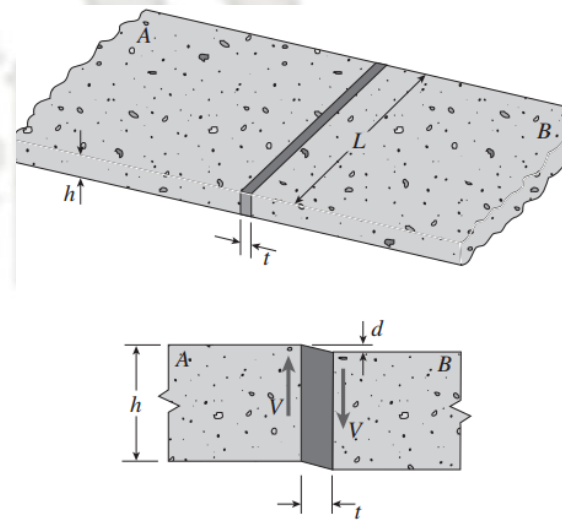
$$\gamma_{xy} = \frac{\pi}{2} - 1.58404 = -0.0132rad$$

EXAMPLE 2-22

A joint between two concrete slabs A and B is filled with a flexible epoxy that bonds to the concrete as shown in the Figure. The height of the joint is h=100 mm, its length is L=1000 mm, and its thickness is t=12.5 mm. Under the action of shear forces V, the slabs displace vertically through the distance d=0.05 mm relative to each other.

(a) What is the average shear strain in the epoxy?

(b) What is the magnitude of the forces V if the shear modulus of elasticity G for the epoxy is 965 MPa?



$$\gamma = \frac{d}{t} = \frac{0.05}{12.5} = 0.004$$

$$\tau = \gamma \times G$$

$$V = \tau \times h \times L$$

$$\therefore V = \gamma \times G \times h \times L$$

$$V = 0.004 \times 965 \times 100 \times 1000 \times 10^{-3} = 386kN$$