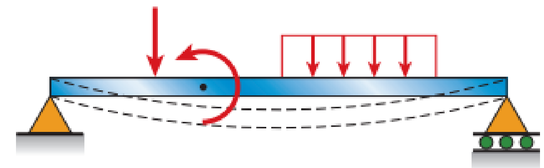
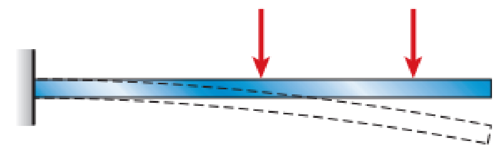
CHAPER THREE – AXIAL FORCE, SHEAR AND BENDING MOMENT



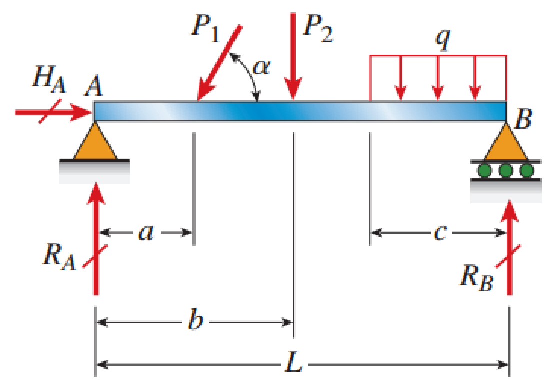


Introduction:

Structural members are usually classified according to the types of loads that they support. For instance, an axially loaded bar supports forces along the axis of the bar, and a bar in torsion supports torques along the axis. Members that are slender and support loadings that are applied perpendicular to their longitudinal axis are called **beams.**

Types of Beams, Load, and Reactions:

Beams are usually described by the manner in which they are supported.

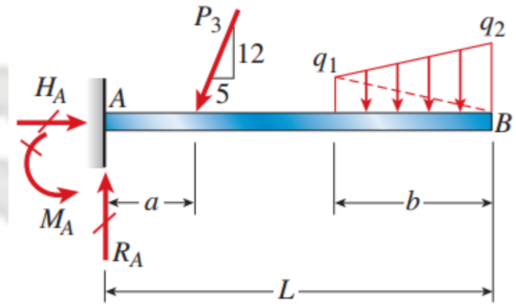


1- Simply supported beam:

A beam with a pin support at one end and a roller support at the other called a **simply supported beam** or a **simple beam**. The essential feature of a **pin support** is that it prevents translation at the end of a beam but does not prevent rotation. Thus, end A of the beam cannot move horizontally or vertically but the axis of the beam can rotate in the plane of the figure. Consequently, a pin support is capable of developing a force reaction with both horizontal and vertical components (HA and RA), but it cannot develop a moment reaction. At end B of the beam, the **roller support** prevents translation in the vertical direction but not in the horizontal direction; hence this support can resist a vertical force (RB) but not a horizontal force. The axis of the beam is free to rotate at B just as it is at A.

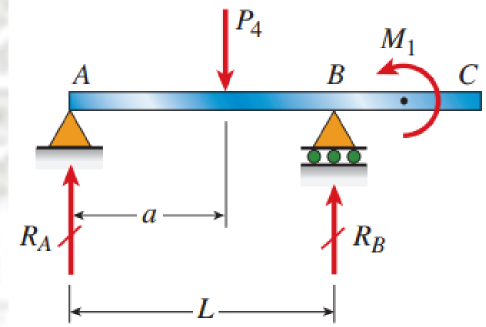
2- Cantilever beam :

The beam is fixed at one end and free at the other, is called a **cantilever beam**. At the **fixed support** (or clamped support) the beam can neither translate nor rotate, whereas at the free end it may do both. Consequently, both force and moment reactions may exist at the fixed support.



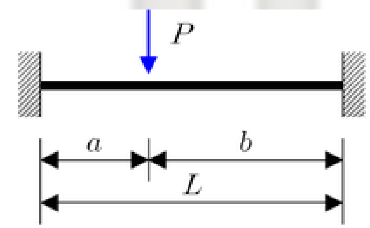
3- Over hanged beam:

This beam is simply supported at points A and B but it also projects beyond the support at B. The overhanging segment BC is similar to a cantilever beam except that the beam axis may rotate at point B.



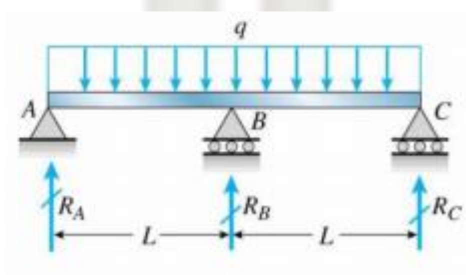
4- Fixed ended beam (clamped beam)

The beam is a statically indeterminate beam



5- Continuous beam

The beam is a statically indeterminate beam



Types of Loads:

1- Concentrated load:

When a load is applied over a very small area it may be idealized as a **concentrated load**, which is a single force. Also a concentrated moment M1 acting on the overhanging beam as a single moment.

2- Distributed load:

When a load is spread along the axis of a beam, it is represented as a **distributed load**, such as the load q.

3- Linearly Varying Load:

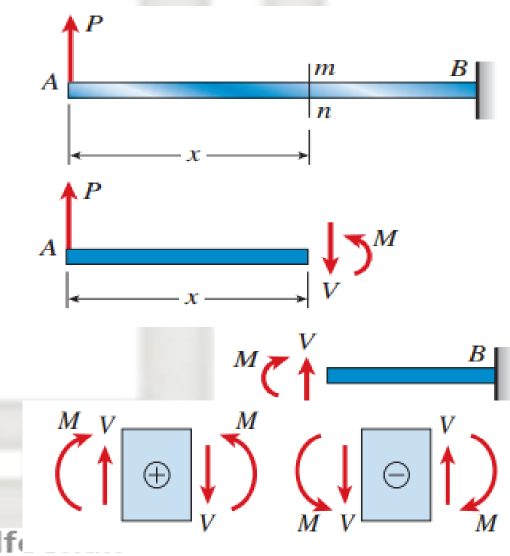
A varying load has an intensity that changes with distance along the axis.

4- Couple:

The couple of moment M1 acting on the overhanging beam.

Internal Forces: There are three types of internal forces in the plane as follows:

- **Axial force (P)**, which algebraically equals the summation of all the axial forces exist on one side of the section.
- **Shear force (V)**, which equals the algebraic summation of all the forces that exist perpendicularly on one side of the section.
- **Bending moment (M)**, which equals the algebraic summation of the moments caused by all the perpendicular forces affect one side of the section.



Sign convention:

Axial force (P)

Tensile **(+)**

Compressive **(-)**

Shear force (V)

At the right-hand side of the segment, downward **(+)**

At the right-hand side of the segment, upward **(-)**

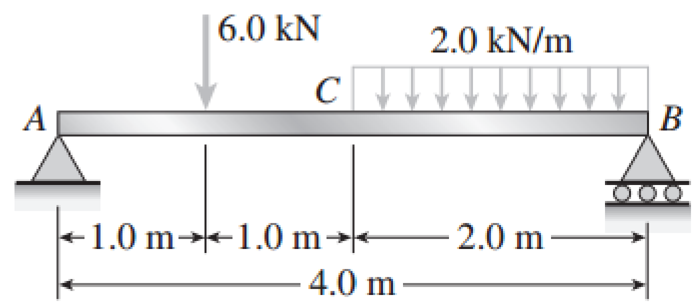
Bending moment (M)

If the moment causes concave, that moment should be considered **(+)**

If the moment causes convex, that moment should be considered **(-)**

EXAMPLE 3-1

Determine the shear force V and bending moment M at the midpoint C of the simple beam AB shown in the figure.



$$\sum M_A = 0$$

$$4R_B = 2\times2\times3+6\times1$$

$$R_B = 4.5kN$$

$$\sum M_B = 0$$

$$4R_A = 2\times2\times1+6\times3$$

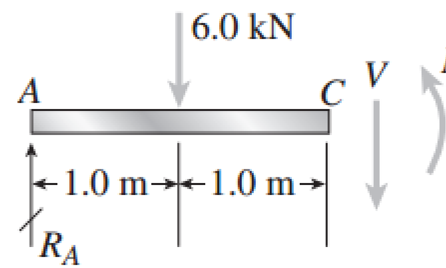
$$R_A = 5.5kN$$

$$+\uparrow\sum F_y = 0$$

$$V+6 = 5.5 \rightarrow V = -0.5kN$$

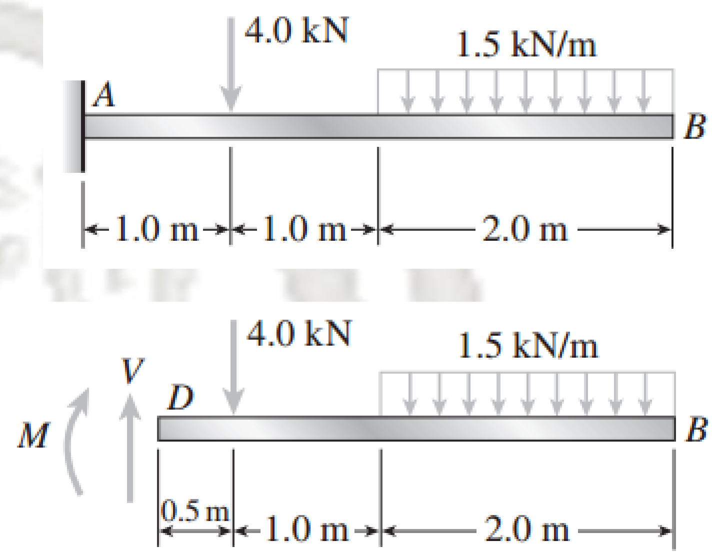
$$\sum M_C = 0$$

$$6\times1+M = 5.5\times2 \rightarrow M = 5kN.m$$



EXAMPLE 3-2

Calculate the shear force V and bending moment M at a cross section located 0.5 m from the fixed support of the cantilever beam AB shown in the figure.



$$+\uparrow \sum F_y = 0$$

$$V = 4 + 1.5 \times 2 = 7kN$$

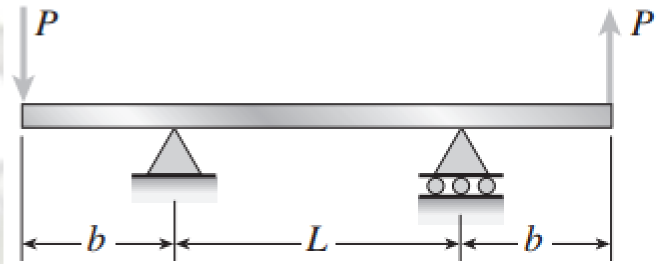
$$\sum M_D = 0$$

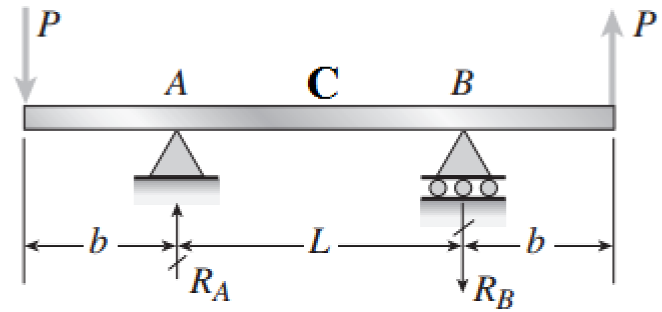
$$M + 4 \times 0.5 + 1.5 \times 2 \times 2.5 = 0$$

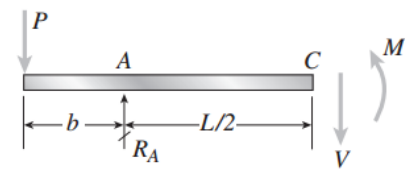
$$M = -9.5kN.m$$

EXAMPLE 3-3

Determine the shear force V and bending moment M at the midpoint of the beam with overhangs. Note that one load acts downward and the other upward.







$$\sum M_B = 0$$

$$R_A \times L = P(L+b) + Pb = PL + 2Pb$$

$$R_A = P(1+\frac{2b}{L}) \uparrow$$

$$\sum M_A = 0$$

$$R_B = P(1+\frac{2b}{L}) \downarrow$$

$$\sum F_y = 0$$

$$V = R_A - P = P(1+\frac{2b}{L}) - P = \frac{2bP}{L}$$

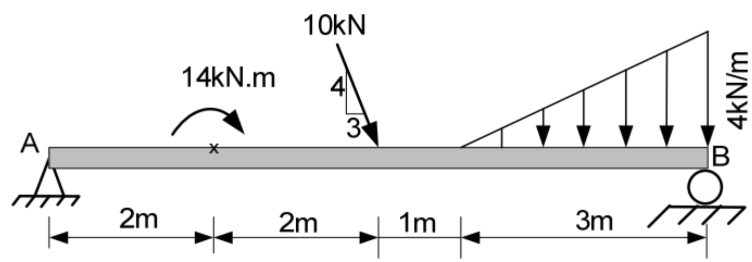
$$\sum M_C = 0$$ **C at mid spam**

$$M = P(1+\frac{2b}{L})(\frac{L}{2}) - P(b+\frac{L}{2})$$

$$M = \frac{PL}{2} + Pb - Pb - \frac{PL}{2} = 0$$

EXAMPLE 3-4

Calculate the reactions on the supports A and B



$$\sum F_x = 0$$

$$A_x = 10 \times \frac{3}{5} = 6kN$$

$$\sum M_A = 0$$

$$R_B \times 8 = \frac{1}{2} \times 3 \times 4 \times 7 + 10 \times \frac{4}{5} \times 4 + 14$$

$$R_B = 11kN$$

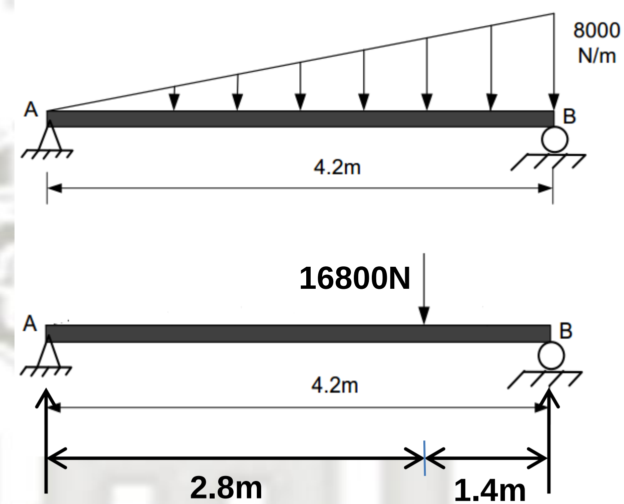
$$\sum F_y = 0$$

$$A_y + 11 = 8 + 6$$

$$A_y = 3kN$$

EXAMPLE 3-5

Calculate the reactions on the supports A and B

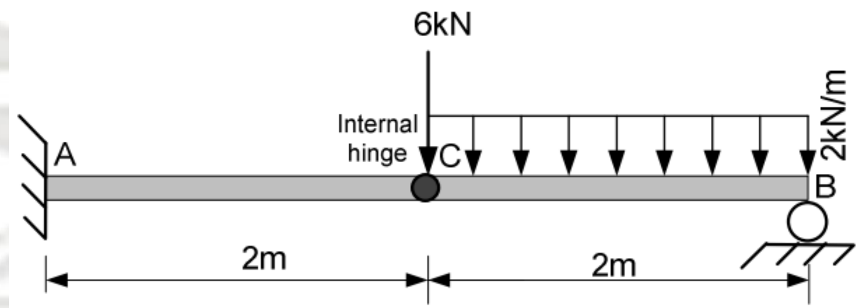


$$R_A = 5600N$$

$$R_B = 11200N$$

EXAMPLE 3-6

Calculate the reactions on the supports A and B



The internal hinge always adds additional equilibrium equation to the three original equilibrium equations that is:

$$\sum M_{hinge} = 0$$

Part BC:

$$\sum M_c = 0$$

$$4 \times 1 = R_B \times 2 \Rightarrow R_B = 2kN$$

$$\sum F_y = 0$$

$$C_y + 2 = 4$$

$$C_y = 2kN$$

Part AC:

$$\sum F_y = 0$$

$$A_y = 6 + 2 = 8kN$$

$$\sum M_A = 0$$

$$M_A = 6 \times 2 + 2 \times 2 = 16kN.m$$

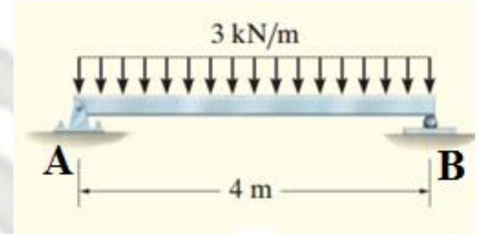
$$\sum F_x = 0$$

$$A_x = 0$$

Shear and Moment Diagrams by Equations:

EXAMPLE 3-7

Draw the shear and moment diagrams for the beam shown in the figure shown.



$$R_A = (3 \times 4)/2 = 6kN$$

$$+\uparrow \sum F_y = 0$$

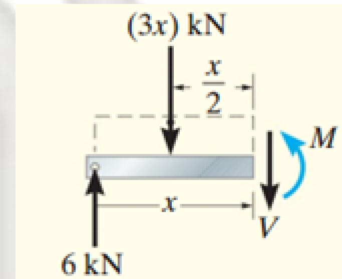
$$6 - 3x - V = 0 \rightarrow$$

$$V = 6 - 3x \rightarrow (1)$$

$$\circlearrowleft + \sum M = 0$$

$$-6x + 3x \times \frac{1}{2} x + M = 0$$

$$M = (6x - 1.5x^2) \rightarrow (2)$$



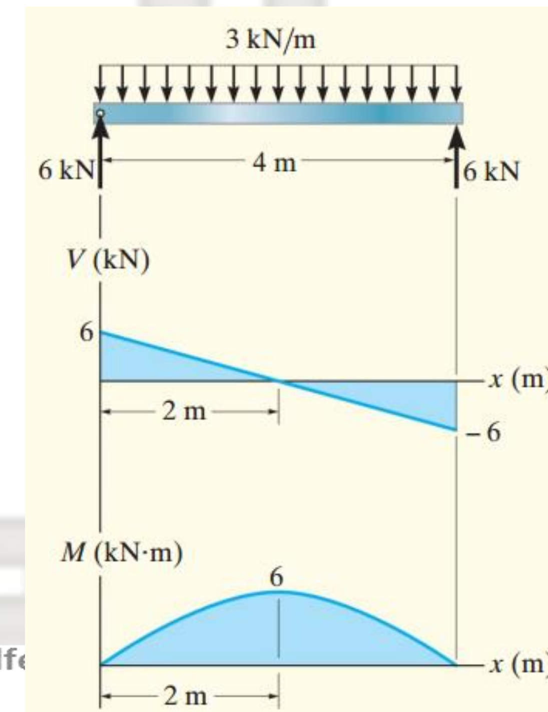
Shear and moments diagrams: The shear and moment diagrams are obtained by plotting Equations 1 and 2. The point of zero shear can be found from Equation 1.

$$V = 6 - 3x = 0$$

$$x = 2$$

From the moment diagram, this value of x represents the point on the beam where the maximum moment occurs. The slope V = dM/dx = 0.

$$M_{\max} = 6 \times 2 - 1.5 \times 2^2 = 6kN.m$$



EXAMPLE 3-8

Draw the shear and moment diagrams for the beam shown in the figure shown.

$$+\uparrow \sum F_y = 0$$

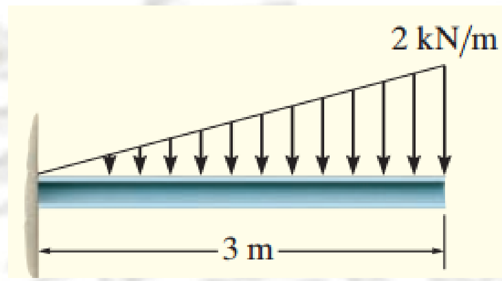
$$3 - \frac{1}{2}\left(\frac{2}{3}x\right)x - V = 0$$

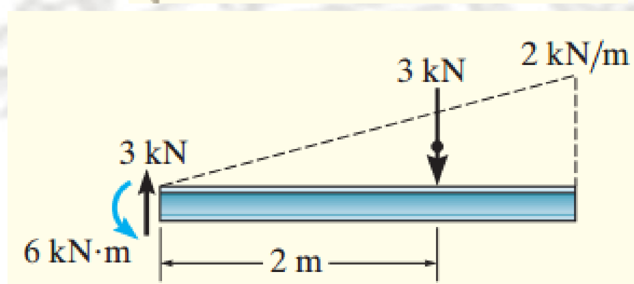
$$V = 3 - \frac{1}{3}x^2$$

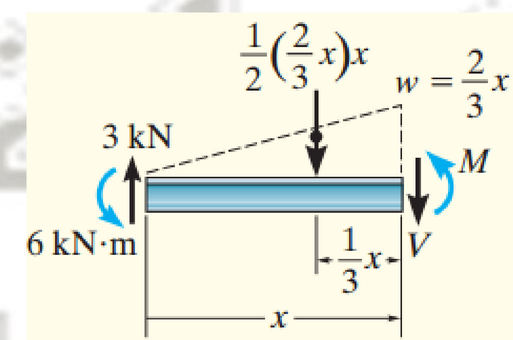
$$\sum M = 0$$

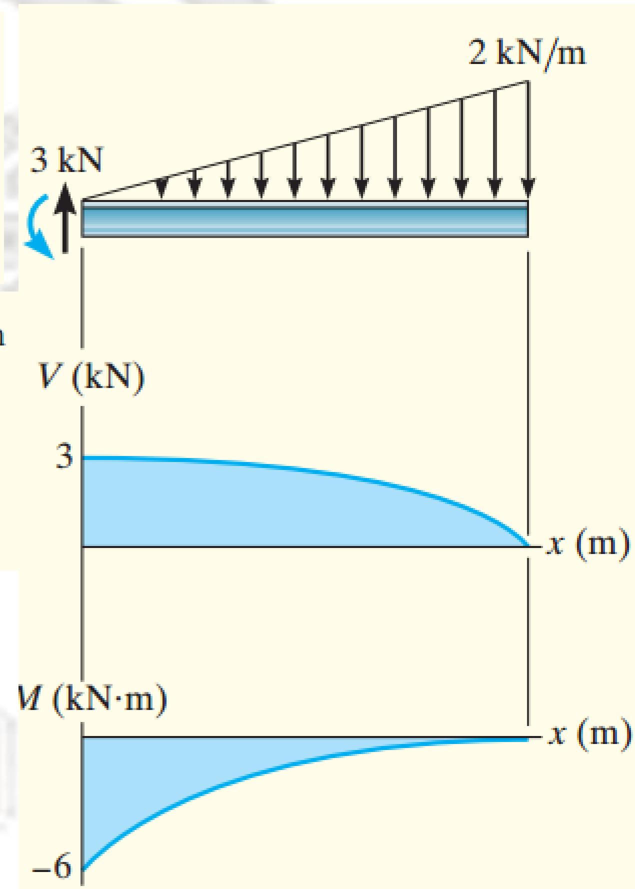
$$6 - 3x + \frac{1}{2}\left(\frac{2}{3}x\right)x\left(\frac{1}{3}x\right) + M = 0$$

$$M = -6 + 3x - \frac{1}{9}x^3$$









EXAMPLE 3-9

Draw the shear and moment diagrams for the beam shown in the figure shown.

$$0 \le x_1 < 5:$$

$$+\uparrow \sum F_y = 0$$

$$5.75 - V = 0 \Rightarrow V = 5.75kN$$

$$\sum M = 0$$

$$-80 - 5.75x_1 + M = 0$$

$$M = 5.75x_1 + 80$$

$$5 < x_2 \le 10:$$

$$+\uparrow \sum F_y = 0$$

$$5.75 - 15 - 5(x_2 - 5) - V = 0$$

$$V = 15.75 - 5x_2$$

$$\sum M = 0$$

$$-80 - 5.75x_2 + 15(x_2 - 5) + 5(x_2 - 5)(\frac{x_2 - 5}{2}) + M = 0$$

$$M = -2.5x_2^2 + 15.75x_2 + 92.5$$