Shear and Moment Diagrams by Graphical Method:

Method based on two differential relations, one that exists between the distributed load and shear, and the other between the shear and moment is a simpler method for constructing the shear and moment diagrams. For region of distributed load:

$$+\uparrow \sum F_y = 0$$

$$V + w\Delta x - (V + \Delta V) = 0$$

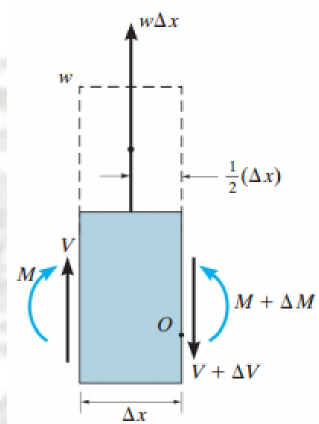
$$\Delta V = w\Delta x$$

$$\sum M = 0$$

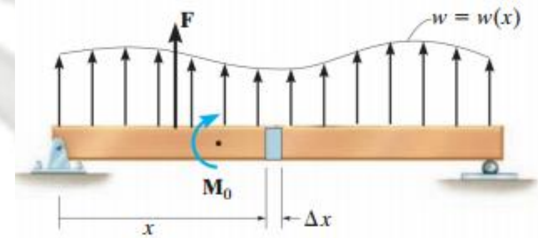
$$-V\Delta x - M - w\Delta x(\frac{1}{2}\Delta x) + (M + \Delta M) = 0$$

$$\Delta M = V\Delta x + w\frac{1}{2}(\Delta x)^2$$

$$\Delta M = V\Delta x + w\frac{1}{2}(\Delta x)^2$$



Free-body diagram of segment Δx



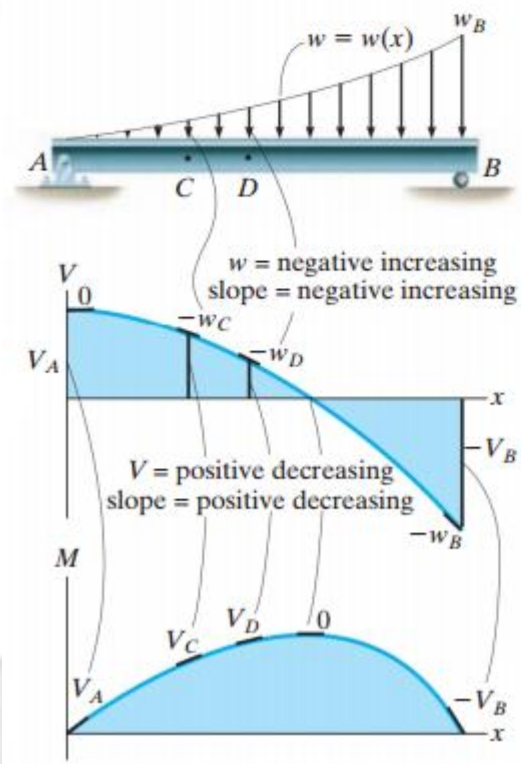
Dividing by x and taking the limit as $\Delta x \to 0$, the above two equations become:

$$\frac{dV}{dx} = w \quad \text{and} \quad \frac{dM}{dx} = V \quad \text{or} \quad \frac{d^2M}{dx^2} = \frac{dV}{dx} = w$$

slope of shear diagram at each point	distributed load intensity at each point	slope of moment diagram at each point	shear at each point

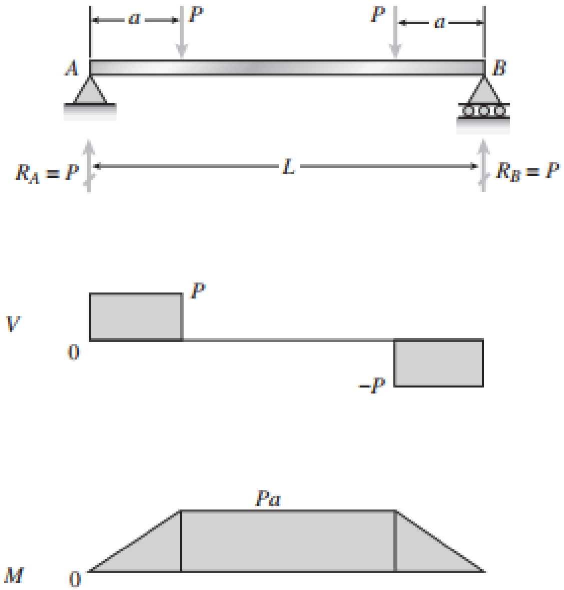
Or : $\Delta V = \int w dx + C_1$ $\quad \Delta M = \int V dx + C_2$

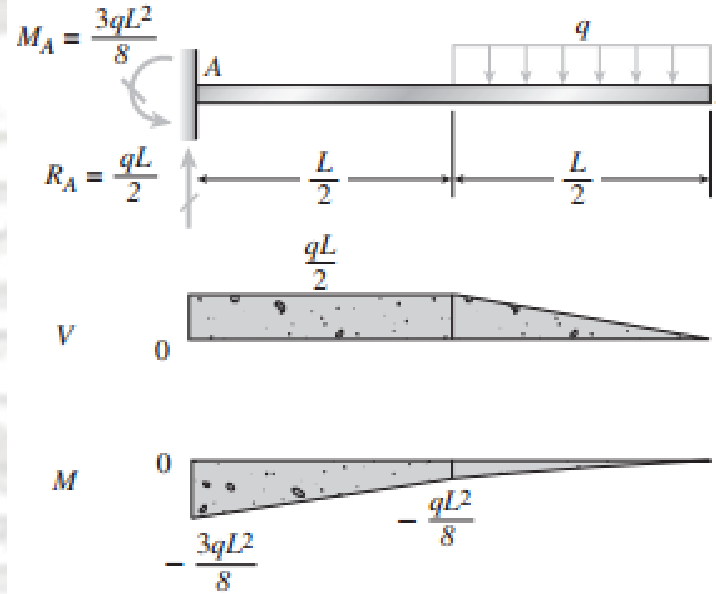
change in shear	area under distributed loading	change in moment	area under shear diagram

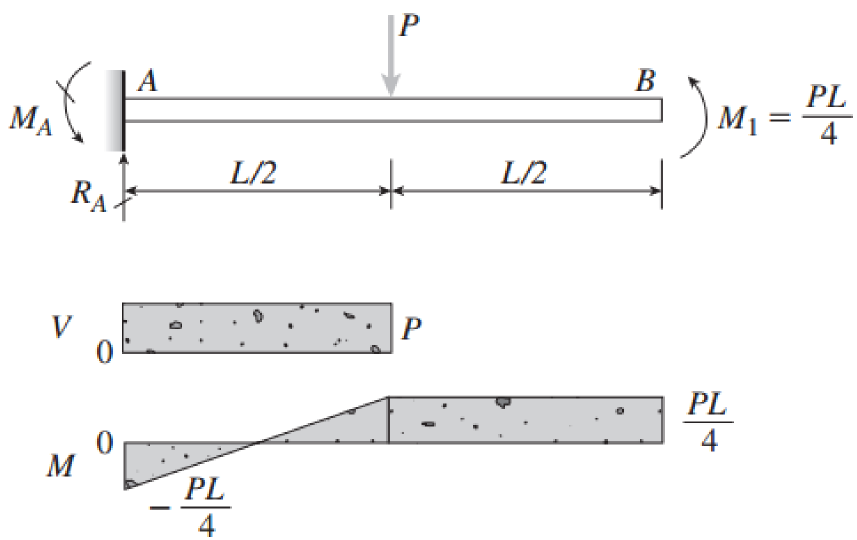


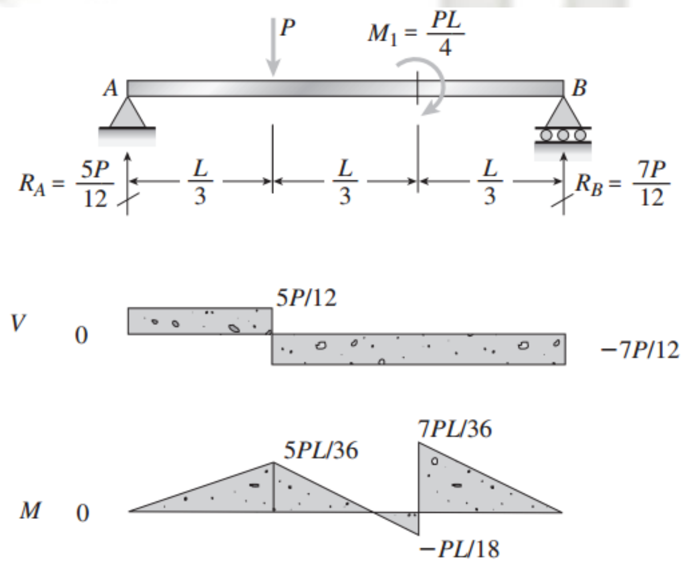
EXAMPLE 3-10

Draw the shear and moment diagrams for the beams shown in the figures.



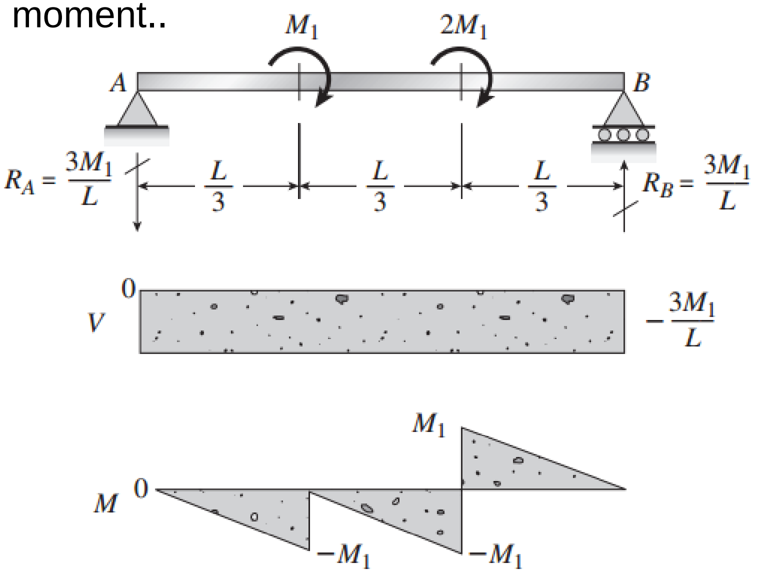




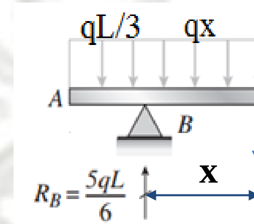


EXAMPLE 3-11

Draw the shear and moment diagrams for the beams shown in the figures and find the position of max. bending moment..



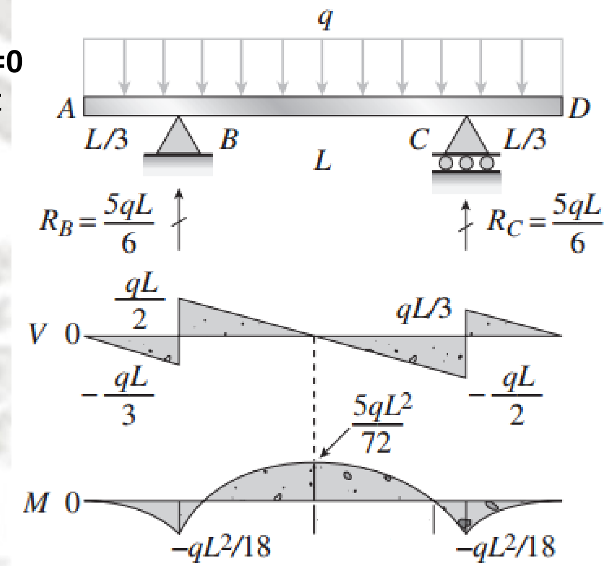
Maximum bending moment occurs when V=0 or in the interior support

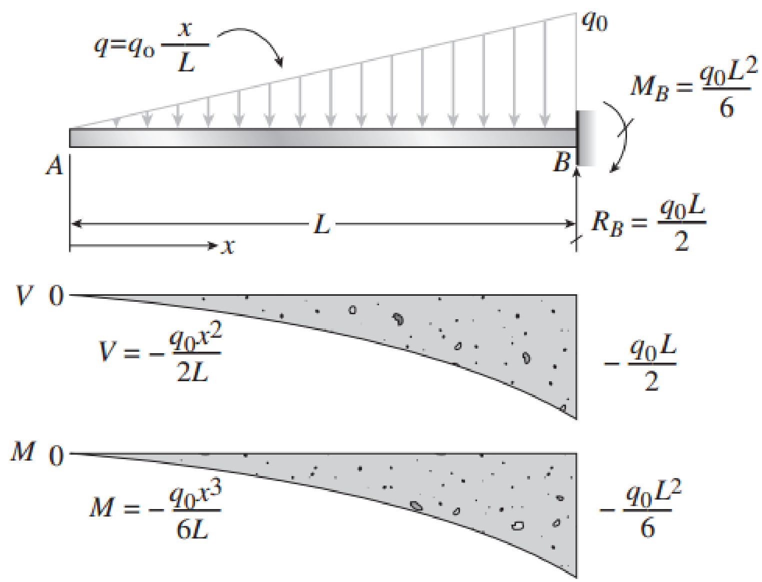


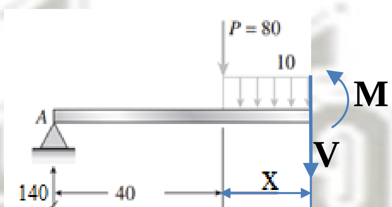
$$\sum Fy = 0$$

$$\frac{qL}{3} + qx + V = \frac{5qL}{6}$$

$$V = 0 \Rightarrow \frac{qL}{2} = qx \Rightarrow x = \frac{L}{2}$$



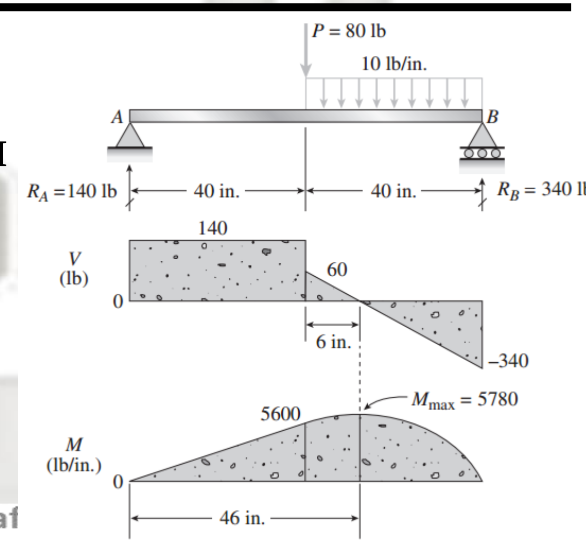




$$\sum Fy = 0$$

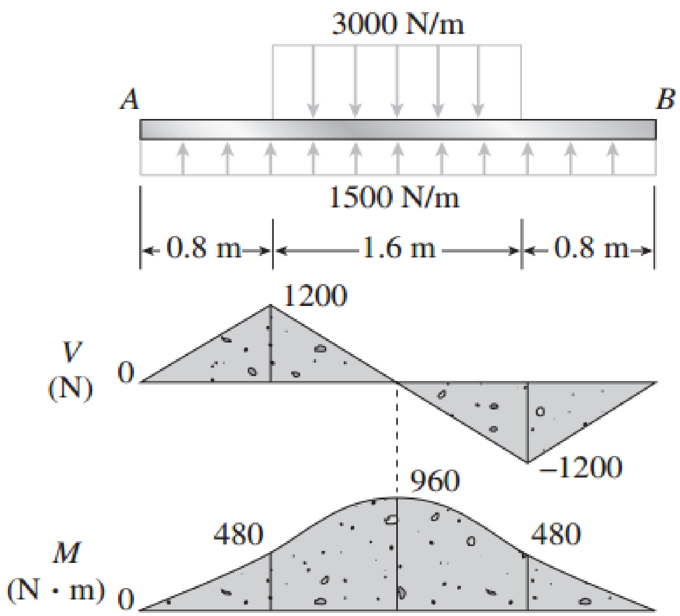
$$80 + 10x + V = 140$$

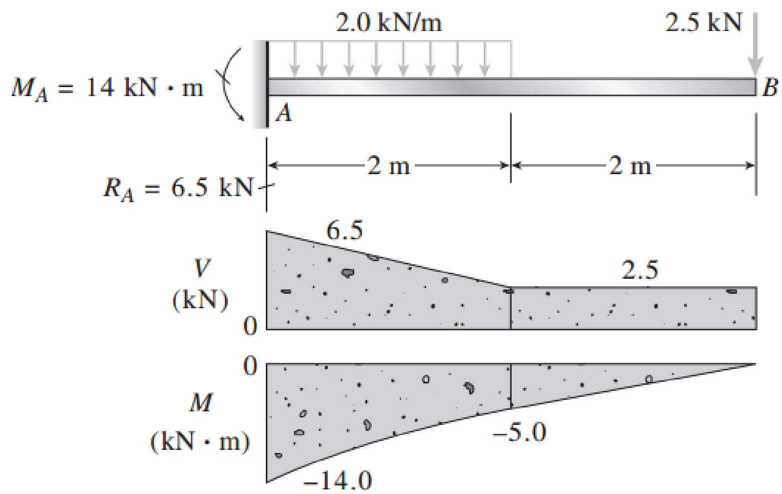
$$x = 6in$$

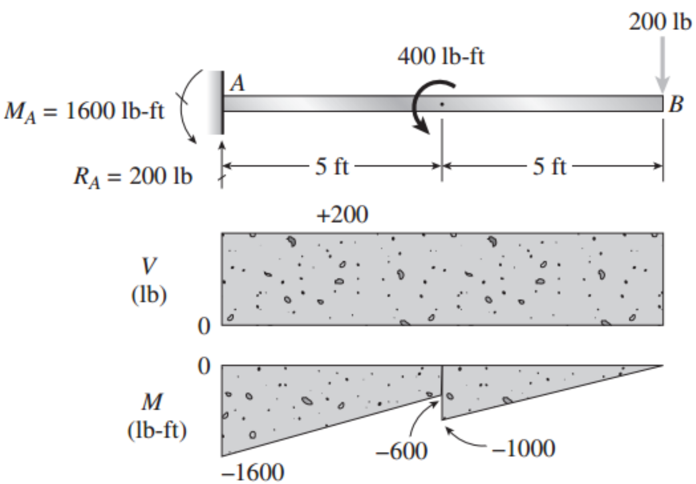


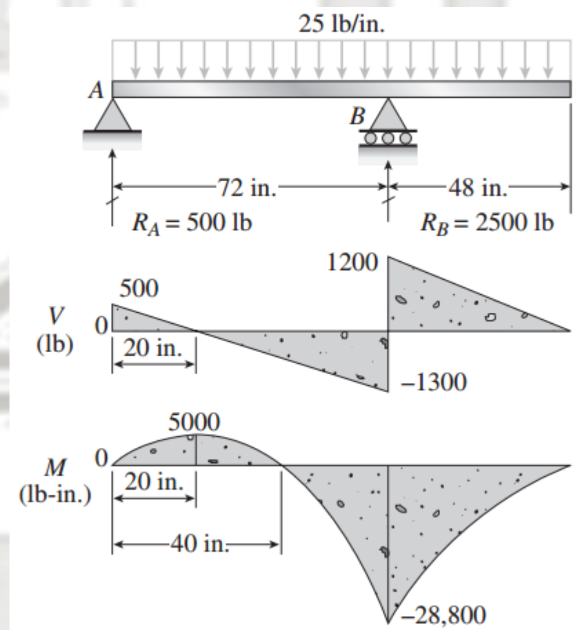
EXAMPLE 3-12

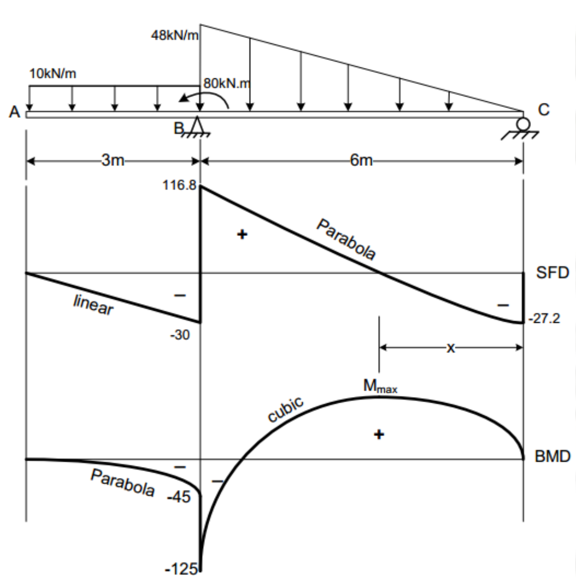
Draw the shear and moment diagrams for the beams shown in the figures and find the position of max. bending moment..

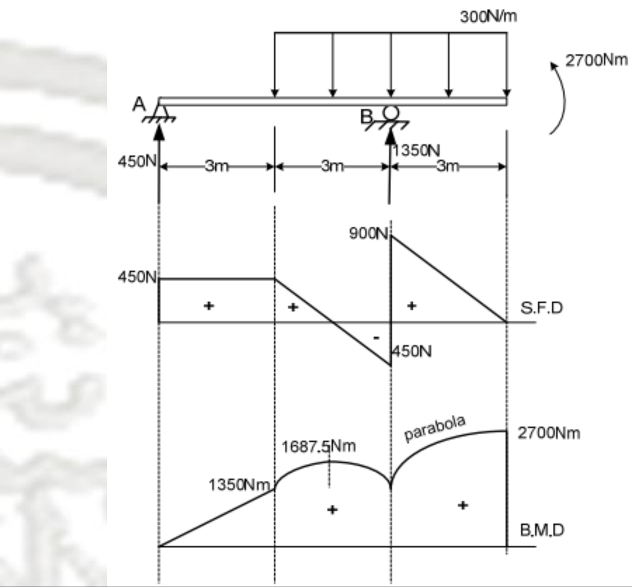


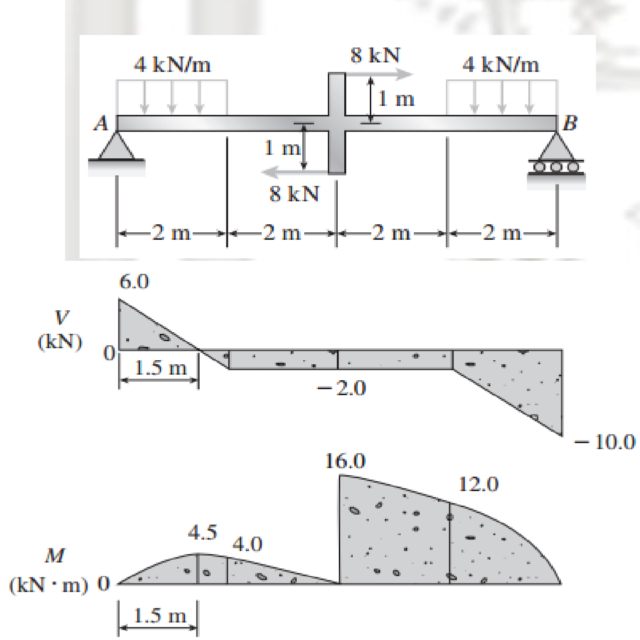


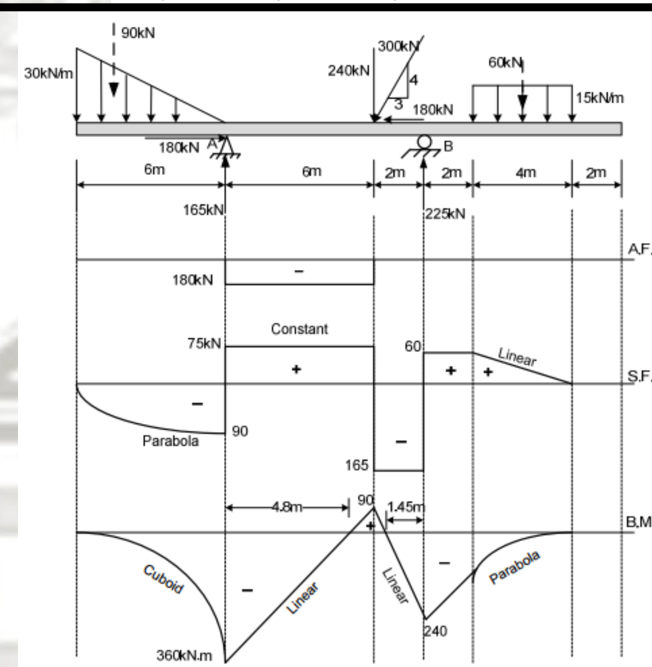





48kN/m
10kN/m
80kN.m
A
B
C
3m
6m
116.8
Parabola
SFD
linear
-30
-27.2
x
Mmax
cubic
BMD
Parabola
-45
-125

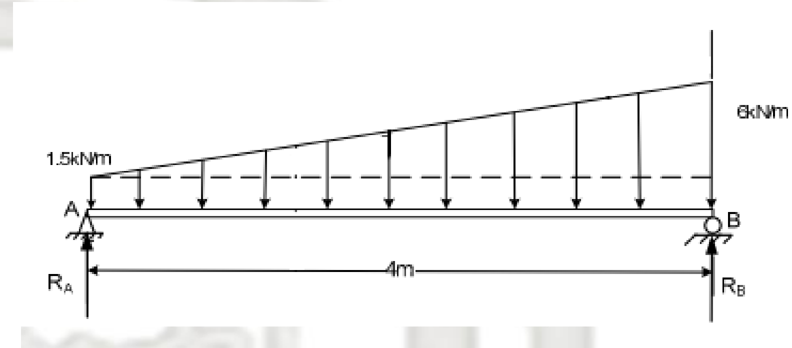

300N/m
2700Nm
A
B
450N
1350N
3m
3m
3m
900N
450N
S.F.D
450N
1687.5Nm
parabola
2700Nm
1350Nm
B.M.D

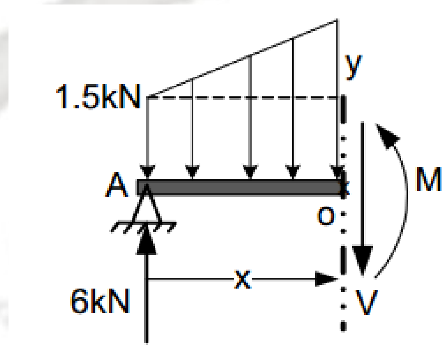

4 kN/m
8 kN
4 kN/m
1 m
A
B
1 m
8 kN
2 m
2 m
2 m
2 m
6.0
V
(kN)
0
1.5 m
−2.0
−10.0
16.0
12.0
4.5
4.0
M
(kN · m)
0
1.5 m


90kN
30kN/m
300kN
240kN
4
3
180kN
60kN
15kN/m
180kN
A
B
6m
6m
2m
2m
4m
2m
165kN
225kN
A.F.D
180kN
75kN
Constant
60
Linear
S.F.D
Parabola
90
165
4.8m
90
1.45m
B.M.D
Cuboid
Linear
Linear
Parabola
240
360kN.m

EXAMPLE 3-13

Draw the shear and moment diagrams for the beam shown in the figure and find the position of max. bending moment..





$$\sum Fy = 0$$

$$6 = 1.5x + \frac{4.5x^2}{8} + V$$

$$V = 0$$

$$6 = 1.5x + \frac{4.5x^2}{8}$$

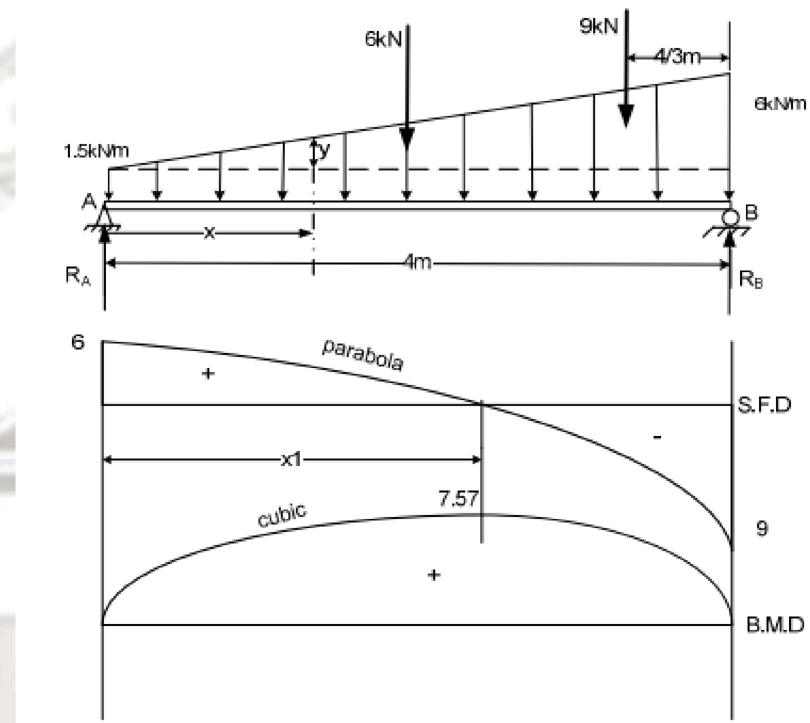
$$0.563x^2 + 1.5x - 6 = 0$$

$$x = \frac{-1.5 \pm \sqrt{(-1.5)^2 + 4 \times 0.563 \times 6}}{2 \times 0.563}$$

$$x = 2.2m$$

$$\sum M_o = 0$$

$$M + \frac{1.5x^2}{2} + \frac{4.5x}{4} \times \frac{x}{2} \times \frac{x}{3} - 6x = 0$$



EXAMPLE 3-14

Draw the shear and moment diagrams for the beam shown in the figure and find the position of max. bending moment..