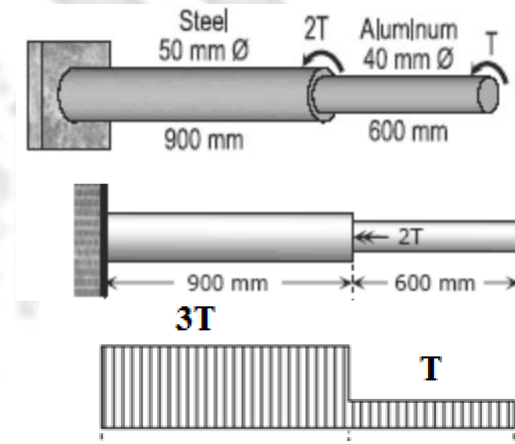
EXAMPLE 4-11

A compound shaft consisting of a steel segment and an aluminum segment is acted upon by two torques as shown in the figure. Determine the maximum permissible value of T subject to the following conditions: τ_{st}= 83 MPa, τ_{al}= 55 MPa, and the angle of rotation of the free end is limited to 6°. For steel, G = 83 GPa and for aluminum, G = 28 GPa.



Based on the maximum shearing stress:

$$\tau_{st}=\frac{T_{\max}r}{J}=\frac{2T_{\max}}{\pi\times r^3}$$

$$83=\frac{2\times 3T}{\pi\times 25^3}$$

$$T=679042.16N.mm\Rightarrow 679.04N.m$$

$$\tau_{al}=\frac{Tr}{J}=\frac{2T}{\pi\times r^3}$$

$$55=\frac{2\times T}{\pi\times 20^3}$$

$$T=691150.38N.mm\Rightarrow 691.15N.m$$

Based on the maximum angle of twist:

$$\phi=\phi_{st}+\phi_{al}=(\frac{TL}{GJ})_{st}+(\frac{TL}{GJ})_{al}$$

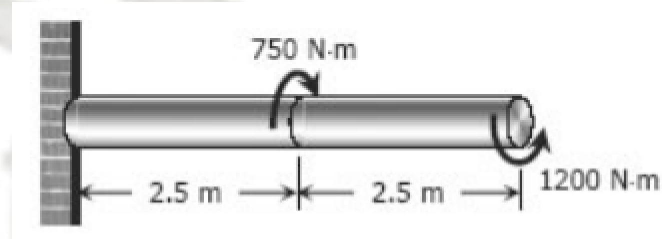
$$6\times\frac{\pi}{180}=\frac{3T\times 900}{\frac{\pi}{2}\times 25^4\times 83000}+\frac{T\times 600}{\frac{\pi}{2}\times 20^4\times 28000}$$

$$T=757316.32N.mm=757.32N.m$$

$$\therefore T=679.04N.m$$

EXAMPLE 4-12

A solid steel shaft is loaded as shown in figure. Using G = 83 GPa, determine the required diameter of the shaft if the shearing stress is limited to 60 MPa and the angle of rotation at the free end is not to exceed 4 deg.



Based on the maximum allowable shearing stress:

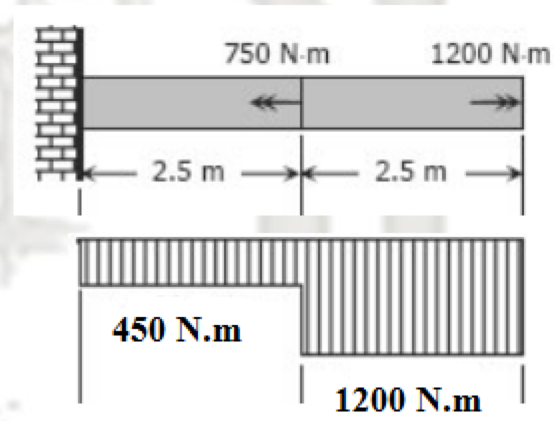
$$\tau_{\max} = \frac{T_{\max} r}{J} = \frac{2T_{\max}}{\pi \times r^3}$$

$$60 = \frac{2 \times 450 \times 10^3}{\pi \times r^3}$$

$$r = 16.838mm \Rightarrow D = 33.677mm$$

$$60 = \frac{2 \times 1200 \times 10^3}{\pi \times r^3}$$

$$r = 23.35mm \Rightarrow D = 46.7mm$$



Based on the maximum angle of twist:

$$\phi = \sum \frac{TL}{GJ}$$

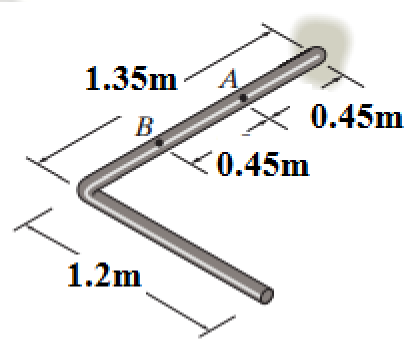
$$4 \times \frac{\pi}{180} = \frac{(450 + 1200) \times 10^3 \times 2500}{\frac{\pi}{2} \times r^4 \times 83000}$$

$$r = 25.94mm \Rightarrow D = 51.89mm$$

$$\therefore D = 51.89mm$$

EXAMPLE 4-13

The rod has a diameter of 25mm and a weight of 15 N/m. Determine the maximum torsional stress in the rod at a section located at B due to the rod's weight



$$\sum M_x = 0$$

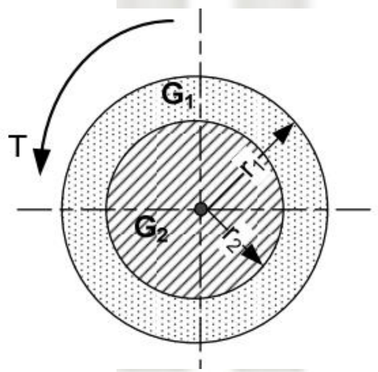
$$T_B = 15\times1.2\times0.6 = 10.8\,N.m$$

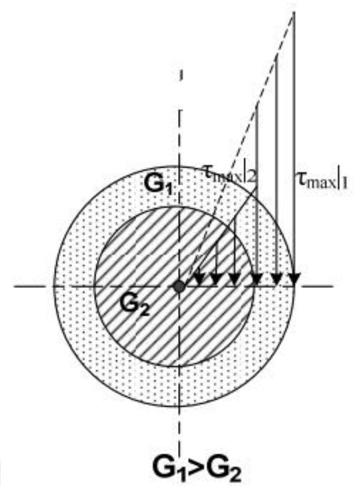
$$J = \frac{\pi}{2}r^4 = \frac{\pi}{2}12.5^4 = 38.35mm^4$$

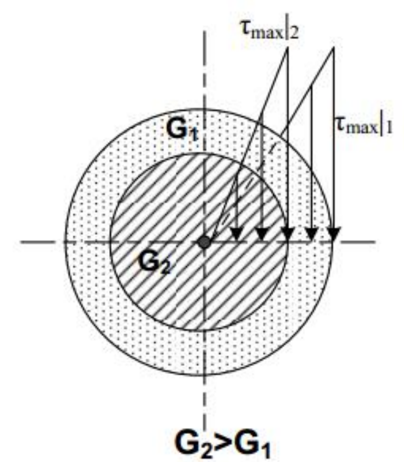
$$\tau_{\max} = \frac{Tr}{J} = \frac{10.8\times10^3\times12.5}{38.35} = 3.52MPa$$

Statically Indeterminate Problems

Equilibrium Equation: $T_1 + T_2 = T \rightarrow (1)$ **Compatibility Equation:** $\phi_1 = \phi_2 \rightarrow (2)$

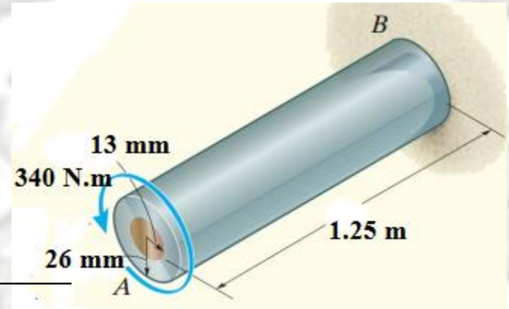


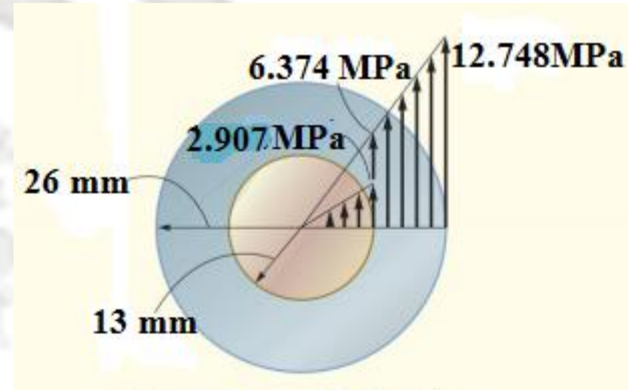




EXAMPLE 4-14

The shaft shown in figure is made from a steel tube, which is bonded to a brass core. If a torque of T = 340 N.m is applied at its end, plot the shear-stress distribution along a radial line on its cross section. Take Gst = 78600 MPa and Gbr =35850.





Shear–stress distribution

$$T_{st} + T_{br} = 340 \rightarrow (1)$$

$$\phi = \phi_{st} = \phi_{br} \Rightarrow \left(\frac{TL}{GJ}\right)_{st} = \left(\frac{TL}{GJ}\right)_{br}$$

$$\frac{T_{st} \times L}{\frac{\pi}{2} \times (26^4 - 13^4) \times 78600} = \frac{T_{br} \times L}{\frac{\pi}{2} \times 13^4 \times 35850}$$

$$T_{st} = \frac{T_{br} \times (26^4 - 13^4) \times 78600}{13^4 \times 35850} \Rightarrow T_{st} = 32.887 T_{br} \rightarrow (2)$$

$$\therefore T_{br} = 10.033 N.m$$

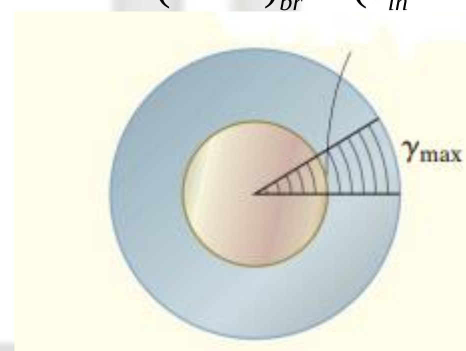
$$T_{st} = 329.966 N.m$$

$$\gamma_{br} = (\gamma_{st})_{\min}$$

$$(\tau \times G)_{br} = (\tau_{in} \times G)_{st}$$

$$(\tau_{br})_{\max} = \frac{T_{br} r}{J} = \frac{2T_{\max}}{\pi \times r^3} = \frac{2 \times 10.033 \times 10^3}{\pi \times 13^3} = 2.907 MPa$$

$$(\tau_{st})_{\max} = \frac{T_{st} r_2}{J} = \frac{T_{st} r_2}{\frac{\pi}{2}(r_2^4 - r_1^4)} = \frac{329.966 \times 10^3 \times 26}{\frac{\pi}{2}(26^4 - 13^4)} = 12.748 MPa$$

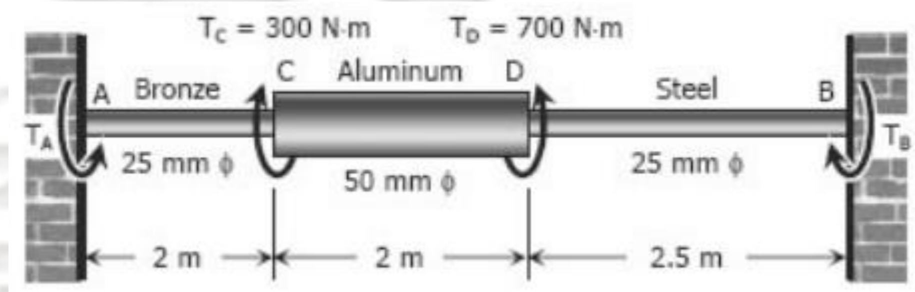


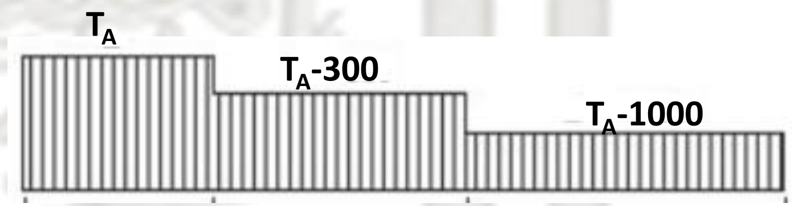
Shear–strain distribution

$$(\tau_{st})_{\min} = \frac{T_{st} r_1}{J} = \frac{T_{st} r_1}{\frac{\pi}{2}(r_2^4 - r_1^4)} = \frac{329.966 \times 10^3 \times 13}{\frac{\pi}{2}(26^4 - 13^4)} = 6.374 MPa$$

EXAMPLE 4-15

A shaft composed of segments AC, CD, and DB is fastened to rigid supports and loaded as shown in figure. For bronze, G = 35 GPa; aluminum, G = 28 GPa, and for steel, G = 83 GPa. Determine the maximum shearing stress developed in each segment.





$$\phi_{A/B} = 0$$

$$\sum\left(\frac{TL}{GJ}\right)_{A/B} = 0$$

$$\frac{T_A \times 10^3 \times 2000}{\frac{\pi}{2}12.5^4 \times 35000} + \frac{(T_A - 300)\times 10^3 \times 2000}{\frac{\pi}{2}25^4 \times 28000} + \frac{(T_A - 1000)\times 10^3 \times 2500}{\frac{\pi}{2}12.5^4 \times 83000} = 0$$

$$\frac{2T_A}{12.5^4 \times 35} + \frac{2(T_A - 300)}{25^4 \times 28} + \frac{2.5(T_A - 1000)}{12.5^4 \times 83} = 0$$

$$\frac{16T_A}{35} + \frac{(T_A - 300)}{28} + \frac{20(T_A - 1000)}{83} = 0$$

$$\frac{16}{35}T_A + \frac{1}{28}T_A - \frac{25}{7} + \frac{20}{83}T_A - \frac{20000}{83} = 0$$

$$T_A = 342.97\,N.m$$

$$\sum M_x = 0$$

$$T_A + T_B = 300 + 700$$

$$342.97 + T_B = 1000 \rightarrow T_B = 657.03 N.m$$

$$T_{br} = 342.97 N.m, T_{al} = 342.97 - 300 = 42.97 N.m, T_{st} = 342.97 - 1000 = -657.03 N.m$$

$$\tau_{\max} = \frac{Tr}{J} = \frac{2T}{\pi \times r^3}$$

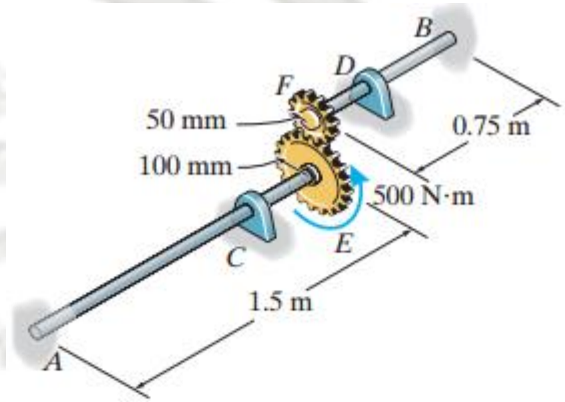
$$\tau_{br} = \frac{2 \times 342.97 \times 10^3}{\pi \times 12.5^3} = 111.79 MPa$$

$$\tau_{al} = \frac{2 \times 42.97 \times 10^3}{\pi \times 25^3} = 1.75 MPa$$

$$\tau_{st} = \frac{2 \times 657.03 \times 10^3}{\pi \times 12.5^3} = 214.16 MPa$$

EXAMPLE 4-15

The two shafts are made of steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. If a torque of is applied to the gear at E as shown, determine the reactions at A and B and the angle of twist at point E. (G=75GPa)



$$T_A + F \times 0.1 - 500 = 0$$

$$T_B - F \times 0.05 = 0$$

$$T_A + 2T_B - 500 = 0 \rightarrow (1)$$

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A \times 1.5}{GJ} = 0.5 \times \frac{T_B \times 0.75}{GJ}$$

$$T_A = 0.25T_B$$

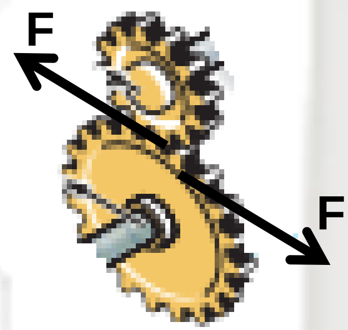
$$T_A = 55.56 N.m$$

$$T_B = 222.22 N.m$$

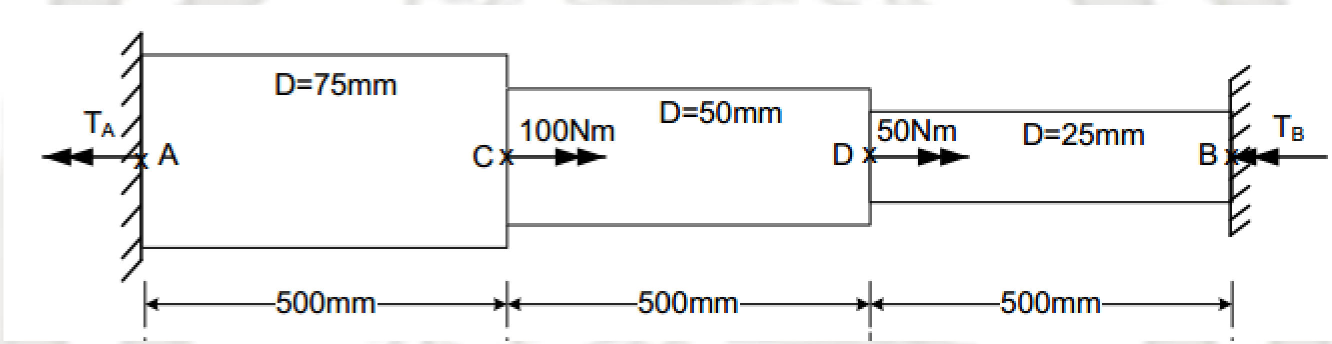
$$\phi_E = \frac{T_A \times L}{GJ}$$

$$\phi_E = \frac{55.56 \times 10^3 \times 1500}{\frac{\pi}{2} \times 12.5^4 \times 75000}$$

$$\phi_E = 0.02897 rad. = 1.66^\circ$$



H.W: A stepped solid circular steel shaft has the shape shown in the figure below and is having G=80*103MPa. The region AC is having D=75mm region CD having D=50mm, and region BD having D=25mm. Determine the maximum shearing stress occurs in the shaft as well as the angle of twist at C where a torsional load of 100N m is applied. Ends A and B are rigidly clamped.



H.W: The shaft is made of L2 tool steel, has a diameter of 40 mm, and is fixed at its ends A and B. If it is subjected to the torque, determine the maximum shear stress in regions AC and CB.