Al-Mustansiriyah University/ Faculty of Engineering
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Mathematics Lecture
2^{nd} class

Q1/ Solve the following first order differential equation:

$$-ydx + \left(x + \sqrt{xy}\right)dy = 0$$

Test for homogenous

$$\frac{dy}{dx} = \frac{y}{x+\sqrt{xy}} = \frac{\lambda y}{\lambda x + \sqrt{\lambda x \lambda y}} = \frac{\cancel{\lambda} y}{\cancel{\lambda} x + \cancel{\lambda}\sqrt{xy}}$$

$$\frac{dy}{dx} = \frac{y}{x+\sqrt{xy}} \qquad \therefore \text{homogenous}$$

let $\frac{y}{x} = v \longrightarrow y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{\frac{y}{x}}{\frac{x}{x} + \frac{\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}}\frac{\sqrt{y}}{\sqrt{x}}} = \frac{\frac{y}{x}}{1+\sqrt{\frac{y}{x}}}$$

$$v + x\frac{dv}{dx} = \frac{v}{1+\sqrt{v}}$$

$$x\frac{dv}{dx} = \frac{v}{1+\sqrt{v}} - v$$

$$x\frac{dv}{dx} = -\frac{v\sqrt{v}}{1+\sqrt{v}}\, dv$$

$$-\frac{dx}{x} = 1 + \frac{1+\sqrt{v}}{v\sqrt{v}}$$

$$-\frac{dx}{x} = \left(\frac{1}{v\sqrt{v}} + \frac{\cancel{\sqrt{v}}}{v\cancel{\sqrt{v}}}\right) dv$$

$$-\frac{dx}{x} = \left(v^{-3/2} + \frac{1}{v}\right) dv$$

$$-\ln x = \frac{-2}{\sqrt{v}} + \ln v + c$$

$$-\ln x - \ln v = -\frac{2}{\sqrt{v}} + c$$

$$-\ln x - \ln\left(\frac{y}{x}\right) = -\frac{2}{\sqrt{\frac{y}{x}}} + c$$

$$-\cancel{\ln x} - \ln y + \cancel{\ln x} = -\frac{2\sqrt{x}}{\sqrt{y}} + c$$

$$(\ln y + c)\sqrt{y} = 2\sqrt{x}$$

$$4x = (\ln y + c)^2 y$$

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Q1/ Solve the following first order differential equation:

$$2x + x\,e^{xy}y' + ye^{xy} = -2yy'$$

$$2x + \underbrace{xe^{xy}y'}_{} + ye^{xy} + \underbrace{2yy'}_{} = 0$$

$$(2x + ye^{xy}) + (2y + xe^{xy})\frac{dy}{dx} = 0$$

$$\underbrace{(2x + ye^{xy})}_{M}dx + \underbrace{(2y + xe^{xy})}_{N}dy = 0$$

$$\frac{\partial M}{\partial y} = 0 + y\cdot xe^{xy} + e^{xy}\cdot 1 = xye^{xy} + e^{xy}$$

$$\frac{\partial N}{\partial x} = 0 + x\cdot ye^{xy} + e^{xy}\cdot 1 = xye^{xy} + e^{xy}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \therefore \text{ exact}$$

① $M = \frac{\partial f}{\partial x} \longrightarrow f = \int M\,dx$

$$f = \int (2x + ye^{xy})\,dx = x^2 + e^{xy} + g(y) + c$$

② $N = \frac{\partial f}{\partial y}$

$$2y + \cancel{xe^{xy}} = 0 + \cancel{xe^{xy}} + g'(y) + 0$$

$$g'(y) = 2y \longrightarrow g(y) = \int g'(y)\,dy = \int 2y\,dy = y^2$$

$$f = x^2 + e^{xy} + y^2 + c$$

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