Higher order Differential Equation.

Ex1/ solve:

$$y''' + y'' - 6y' + 4y = 0$$

let $y = e^{mx}$

$$y' = me^{mx}$$

$$y'' = m^2 e^{mx}$$

$$y''' = m^3 e^{mx}$$

$$m^3 e^{mx} + m^2 e^{mx} - 6m e^{mx} + 4e^{mx} = 0$$

$$(m^3 + m^2 - 6m + 4)\, e^{mx} = 0 \quad (e^{mx} \neq 0)$$

$$m^3 + m^2 - 6m + 4 = 0$$

$$\begin{array}{r|l} & m^2 + 2m - 4 \\ \hline m - 1 & m^3 + m^2 - 6m + 4 \\ & -m^3 \pm m^2 \\ \hline & 2m^2 - 6m + 4 \\ & -2m^2 \pm 2m \\ \hline & -4m + 4 \\ & -4m \mp 4 \\ \hline & 0 \quad 0 \end{array}$$

$$(m-1)(m^2+2m-4)=0$$

either $m-1=0 \longrightarrow m_1=1$

or $(m^2+2m-4)=0$

$$m_{2,3}=\frac{-2\mp\sqrt{4+16}}{2}=-1\mp\sqrt{5}$$

$$m_2=-1+\sqrt{5}$$

$$m_3=-1-\sqrt{5}$$

$$y=c_1e^{x}+c_2e^{(-1+\sqrt{5})x}+c_3e^{(-1-\sqrt{5})x}$$

Ex2: solve:

$$y''' - 3y'' + 3y' - y = 10x e^{2x}$$

homogenous part:

$$y''' - 3y'' + 3y' - y = 0$$

$$\text{let } y = e^{mx} \rightarrow y' = me^{mx} \rightarrow y'' = m^2 e^{mx} \rightarrow y''' = m^3 e^{mx}$$

$$m^3 e^{mx} - 3m^2 e^{mx} + 3me^{mx} - e^{mx} = 0$$

$$(m^3 - 3m^2 + 3m - 1)\, e^{mx} = 0 \qquad e^{mx} \neq 0$$

$$m^3 - 3m^2 + 3m - 1 = 0$$

$$\begin{array}{r|l} m^3 - 3m^2 + 3m - 1 & m - 1 \\ -m^3 \pm m^2 & \hline m^2 - 2m + 1 \\ \hline -2m^2 + 3m - 1 & \\ -2m^2 + 2m & \\ \hline m - 1 & \\ -m \pm 1 & \\ \hline 0 \quad 0 & \end{array}$$

$$(m-1)(m^2-2m+1)=0$$

$$(m-1)(m-1)(m-1)=0$$

either $m-1=0 \longrightarrow m_1=1$
or $m-1=0 \longrightarrow m_2=1$
or $m-1=0 \longrightarrow m_3=1$

$$y_h = c_1 e^x + c_2 x e^x + c_3 x^2 e^x$$

Particular part:

$$y_p = (a_0 x + a_1) e^{2x}$$

$$y_p = a_0 x e^{2x} + a_1 e^{2x}$$

$$y_{p_1}' = a_0 x \cdot 2e^{2x} + a_0 e^{2x} + 2a_1 e^{2x}$$

$$y_{p_1}' = 2a_0 x e^{2x} + a_0 e^{2x} + 2a_1 e^{2x}$$

$$y_{p_1}'' = 2a_0 x \cdot 2e^{2x} + 2a_0 \cdot e^{2x} + 2a_0 e^{2x} + 4a_1 e^{2x}$$

$$y_{p_1}'' = 4a_0 x e^{2x} + 4a_0 e^{2x} + 4a_1 e^{2x}$$

$$y_{p_1}''' = 4a_0 x \cdot 2e^{2x} + 4a_0 e^{2x} + 8a_0 e^{2x} + 8a_1 e^{2x}$$

$$y_{p_1}''' = 8a_0 x e^{2x} + 12a_0 e^{2x} + 8a_1 e^{2x}$$

$$8a_0 x e^{2x} + 12a_0 e^{2x} + 8a_1 e^{2x}$$

$$- 3(4a_0 x e^{2x} + 4a_0 e^{2x} + 4a_1 e^{2x})$$

$$+ 3(2a_0 x e^{2x} + a_0 e^{2x} + 2a_1 e^{2x})$$

$$- a_0 x e^{2x} + a_1 e^{2x} = 10 x e^{2x}$$

$$8a_0 - 12a_0 + 6a_0 - a_0 = 10$$

$$a_0 = 10$$

$$12a_0 + 8a_1 - 12a_0 - 12a_1 + 3a_0 + 6a_1$$

$$- a_1 = 0$$

$$3a_0 + a_1 = 0$$

$$a_1 = -30$$

$$y_p = 10 x e^{2x} - 30 e^{2x}$$

$$y = y_h + y_p$$

$$= c_1 e^{x} + c_2 x e^{x} + c_3 x^2 e^{x} + 10 x e^{2x} - 30 e^{2x}$$

EX3: Solve

$$y''' - y' = x^2$$

homogenous part:

$$y''' - y' = 0$$

Let $y = e^{mx} \rightarrow y' = me^{mx} \rightarrow y'' = m^2 e^{mx} \rightarrow y''' = m^3 e^{mx}$

$$m^3 e^{mx} - me^{mx} = 0$$

$$(m^3 - m)e^{mx} = 0 \qquad e^{mx} \neq 0$$

$$m^3 - m = 0$$

$$m(m^2 - 1) = 0$$

either $m = 0$

or $m^2 - 1 = 0$

$$m^2 = 1$$

$$m_{2,3} = \mp 1$$

$$y_h = c_1 + c_2 e^{x} + c_2 e^{-x}$$

particular part:

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$$y_p = X(a_0 X^2 + a_1 X + a_2)$$

$$y_p = a_0 X^3 + a_1 X^2 + a_2 X$$

$$y_p' = 3a_0 X^2 + 2a_1 X + a_2$$

$$y_p'' = 6a_0 X + 2a_1$$

$$y_p''' = 6a_0$$

$$6a_0 - (3a_0 X^2 + 2a_1 X + a_2) = X^2$$

$$6a_0 - 3a_0 X^2 - 2a_1 X - a_2 = X^2$$

$-3a_0 = 1 \longrightarrow a_0 = -\frac{1}{3}$ معامل X^2

$-2a_1 = 0 \longrightarrow a_1 = 0$ معامل X

$6a_0 - a_2 = 0$ الثوابت

$6(-\frac{1}{3}) + a_2 = 0 \longrightarrow a_2 = 2$

$$y_p = -\frac{1}{3} X^3 + 2X$$

$$y = y_h + y_p = c_1 + c_2 e^X + c_3 e^{-X} - \frac{1}{3} X^3 + 2X$$