Al-Mustansiriyah University Faculty of Engineering Mech. Eng. Dept. Graduate studies Exam :1		**Subject: Math. II** **Max. Time: 100 Mins.** **Class:2nd** **Date: 24 / 12 / 2017**
	2017 – 2018	**Max.Mark:100**

<u>Anser only Two quastions</u>

Q1 A / Solve the following differential equation

$$[e^x \cos(y) + (1-x)\sin(y)]\frac{dy}{dx} + e^x(1+\sin(y)) + \cos(y) = 0$$

B/ Solve the following differential equation using **(variation of parameter method)**

$$\bar{\bar{y}} - 6\bar{y} + 9y = x^{-3}e^{3x}$$

…………….. (50mark)

Q2 A / Solve the following differential equation

$$3x\bar{y} - y = \ln(x) + 1$$

B/ Solve the following differential equation using **(undetermined coefficient method)**

$$\bar{\bar{y}} + 9y = 9x - \cos(x)$$

…………….. (50mark)

Q3 A / Solve the following differential equation

$$(2xy + 3y^2)dx - (x^2 + 2xy)dy = 0$$

B/ Find the general solution of

$$y^{\equiv} + 3y^{=} + 3y^{-} + y = 4x^3 + 3e^x$$

…………….. (50mark)

Examiner: ***Asst. Prof. Dr. Hayder M. Jaffal*** **Head of Department:** ***Prof. Dr. Adnan A. Abdul Rasool***

Q1/A

$$[\underbrace{e^x\cos y+(1-x)\sin y}_{N}]dy+[\underbrace{e^x(1+\sin y)+\sin y}_{M}]dx=0$$

$$M=e^x(1+\sin y)+\cos y$$

$$N=e^x\cos y+(1-x)\sin y$$

$$\left.\begin{array}{l}\dfrac{\partial M}{\partial y}=e^x\cos y-\sin y\\[2ex] \dfrac{\partial N}{\partial x}=e^x\cos y-\sin y\end{array}\right]\therefore \text{exact}$$

① $M=\dfrac{\partial f}{\partial x}\longrightarrow f=\int M\,dx$

$$f=\int[e^x(1+\sin y)+\cos y]\,dx$$

$$=e^x(1+\sin y)+\cos y\,x+g(y)+C$$

② $N=\dfrac{\partial f}{\partial y}$

$$\cancel{e^x\cos y}+\sin y-\cancel{x\sin y}=\cancel{e^x\cos y}-\cancel{x\sin y}$$
$$+g'(y)+0$$

$g'(y) = \sin y$

$g(y) = \int g'(y)\,dy = \int \sin y\,dy = -\cos y$

$\therefore f = e^x(1+\sin y) + \cos y\, x - \cos y + c$

$= e^x(1+\sin y) + (x-1)\cos y + c$

Q1 / B

$$y'' - 6y' + 9y = x^{-3} e^{3x}$$

homogenous part:

$$y'' - 6y' + 9y = 0$$

let $y = e^{mx} \longrightarrow y' = m e^{mx} \longrightarrow y'' = m^2 e^{mx}$

$$m^2 e^{mx} - 6m e^{mx} + 9 e^{mx} = 0$$

$$(m^2 - 6m + 9) e^{mx} = 0 \qquad e^{mx} \neq 0$$

$$m^2 - 6m + 9 = 0$$

$$(m-3)(m-3) = 0$$

either $m - 3 = 0 \longrightarrow m_1 = 3$

or $m - 3 = 0 \longrightarrow m_2 = 3$

$$y_h = c_1 \underbrace{e^{3x}}_{u_1} + c_2 \underbrace{x e^{3x}}_{u_2}$$

Non-homogneuas part:

$$w = \begin{vmatrix} u_1 & u_2 \\ u_1' & u_2' \end{vmatrix} = \begin{vmatrix} e^{3x} & x e^{3x} \\ 3e^{3x} & e^{3x} + 3x e^{3x} \end{vmatrix}$$

$$= e^{6x} + \cancel{3x e^{6x}} - \cancel{3x e^{6x}} = e^{6x}$$

$$w_1 = \begin{vmatrix} 0 & u_2 \\ f(x) & u_2' \end{vmatrix} = \begin{vmatrix} 0 & xe^{3x} \\ x^{-3}e^{3x} & e^{3x}+3xe^{3x} \end{vmatrix}$$

$$w_1 = -x^{-2}e^{6x}$$

$$w_2 = \begin{vmatrix} u_1 & 0 \\ u_1' & f(x) \end{vmatrix} = \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & x^{-3}e^{3x} \end{vmatrix}$$

$$w_2 = x^{-3}e^{6x}$$

$$v_1' = \frac{w_1}{w} = \frac{-x^{-2}e^{6x}}{e^{6x}} = -x^{-2}$$

$$v_1 = \int v_1'\,dx = \int -x^{-2}\,dx = \frac{1}{x}$$

$$v_2' = \frac{w_2}{w} = \frac{x^{-3}e^{6x}}{e^{6x}} = x^{-3}$$

$$v_2 = \int v_2'\,dx = \int x^{-3}\,dx = -\frac{x^{-2}}{2}$$

$$y_p = u_1v_1 + u_2v_2$$

$$= e^{3x}\cdot\frac{1}{x} + (xe^{3x})\left(-\frac{x^{-2}}{2}\right)$$

$$= \frac{e^{3x}}{x} - \frac{e^{3x}}{2x} = \frac{e^{3x}}{2x}$$

$$y = y_h + y_p$$

$$y = c_1e^{3x} + c_2xe^{3x} + \frac{e^{3x}}{2x}$$

Q2 A

$$3xy' - y = \ln x + 1$$

$$y' \underbrace{- \frac{1}{3x}}_{P(x)} y = \frac{\ln x + 1}{3x} \quad Q(x) \qquad \text{linear}$$

$$I\cdot F = e^{\int P(x)\,dx}$$

$$\int P(x)\,dx = \int -\frac{1}{3x}\,dx = -\frac{1}{3}\ln x = \ln x^{-\frac{1}{3}}$$

$$I\cdot f = e^{\ln x^{-\frac{1}{3}}} = x^{-\frac{1}{3}}$$

$$y = \frac{1}{I\cdot F}\int I\cdot f\, Q(x)\,dx + \frac{c}{I\cdot f}$$

$$y = x^{\frac{1}{3}} \int \left[x^{-\frac{1}{3}} \cdot \frac{\ln x + 1}{3x} \right] dx + c\,x^{\frac{1}{3}}$$

$$= x^{\frac{1}{3}} \int \frac{1}{3}[\ln x + 1]\, x^{-4/3}\,dx + c\,x^{\frac{1}{3}}$$

$$= x^{\frac{1}{3}} \left[\frac{1}{3} \int (\ln x \cdot x^{-\frac{4}{3}} + x^{-\frac{4}{3}})\,dx + c\,x^{\frac{1}{3}} \right.$$

$$= x^{\frac{1}{3}} \left[\frac{1}{3} \left[\int \underbrace{\ln x \cdot x^{-\frac{4}{3}}}_{u\,dv}\,dx + \int x^{-\frac{4}{3}}\,dx \right] \right] + c\,x^{\frac{1}{3}}$$

$$\text{let } u = \ln x \longrightarrow du = \frac{1}{x}\,dx$$

$$dv = x^{-\frac{4}{3}}\,dx \longrightarrow v = -3\,x^{-\frac{1}{3}}$$

$$v \cdot u - \int v\,du = -3x^{-\frac{1}{3}} \ln x - \int \frac{-3x^{-\frac{1}{3}}}{x} dx$$

$$= -3x^{-\frac{1}{3}} \ln x - 9x^{-\frac{1}{3}}$$

$$\int x^{-\frac{4}{3}} dx = -3x^{-\frac{1}{3}}$$

$$y = x^{\frac{1}{3}} \left[\frac{1}{3} \left(-3x^{-\frac{1}{3}} \ln x - 9x^{-\frac{1}{3}} - 3x^{-\frac{1}{3}} \right) \right] + c\,x^{\frac{1}{3}}$$

$$y = \cancel{x^{\frac{1}{3}}} \left[-(\ln x + 4)\,\cancel{x^{-\frac{1}{3}}} \right] + c\,x^{\frac{1}{3}}$$

$$y = -(\ln x + 1) + c\,x^{\frac{1}{3}}$$

Q2 /B

yp_1 yp_2

$$y'' + 9y = 9x - \cos x$$

homogenuous Part:

$$y'' + 9y = 0$$

let $y = e^{mx} \longrightarrow y' = me^{mx} \longrightarrow y'' = m^2 e^{mx}$

$$m^2 e^{mx} + 9e^{mx} = 0$$

$$(m^2 + 9) e^{mx} = 0 \qquad e^{mx} \neq 0$$

$$m^2 + 9 = 0$$

$$m^2 = -9$$

$$m_{1,2} = \mp 3i$$

$$yh = \cancel{e^{0x}} (c_1 \sin 3x + c_2 \cos 3x)$$

$$yh = c_1 \sin 3x + c_2 \cos 3x$$

Non-homogenuous Part:

$$yp = yp_1 + yp_2$$

$$yp_1 = a_0 x + a_1$$

$$yp_1' = a_0$$

$$yp_1'' = 0$$

$$0 + 9(a_0 x + a_1) = 9x$$

$$9a_0 x + 9a_1 = 9x$$

$$9a_0 = 9 \longrightarrow a_0 = 1$$ مشابهة معاملات X

$$9a_1 = 0 \longrightarrow a_1 = 0$$ الثوابت

$$y_{p1} = x$$

$$y_{p2} = a_2 \sin x + a_3 \cos x$$

$$y_{p2}' = a_2 \cos x - a_3 \sin x$$

$$y_{p2}'' = -a_2 \sin x - a_3 \cos x$$

$$-a_2 \sin x - \underline{a_3 \cos x} + 9a_2 \sin x$$
$$+ 9\underline{a_3 \cos x} = \underline{-\cos x}$$

$$\underline{8a_2 \sin x} + 8\underline{a_3 \cos} x = -\cos x$$

$$8a_3 = -1 \longrightarrow a_3 = -\frac{1}{8}$$ مشابهة معاملات cos

$$8a_2 = 0 \longrightarrow a_2 = 0$$ مشابهة معاملات sin

$$y_{p2} = -\frac{1}{8}\cos x$$

$$y_p = x - \frac{1}{8}\cos x$$

$$y = y_h + y_p$$

$$y = c_1 \sin 3x + c_2 \cos 3x + x - \frac{1}{8}\cos x$$

Q3/A

$$(2xy+3y^2)dx-(x^2+2xy)dy=0$$

$$\frac{dy}{dx}=\frac{2xy+3y^2}{x^2+2xy}$$ test for homogenous

put $x=\lambda x$, $y=\lambda y$

$$\frac{dy}{dx}=\frac{2\lambda x\lambda y+3\lambda^2y^2}{\lambda^2x^2+2\lambda x\lambda y}$$

$$\frac{dy}{dx}=\frac{\cancel{\lambda^2}(2xy+3y^2)}{\cancel{\lambda^2}(x^2+2xy)}$$ ∴ homogenous

let $\frac{y}{x}=v$, $y=xv$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=\frac{2xy+3y^2}{x^2+2xy}$$

$$v+x\frac{dv}{dx}=\frac{2\frac{y}{x}+3\frac{y^2}{x^2}}{1+\frac{2y}{x}}$$

$$x\frac{dv}{dx}=\frac{2v+3v^2}{1+2v}-v$$

$$=\frac{2v+3v^2-v-2v^2}{1+2v}$$

$$x \frac{dv}{dx} = \frac{v + v^2}{1 + 2v}$$

$$\frac{1+2v}{v+v^2} \, dv = \frac{dx}{x}$$

$$\ln(v+v^2) = \ln x + c$$

$$\ln\left(\frac{y}{x} + \frac{y^2}{x^2}\right) = \ln x + c$$

Q3/B

$$y''' + 3y'' + 3y' + y = \underbrace{4x^3}_{y_{P1}} + \underbrace{3e^{-x}}_{y_{P2}}$$

homogenous part:

$$y''' + 3y'' + 3y' + y = 0$$

let $y = e^{mx} \longrightarrow y' = me^{mx} \longrightarrow y'' = m^2e^{mx} \longrightarrow y''' = m^3e^{mx}$

$$m^3e^{mx} + 3m^2e^{mx} + 3me^{mx} + e^{mx} = 0$$

$$(m^3 + 3m^2 + 3m + 1)\,e^{mx} = 0 \qquad e^{mx} \neq 0$$

$$m^3 + 3m^2 + 3m + 1 = 0$$

$$
\begin{array}{r|l}
 & m^2 + 2m + 1 \\
\hline
m+1 & m^3 + 3m^2 + 3m + 1 \\
 & -\;m^3 \mp m^2 \\
\hline
 & \quad 2m^2 + 3m + 1 \\
 & -\;2m^2 \mp 2m \\
\hline
 & \qquad\quad m + 1 \\
 & \qquad -\;m \mp 1 \\
\hline
 & \qquad\quad 0 \quad 0
\end{array}
$$

$$(m+1)(m^2+2m+1)=0$$

$$(m+1)(m+1)(m+1)=0$$

either $m+1=0 \longrightarrow m_1=-1$

or $m+1=0 \longrightarrow m_2=-1$

or $m+1=0 \longrightarrow m_3=-1$

$$y_h = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x}$$

non-homogenous Part!

$$y_p = y_{p1} + y_{p2}$$

$$y_{p1} = a_0 x^3 + a_1 x^2 + a_2 x + a_3$$

$$y_{p_1}' = 3a_0 x^2 + 2a_1 x + a_2$$

$$y_{p_1}'' = 6a_0 x + 2a_1$$

$$y_{p1} = 6a_0$$

$$6a_0 + 3(6a_0 x + 2a_1) + 3(3a_0 x^2 + 2a_1 x + a_2)$$

$$+ a_0 x^3 + a_1 x^2 + a_2 x + a_3 = 4x^3$$

$$6a_0 + 18a_0X + 6a_1 + 9a_0X^2 + 6a_1X + 3a_2$$
$$+ a_0X^3 + a_1X^2 + a_2X + a_3 = 4X^3$$

$$a_0X^3 + (a_1 + 9a_0)X^2 + (18a_0 + 6a_1 + a_2)X$$
$$+ (6a_0 + 6a_1 + 3a_2 + a_3) = 4X^3$$

$a_0X^3 = 4X^3 \longrightarrow a_0 = 4$ معاملات X^3

$a_1 + 9a_0 = 0$ معاملات X^2

$a_1 + 9(4) = 0 \longrightarrow a_1 = -36$

$18a_0 + 6a_1 + a_2 = 0$ معاملات X

$18(4) + 6(-36) + a_2 = 0 \longrightarrow a_2 = 144$

$6a_0 + 6a_1 + 3a_2 + a_3 = 0$ الثوابت

$6(4) + 6(-36) + 3 * 144 + a_3 = 0 \longrightarrow a_3 = -240$

$$y_{p1} = 4X^3 - 36X^2 + 144X - 240$$

$y_{p2} = a_4 e^{x}$

$y_{p2}' = a_4 e^{x}$

$y_{p2}'' = a_4 e^{x}$

$y_{p2}''' = a_4 e^{x}$

$a_4 e^{x} + 3a_4 e^{x} + 3a_4 e^{x} + a_4 e^{x} = 3e^{x}$

$8a_4 e^{x} = 3e^{x}$

$a_4 = \frac{3}{8}$

$y_{p2} = \frac{3}{8} e^{x}$

$$y_p = y_{p1} + y_{p2} = 4x^3 - 36x^2 + 144x - 240 + \frac{3}{8} e^{x}$$

$y = y_h + y_p$

$$= c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x} + 4x^3 - 36x^2 + 144x - 240 + \frac{3}{8} e^{x}$$